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Possible CP-violation effects in core-collapse supernovae

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Abstract

We study CP-violation effects when neutrinos are present in dense matter, such as outside the proto-neutron star formed in a core-collapse supernova. Using general arguments based on the Standard Model, we confirm that there are no CP-violating effects at the tree level on the electron neutrino and anti-neutrino fluxes in a core-collapse supernova. On the other hand significant effects can be obtained for muon and tau neutrinos even at the tree level. We show that CP-violating effects can be present in the supernova electron (anti-)neutrino fluxes as well, if muon and tau neutrinos have different fluxes at the neutrinosphere. Such differences could arise due to physics beyond the Standard Model, such as the presence of flavor-changing interactions.

Keywords: CP-violation in neutrino sector; r-process nucleosynthesis; Core-collapse supernova

1. Introduction

Recent results from solar, atmospheric and reactor experiments have significantly improved our knowledge of the neutrino mass differences and of two of the mixing angles. If the remaining mixing angle, θ_{13}, is relatively large there is a possibility that violation of CP symmetry may be observable in the neutrino sector. Currently planned and future experiments will have improved sensitivities to the value of this angle (see, e.g., [1–3]). Effects of CP-violation in accelerator neutrino oscillation experiments have been extensively investigated [4–9]. The discovery of a non-zero Dirac delta phase might help our understanding of the observed matter–antimatter asymmetry of the universe [10–13]. Besides studies on terrestrial experiments with man-made sources, a few recent works have addressed CP-violation with neutrinos from astrophysical sources (see, e.g., [14,15]). The purpose of the present paper is to explore possible effects coming from the CP-violating phase in dense matter, such as that encountered in core-collapse supernovae.

Core-collapse supernovae occur following the stages of nuclear burning during stellar evolution after an iron core is formed. The iron cores formed during the evolution of massive stars are supported by the electron degeneracy pressure and hence are unstable against a collapse during which most of the matter is neutronized. Once the density exceeds the nuclear density this collapse is halted. Rebounding pressure waves break out into a shock wave near the sonic point where the density reaches the nuclear density. Evolution of this shock wave and whether it produces an explosion is a point of current investigations. However, it is observed that the newly-formed hot proto-neutron star cools by neutrino emission. Essentially the entire gravitational binding energy of eight or more solar mass star is radiated away in neutrinos. Although the initial collapse is a very orderly (i.e., low entropy) process, during the cooling stage at later times a neutrino-driven wind heats the neutron-rich material to high entropies [16–18].

Neutrino interactions play a very important role in the evolution of core-collapse supernovae and in determining the element abundance [19]. Neutrino heating is a possible mechanism for reheating the stalled shock [20]. A good fraction of the heavier nuclei were formed in the rapid neutron capture (r-process) nucleosynthesis scenario [21]. Core-collapse supernovae are one

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of the possible sites for the r-process nucleosynthesis. A key quantity for determining the r-process yields is the neutron-to-seed nucleus ratio, determined by the neutron-to-proton ratio, which is controlled by the neutrino fluxes. In addition, recent work indicates that neutrino–neutrino interactions plays a potentially very significant role in supernovae [22–27].

In this Letter we study CP-violation aspects in the core-collapse supernova environment. We first analyze analytically and in general terms, how the neutrino propagation equations and the evolution operator are modified in matter, in presence of a non-zero Dirac delta phase. We obtain a general formula which is valid for any matter density profile.[1] In particular we demonstrate that, as in vacuum, the electron (anti-)neutrino fluxes are independent of the phase δ, if mu and tau neutrinos have the same fluxes at the neutrinosphere in the supernova.[2] On the other hand the electron (anti-)neutrino fluxes will depend on δ, if mu and tau neutrinos have different fluxes at the neutrinosphere, at variance with what was found in [14]. We present numerical calculations on possible CP violation effects on the mu and tau neutrino fluxes as well as on the electron (anti-)neutrino flux and the electron fraction. The latter can only appear if physics beyond the Standard Model, such as flavor changing interactions, induces differences on the mu and tau neutrino initial total luminosities and/or temperatures. Finally we calculate the effects from the CP-violating phase on the number of events in an observatory on earth.

The plan of this Letter is as follows. In Section 2 we present the general formalism to describe the neutrino evolution in presence of the δ phase. The formulas concerning neutrino fluxes and the electron fraction in the supernova environment are recalled in Section 3. Numerical results on these quantities are presented in Section 4. Conclusions are made in Section 5.

2. Neutrino mixing in the presence of CP-violating phases

2.1. Neutrino mixing in ordinary matter in presence of CP-violating phases

The neutrino mixing matrix is $U_{\alpha i}$ where α denotes the flavor index and i denotes the mass index:

$$\Psi_\alpha = \sum_i U_{\alpha i}\Psi_i. \tag{1}$$

For three neutrinos we take

$$U_{\alpha i} = T_{23}T_{13}T_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -S_{13}e^{i\delta} & 0 & C_{13} \end{pmatrix} \times \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where C_{13}, etc., is the short-hand notation for $\cos\theta_{13}$, etc., and δ is the CP-violating phase. The MSW equation is

$$i\frac{\partial}{\partial t}\begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix} = \left[T_{23}T_{13}T_{12}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} T_{12}^\dagger T_{13}^\dagger T_{23}^\dagger + \begin{pmatrix} V_c + V_n & 0 & 0 \\ 0 & V_n & 0 \\ 0 & 0 & V_n \end{pmatrix}\right] \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix}, \tag{3}$$

where

$$V_c(x) = \sqrt{2}G_F N_e(x) \tag{4}$$

for the charged-current and

$$V_n(x) = -\frac{1}{\sqrt{2}}G_F N_n(x) \tag{5}$$

for the neutral current. Since V_n only contributes an overall phase to the neutrino evolution we ignore it.[3] Following Refs. [30] and [31] we introduce the combinations

$$\tilde{\Psi}_\mu = \cos\theta_{23}\Psi_\mu - \sin\theta_{23}\Psi_\tau, \tag{6}$$

$$\tilde{\Psi}_\tau = \sin\theta_{23}\Psi_\mu + \cos\theta_{23}\Psi_\tau, \tag{7}$$

which corresponds to multiplying the neutrino column vector in Eq. (3) with $T_{23}^\dagger$ from the left. Eq. (3) then becomes

$$i\frac{\partial}{\partial t}\begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix} = \left[T_{13}T_{12}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} T_{12}^\dagger T_{13}^\dagger + \begin{pmatrix} V_c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] \begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix}. \tag{8}$$

