Alexandria Engineering Journal (2016) 55, 3145–3151

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ORIGINAL ARTICLE

Natural convection flow in a vertical tube inspired by time-periodic heating

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Received 15 May 2016; revised 15 August 2016; accepted 25 August 2016
Available online 13 September 2016

KEYWORDS

Natural convection;
Fully developed flow;
Time-periodic heating;
Vertical tube

Abstract This paper theoretically analyzes the flow formation and heat transfer characteristics of fully developed natural convection flow in a vertical tube due to time periodic heating of the surface of the tube. The mathematical model responsible for the present physical situation is presented under relevant boundary conditions. The essential features of natural convection flow formation and associated heat transfer characteristics through the vertical tube are clearly highlighted by the variation in the dimensionless velocity, dimensionless temperature, skin-friction, mass flow rate and rate of heat transfer. Moreover, the effect of Prandtl number and Strouhal number on the momentum and thermal transport characteristics is discussed thoroughly. The study reveals that flow formation, rate of heat transfer and mass flow rate are appreciably influenced by Prandtl number and Strouhal number.

1. Introduction

In the course of modeling a real life situation of fluid flow, periodic heat input in a channel has captured the attention of many researchers due to its everyday applications in electrical and electronic appliances such as oven, drying machines, thermostat component, automatic control and microchips. A number of researchers have studied the effect of periodic heat input on time dependent free convection flow in a channel. Chung and Anderson [1] examined unsteady laminar free convection in a channel. Yang et al. [2] studied laminar natural convection with oscillatory surface temperature using the finite difference approach. From miniaturization of electrical and electronic panels, Bar-Cohen and Rohsenow [3] analyzed fully developed natural convection between two periodically heated parallel plates. Wang [4] investigated free convection flow between vertical plates with periodic heat input. He presented the solutions for the temperature and velocity field of the fluid as function of Prandtl number, Strouhal number and indirectly Rayleigh number. He concluded that increasing Strouhal number decreases the unsteady temperature and velocity of the fluid.

In recent past, Jha and Ajibade [5] analyzed free convection flow between vertical porous plates with periodic heat input being an extension of [4] by including suction/injection on the vertical porous plates. They concluded that inclusion of suction/injection has distorted the symmetric nature of the flow considered by [4]. Other research work on periodic heat input includes Jha and Ajibade [6] who investigated the effects of heat generating/absorbing fluid between vertical porous

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.08.025
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Nomenclature

a	radius of the tube
A	steady velocity profile
B	periodic velocity profile
F	steady temperature profile
G	periodic temperature profile
g	acceleration due to gravity
I_n	modified Bessel's function of first kind of order n. Where $n = 0, 1, 2, 3, \ldots$
M	mass flow rate
Pr	Prandtl number
r	dimensional radial coordinate
R	dimensionless radial coordinate
St	Strouhal number
t	time
T	temperature of the fluid
T_0	initial temperature
T_1	steady temperature at the tube
T_2	amplitude of periodic temperature at the tube
u	velocity of fluid

Greek alphabets

α	thermal diffusivity
β	coefficient of thermal expansion
κ	thermal conductivity
ρ	fluid density
ψ	phase of temperature
χ	phase of velocity
τ	skin friction
ν	kinematic viscosity
ω	frequency

plates with periodic heat input. Sparrow and Gregg [7] studied nearly Quasi-steady free convection heat transfer in gases. Menold and Yang [8], Nanda and Sharma [9] and Muhuri and Gupta [10] investigated the effect of periodic heating on a single vertical plate with no edge on the boundary layer development. A coupled stress fluid modeling on free convection oscillatory hydromagnetic flow in inclined rotating channel was studied by Ahmed et al. [11] to see the effect of periodic heat input on microchannel. Adesanya [12] studied free convective flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump. He concluded that increase in slip and temperature jump parameters increases the flow velocity and fluid temperature respectively. Also, Adesanya et al. [13] investigated the effect of a transverse magnetic field on the flow of viscous incompressible fluid flowing through a channel subjected to periodic heating using Adomian decomposition method. They found that both skin friction and Nusselt number decrease at the wall with increasing value of magnetic field parameter. Hossain and Floryan [14] studied pressure-driven flow in a horizontal channel exposed to thermal modulations and concluded that the heating results in a significant reduction in the pressure gradient required to drive the flow when compared to a similar flow in an isothermal channel.

