Stochastic Processes and their Applications 19 (1985) 1-61
North-Holland

FOURTEENTH CONFERENCE ON STOCHASTIC PROCESSES AND THEIR APPLICATIONS

Gothenburg, Sweden, 12–16 June 1984

INTRODUCTION

The fourteenth Conference on Stochastic Processes and their Applications was held on the campus of the Chalmers University of Technology in Gothenburg, Sweden, over the period 12-16 June 1984. The Conference was arranged under the auspices of the Committee for Conferences on Stochastic Processes of the ISI's Bernoulli Society for Mathematical Statistics and Probability and was supported by the Swedish Natural Sciences Research Council, the Swedish Ministry of Education, Chalmers University of Technology, the Swedish Institute of Applied Mathematics, Swedish university departments of mathematics and mathematical statistics and Swedish industry.

The Conference was attended by 214 participants representing 29 different countries. The scientific program consisted of 18 invited and 112 contributed papers. The following are the abstracts of the papers presented.

ABSTRACTS

1. INVITED PAPERS

Tail Behaviour for the Suprema of Empirical Processes

Robert J. Adler, *Technion - Israel Institute of Technology, Haifa, Israel*

We consider multi-variate empirical processes $X_n(t) := \sqrt{n}(F_n(t) - F(t))$, where F_n is an empirical distribution function based on i.i.d. variables with distribution function F, and $t \in \mathbb{R}^k$. For X_F the weak limit of X_n, it is shown that

$$c(F, k)\lambda^{2(k-1)} e^{-2\lambda^2} \leqslant P\{\sup_t X_F(t) > \lambda\} \leqslant C(k)\lambda^{2(k-1)} e^{-2\lambda^2}$$

for large λ and appropriate constants c, C. When $k = 2$ these constants can be identified, thus permitting the development of Kolmogorov-Smirnov tests for bivariate problems. For general k the bound can be used to obtain sharp upper-lower class results for the growth of $\sup_t X_n(t)$ with n.

The methodology used to establish this bound can also be employed to study the tail behaviour of general Gaussian processes, $Y(t)$, and to remove the ε in the Fernique-Landau-Marcus-Shepp result that for all $\varepsilon > 0$

$$\log(P\{\sup_t Y(t) > \lambda\}) < \lambda^2\{\varepsilon - \tfrac{1}{2}[\sup_t \operatorname{var} Y(t)]^{-1}\}$$

for large enough λ, and a large class of Y.

Sojourns and Maxima of Stochastic Processes: Nonstationary Gaussian and Stable Processes

Simeon M. Berman, *Courant Institute of Mathematical Sciences, New York University, NY, USA*

Let $X(s)$, $s \geqslant 0$, be a real separable, measurable stochastic process. For fixed $t > 0$ and $u > 0$, put $L_u = \operatorname{mes}(s\colon 0 \leqslant s \leqslant t,\ X(s) > u)$ and $M = \sup(X(s)\colon 0 \leqslant s \leqslant t)$. In previous work, we obtained limit theorems for the ratios

$$\frac{P(vL_u > x)}{E(vL_u)} \quad \text{and} \quad \frac{P(M > u)}{E(vL_u)},$$

for $u \to \infty$, for some positive function $v = v(u)$, under the hypothesis that X is stationary. In the present paper, the methods are extended to apply to a large class of stochastic processes which are distinguished by their nonstationarity. In particular we consider processes with stationary increments. The limit theorems of the type given above are proved for processes with stationary Gaussian increments and with stationary independent symmetric stable increments. As an example of one of our results, we mention the following. If X has stationary Gaussian increments with mean 0, and if $\sigma^2(t) = EX^2(t)$ satisfies the following conditions: $\sigma^2(0) = 0$, $\sigma^2(t) = o(t)$ for $t \to 0$, and $\sigma^2(t)$ is convex for $t > 0$, then, for every $t > 0$, $P(M > u) \sim P(X(t) > u)$, for $u \to \infty$.

Ergodic Properties of Stationary Stable Processes

Stamatis Cambanis, Clyde D. Hardin, Jr. and Aleksander Weron, *University of North Carolina, Chapel Hill, NC, USA*

We derive spectral necessary and sufficient conditions for stationary symmetric stable processes to be metrically transitive, mixing, and to satisfy a law of large numbers, and we show how to estimate their covariation functions. We then consider some important special classes of stationary stable processes: Sub-Gaussian stationary processes are never metrically transitive, and we identify their ergodic decomposition. Stationary stable solutions of stochastic differential equations are strongly mixing. Stationary stable processes with a harmonic spectral representation are never metrically transitive, in sharp contrast with the Gaussian case, and some intriguing facts concerning their ergodic decomposition are derived; also stable processes with a harmonic spectral representation satisfy a strong law of large numbers even though they are not generally stationary. For doubly stationary stable processes, sufficient conditions are derived for metric transitivity and mixing, and necessary and sufficient conditions for a law of large numbers.

Some Remarks on the Early Development of the Theory of Stochastic Processes

Harald Cramér, *Stockholm, Sweden*

This lecture gives some personal recollections of the author from the time of the origin and early development of the theory of stochastic processes.

During the twenty years between the two world wars, mathematical probability theory passed through a period of intense development, a particular feature of which was the first mathematically rigorous treatment of problems connected with stochastic processes. Four names should be mentioned as leaders of this development: Khintchine and Kolmogorov in Moscow, Lévy in Paris, and Wiener at MIT in the United States.

The first to treat a stochastic process by modern mathematical methods was Norbert Wiener, who in the early 1920's gave a theory of molecular Brownian motion, where he introduced a probability distribution in a space of functions of a real variable. So far, this was only concerned with one particular case.

A general theory of stochastic processes appeared in some famous works of Khintchine and Kolmogorov during the years 1931–33. In his book "Grundbegriffe der Wahrscheinlichkeitsrechnung" of 1933 Kolmogorov introduced new foundations of the whole subject, based on the now familiar concepts of a probability space and probability measure. He also gave some basic general theorems on stochastic processes, regarded as probability distributions in spaces of functions.

Further work on stochastic processes was due to Lévy and Khintchine. The important class of stationary processes was introduced by Khintchine. In our probabilistic group at the University of Stockholm we followed all this work with a keen interest. Among our members, Feller generalized the work of Kolmogorov, while Wold and the present author gave some contributions to the theory and applications of stationary processes.

When, after the isolation by the war, international scientific relations became possible, it appeared that important applications of stochastic processes had been made in military and industrial war work. In this connection, Kolmogorov had pointed out that Hilbert space theory could be successfully applied to the study of stationary stochastic processes. After the war, further important work on this line, also for other classes of processes, was done by many authors, of whom I will mention Karhunen and Grenander, both associated with our Stockholm group.

And then, in the beginning of the 1950's, time was ripe for the publications of the first general treatise of the theory of stochastic processes, the famous book by Doob. The theory had outgrown its early stage, and reached maturity.

The Intersection of Classical Potential Theory with Martingale Theory

J.L. Doob, *University of Illinois, Urbana, USA*

Classical potential theory and martingale theory use identical methods to obtain parallel results. The following examples will be discussed.