We define

$$\tilde{H} = \left[T_{13}T_{12}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} T_{12}^\dagger T_{13}^\dagger + \begin{pmatrix} V_c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]. \tag{9}$$

The Hamiltonian $\tilde{H}$ depends on the CP-violating phase, δ. It is lengthy but straightforward to show that

$$\tilde{H}(\delta) = S^\dagger \tilde{H}(\delta = 0) S, \tag{10}$$

where

$$S^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}. \tag{11}$$

We are interested in solving the evolution equation corresponding to Eq. (8):

$$i\hbar\frac{d\hat{U}}{dt} = \tilde{H}\hat{U}. \tag{12}$$

[1] Such findings are in agreement with what was found in Ref. [28].

[2] A remark on this aspect was made in [29].

[3] The results we show in the present work do not include the difference in the mu and tau refractive indices which appear at one loop level due to different muon and tau lepton masses, $V_{\mu\tau}$ [41], which is $10^{-5}V_c$. In fact, we have tested that the inclusion of this correction modifies very little the numerical results presented in Section 3.

It is important to recall that we need to solve this equation with the initial condition

$$\hat{U}(t=0)=1. \tag{13}$$

Defining

$$\mathcal{U}_0 = S\hat{U}, \tag{14}$$

and using the relation in Eq. (10) we get

$$i\hbar\frac{d\mathcal{U}_0}{dt} = \tilde{H}(\delta=0)\,\mathcal{U}_0, \tag{15}$$

i.e., $\mathcal{U}_0$ provides the evolution when the CP-violating phase is set to zero. Using Eq. (13) we see that the correct initial condition on $\mathcal{U}_0$ is $\mathcal{U}_0(t=0)=S$. However, Eq. (15) is nothing but the neutrino evolution equation with CP-violating phase set equal to zero. If we call the solution of this equation with the standard initial condition $\hat{U}_0(t=0)=1$ to be $\hat{U}_0$, we see that we should set $\mathcal{U}_0=\hat{U}_0 S$, which yields

$$\hat{U}(\delta) = S^{\dagger}\hat{U}_0 S. \tag{16}$$

Eq. (16) illustrates how the effects of the CP-violating phase separate in describing the neutrino evolution. It is valid both in vacuum and in matter. This result is in agreement with [28]. It is easy to verify that this result does not depend on the choice of the parametrization for the neutrino mixing matrix.

Using Eq. (16) it is possible to relate survival probabilities for the two cases with $\delta=0$ and $\delta\neq0$. We define the amplitude for the process $\nu_x\to\nu_y$ to be A_{xy} when $\delta\neq0$ and to be B_{xy} when $\delta=0$ so that

$$P(\nu_x\to\nu_y,\delta\neq0)=|A_{xy}|^2 \tag{17}$$

and

$$P(\nu_x\to\nu_y,\delta=0)=|B_{xy}|^2. \tag{18}$$

Using Eq. (16) one can immediately see that the electron neutrino survival probability does not depend on the CP-violating phase. One can further write

$$c_{23}A_{\mu e}-s_{23}A_{\tau e}=c_{23}B_{\mu e}-s_{23}B_{\tau e},$$
$$s_{23}A_{\mu e}+c_{23}A_{\tau e}=e^{-i\delta}[s_{23}B_{\mu e}+c_{23}B_{\tau e}].$$

By solving these equations one gets

$$A_{\mu e}=\left(c_{23}^2+s_{23}^2e^{-i\delta}\right)B_{\mu e}+c_{23}s_{23}\left(e^{-i\delta}-1\right)B_{\tau e}, \tag{19}$$

and

$$A_{\tau e}=c_{23}s_{23}\left(e^{-i\delta}-1\right)B_{\mu e}+\left(s_{23}^2+c_{23}^2e^{-i\delta}\right)B_{\tau e}. \tag{20}$$

Clearly the individual amplitudes in Eqs. (19) and (20) depend on the CP-violating phase. However, taking absolute value squares of Eqs. (19) and (20), after some algebra one obtains that

$$|A_{\mu e}|^2+|A_{\tau e}|^2=|B_{\mu e}|^2+|B_{\tau e}|^2, \tag{21}$$

or equivalently

$$P(\nu_\mu\to\nu_e,\delta\neq0)+P(\nu_\tau\to\nu_e,\delta\neq0)$$
$$=P(\nu_\mu\to\nu_e,\delta=0)+P(\nu_\tau\to\nu_e,\delta=0). \tag{22}$$

Since $P(\nu_e\to\nu_e)$ does not depend on δ, Eq. (22) implies that if one starts with identical spectra with tau and mu neutrinos, one gets the same electron neutrino spectra no matter what the value of the CP-violating phase is (also see Eq. (24)). This result was first established in [14] with a different derivation. A remark on this aspect is also made in [29].

Some differences in the muon/tau neutrino fluxes at emission can arise at the level of the Standard Model, from example from radiative corrections to the muon and tau neutrino cross sections [41]. On the other hand, if physics beyond the Standard Model operates during the infall and the shock-bounce stages of the supernova evolution, mu and tau neutrino fluxes can differ and induce CP-violating effects in the supernova environment. For example, generic neutrino-flavor changing interactions can give rise to significant net mu and tau lepton numbers [32]. In particular, if there are flavor changing interactions involving charged leptons (e.g., a large scale conversion in the $e^-\to\mu^-$ channel) one could also end up with significantly different mu and tau neutrino fluxes. In such cases one could have effects from the CP-violating phase on the electron (anti-)neutrino fluxes as well.

Note that our findings are at variance with those of Ref. [14]. In fact, there the authors conclude that even if mu and tau neutrino fluxes are different, CP-violation effects cannot be observed. Such a difference arises from the fact that different initial conditions are taken in our calculations compared to those used in Eq. (47) of [14]. Indeed since the initial neutrino states should be those at the neutrinosphere, the neutrino conversion probability $P_{i\alpha}$ should depend on the δ phase (see Section 3.4 of [14]).