In other related works, Shiu and Wu [15] discussed transient heat transfer in annular fins of various shapes with their bases subjected to a heat flux varying as a sinusoidal function of time. Furthermore, Cole and Crittenden [16] investigated Steady-Periodic heating of cylinder using Green's function approach.

The objective of the present investigation was to present a theoretical analysis of natural convection flow in a vertical tube inspired by time-periodic heating of the surface of the tube. Analytical solution of the momentum and energy equations is derived in terms of modified Bessel's function of first kind. Line and contour graphs are plotted to investigate the effect of periodic heating as well as Prandtl number.

2. Mathematical analysis

Consider a natural convection flow of a viscous incompressible fluid in a vertical tube as shown in Fig. 1. Initially, the surface temperature of the tube and fluid is at T_0 and then assumed to be heated periodically to $T(r,t) = T_1 + T_2\cos(\omega t)$. The natural convection flow formation is due to the temperature gradient between the surface of the tube and fluid. Every other parameters are assumed constant except otherwise stated and are presented in nomenclature. The flow is assumed to be fully developed and the viscous dissipation term in the energy equation is also assumed to be negligible.

For small temperature difference, the density of the fluid in the buoyancy term in the momentum equation considered varies with temperature whereas the density appearing elsewhere in these equations is considered constant (Boussinesq's approximation). Under the usual Boussinesq's approximation, the equation of state is assumed to be

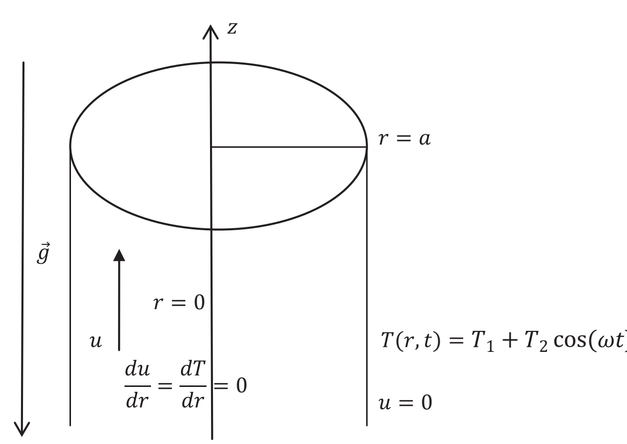

Figure 1 Schematic diagram of the problem.

$$\rho = \rho_0[1 - \beta(T - T_0)] \tag{1}$$

Using Boussinesq's approximation, the governing continuity, momentum and energy equations describing the present physical situation can be written respectively in dimensional form as follows:

$$\frac{\partial u}{\partial r} = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] + g\beta(T - T_0) \tag{3}$$

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] \tag{4}$$

subject to the following relevant boundary conditions:

$$\begin{array}{lll} \frac{\partial u}{\partial r} = 0 & \frac{\partial T}{\partial r} = 0 & r = 0 \\ u = 0 & T = T_1 + T_2\cos(\omega t) & r = a \end{array} \tag{5}$$

In order to solve Eqs. (2)–(5), we use the following expressions to separate the velocity and temperature respectively into steady and periodic parts respectively by Wang [4]:

$$u(r,t) = \frac{g\beta a^2}{\nu}[(T_1 - T_0)A(R) + T_2B(R)e^{i\omega t}] \tag{6}$$

$$T(r,t) = T_0 + [(T_1 - T_0)F(R) + T_2G(R)e^{i\omega t}] \tag{7}$$

where $A(R), F(R)$ represent the steady parts and $B(R), G(R)$ represent the unsteady parts of the velocity and temperature respectively.