Greatest harmonic (martingale) minorant of a superharmonic function (supermartingale).

Orthogonal decompositions of the vector lattice of positive superharmonic function (supermartingale) differences.

The measure associated with a superharmonic function (supermartingale) and the domination principle.

The reduction operation on superharmonic functions (supermartingales), and the continuity properties it yields in each context.

Disordered Markov Fields and Percolation

Hans-Otto Georgii, *University of Munich, FR Germany*

We consider a model of a disordered ferromagnet. At each site i of the square lattice $\mathbb{Z}^2$ there is a bounded real-valued spin σ_i. These spins interact as follows.

There is no interaction between non-adjacent spins. If $|i-j|=1$ then the interaction energy between σ_i and σ_j is given by $-\beta J_{\{i,j\}}\sigma_i\sigma_j$; here β is proportional to the inverse temperature, and the $J_{\{i,j\}}$'s are independent non-negative random variables with an identical distribution α. For fixed values of the coupling variables $J_{\{i,j\}}$ the conditional joint distribution of the spins is assumed to be the maximal Gibbs measure for these interaction values and some even a priori measure on the real line.

We investigate the region in the (α, β)-space where spontaneous magnetization occurs in the sense that $\mathbb{E}\sigma_0 > 0$. Up to boundary points, this region is of the form

$$\{(\alpha, \beta)\colon \alpha(\{0\}) < \tfrac{1}{2},\ \beta > \beta_c(\alpha)\}.$$

$\beta_c(\alpha)$ is called the inverse critical temperature; $\frac{1}{2}$ is the threshold value for Bernoulli bond percolation on $\mathbb{Z}^2$. We present upper and lower bounds of $\beta_c(\alpha)$ which provide an estimate of the singularity of $\beta_c(\alpha)$ when $\alpha(\{0\})$ tends to $\frac{1}{2}$. These bounds are obtained by an analysis of a suitable site-bond percolation problem.

A Machine for Pattern Inference

Ulf Grenander, *Brown University, USA*

A general purpose logic machine has been designed for carrying out pattern inferences for incompletely known patterns. Its application to image restoration and pattern recognition will be discussed. This is based on random geometries in the sense of pattern theory, and a number of concrete examples will be presented. The machine computes Bayesian solutions to problems of inference in Markov processes on graphs.

The construction of the machine has been motivated to some extent by recent advances in computer architecture. We shall discuss its relation to highly parallel computers.

Stochastic Differential Equations in Infinite Dimensions

K. Itô, *Gakushuin University, Tokyo, Japan*

A typical example of stochastic differential equations is of the form

$$\mathrm{d}X_t = \sigma(X_t)\,\mathrm{d}B_t + a(X_t)\,\mathrm{d}t,$$

where $\{X_t\}$ is an $\mathbb{R}^r$-valued sample continuous process and $\{B_t\}$ is an $\mathbb{R}^d$-valued Wiener process. Our purpose is to discuss an infinite dimensional extension of this equation where the state spaces of X_t and B_t are Schwartz spaces of distributions, because practically all problems appearing in applications can be discussed in this frame-work.

In this note we will discuss basic concepts such as random variables, stochastic processes, Wiener processes, stochastic integrals and stochastic differential equations in infinite dimensions and deal with concrete stochastic differential equations from our view-point.

Cooptional Times and Invariant Measures for Transient Markov Chains

Martin Jacobsen, *University of Copenhagen, Denmark*

The classical necessary and sufficient condition for an irreducible, transient Markov chain $X = (X_n)_{n \geq 0}$ on a discrete state space J and with stationary transitions, to possess an invariant measure, was established by Harris [1] and Veech [3] more than twenty years ago.

It is the purpose of the present paper to review the Harris–Veech condition using some recent results, Jacobsen [2], on cooptional (generalized last-exit) times, and also by means of the same results, to present some new necessary and sufficient conditions for the existence of an invariant measure.

The main theorem states the following, writing P for the transition matrix and $G = \sum_{n=0}^{\infty} P^n$ for the Green's function of X. In order that P has an invariant measure μ, it is necessary and sufficient that there exists a sequence (ν_n) of finite measures on J such that

$$\text{(i)}\ \nu_n \to 0, \qquad \text{(ii)}\ \nu_n G \to \mu, \qquad \text{(iii)}\ \nu_n GP \to \mu P \text{ pointwise as } n \to \infty.$$

Here it is absolutely trivial that if (ν_n) can be found such that (i), (ii), (iii) hold with $0 < \mu < \infty$, then μ is invariant. For the proof of the other half the (ν_n) are constructed explicitly, using suitable last-exit times.

The first link to the classical theory is then provided by showing that if an invariant μ exists, one may always choose ν_n of the form $G^{-1}(x_n, a)\varepsilon_{x_n}$ with a a given state and (x_n) an infinite sequence of distinct states. From here, the gap to the Harris–Veech condition is bridged fairly easily, using properties of cooptional times.

References

[1] T.E. Harris, Transient Markov chains with stationary measures, Proc. Amer. Math. Soc. 8 (1957), 937–942.
[2] M. Jacobsen, Two operational characterizations of cooptional times, Ann. Probab. (to appear, 1984).
[3] W. Veech, The necessity of Harris' condition for the existence of a stationary measure, Proc. Amer. Math. Soc. 14 (1963) 856–860.

Malliavin Calculus for Jump Processes

Jean Jacod, *University of Paris, France*

One of the purposes of Malliavin calculus is to give a probabilistic proof of various regularity results, like Hörmander's theorem, for the solution of elliptic operators.

If, instead of starting from a continuous diffusion, one starts with a diffusion 'with jumps', one can solve similar problems for the following integro-differential operator:

$$Yf(x)=\sum a^i(x)D_i f(x)+\tfrac{1}{2}\sum \beta^{i,j}(x)D^2_{i,j}f(x)$$
$$+\int K(x,\mathrm{d}y)[f(x+y)-f(x)-\sum y^i D_i f(x)].$$

As a matter of fact, the regularity conditions are much more easily expressed on the coefficients of the corresponding stochastic equation

$$\mathrm{d}X_t=a(X_t)\,\mathrm{d}t+b(X_t)\,\mathrm{d}W_t+c(X_{t-},y)\tilde{\mu}(\mathrm{d}t,\mathrm{d}y)$$

($\tilde{\mu}$ = compensated Poisson measure, $\beta=bb^t$ and $K(x,\cdot)$ is the image of the intensity measure of $\tilde{\mu}$ by the mapping $c(x,\cdot)$).

We will give some ('elliptic' type) conditions for the random variable X_t to have a density, which is regular under some 'strictly elliptic' type conditions.

On Previsible Sampling from Finite Populations, with Applications to Gambling and Stochastic Integration

Olav Kallenberg, *Chalmers University of Technology, Göteborg, Sweden*

Some recent developments in exchangeability theory are concerned with the interplay between exchangeability and some notions of modern process theory, notably stopping times, martingales and stochastic integration (cf. [2]). In the present talk, we shall focus our attention on a general invariance property for exchangeable sequences and processes, and we shall illustrate its use by discussing some rather surprising (sometimes amusing) applications to gambling and stochastic integration.