3. Neutrino fluxes and electron fraction in supernovae

In this work we will discuss possible effects induced by the Dirac CP-violating phase on two particular observables in the core-collapse supernova environment: the neutrinos fluxes ϕ_ν and the electron fraction Y_e. Note that the impact of the neutrino magnetic moment on such observables was studied in [33]. According to supernova simulations, the neutrino fluxes at the neutrinosphere are quite well described by Fermi–Dirac distributions [34] or power-law spectra [35]. Neutrino masses and mixings modify this simple pattern by mixing the spectra during neutrino evolution. Since muon and tau neutrinos only undergo neutral current interactions, they decouple deeper in the star. Electron (anti-)neutrinos experience both charged and neutral current interactions, the anti-neutrino cross sections being weaker than for neutrinos and matter being neutron-rich. As a result a neutrino hierarchy of temperatures is expected, $\langle E_{\nu_e}\rangle<\langle E_{\bar\nu_e}\rangle<\langle E_{\nu_\tau}\rangle$ with typical ranges of 10–13, 13–18 and 18–23 MeV respectively [35]. In order to show possible CP-violating effects on the ν_i fluxes, we will use the ratio:

$$R_{\nu_i}(\delta)=\frac{\phi_{\nu_i}(\delta)}{\phi_{\nu_i}(\delta=0^\circ)}, \tag{23}$$

where the neutrino fluxes are given by

$$\phi_{\nu_i}(\delta) = L_{\nu_i} P(\nu_i \to \nu_i) + L_{\nu_j} P(\nu_j \to \nu_i) + L_{\nu_k} P(\nu_k \to \nu_i), \tag{24}$$

with the luminosities

$$L_{\nu_i}(r, E_\nu) = \frac{L^0_{\nu_i}}{4\pi r^2 (kT)^3 \langle E_\nu \rangle F_2(\eta)} \frac{E^2_\nu}{1 + \exp(E_\nu/T_\nu - \eta)}, \tag{25}$$

where $F_2(\eta)$ is the Fermi integral, L_0 is the luminosity that we take as 6×10^{51} erg/s as an example and r is the distance from the proto-neutron star. We consider the Fermi–Dirac distribution as typical example. The quantities $P(\nu_i \to \nu_{i/j})$ correspond to the survival/appearance neutrino probability during the evolution in matter.

The dominant reactions that control the neutron-to-proton ratio outside the hot proto-neutron star is the capture reactions on free nucleons

$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-, \tag{26}$$

and

$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+. \tag{27}$$

We designate the rates of the forward and backward reactions in Eq. (26) to be λ_{ν_e} and λ_{e^-} and the rates of the forward and backward reactions in Eq. (27) to be $\lambda_{\bar{\nu}_e}$ and λ_{e^+}. The electron fraction, Y_e, is the net number of electrons (number of electrons minus the number of positrons) per baryon:

$$Y_e = (n_{e^-} - n_{e^+})/n_B, \tag{28}$$

where n_{e^-}, n_{e^+}, and n_B are number densities of electrons, positrons, and baryons, respectively. If no heavy nuclei are present we can write the rate of change of Y_e as

$$\frac{dY_e}{dt} = \lambda_n - (\lambda_p + \lambda_n) Y_e + \frac{1}{2}(\lambda_p - \lambda_n) X_\alpha, \tag{29}$$

where we introduced the alpha fraction X_α, the total proton loss rate $\lambda_p = \lambda_{\bar{\nu}_e} + \lambda_{e^-}$ and the total neutron loss rate $\lambda_n = \lambda_{\nu_e} + \lambda_{e^+}$. From Eq. (29) one can write the equilibrium value of the electron fraction

$$Y_e = \frac{\lambda_n}{\lambda_p + \lambda_n} + \frac{1}{2}\frac{\lambda_p - \lambda_n}{\lambda_p + \lambda_n} X_\alpha. \tag{30}$$

As the alpha particle mass fraction increases more and more free nucleons get bound in alpha particles [36]. This phenomenon, called alpha effect, pushes the electron fraction towards the value 0.5 (cf. Eq. (30)). Since it reduces available free neutrons, alpha effect is a big impediment to r-process nucleosynthesis [37]. At high temperatures, alpha particles are absent and the second term in Eq. (30) can be omitted. Since electron and positron capture rates are very small, the electron fraction can be rewritten as

$$Y_e^{(0)} = \frac{1}{1 + \lambda_p/\lambda_n}, \tag{31}$$

with the capture rates on $x = p, n$ given by

$$\lambda_{n,p} = \int \sigma_{\nu_e n, \bar{\nu}_e p}(E_\nu) \phi(E_\nu)\, dE_\nu \tag{32}$$

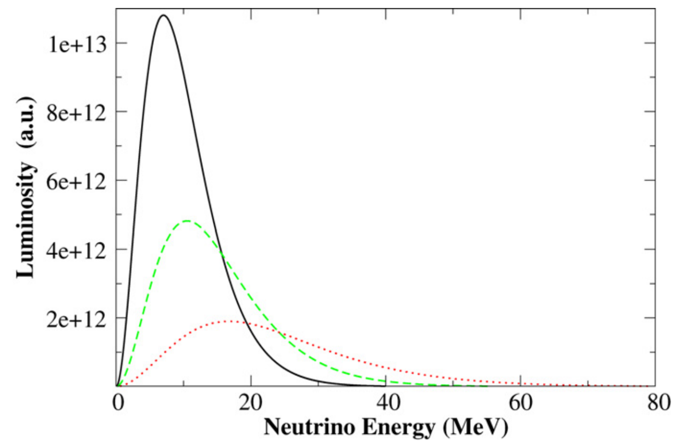

Fig. 1. Neutrino fluxes at the neutrinosphere: the curves show the Fermi–Dirac distributions used for electron neutrinos with $T_{\nu_e} = 3.2$ MeV (solid), electron anti-neutrinos $T_{\bar{\nu}_e} = 4.8$ MeV (dashed) and for the other flavors $T_{\nu_x} = 7.6$ MeV (with $\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$) (dotted line).

and $\sigma_{\nu_e n, \bar{\nu}_e p}$ being the reaction cross sections for the corresponding processes Eqs. (26)–(27).