Substituting Eqs. (6) and (7) into the momentum and energy equations (3) and (4), we obtain the following ordinary differential equations in dimensionless form:

$$\begin{aligned} &\frac{d^2A(R)}{dR^2} + \frac{1}{R}\frac{dA(R)}{dR} = -F(R) \\ &\frac{d^2B(R)}{dR^2} + \frac{1}{R}\frac{dB(R)}{dR} - iStB(R) = -G(R) \\ &\frac{d^2F(R)}{dR^2} + \frac{1}{R}\frac{dF(R)}{dR} = 0 \\ &\frac{d^2G(R)}{dR^2} + \frac{1}{R}\frac{dG(R)}{dR} - iStPrG(R) = 0 \end{aligned} \tag{8}$$

subject to the following boundary conditions

$$\begin{aligned} &\frac{dA}{dR} = \frac{dB}{dR} = \frac{dF}{dR} = \frac{dG}{dR} = 0 \quad \text{at} \quad R = 0 \\ &A(1) = B(1) = 0 \quad \text{at} \quad R = 1 \\ &F(1) = G(1) = 1 \quad \text{at} \quad R = 1 \end{aligned} \tag{9}$$

The dimensionless quantities used in the above equations are defined as follows:

$$R = \frac{r}{a}, \quad Pr = \frac{\nu}{\alpha}, \quad St = \frac{a^2\omega}{\nu} \tag{10}$$

The physical quantities used in the above equations are defined in the nomenclature.

The solution of Eq. (8) with boundary conditions (9) is given below:

$$G(R) = \frac{I_0(R\sqrt{iPrSt})}{I_0(\sqrt{iPrSt})} \tag{11}$$

$$F(R) = 1 \tag{12}$$

$$B(R) = \frac{1}{iSt(Pr-1)}\left[\frac{I_0(R\sqrt{iSt})}{I_0(\sqrt{iSt})} - \frac{I_0(R\sqrt{iPrSt})}{I_0(\sqrt{iPrSt})}\right] \tag{13}$$

$$A(R) = \frac{1}{4}[1 - R^2] \tag{14}$$

where I_0 is the modified Bessel's function of first kind of order zero.

The phase (ψ) of the periodic temperature and the phase (χ) of the periodic velocity are obtained by [5]

$$\psi = \tan^{-1}\left(\frac{Imag(G(R))}{Real(G(R))}\right) \tag{15}$$

$$\chi = \tan^{-1}\left(\frac{Imag(B(R))}{Real(B(R))}\right) \tag{16}$$

While the amplitudes of the periodic temperature and periodic velocity are given respectively by

$$|G| = \sqrt{(Real(G(R))^2 + Imag(G(R))^2)} \tag{17}$$

$$|B| = \sqrt{(Real(B(R))^2 + Imag(B(R))^2)} \tag{18}$$

The phase of the velocity χ is the angle between the velocities B_i and B_R while the amplitude $|B|$ denotes the absolute velocity at any given point in the tube.

The skin-friction (τ) at the tube surface is obtained by differentiating the velocity (ref. [17]) as follows:

$$\begin{aligned} \tau &= \frac{dB(R)}{dR}\Big|_{R=1} \\ &= \frac{1}{\sqrt{iSt}(Pr-1)}\left[\frac{I_1(\sqrt{iSt})}{I_0(\sqrt{iSt})} - \frac{\sqrt{Pr}I_1(\sqrt{iPrSt})}{I_0(\sqrt{iPrSt})}\right] \end{aligned} \tag{19}$$

While the rate of heat transfer at the tube wall is obtained by differentiating the temperature as follows:

$$Nu = \frac{dG(R)}{dR}\Big|_{R=1} = \frac{\sqrt{iPrSt}I_1(\sqrt{iPrSt})}{I_0(\sqrt{iPrSt})} \tag{20}$$

Also, the amount of fluid passing through the tube is given by the following:

$$\begin{aligned} M &= \int_0^1 RB(R)dR \\ &= \frac{1}{(iSt)^{3/2}(Pr-1)}\left[\frac{I_1(\sqrt{iSt})}{I_0(\sqrt{iSt})} - \frac{I_1(\sqrt{iPrSt})}{\sqrt{Pr}I_0(\sqrt{iPrSt})}\right] \end{aligned} \tag{21}$$

3. Results and discussion

The basic parameters controlling the present physical situations are Strouhal number (St) and the Prandtl number (Pr). The Strouhal number is directly proportional to the frequency of the periodic heating and Pr is the Prandtl number which varies directly to kinematic viscosity and inversely proportional to thermal diffusivity.

For small Strouhal number, Eqs. (11) and (13) reduce to

$$G(R) \cong 1 \tag{22}$$

$$B(R) \cong 0 \tag{23}$$

This is true because $I_0(x) \cong 1$ for small value of x [18]. These results in Eqs. (22) and (23) correspond to the conclusion of Wang [4] that for small Strouhal number, the periodic regime corresponds to steady state and hence the simple steady state solution of Eqs. (12) and (14) can be used for velocity and temperature respectively.