To describe the invariance property in the simplest possible case, let $\xi_1,\ldots,\xi_n$ be the sequence of elements obtained by random sampling without replacement from a population of size n. Further suppose that $T_1,\ldots,T_n$ are previsible stopping times constituting a random permutation of $(1,\ldots,n)$. (Independent randomizations are allowed!) Then the permuted sequence $(\xi_{T_1},\ldots,\xi_{T_n})$ has the same distribution as $(\xi_1,\ldots,\xi_n)$.

The continuous time analogue is less elementary, and requires stochastic integrals for its formulation. Let X be an exchangeable process on $[0,1]$ with $X_0=0$. (This means that X has exchangeable increments over disjoint intervals of equal length. The structure of such processes is completely known, cf. [1]. Note in particular that they are semi-martingales.) Let V be a previsible process whose paths are Lebesgue measure preserving transformations of $[0,1]$, and define the process Y by

$$Y_t=\int_0^1 1\{V_s\leq t\}\,\mathrm{d}X_s,\quad t\in[0,1].$$

Then suitable versions of X and Y have the same distribution.

References

[1] O. Kallenberg, Canonical representations and convergence criteria for processes with interchangeable increments, ZfW 27 (1973) 23–36.
[2] O. Kallenberg, Characterizations and embedding properties in exchangeability, ZfW 60 (1982) 249–281.

On Extremal Theory and Practice – An Overview and Recent Developments

M.R. Leadbetter, *University of North Carolina, Chapel Hill, NC, USA*

Much of the practical success of classical extreme value theory is due to the fact that it is 'robust' against certain departures from independence. Indeed, an extensive 'working theory' is now available to handle extremes for dependent cases both in discrete and continuous time. This paper gives (i) a brief overview of the theory, (ii) recent results concerning the extent of the 'robustness' of the classical theory and the further types of behaviour possible under dependence, and (iii), comments on estimation of certain functions central in applications.

The Stable Pedigrees of Supercritical Branching Populations in General Type Spaces

Olle Nerman, *Chalmers University of Technology, Göteborg, Sweden*

Consider a branching population of individuals of different types $\gamma \in \Gamma$. Denote by $\mu(\gamma, \cdot)$ the expected reproduction measure on $[0,\infty)\times\Gamma$ (age $\times$type) of an individual of type γ. Let μ^n be the n'th iterate of μ (in the Markov renewal sense). Then

$$\nu(\gamma, \cdot) = \sum_{n=0}^{\infty} \mu^n(\gamma, \cdot)$$

is the expected total population measure.

Assume that there is an $h : \Gamma \to (0,\infty)$ and an $\alpha > 0$ such that

$$h(\gamma) = \int_{\Gamma} h(\gamma')\, \mathrm{e}^{-\alpha u} \mu(\gamma, \mathrm{d}u \times \mathrm{d}\gamma').$$

Then the transformed kernel

$$\mu_\alpha(\gamma, \mathrm{d}u \times \mathrm{d}\gamma') = h(\gamma')\, \mathrm{e}^{-\alpha u} \mu(\gamma, \mathrm{d}u \times \mathrm{d}\gamma')/h(\gamma)$$

defines a Markov renewal law with Markov renewal kernel

$$\nu_\alpha(\gamma, \mathrm{d}u \times \mathrm{d}\gamma') = h(\gamma')\, \mathrm{e}^{-\alpha u} \nu(\gamma, \mathrm{d}u \times \mathrm{d}\gamma')/h(\gamma).$$

This kernel can be utilized to study the asymptotics of $\nu(\gamma, t + \mathrm{d}u \times \mathrm{d}\gamma')$ as $t \to \infty$. Under natural conditions the expected population of new-borns ultimately grows exponentially with rate α and a time invariant type distribution.

Under some more restrictions the empirical composition of the population can be seen to stabilize on the set of non-extinction. This can be expressed by the notion of stable pedigree measures. These arise at time limits of the empirical pedigrees corresponding to sampling of a central individual (ego) from natural populations (like all born individuals or all living individuals) and consideration of the lives and relations of ego's relatives. The stable pedigrees have nice structures, e.g. the types and successive generation spans of ego's ancestors are governed by the time reversed Markov renewal law corresponding to $\mu_\alpha(\gamma, \cdot)$.

On Non-Singular Semigroups of Transition Kernels

Esa Nummelin, *University of Helsinki, Finland*

Let $(P_t) = (P_t(x, A);\ x \in E, A \in \mathscr{E}, t \geq 0)$ be a measurable semigroup of positive transition kernels on a countably generated measurable space $(E, \mathscr{E})$. Let ψ be a σ-finite reference measure on $(E, \mathscr{E})$, and assume that the semigroup (P_t) is *non-singular* in the sense that

$$\int_E \int_0^\infty p_t^\psi(x, y)\psi(\mathrm{d}y)\,\mathrm{d}t > 0,$$

where $p_t^\psi(x, y)$ denotes the density of $P_t(x, \mathrm{d}y)$ w.r.t. ψ.

One can prove that there then exists an embedded non-singular renewal measure 'in' the semigroup, which in the stochastic case corresponds to a certain embedded regeneration structure. This device gives new insight into the recurrence and limit structure of general positive semigroups. For example one can prove the following monotone limit theorem which shows that (under a suitable recurrence hypothesis) the absolutely continuous part of the semigroup 'dominates' as $t \to \infty$.

Theorem. *Let (P_t) be a non-singular semigroup of transition probabilities of a positive ψ-recurrent Markov process (X_t) on $(E, \mathscr{E})$ having invariant probability measure $\pi \gg \psi$. Then there exists a function $c(t) \uparrow 1$ and sets $E(t) \uparrow E$ as $t \to \infty$ such that*

$$p_t^\pi(x, y) \geq c(t) \quad \textit{for all } x, y \in E(t), \textit{ all } t \geq 0.$$

(As a corollary we get Orey's convergence theorem: $P_t(x, \cdot) \to \pi(\cdot)$ in total variation norm as $t \to \infty$, for all $x \in E$.)

Reference

S. Niemi and E. Nummelin, On non-singular renewal kernels with an application to semigroups of transition kernels, Preprint (University of Helsinki, 1984).

On the Additive Functionals of Branching Processes

V.M. Shurenkov, *Institute of Mathematics, Kiev, USSR*

We define the additive functionals on the trajectories of branching processes. It is shown that in the critical and supercritical cases under the condition of non-extinction the value of the additive functional is asymptotically linearly dependent on the number of particles. Some examples are given.

Error Rates and Estimation Errors in Extreme Value Theory

Richard L. Smith, *Imperial College, London, England*

The theory of regularly varying functions is of great importance in the limit theorems of extreme value theory. If the condition defining regular variation is strengthened to include a remainder term, then one can determine rates of convergence in these limit theorems (Smith 1982, Cohen 1982). In this talk I shall survey these ideas, and then discuss more recent work relating to the statistical estimation of extremes. In particular, for the case that $1-F(x)$ (for some d.f. F) is regularly varying with index $-\alpha(\alpha>0)$, we construct estimators of α and show how the estimation error is related to the size of the remainder term.