4. Possible CP-violation effects: Numerical results

It is the goal of this section to investigate numerically effects induced by the Dirac δ phase: (i) on the muon and tau neutrino fluxes when their fluxes at the neutrinosphere are supposed to be equal; (ii) on the electron, muon, tau (anti-)neutrino fluxes, when the muon and neutrino fluxes differ at the neutrinosphere. In fact, Eqs. (1)–(22) and (24) show that in the latter case the electron (anti-)neutrino fluxes become sensitive to the CP-violating phase. We have performed calculations for several values of the phase. The effects discussed here are present for any value and maximal for $\delta = 180°$. For this reason most of the numerical results we show correspond to this value.

We have calculated the neutrino evolution outside the supernova core using Eqs. (2)–(5) and determined the neutrino fluxes Eqs. (23)–(25) and the electron fraction (31)–(32). The numerical results we present are obtained with a supernova density profile having a $1/r^3$ behavior (with the entropy per baryon, $S = 70$ in units of Boltzmann constant), that fits the numerical simulations shown in [24]. The neutrino fluxes at the neutrinosphere are taken as Fermi–Dirac distributions with typical temperatures of $T_{\nu_e} = 3.17$ MeV, $T_{\bar{\nu}_e} = 4.75$ MeV and $T_{\nu_x} = 7.56$ MeV (with $\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$) (Fig. 1) (the chemical potentials are assumed to be zero for simplicity). The oscillation parameters are fixed at the present best fit values [38], namely $\Delta m^2_{12} = 8 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{12} = 0.83$ and $\Delta m^2_{23} = 3 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$ for the solar and atmospheric differences of the mass squares and mixings, respectively. For the third still unknown neutrino mixing angle θ_{13}, we take either the present upper limit $\sin^2 2\theta_{13} = 0.19$ at 90% C.L. (L) or a very small value of $\sin^2 2\theta_{13} = 3 \times 10^{-4}$ (S) that might be attained at the future (third generation) long-baseline experiments [9]. Note that the value of θ_{13} determines the adiabaticity of the first MSW resonance at high density [39,40], while θ_{12} governs the second (adiabatic) one at low density.

Since the sign of the atmospheric mixing is unknown, we consider both the normal (N) and inverted (I) hierarchy. In the former (latter) case (anti-)neutrinos undergo the resonant conversion. We will denote results for the normal hierarchy and $\sin^2 2\theta_{13} = 0.19$ (N–H), inverted and $\sin^2 2\theta_{13} = 0.19$ (I–H), normal hierarchy and $\sin^2 2\theta_{13} = 3 \times 10^{-4}$ (N–S), inverted and $\sin^2 2\theta_{13} = 3 \times 10^{-4}$ (I–S).

Figs. 2 and 3 show the $\bar{\nu}_\mu, \bar{\nu}_\tau$ and ν_μ, ν_τ flux ratios Eq. (23) for $\delta = 180°$ over for $\delta = 0°$. One can see that large effects, up to 60% are present for low neutrino energies in the anti-neutrino case; while smaller effects, of the order of a few percent, appear in the neutrino case. The effect of a non-zero delta over the ν_μ, ν_τ fluxes as a function of neutrino energy is shown in Fig. 4 at a distance of 1000 km. We see that an increase as large as a factor of 8 (4) can be seen at low energies in the ν_μ (ν_τ) spectra. A similar behavior is found in the anti-neutrino case.

In most of supernova simulations, the ν_μ and ν_τ luminosities are approximately equal, because these particles interact via neutral current only, at the low energies possible at supernova.[4] Since the $\nu_e, \bar{\nu}_e$ appearance probabilities are independent of δ and as long as the ν_μ and ν_τ luminosities are taken to be equal, using Eqs. (22) and (24) one can show that the ν_e and $\bar{\nu}_e$ fluxes are independent of the CP-violating phase. Practically all the literature concerning the neutrino evolution in core-collapse supernovae ignore the Dirac phase, for simplicity. Our results justify this assumption if such calculations make the hypothesis that the ν_μ ($\bar{\nu}_\mu$) and ν_τ ($\bar{\nu}_\tau$) luminosities are equal and neglect the $V_{\mu\tau}$.

On the other hand, the situation is different if the muon and tau neutrino fluxes are different at the neutrinosphere either because of the corrections within the Standard Model and/or because of physics beyond the Standard Model, such as flavor changing interactions [32] which might populate the ν_μ, ν_τ fluxes differently and differentiate their temperatures at decoupling. Our aim is to show the CP-violating effects in this case. We have explored various differences between the ν_μ, ν_τ luminosities. We present here for example results corresponding to the ν_μ, ν_τ total luminosities Eq. (24) different by 10%, e.g., $L^0_{\nu_\tau} = 1.1 L^0_{\nu_\mu}$ or $T_{\nu_\tau} = 8.06$ MeV while $T_{\nu_\mu} = 7.06$ MeV. Fig. 5 presents as an example the evolved ν_e, ν_μ neutrino fluxes, at 1000 km from the neutron star surface, when $T_{\nu_\mu} \neq T_{\nu_\tau}$. The different curves show results for the two hierarchies and the two values of θ_{13}. Similarly to the case where $T_{\nu_\mu} = T_{\nu_\tau}$, while for the N–L case the first resonance is adiabatic and the electron neutrinos get a hotter spectrum, for all other cases the spectra keep very close to the Fermi–Dirac distributions (Fig. 1). The situation is obviously reversed for the muon neutrino flux. Figs. 6–8 show the ratios of the ν_e and ν_μ fluxes for a non-zero over a zero delta phase, as a function of neutrino energy, at different distances from the neutron star surface. One can see that effects up to a factor of 2–4 on the ν_e and up to 10% on ν_μ are present. A similar behavior is found for the $\bar{\nu}_e$ and ν_τ fluxes.