For large Strouhal number, we obtained the following expressions for unsteady temperature and velocity field respectively reduces Eqs. (11) and (13)

$$G(R) \cong \frac{e^{(R-1)\sqrt{iPrSt}}}{R^{1/2}} \tag{24}$$

$$B(R) \cong \frac{1}{iSt(Pr-1)R^{1/2}}\left(e^{(R-1)\sqrt{iSt}} - e^{(R-1)\sqrt{iPrSt}}\right) \tag{25}$$

A singularity point is seen to exist when $(Pr = 1)$ in the velocity Eq. (13), skin-friction Eq. (19) and mass flow rate Eq. (21). In order to remove this singularity point, unlike the method used in [19], we take limit as Pr tends to one for Eqs. (13), (19), and (21) and obtained respectively;

$$B(R) = \frac{I_0\left(R\sqrt{iSt}\right)I_1\left(\sqrt{iSt}\right) - RI_1\left(R\sqrt{iSt}\right)I_0\left(\sqrt{iSt}\right)}{2\sqrt{iSt}I_0\left(\sqrt{iSt}\right)} \tag{26}$$

$$\tau = \left[\frac{I_1^2(\sqrt{iSt})}{2I_0^2(\sqrt{iSt})} - \frac{1}{2}\right] \tag{27}$$

$$M = \frac{i}{2St}\left[1 - \frac{I_1^2(\sqrt{iSt})}{I_0^2(\sqrt{iSt})} - \frac{2I_1(\sqrt{iSt})}{\sqrt{iSt}I_0(\sqrt{iSt})}\right] \tag{28}$$

Table 1 presents a comparison between steady and unsteady temperature and velocity for different values of St and this gives an excellent comparison for small Strouhal number (St).

To reveal the influence of pertinent parameters, Figs. 2–11 depict line and contour graphs of unsteady temperature profile, velocity profile, skin-friction and rate of heat transfer.

Figs. 2 and 3 reveal the effect of Prandtl number (Pr) on temperature and velocity profiles respectively in the tube for fixed value of Strouhal number (St). It is found that as Prandtl number (Pr) increases, the temperature distributions as well as fluid velocity decreases. This is due to the fact that an increase in Prandtl number (Pr), decreases thermal diffusivity of the fluid which in turn leads to weak convection current. The same result is also noticed for the velocity profile in Fig. 3. This is due to the fact that the momentum equation depends on temperature which leads to decrease in fluid velocity. Also, for unity Prandtl number $(Pr = 1)$, the periodic regime velocity is seen to be discontinuous; this explains the singularity discussed in Eq. (13). In addition, for $(Pr > 1)$, the periodic regime velocity is seen to have an adverse effect on the overall velocity.

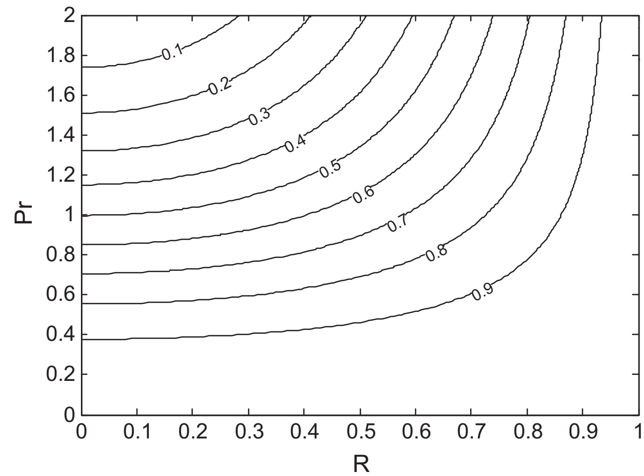

Figure 2 Temperature profiles $Real(G(R))$ for different values of Pr at $St = 5.0$.