References

R.L. Smith, Uniform rates of convergence in extreme value theory. Adv. Appl. Prob. 15 (1982) 600–622.

J.P. Cohen, Convergence rates for the ultimate and penultimate approximations in extreme value theory, Adv. Appl. Probab. 15 (1982) 833–854.

A Genealogical View of some Stochastic Models in Population Genetics

Simon Tavaré, *Colorado State University, CO, USA*

Many classical population genetics problems are studied by a direct analysis of stochastic gene frequency models. Such methods have obscured the central role played by genealogy, the relationship between individuals in the population.

This paper reviews some recent results about the stochastic structure of the genealogy of large haploid populations. Applications of the results to an important class of population genetics models are given.

Reflected Brownian Motion in a Polyhedral Region

R.J. Williams, *Courant Institute of Mathematical Sciences, New York University, USA*

This work is concerned with the study of Brownian motion in a three-dimensional polyhedral region with instantaneous reflection at the boundary in given constant directions on each boundary face. Under certain restrictions on the directions of reflection and geometry of the region, such processes arise naturally as diffusion approximations to queueing networks and storage systems, and can be represented as semimartingales. However, in general, with given data, a process with instantaneous reflection need not exist or be representable as a semimartingale.

The problems of existence and uniqueness, characterization, and semimartingale representation for given data will be addressed. Central to the study of these problems, and to others of practical importance such as recurrence classification and simulation by regeneration, is the question of whether the process hits edges or corners, these being parts of the boundary where there are discontinuities in the directions of reflection and non-smoothness of the boundary. Criteria for determining the answer to this question will be given.

2. CONTRIBUTED PAPERS

2.1. *Branching, population and epidemic models*

The Threshold Behaviour of Epidemic Models

Frank Ball, *University of Nottingham, England*

A simple coupling argument is used to construct a sequence of general stochastic epidemics, indexed by the initial number of susceptibles, from a time-homogeneous birth-and-death process. This construction provides us with a simple proof of the threshold behaviour of the general stochastic epidemic. The method is quite general and extensions to other epidemic models, such as chain-binomial and host-vector models, are briefly described.

Asymptotic Properties of an Estimator in the Multitype Branching Process

Maria Lucilia Carvalho, *Departamento de Estadistica da F.C.L., Lisbon, Portugal*

Let Z_n be a positive regular, supercritical, r-type Galton–Watson process. Let ρ and u, respectively, be the principal eigenvalue and the corresponding right eigenvector of the reproduction mean matrix.

Let $\tilde{\rho}_1 = \sum_{n=1}^{N} Z_n \cdot u / \sum_{n=1}^{N} Z_{n-1} \cdot u$ be the estimator proposed by Asmussen and Keiding based on the knowledge of u and the generation vectors Z_n.

Using exactly the same information we give an estimator $\tilde{V}_n$ of V, the asymptotic variance of $\tilde{\rho}_1$. Conditions for the weak and strong convergence of $\tilde{V}_n$ to V are established.

Limit Theorems for Infinite Particle Branching Brownian Motions with Immigration

Luis G. Gorostiza, *Centro de Investigación y de Estudios Avanzados, México*

We consider an infinite system of independent branching Brownian motions on $\mathbb{R}^d$, such that the initial particles are distributed according to a homogeneous Poisson random field, and particles from an external source immigrate into $\mathbb{R}^d$ according to a homogeneous space-time Poisson field, each one generating an independent branching Brownian motion. We discuss the limit behavior of the fluctuations of the system with respect to the mean under different scalings: high-density, space scaling, space-time scaling. The fluctuation limits are generalized Gauss–Markov processes. In particular we describe the time evolution corresponding to the fixed-time spatial central limit theorem proved by G. Ivanoff ("The branching diffusion with immigration", J. Appl. Prob. 12 (1980) 825-847).

Population Momentum – A Formulation Based on a Stochastic Population Process

C.J. Mode, *Drexel University, Philadelphia, USA*

The momentum of population growth is studied within a unifying framework based on a stochastic population process with time homogeneous laws of evolution. The population process, in turn, is based on a generalized age-dependent branching process in discrete time. A discrete time formulation has been preferred as an aid to computer implementation. Within the discrete time formulation, the mean age structure, representing sums of solutions to renewal type equations, may be computed recursively, using a Leslie-matrix type algorithm. The role of the Fisherian reproductive value in population momentum will be discussed. An example is provided whereby a population would continue to grow for about 30 years even if there were an abrupt change to a fertility regime in which mean family size was one offspring.

Asymptotic Distribution of the Final Size in some Epidemic Models

Gianpaolo Scalia-Tomba, *Stockholm University, Stockholm, Sweden*

The Reed–Frost chain-binomial process is one of the classical models for the spread of an infectious disease in a closed, stable population. In that model, it is assumed that infection probabilities remain constant during the course of the epidemic and that they are equal for all pairs of individuals (homogeneous mixing).

Some extensions of the basic Reed–Frost model, to multitype populations and to density dependent infection probabilities, will be presented and the asymptotic distribution, as the initial population size tends to infinity, of the final size of the epidemic in these extended models will be derived.

2.2. Gaussian processes

Approximating Narrow Band Stationary Gaussian Processes by Trigonometric Polynomials

A.M. Hasofer, *University of New South Wales, Sydney, Australia*

A simple practical procedure for approximating a Narrow Band Gaussian Stationary Process over a finite interval by a trigonometric polynomial is described. The approximation is used to calculate the upper tail of the distribution of the maximum. Exact bounds for the approximation error and numerical results are given.

Exponents of Entropy and ε-Entropies for Dominated Families of Gaussian Measures on a Hilbert Space

T. Koski, *Åbo Academy, Finland*

A zero mean Gaussian measure μ on a real separable Hilbert space H with scalar product (x, y) is determined by a dense subspace H_μ of H, which is also a Hilbert space under the scalar product $(W^{1/2}x, W^{1/2}y)_\mu = (x, y)$, where W is the covariance operator corresponding to μ. We shall first consider the unit ball U_1 in H_μ, i.e. the variance ellipsoid of μ. It is known that the exponent of entropy r of U_1, defined e.g. in [1], is equal to the exponent of convergence p of the spectrum of $W^{1/2}$. We show, under certain conditions, that the exponents of entropy of a dominated family of Gaussian measures on H are all equal to r for the Gaussian reference measure. This is seen in a particularly simple fashion when the covariance operators are commuting. As r is equal to p we have here a fact about the supports of measures induced on l_2 on account of a result in [2].

In addition we show that r governs the ε-entropies (in the sense of Pinsker) of the underlying H-valued Gaussian variables. Thus the equality of the exponents of entropy explained above implies a partial generalization of the main theorem in [3]. All these concepts are applicable to the transition probabilities induced by certain H-valued linear, time invariant stochastic differential equations, [4].