The behavior of the flux ratios shown in Fig. 7 is easy to understand. From Eqs. (19) and (20) one can write

$$\phi_{\nu_e}(\delta) = \phi_{\nu_e}(\delta = 0) + \sin 2\theta_{23} \sin\frac{\delta}{2}(L_{\nu_\tau} - L_{\nu_\mu}) \times \left[\sin 2\theta_{23} \sin\frac{\delta}{2}\left(|B_{\mu e}|^2 - |B_{\tau e}|^2\right) + \left[\left(\cos 2\theta_{23} \sin\frac{\delta}{2} - i\cos\frac{\delta}{2}\right)\left(B_{\mu e}B^*_{\tau e}\right) + \text{h.c.}\right]\right]. \quad (33)$$

Clearly the ratios calculated in these figures would be identity at the value of the energy where ν_μ and ν_τ spectra would cross (i.e., $L_{\nu_\tau} = L_{\nu_\mu}$). Away from this energy one expects an oscillatory behavior due to the additional terms in Eq. (33) as the figure indicates. Note that even for $\delta = 0$ the neutrino fluxes could also exhibit an oscillatory behavior. Concerning Fig. 8, one can see that the effects due to $\delta \neq 0$ and induced by taking different temperatures or luminosities are small, compared to the case with $\delta \neq 0$ only (Fig. 4).

Fig. 9 shows results on the electron fraction Y_e. Note that if $\delta \neq 0$ there are no CP-violation effects on Y_e since this quantity depends on the electron neutrino and anti-neutrino fluxes only Eqs. (31), (32). Our results show that the effects due to $\delta \neq 0$ are small (of the order of 0.1%) in all the studied cases with different muon and tau total luminosities and/or temperatures.

Finally, we discuss the effects induced by the CP violating phase δ on the supernova neutrino signal in a terrestrial observatory. Fig. 10 presents the expected number of events associated to electron anti-neutrino scattering on protons for different δ values. This is calculated by convoluting the fluxes from Eqs. (24)–(25) by the relevant anti-neutrino proton cross section [42]. A water Čerenkov detector such as super-Kamiokande (22.5 kt) is considered as an example. We assume 100% efficiency. Note that the neutral current signal which is sensitive to all fluxes turns out to be δ independent as well, as can be shown by adding the three fluxes Eq. (24). One can see that δ phase in-

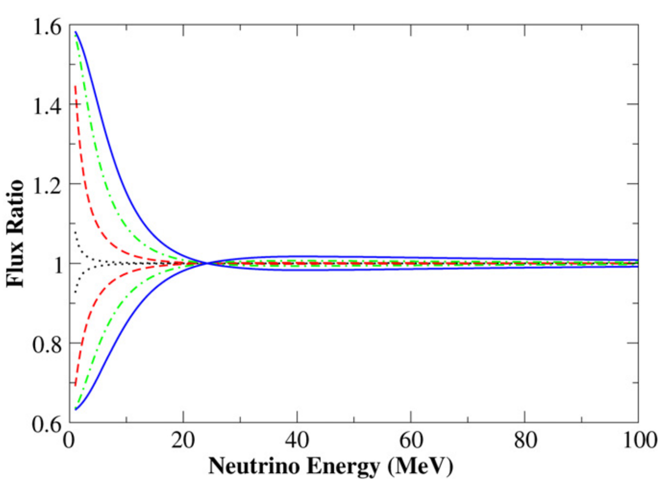

Fig. 2. $\bar{\nu}_\mu$ (lower curves) and $\bar{\nu}_\tau$ (upper curves) flux ratios for a CP violating phase $\delta = 180°$ over $\delta = 0°$ Eq. (23), as a function of neutrino energy. Results at different distances from the neutron-star surface are shown, namely 250 km (dotted), 500 km (dashed), 750 km (dot-dashed) and 1000 km (solid line). The curves correspond to the normal hierarchy and $\sin^2 2\theta_{13} = 0.19$.

[4] Note however that, even at the level of the Standard Model, some differences can arise for example from loop corrections [41].

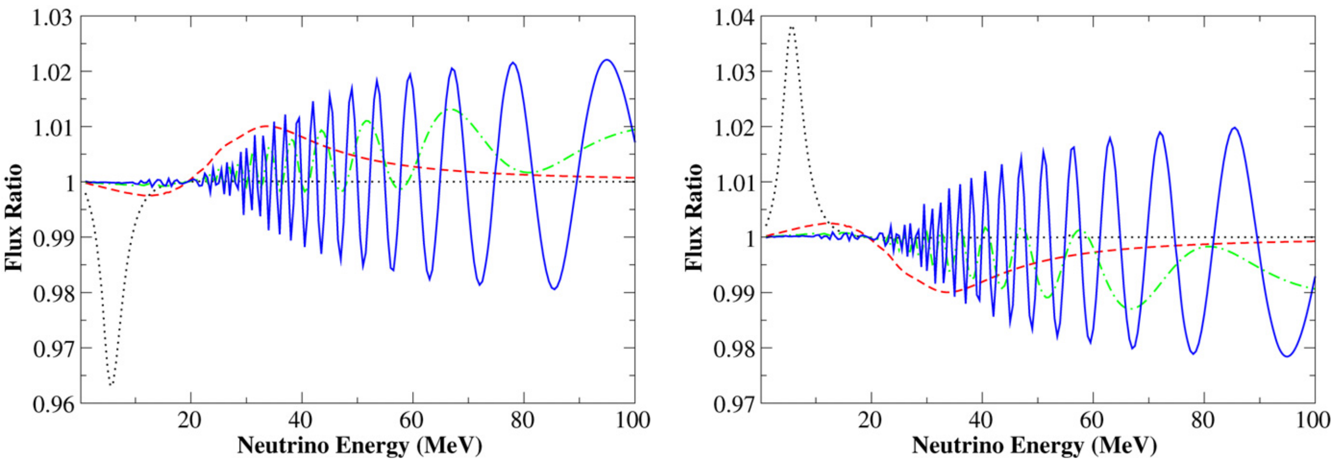

Fig. 3. Same as Fig. 2 for ν_μ (left) and ν_τ (right) fluxes.