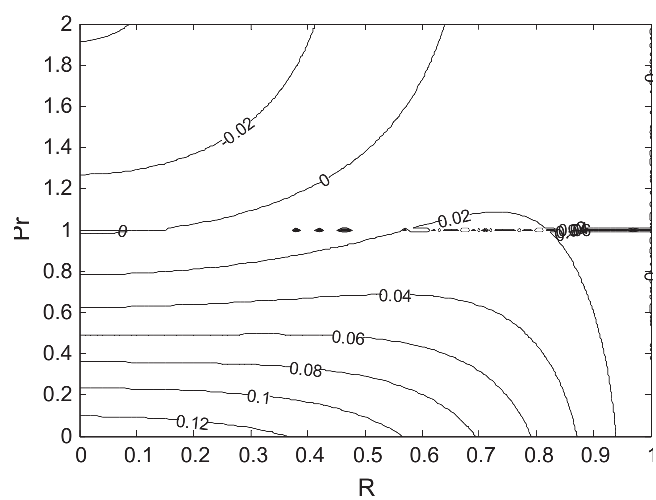

Figure 3 Velocity profiles $Real(B(R))$ for different values of Pr at $St = 5.0$.

Table 1 Steady state and Periodic comparison of velocity and temperature for different values of Strouhal number (St) at $Pr = 0.71$.

	$St = 2.0$				$St = 0.1$				$St = 0.01$			
R	$A(R)$	$B(R)$	$F(R)$	$G(R)$	$A(R)$	$B(R)$	$F(R)$	$G(R)$	$A(R)$	$B(R)$	$F(R)$	$G(R)$
0.0	0.2500	0.1868	1.0	0.9110	0.2500	0.2498	1.0	0.9998	0.2500	0.2500	1.0	1.0
0.2	0.2400	0.1804	1.0	0.9156	0.2400	0.2398	1.0	0.9998	0.2400	0.2400	1.0	1.0
0.4	0.2100	0.1604	1.0	0.9291	0.2100	0.2099	1.0	0.9998	0.2100	0.2100	1.0	1.0
0.6	0.1600	0.1254	1.0	0.9498	0.1600	0.1599	1.0	0.9999	0.1600	0.1600	1.0	1.0
0.8	0.0900	0.0729	1.0	0.9747	0.0900	0.0900	1.0	0.9999	0.0900	0.0900	1.0	1.0
1.0	0	0.0000	1.0	1.0000	0	0.0000	1.0	1.0000	0	0	1.0	1.0

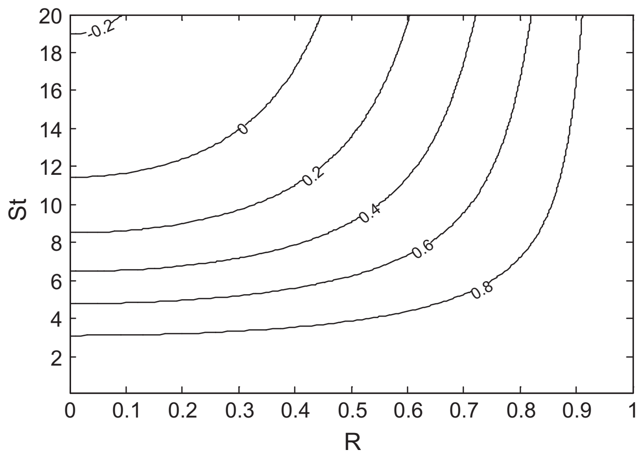

Figure 4 Temperature profiles $Real(G(R))$ for different values of St at $Pr = 0.71$.

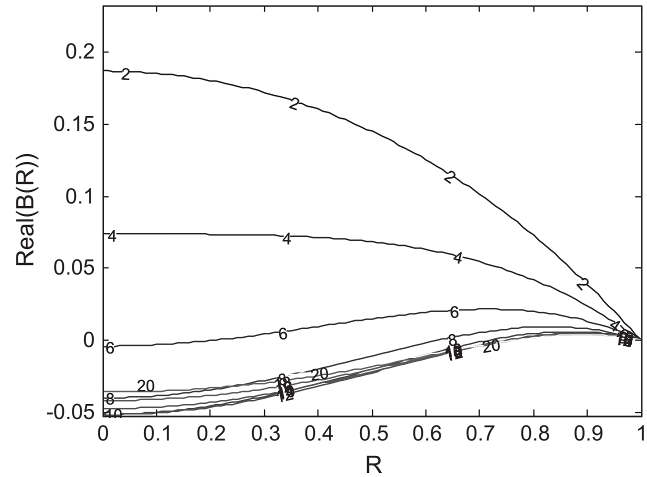

Figure 7 Velocity profile $Real(B(R))$ for different values of St at $Pr = 0.71$.