References

[1] R.M. Dudley, The sizes of compact subsets of Hilbert space and continuity of Gaussian processes; Journal of Functional Analysis (1967) 290–330.
[2] U. Grenander, Abstract Inference (Wiley, New York, 1981).
[3] S. Ihara, On ε-entropy of equivalent Gaussian processes, Nagoya Math. Journal (1970) 121–130.
[4] J. Zabczyk, Structural properties and limit behaviour of linear stochastic systems in Hilbert spaces (Banach Center Publ., 1983).

Optimal Prediction of Level Crossings in Gaussian Processes

Georg Lindgren, *University of Lund, Sweden*

Let $\eta(t)$ be a stationary Gaussian process and suppose we want to predict, from data available at time t, future upcrossings of a given level u. We shall investigate certain criteria which ought to be met by a good level crossing predictor, and in particular restate, in simple form, a result by de Maré on optimal prediction. Let $\hat{\eta}_t(t+h)$ be the predictor of $\eta(t+h)$ at time t, and $\hat{\zeta}_t(t+h)$ the conditional predictor of $\eta'(t+h)$ given observed data and given that $\eta(t+h)=u$. The 'optimal' level crossing predictor gives an 'alarm' at time t for an upcrossing at time $t+h$ if $\hat{\eta}_t(t+h)$ differs from u by a quantity that is a function of the expected rate of increase $\hat{\zeta}_t(t+h)$. In fact, an alarm is given if

$$\sigma_h^{-2}(u-\hat{\eta}_t(t+h))^2 < 2\log \Psi(\hat{\zeta}_t(t+h)/\sigma_h') + C_h,$$

where σ_h and σ'_h are the standard deviations of the predictors, C_h a constant, and $\Psi'(x) = \Phi(x)$. This procedure has the highest detection probability among all procedures with the same total alarm probability. Formulas are given for the upcrossing risk after alarm, the detection probability, and the total time alarm is given. The optimal alarm is compared to the naive alarm that predicts an upcrossing if $\hat{\eta}_t(t+h)$ differs from u by a fixed quantity. It is shown that the optimal alarm locates the correct time of the upcrossings more precisely and at an earlier stage than the naive alarm.

Some Sojourn Time Problems for Two-Dimensional Gaussian Processes

Makoto Maejima, *Keio University, Yokohama, Japan*

Let $\{X(t) = (X_1(t), X_2(t)),\ t \geqslant 0\}$ be a two-dimensional measurable separable zero-mean stationary Gaussian process with some long-range dependence. Two components $X_1(t)$ and $X_2(t)$ are not necessarily assumed to be independent.

In this talk, the limiting distributions of sojourn functionals

$$M(t) = \int_0^t I[X(s) \in D]\,\mathrm{d}s, \quad D \subset \mathbb{R}^2,$$

will be discussed, where $I[\cdot]$ is the indicator function. Especially, the following types of D's are considered:

$$D = \{(x_1, x_2) \mid a^2x_1^2 + b^2x_2^2 \leqslant 1\}, \quad 0 < a \leqslant b < \infty,$$

or

$$D = \{(x_1, x_2) \mid a^2 \leqslant x_1^2 + x_2^2 \leqslant b^2\}, \quad 0 < a < b < \infty.$$

The main purpose of this study is to see how the limiting distributions depend on the shape of D or the dependence of the components of $X_1(t)$ and $X_2(t)$. To get the limiting distributions, the non-central limit theorem by Dobrushin–Major–Taqqu [1, 2] is applied.

References

[1] R.L. Dobrushin and P. Major, Non-central limit theorems for non-linear functionals of Gaussian fields, ZfW 50 (1979) 27–52.
[2] M.S. Taqqu, Convergence of integrated processes of arbitrary Hermite rank, ZfW 50 (1979) 53–83.

2.3. Markov and semi-Markov processes

Two-Sided Markov Chains

Bruce W. Atkinson, *University of Florida, USA*

Let S be a countable set, $a, b \notin S$, and Ω the space of functions $\omega : Z \to S \cup \{a, b\}$ so that $\omega(n) \in S$ for some n, $\omega(n) = a \Rightarrow \omega(m) = a$ for $m < n$, and $\omega(n) = b \Rightarrow$

$\omega(m)=b$ for $m>n$. On Ω let $x(n)$ denote the n-th coordinate and $F=\sigma(x(n)\colon n\in Z)$. A measure P on (Ω, F), such that $P(x(n)=j)<\infty$ for all $n\in Z$ and $j\in S$, is called a two-sided Markov chain if P is Markov and if there exist substochastic matrices $\hat{p}(i,j)$ and $p(i,j)$ on S so that for every n,

$$P(x(n)=i,\, x(n+1)=j)=\pi_n(i)p(i,j)=\pi_{n+1}(j)\hat{p}(j,i)\ \ \forall i,j\in S$$

where $\pi_k(r)=P(x(k)=r)\ \forall k\in Z$ and $r\in S$. In other words P is time-homogeneous in both time directions with p as the one-step forward transition matrix and $\hat{p}$ as the one-step backward transition matrix. This paper explores conditions for the existence of such processes.

It is shown that if $p(i,j)>0$ for all i,j then there exists $\rho>0$ such that $P(x(n+1)=i)=\rho P(x(n)=i)\ \forall n\in Z$ and $i\in S$, and such that

$$\sum_{i\in S}\pi(i)p(i,j)\leq\rho\pi(j)\quad \forall j\in S \tag{$\#$}$$

where $\pi=\pi_0$. Also, given a substochastic matrix p (not necessarily >0) and a measure (i.e. non-negative row vector)π satisfying ($\#$) for some $\rho>0$, then there exists a two-sided Markov chain with p as the one-step forward transition matrix and with $\pi_n=\rho^n\pi\ \forall n\in Z$. Of special interest is the case where $\rho=1$, which corresponds to stationary two-sided Markov chains. (Note: For processes with continuous time and space such stationary two-sided processes have played an important role in recent developments in probabilistic potential theory.)

On Exponential Ergodicity for Birth–Death Processes

Erik A. van Doorn, *Centre for Mathematics and Computer Science, Amsterdam, The Netherlands*

This paper is addressed to two problems in connection with exponential ergodicity for birth–death processes on the nonnegative integers. The first is to determine from the birth and death rates whether exponential ergodicity prevails, i.e., whether there exists a positive number a such that for all i, j

$$p_{ij}(t)-p_{ij}(\infty)=\mathrm{O}(\exp(-at))\quad \text{as } t\to\infty, \tag{1}$$

where the $p_{ij}(t)$ denote transition probabilities. We give some necessary and some sufficient conditions which suffice to settle the question for most processes encountered in practice. In particular, a complete solution has been obtained for processes where, from some state onwards, the rates are rational functions of the state.

The second, more difficult problem is to evaluate the decay parameter of a birth–death process, which is the supremum of the a's satisfying (1) for all i, j. Our contribution to the solution of this problem consists of a number of upper and lower bounds.

Markov Chain Ergodicity and Time Series Models

Paul D. Feigin, *Technion, Haifa, Israel*
R.L. Tweedie, *Siromath, Sydney, Australia*

Simple yet practically efficient conditions for the ergodicity of a Markov chain on a general state space have recently been developed. We illustrate their application to non-linear time series models and, in particular, to random coefficient autoregressive models.