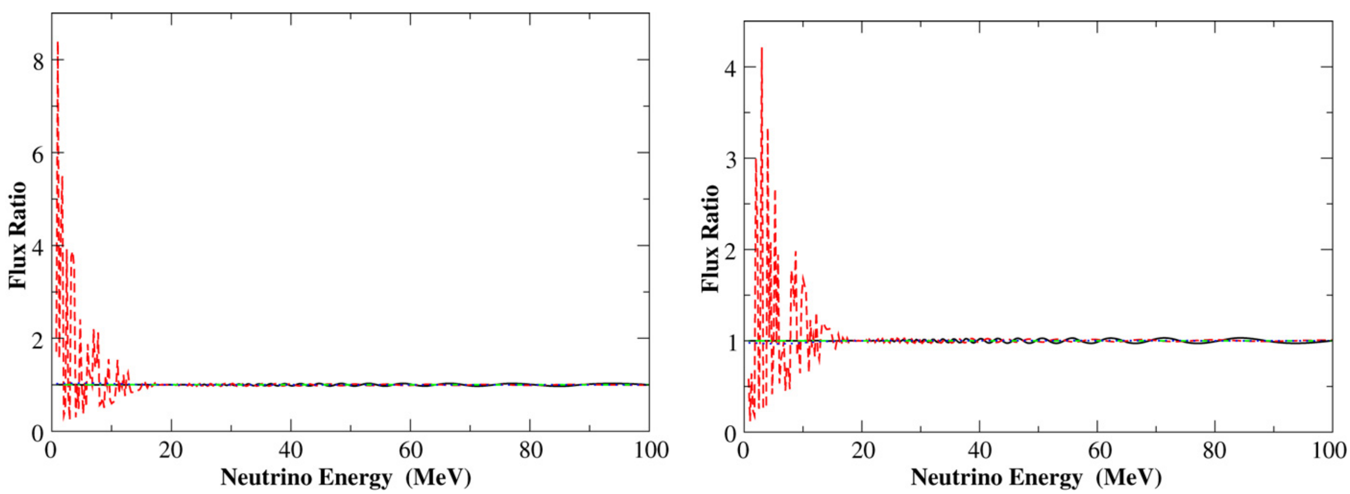

Fig. 4. Ratio of the ν_μ (left) and ν_τ (right) fluxes for $\delta = 180^\circ$ over $\delta = 0^\circ$ at a distance of 1000 km from the neutron-star surface. The curves correspond to N–L (solid), N–S (dashed), I–L (dot-dashed), I–S (dotted).

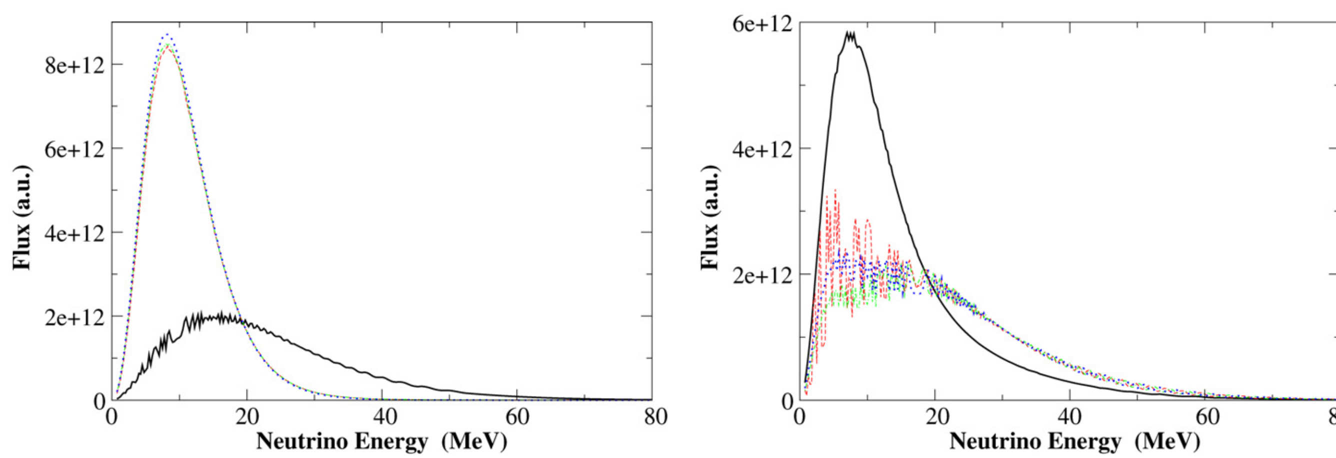

Fig. 5. Electron (left) and muon (right) neutrino fluxes Eq. (24) at 1000 km from the neutron star surface, N–L (solid), N–S (dashed), I–L (dot-dashed), I–S (dotted). In the N–L case, the first resonance is adiabatic and the Fermi–Dirac ν_e distributions at the neutrinospere (Fig. 1) are completely swapped with ν_x. The situation is reversed for ν_μ. These results are obtained by fixing T_{ν_τ} larger than T_{ν_μ} by 1 MeV, as an example of the difference that could be induced by the presence of flavor-changing interactions in the neutrinosphere (see text).