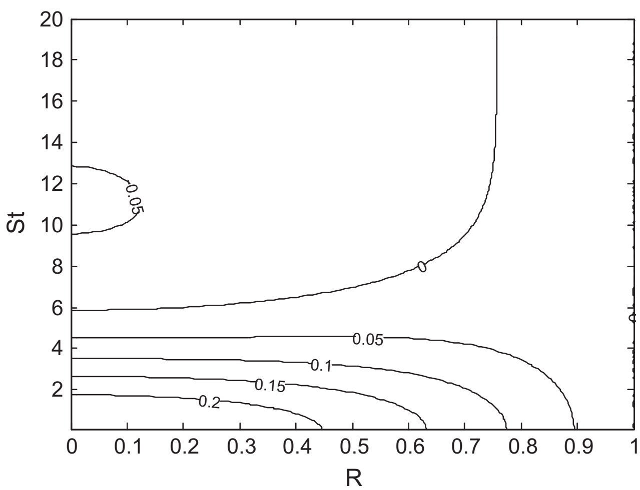

Figure 5 Velocity profiles $Real(B(R))$ for different values of St at $Pr = 0.71$.

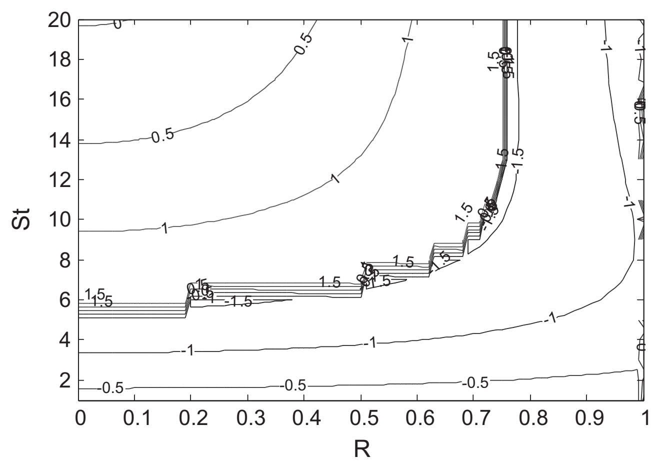

Figure 8 Phase of velocity (χ) for different values of St at $Pr = 0.71$.

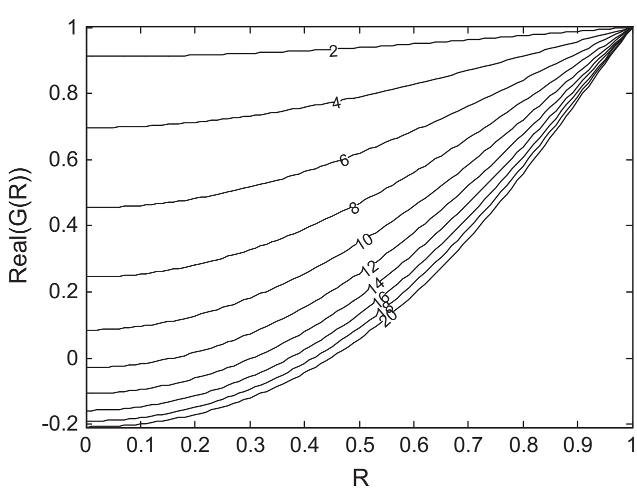

Figure 6 Temperature profiles $Real(G(R))$ for different values of St at $Pr = 0.71$.

Figs. 4 and 5 on the other hand depict the influence of Strouhal number (St) on temperature and velocity distribution respectively. In both figures, it is seen that the temperature as well as the velocity is decreasing function of Strouhal number (St). This is physically possible, since as Strouhal number increases, frequency of heating also increases which in turn decreases the temperature as well as velocity. In addition, flow reversal is observed for large value of Strouhal number (St) which is caused as a result of the frequency of heating, and this result agrees with the findings of [5].

Figs. 6 and 7 give a clearer view and understanding of Figs. 4 and 5. It can be seen from these figures that for large Strouhal number, the periodic regime velocity profile contributes a negative impact on the overall fluid velocity.