As well as ensuring the existence of a unique stationary distribution, geometric rates of convergence to stationarity are ensured. Moreover, sufficient conditions for the existence and convergence of moments can be determined by a closely related method. The latter conditions, in particular, are new.

A Boltzmann H-Theorem for some Non-Linear Processes (Including McKean Jr.'s Tagged Molecule)

Gaston Giroux, *University of Sherbrooke, Quebec, **Canada***

In the Proceedings of the N.A.S. of 1966 McKean Jr. has introduced homogeneous Markov processes which do not satisfy the linearity property ($\mathbb{P}_\mu \neq \int \mu(\mathrm{d}x)\mathbb{P}_x$). We want to point out that for a class of homogeneous processes, including McKean Jr.'s tagged molecule but not necessarily Markov, we can deduce an H-Theorem as soon as they possess a 'strong' equilibrium of Gibbs type.

Probabilistic Properties of a General Cyclic Markovian Model in Discrete Time

Christine Jacob, *INRA, 78350 Jouy-en-Josas, France*

In C. Jacob [1], the intuitive idea of a stochastic cycle is mathematically defined in discrete time from a generalization of Orey's definition [3]. We also defined a stochastic limit cycle and the stability and self-exciting properties of such a cycle. We then built a general Markovian time series model possessing a stochastic cycle. An example was given. It concerned the well-known Canadian lynx data series.

Here we study the probabilistic properties of the general cyclic Markovian model:

$$x_{n+1} = f(x_n) + \sum_{j=1}^{d} 1_{\mathscr{C}_j}(x_n) 1_j(\phi_n (\mathrm{mod}\ d))\xi_{n+1,j+1}$$

$$\phi_{n+1} = \phi_n + 1$$

$$x_0 \in \mathscr{C}_{j_0}; \quad \phi_0 = j_0; \quad j_0 \in (1, \ldots, d)$$

where $(\mathscr{C}_1, \ldots, \mathscr{C}_d)$ denotes the stochastic cycle, ϕ_n represents the cycle phasing, and $(\xi_{n+1,j+1})_n$ is a white noise taking values in $\Omega_{j+1} = \bigcap_{x \in \mathscr{C}_j} \mathscr{C}_{j+1} - f(x)$. This 'innovation' can be interpreted as a 'regulation noise': in order to have oscillations,

$\xi_{n+1,j+1}$ has to be chosen in such a way that x_{n+1} takes values in $\mathscr{C}_{j+1}$ when x_n is in $\mathscr{C}_j$. When $\mathscr{C}_j = (-\infty, +\infty)$ for all j, the model is reduced to the classical nonlinear autoregressive model, studied by Doukhan and Ghindès [1]: $x_{n+1} = f(x_n) + \xi_{n+1}$.

We give conditions under which the process is ϕ-irreducible, ϕ-recurrent, ergodic, continuous and asymptotically stationary where ϕ is an associated measure. We prove for example, that if $F_j \gg \lambda 1_{\Omega_j} \forall j$, where F is the law of the $\xi_{n,j}$ and $\lambda 1_{\Omega_j}$ is the Lebesgue measure restricted to Ω_j, and if

$$\mathscr{C}_j = f(\mathscr{C}_{j-1}) \quad \text{and} \quad f^{-ld}(\mathscr{B}_j) \xrightarrow[l \to \infty]{} \mathscr{C}_j \quad \forall \mathscr{B}_j \subset \mathscr{C}_j$$

such that $\lambda(\mathscr{B}_j) > 0$, then the process is ϕ-irreducible. If, in addition, the $\mathscr{C}_j$ are compact, f is continuous and $F_j \Leftrightarrow \lambda l_{\Omega_j}$, then the process is uniformly ϕ-recurrent.

References

[1] P. Doukhan and M. Ghindes, Etude des processus $X_n = f(X_{n-1}) + \varepsilon_n$, Thèse de 3ème cycle, Université Paris-Sud, Département de Mathématique, Orsay (1980).

[2] C. Jacob, Random oscillations modelling. In: O.D. Anderson, ed. Time Series Analysis: Theory and Practice (North-Holland, Amsterdam, NY, 1984).

[3] S. Orey, Lecture Notes on Limit Theorems for Markov Chain Transition Probabilities (Van Nostrand Reinhold mathematical studies. London, 1971).

Stochastic Increase of Quasi-Stationary Distributions in Birth–Death Processes

Julian Keilson and Ravi Ramaswamy, *University of Rochester, USA*

Let $N(t)$ be a birth-death process on the lattice of non-negative integers $\{0, 1, 2, \ldots\}$ with state zero reflecting ($\mu_0 = 0$). Let q_k^{T} be the quasi-stationary distribution with support on the set $\{0, 1, \ldots, k\}$ for $k = 1, 2, \ldots$. It is shown that the sequence (q_k^{T}) increases stochastically with k. Some interesting results on the bivariate Markov chain $(M(t), N(t))$, where $M(t) = \max_{0 \leq \tau \leq t} N(\tau)$, are derived as a stepping stone to the proof of the main theorem. An alternate characterization of the quasi-stationary distribution for birth-death processes is given thereby.

Duality and Semi-Regenerative Systems

Joanna B. Mitro, *University of Cincinnati, OH, USA*

Starting with a pair of dual Markov processes X and $\hat{X}$, we consider 'dual' semi-regenerative processes corresponding to the closed random sets $M = \{(t, \omega): (X_{t-}, X_t) \in \Gamma\}$ and $\hat{M} = \{(t, \omega): (\hat{X}_t, \hat{X}_{t-}) \in \Gamma\}$. We describe the duality relationship between the 'entrance' and 'exit' processes (R_t, X_{D_t}) and $(\hat{A}_t, \hat{X}_{\hat{G}_t})$, where $D_t = \inf\{s > t: s \in M\}$, $G_t = \sup\{s \leq t: s \in M\}$, $R_t = D_t - t$, $A_t = t - G_t$. (The dual objects for $\hat{X}$ are denoted with '$\wedge$'.) A definition for duality of general semi-regenerative processes is proposed.

Maxima and Exceedances of Stationary Markov Chains

Holger Rootzén, *University of Copenhagen, Denmark*

Let $\{X_t;\, t=0,1,\ldots\}$ be a stationary Markov chain on a general state space $(E, \mathscr{E})$, let $\xi_t = f(X_t)$ be an instantaneous real function of X_t, and define $M_k = \max\{\xi_1, \ldots, \xi_k\}$. If $\{X_t\}$ has a 'regeneration set', then the three extreme value types are the only possible limit laws of M_n under linear normalization. Further, if in addition $\{u_n\}$ are 'levels' such that $nP(\xi_0 > u_n) \to \tau > 0$ and if there exists a $\theta \in (0, 1]$ with $\limsup_{n\to\infty} |P(M_{[n\varepsilon]} \le u_n \mid \xi_0 > u_n) - \theta| \to 0$, as $\varepsilon \downarrow 0$, then $P(M_n \le u_n) \to \mathrm{e}^{-\theta\tau}$ as $n \to \infty$, and exceedances of u_n come in small clusters, where asymptotically the locations of the clusters form a Poisson process. The case $\theta = 1$, corresponding to 'clusters' of size one, is of particular interest, and we give sufficient conditions for this in terms of the one-step transition probabilities. Autoregressive processes, a certain queueing process and 'variables defined on a Markov chain' are given as examples where this theory applies.