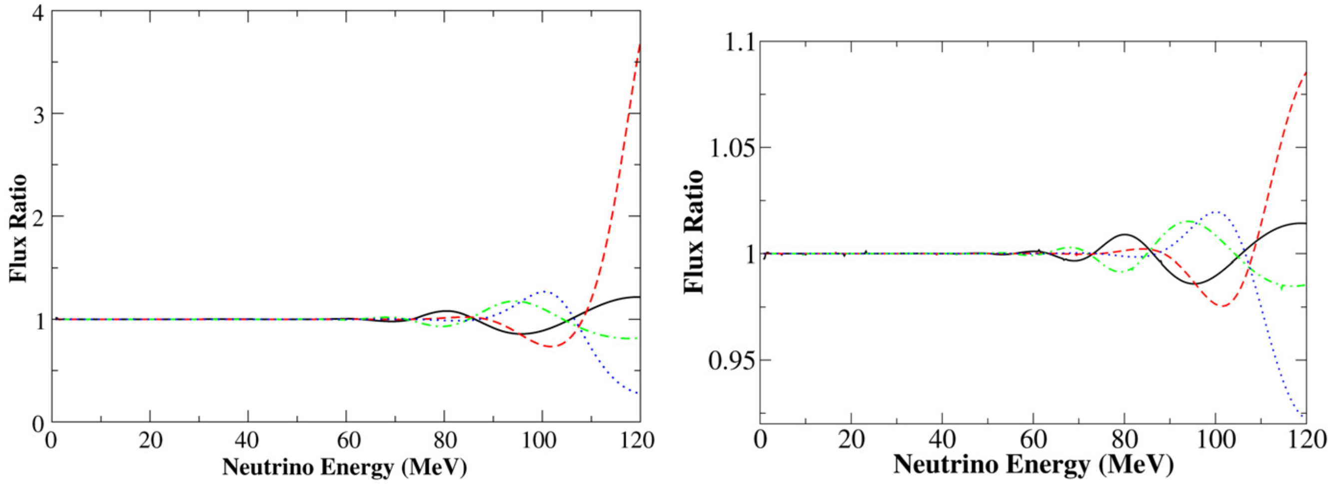

Fig. 6. Ratios of the ν_e flux $\delta = 180^\circ$ over for $\delta = 0^\circ$ at 200 km from the neutron star surface, obtained by taking $L^0_{\nu_\tau} = 1.1 L^0_{\nu_\mu}$ (right) or $T_{\nu_\tau} = 8.06$ MeV and $T_{\nu_\mu} = 7.06$ MeV (left) (see text). The curves correspond to N–L (solid), N–S (dashed), I–L (dot-dashed), I–S (dotted).

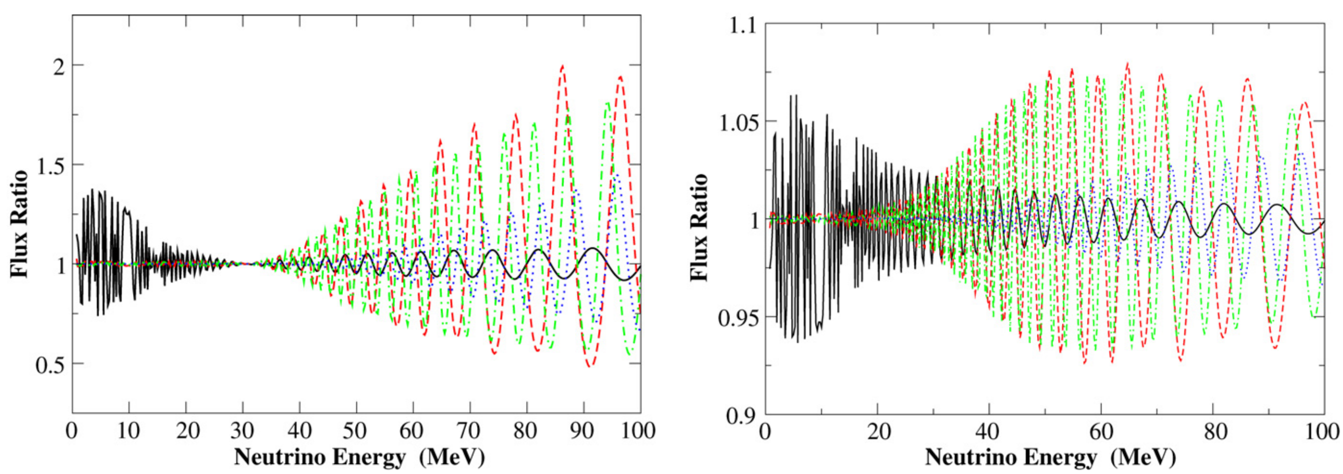

Fig. 7. Same as Fig. 6 but at 1000 km.

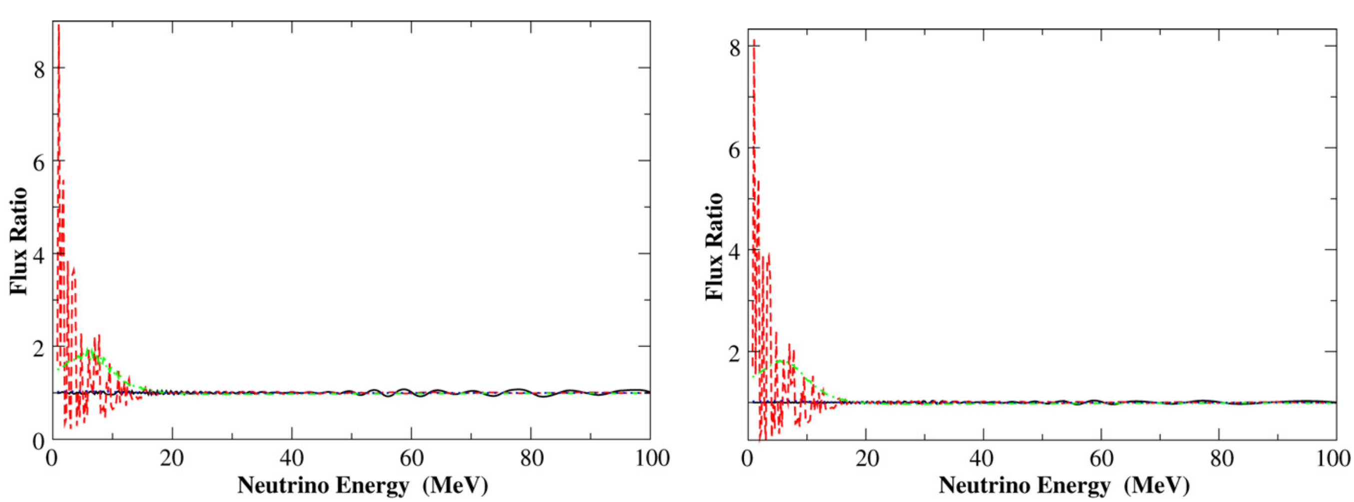

Fig. 8. Same as Fig. 7 but for the ν_μ flux ratios.