Fig. 8 presents the effect of time-periodic heating (Strouhal number (St)) on the phase of the velocity (angle at which the flow deviates from flow direction). It is observed that the fluid motion is characterized by a frequent phase change for value of Strouhal number greater than five, and this change is part of what results in negative values of periodic velocity in Figs. 5

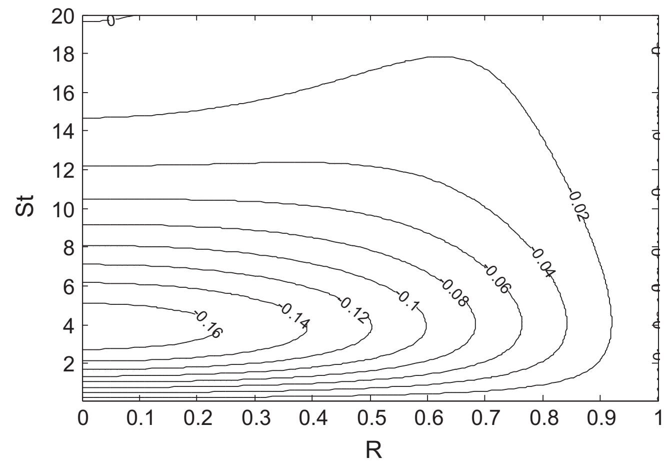

Figure 9 Out-of-phase velocity $Imag(B(R))$ for different values of St at $Pr = 0.71$.

and 6. Furthermore, a critical observation shows that the angle between these velocities is periodic, and this can be attributed to the increase in frequency of heating. Such results are also observed in Jha and Ajibade [5,6].

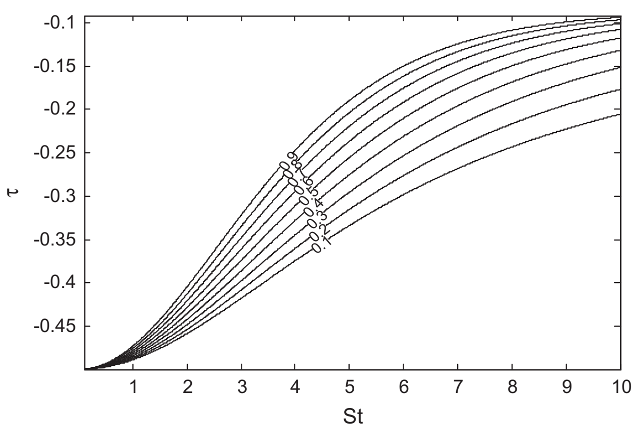

Figure 10 Skin-friction for different values of St and Pr.

The effect of Strouhal number (St) on out-of-phase velocity at fixed value of Prandtl number (Pr) is displayed in Fig. 9. It is found that the velocity profiles tend to zero for high values of St. This may be attributed to the fact that an increase in frequency of heating reduces convection current which in turn results in decrease in out-of-phase velocity.

Fig. 10 shows the combined effect of Prandtl number (Pr) and Strouhal number (St) on the skin-friction at the surface of the tube. It is observed that the effect of Pr and St is to increase the skin-friction. This is because as Strouhal number and Prandtl number increase, frequency of heating as well as kinematic viscosity of the fluid increases respectively which leads to increase in skin-friction at the surface of tube. The same trend is also noticed in Fig. 11 for rate of heat transfer between the fluid and the surface of the tube (Nu). This is true because for higher Prandtl number (Pr), higher temperature gradient exists near the wall thereby causing higher rate of heat transfer. In addition, Strouhal number (St) enhances both skin-friction and rate of heat transfer since increasing frequency of heating results in increase in temperature gradient and therefore increases both the skin-friction and rate of heat transfer.