Markov Approximations of Dynamical Systems

D. Szäsz, *Hungarian Academy of Sciences, Budapest, Hungary*

The Lorentz process, a model of physical Brownian motion, has recently become rigorously tractable through the Markov partition of the Sinai billiard (Bunimovich-Sinai, 1980). It describes the deterministic uniform motion of a point particle among fixed convex scatterers with elastic collisions at them. By choosing the initial point at random, the Lorentz process is believed to behave similarly to a random walk. For $\mathbb{Z}^d$ periodic scatterers, the symbolic dynamics defined by the Markov partition of the billiard appears as a stationary process which can, roughly speaking, be exponentially well approximated by Markov chains uniformly satisfying a lacunary Doeblin condition. This Markov approximation enabled Bunimovich and Sinai (1981) to prove a CLT for the Lorentz process. Since, because of the lacunarity of Doeblin condition, the local CLT seems not to follow from this approximation, Krämli and Szäsz (1984) introduced and proved a quasilocal CLT for the Lorentz process thus concluding its transiency for $d \ge 3$ and showing that, if $d = 2$, the nth collision of the Lorentz process occurs in a ball of radius $\log^{1+\delta} n$ i.o. almost surely. They also interpreted the Lorentz process as follows: while executing a (non-Markovian) random walk on $\mathbb{Z}^d$ the particle changes its internal state according the symbolic dynamics defined by the Markov partition. This enabled them to give a mainly functional analytic calculus to describe the statistical behaviour of the Lorentz process (1983).

Backward Limits of Non-Time-Homogeneous Markov Transition Probabilities

Hermann Thorisson, *Chalmers University of Technology, Göteborg, Sweden*

Consider a Markov chain on a countable state space E and denote by $p_{ij}(m, n)$ the transition probability from state i at time m to state j at time n. It is well known that if the chain is *time-homogeneous* (i.e. $p_{ij}(m, n) = p_{ij}(0, n-m)$) and $i \in E$ is aperiodic and recurrent non-null then, for each $j \in E$,

$$p_{ij}(m, n) \to \pi_{ij} \quad \text{as } n \uparrow \infty \quad \text{(traditional 'forward' limit)} \tag{1}$$

where $\sum_j \pi_{ij} = 1$.

It is clear that (1) can not hold for *non-time-homogeneous* chains without some restrictive conditions such as asymptotic time-homogeneity. Observe, however, that when the chain is time-homogeneous then (1) can be rewritten as

$$p_{ij}(m, n) \to \pi_{ij} \quad \text{as } m \downarrow -\infty \quad \text{(backward limit).} \tag{2}$$

This result (with π_{ij} replaced by $\pi_{ij}(n)$) can be extended to the non-time-homogeneous case under non-trivial regularity conditions such as time-uniform aperiodicity and non-null recurrence. A useful tool is the coupling method which, in particular, yields convergence in the strong sense of total variation.

References

[1] A.N. Kolmogorov, Zur Theorie der Markoffschen Ketten, *Math. Ann.* 112 (1936) 155–160.

[2] H. Thorisson, Backward limits of non-time-homogeneous regenerative processes, Preprint, Department of Mathematics, Göteborg, 1984.

Gram–Charlier Representations in Applied Probability

G.F. Yeo, *Odense University, Denmark*

A Gram–Charlier representation, and a generalization, are considered for a density function $g(x)$ on the whole line, and for the solution of a class of non-linear integral equations arising in storage, population growth and other Markovian models. Typically

$$g(y) = \int l(y, w) g(w) \, \mathrm{d}w \tag{$*$}$$

with the kernel $l(y, w)$ being a conditional density function in y for fixed w.

A taxonomy is given which reviews and extends known results on the convergence of the representation to the function. A convergence result for the bivariate function $l(y, w)$ is used to construct a suitably convergent representation for the solution of $(*)$. Micromodification of the input distributions may be used to improve the speed and the nature of the convergence. Some examples are given.

2.4. *Martingales*

Generalized Martingales, Generalized Harmonic Functions and Generalized Markov Processes

Louis H. Blake, *College of Staten Island, The City University of New York, Staten Island, NY, USA*

A process $\{X_n\}_{n=1}$ is called a weak martingale if $E(X_n \mid X_m) = X_m$ whenever $n > m$. Conditions are given under which a pointwise convergence theorem is obtained for weak martingales. This at once generalizes the martingale theorem and begins to answer an open question of Stout: are there '... any interesting results about almost sure convergence for weak martingales'. Moreover, this generalization of the martingale theorem allows us to define generalized harmonic functions and generalized Markov processes and to obtain some classical results for these generalized functions.

Poincaré-Type Inequalities via Stochastic Integrals

Louis H.Y. Chen, *National University of Singapore*

Let M be a compact Riemannian manifold. The Poincaré inequality states that if $f \in C^1(M)$ is such that $\int_M f = 0$, then $c \int_M f^2 \leqslant \int_M |\nabla f|^2$ where c is a constant depending on the geometry of M. The best possible value of c is the first eigenvalue of the Laplace-Beltrami operator on M and equality is achieved if and only if f is the first eigenfunction. In the case $\partial M \neq \emptyset$, the Laplacian acts on functions which satisfy the Neumann boundary condition. An account of the Poincaré inequality and estimates of the first eigenvalue of the Laplacian is contained in Yau (Sem. Diff. Geometry (1982), 3-71) and also Li (ibid., 73-83).

Let γ be a convolution of probability measures on $\mathbb{R}^k$. By a martingale argument, a Poincaré-type inequality is proved for $f \in C^1(\mathbb{R}^k)$ such that $\int_{\mathbb{R}^k} f \, d\gamma = 0$. In order to be able to establish a necessary and sufficient condition for equality to hold, a stochastic integral representation formula for functionals of the Lévy process is used to prove a Poincaré-type inequality in the case γ is an infinitely divisible distribution. These Poincaré-type inequalities generalize those of Chernoff (1981), Chen (1982) and Cacoullos (1982) and improve those of Cacoullos.

In the case γ is an infinitely divisible distribution, the Poincaré-type inequality yields the generalized Wirtinger inequality on the n-sphere and a weighted Poincaré inequality on each of the dyadic slices of the n-sphere. These weighted Poincaré inequalities include Poincaré inequalities as special cases. As a corollary one obtains the first eigenvalue and eigenfunction of the Laplacian on the n-sphere and on each of the dyadic slices. By extending the method, one computes all the eigenvalues and eigenfunctions of the Laplacian on the n-sphere.