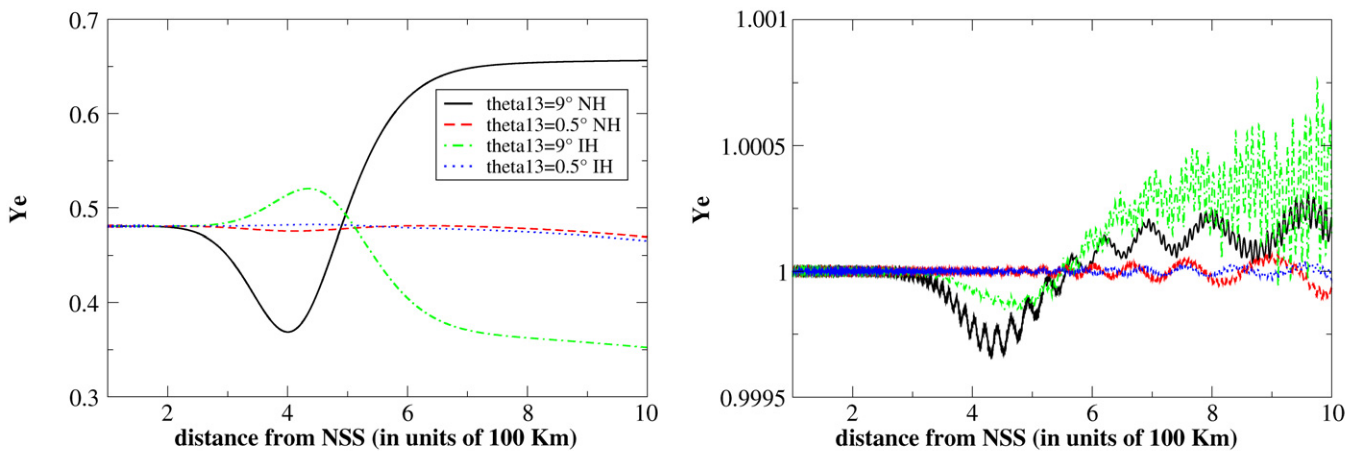

Fig. 9. Electron fraction for $\delta = 0$ (left) and ratios of the electron fraction (right) for $\delta = 180°$ compared to $\delta = 0°$, as a function of the distance from the neutron-star surface. The initial ν_μ, ν_τ fluxes have temperatures which differs by 1 MeV (see text). The results correspond to the normal hierarchy and $\sin^2 2\theta_{13} = 0.19$.

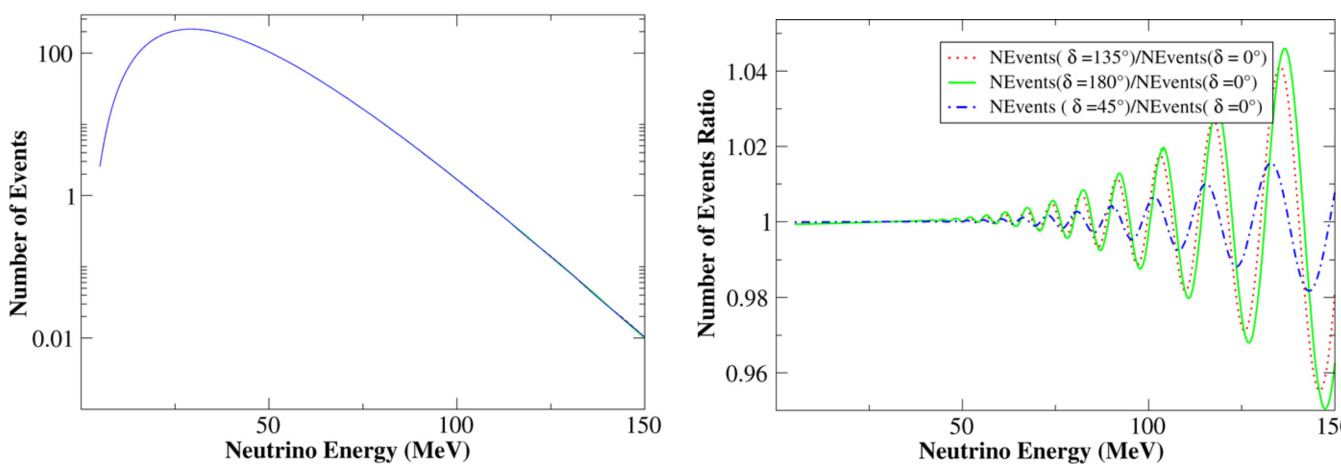

Fig. 10. Number of events associated to $\bar{\nu}_e + p \to n + e^+$ from a possible future supernova explosion at 10 kpc in a detector like super-Kamiokande (22.5 ktons). These results are obtained for inverted hierarchy and large third mixing angle.

duces small modifications up to 5% in the number of events, as a function of neutrino energy, and of the order of 2×10^{-4} on the total number of events. In fact, for a supernova at 10 kpc, we get for inverted hierarchy and large third neutrino mixing angle 7836.1 for $\delta = 45°$, 7837.0 for $\delta = 135°$, 7837.2 for $\delta = 180°$; while it is 7835.9 for $\delta = 0°$. These results are obtained with muon and tau neutrino fluxes having difference temperatures. Similar conclusion are drawn if we take different luminosities. For normal hierarchy and large θ_{13}, effects of the same order are found while for small θ_{13} and inverted/normal hierarchy the effects become as small as 10^{-5}.

5. Conclusions

In this work we have analyzed possible effects induced by the CP-violating Dirac phase in a dense environment such as the core-collapse supernovae. Our major result are that in matter: (i) significant effects are found on the muon and tau neutrino fluxes for a non-zero CP-violating phase; (ii) important effects are also found on the electron (anti-)neutrino fluxes if the ν_μ and ν_τ neutrino fluxes differ at the neutrinosphere. On the other hand the usual assumption of ignoring the CP-violating phase made in the literature is justified if contributions from physics beyond the Standard Model is small and the ν_μ and ν_τ fluxes are equal at emission. We have calculated the events in an observatory on earth and shown that effects at the level of 5% are present on the number of events as a function of neutrino energy.

Recent calculations have shown that the inclusion of neutrino–neutrino interaction introduces new features in the neutrino propagation in supernovae. A detailed study of the neutrino evolution with the CP-violating phase, the neutrino–neutrino interaction as well as loop induced neutrino refractive indices will be the object of further work.

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