Fig. 12 depicts the effect of frequency of heating on the mass flow rate for different Prandtl numbers. The mass flow rate is seen to be a decreasing function of Strouhal number

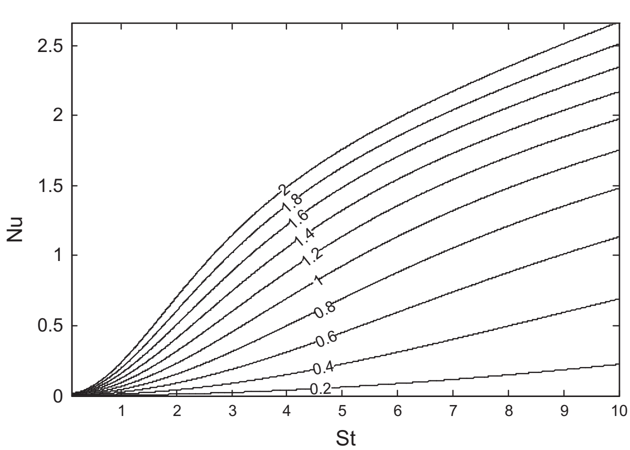

Figure 11 Rate of heat transfer for different values of St and Pr.

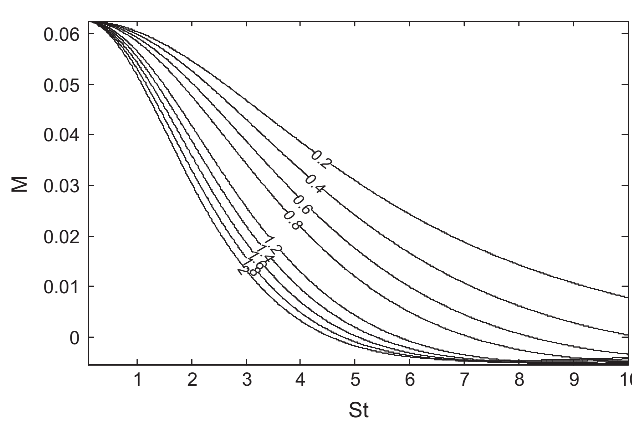

Figure 12 Mass flow rate for different values of St and Pr.

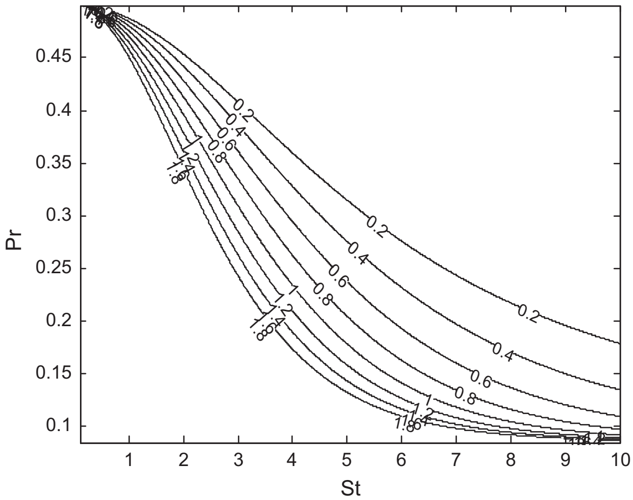

Figure 13 Skin friction for different values of St and Pr.

and Prandtl number. This trend is accompanied by an increase in Prandtl number which increases fluid viscosity and in turn reduces the movability of fluid in the tube, and hence results in decrease in flow rate.

Fig. 13 shows the skin-friction variation with St and Pr. For unity Prandtl number, ($Pr = 1$), there exists discontinuity in the graph. This explains the singularity point of Eq. (19). Therefore, for unity Prandtl number, the solution obtained in Eq. (27) is significant and may be used.

4. Conclusions

This paper presents an analytical solution for time dependent natural convection flow in a vertical tube with surface time-periodic heating. Closed-form expressions for velocity, temperature, skin-friction, mass flow rate and rate of heat transfer which is expressed as Nusselt number are obtained. The role of Strouhal number (St) and Prandtl number (Pr) on flow formation is discussed in Figs. 2–13. Based on the numerical results obtained, we draw the following conclusions:

1. The temperature as well as velocity profile decreases with increase in Strouhal number and Prandtl number.
2. For small Strouhal number (less than 0.1), the temperature, velocity, skin-friction, mass flow rate and Nusselt number correspond to the steady state solutions. This corresponds with the findings of [4–6,12,13].
3. The influence of Strouhal number is found to increase both the skin-friction and rate of heat transfer.
4. For small Strouhal number, the temperature profiles (steady or periodic) are exactly same in tube or parallel plates channel [4]; otherwise, the solution obtained here is significant and different from that of Wang [4].

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