Measure-Valued Random Processes

J. Horowitz, *University of Massachusetts, Amherst, MA, USA*

A measure-valued random process (mvrp) is a random process X_t whose state at each time t is a measure on a measurable space $(E, \mathscr{E})$. Special cases have been studied by various authors, e.g. measure-valued diffusions and measure-valued martingales arising in connection with stochastic integration, but my intention is to develop the theory of mvrps as a tool for modeling certain continuum phenomena such as fluid flow in soil. The present work comprises three parts: (i) General results on the compatibility between measure-theoretic operations (e.g. Lebesgue decomposition) and probabilistic structure. Sample result: The predictable projection of a mvrp is itself a mvrp. (ii) Martingale and related mvrps. Sample result: under a domination condition, the product measure of two square integrable measure-valued martingales is a measure-valued semimartingale. (iii) Accumulation processes: this means that, for each $B \in \mathscr{E}$, $X_t(B)$ is an increasing process (these are called 'random measures' by Dellacherie and Meyer, *Probability and Potentials*, vol. 2). A more or less complete description is given of the Lebesgue decomposition of an accumulation process relative to a fixed measure on $\mathscr{E}$. The results are applied to derive a peculiar geometrical property of Markov processes, and to obtain a recent result of J. Wendel (*Ann. Prob.*, 1980) giving the joint distribution of the hitting time and place of a sphere by Brownian motion.

On the Asymptotic Behaviour of Solutions to Stochastic Differential Equations

Gerhard Keller, *University of Heidelberg, FR Germany*
Götz Kersting, *University of Frankfurt, FR Germany*
Uwe Rösler, *University of Göttingen, FR Germany*

In this paper we study the asymptotic behaviour of the solution of the stochastic differential equation $\mathrm{d}X_t = g(X_t)\,\mathrm{d}t + \sigma(X_t)\,\mathrm{d}W_t$, where σ and g are positive functions and W_t is a Wiener process. We clarify under which conditions X_t may be approximated on $\{X_t \to \infty\}$ by means of a deterministic function. Further the question is treated, whether X_t converges in distribution on $\{X_t \to \infty\}$. We deal with the Itô solution as well as the Stratonovitch solution and compare both.

A Canonical Representation of Filtrations

Frank B. Knight, *University of Illinois, Urbana, USA*

Let $\mathscr{F}_t^0$, $0 \leq t$, be a filtration of countably generated σ-fields on a probability space $(\Omega, \mathscr{F}, P)$, and let $\mathscr{F}_t$ denote the augmentation of $\mathscr{F}_{t+}^0$ by all P-null sets. We assume that $\mathscr{F}_0$ contains only sets of probability 0 or 1. Let $L_0^2(t) = \{X \in \mathscr{F}_t\colon EX = 0, EX^2 < \infty\}$, and $\mathscr{M}_0^2$ be the space of right-continuous $\mathscr{F}_t$-martingales

$M(t) \in L_0^2(t)$. We are concerned with representing all $X \in L_0^2(t)$ in the form

$$X = \sum_{i<n_c+1} \int h_i(u)\, \mathrm{d}B_i(u \wedge T_i(t)) + \sum_{j<n_d+1} \int k_j(u)\, \mathrm{d}P_j(u \wedge R_j(t)) \tag{1}$$

where

(a) $B_i(u)$ are independent Brownian motions, $P_j(u)+u$ are independent Poisson processes, independent of $(B_i(u))$,

(b) $T_i(t)$ (resp. $R_j(t)$) are continuous, nondecreasing, $\mathscr{F}_t$-measurable, with $T_i(0)=0$, $ET_i(t)<\infty$, and if $\mathscr{F}_{\tau_i(u)}$ denote the time-changed filtrations $(\tau_i(u)=\inf\{s\colon T_i(s)>u\})$ then $B_i(u \wedge T_i(t))$ (resp. $P_j(u \wedge R_j(t))$) are martingales of $\mathscr{F}_{\tau_i(u)}$, measurable over $\mathscr{F}_{\tau_i(u-)}$, and

(c) The processes $h_i(u)$ (resp. $k_j(u)$) are $\mathscr{F}_{\tau_i(u)}$-previsible. Only these processes depend on the choice of X in (1), where the series converge in $L_0^2(t)$ if $n_c+n_d=\infty$.

Theorem 1. *Representation* (1) *holds for all X and $t \geq 0$ if and only if $\mathscr{M}_0^2$ does not contain any pure-jump martingales.*

Theorem 2. *If* (1) *holds, let n_c+n_d be as small as possible. Then $n_d = 0$, 1, or ∞, and n_c+n_d cannot be decreased if, in place of B_i and P_j, we allow any independent processes with homogeneous, independent increments.*

An Invariance Principle for Processes with Values in Sobolev Spaces

Michel Métivier, *Ecole Polytechnique, Paris, France*

We give a sufficient condition for weak compactness of the family of laws of a sequence $(M^n)_{n\geq 0}$ of Hilbert valued martingales. This condition is expressed in terms of the sequence of laws of the random variables $(M_0^n)_{n\geq 0}$ and of the processes $\langle M^n \rangle$. An invariance principle is easily derived which is applied to the modeling of distributed systems considered, for example, by L. Arnold, M. Theodosopulu and P. Kotelenez.

On Convergence in Energy

Z.R. Pop-Stojanovic and Murali Rao, *University of Florida, Gainsville, USA*

In several earlier papers the authors have studied the role of energy in probabilistic potential theory. In particular, in dealing with the convergence in energy the authors answered the following problem: For a sequence (s_n) of class (D) potentials which is increasing to a class (D) potential s of finite energy, what can be said about the 'energy functions' $E^{\cdot}[A_{n,\infty}^2]$ and $E^{\cdot}[A_\infty^2]$? It turns out that if the sequence (s_n) converges to s in energy then the energy functions converge in measure to the energy function of s.

On Local Times of a Diffusion

P. Salminen, *Åbo Akademi, Finland*

Local time L_t^y (at a fixed point y) of a transient, regular diffusion X is considered as the dual predictable projection of the indicator process for the last exit time at y. The essential tool is the Doob–Meyer decomposition for submartingales. In particular, it is seen that the dual predictable projection of the process $Z_t = 1_{\{\zeta \leq t\}}$, where ζ is the life time of X, is

$$A_t = \int_I L_t^y k(\mathrm{d}y) + 1_{\{X_{\zeta-} \notin I\}} 1_{\{\zeta \leq t\}},$$

where k and I are the killing measure and the state space of X, respectively. This result can be used, for example, to obtain the well known integral representation for continuous additive functionals.

2.5. Optimization and control

Continuous Solutions of Best Choice Problems

F. Thomas Bruss, *Facultés Universitaires N-D de la Paix-Namur, Belgium*

We shall discuss problems arising in Best Choice modelling with an unknown number of options. If this number is a random variable with known distribution, then optimal strategies may virtually take any form, and their computation may become very difficult. Optimal stopping times may make huge jumps according to only tiny changes within the underlying distribution (stopping islands), and thus these models prove to be inadequate for applications (see e.g. Freeman [2]).

The decision maker would prefer the change of optimal action to be reasonably small if his distribution estimate error is small, but we shall see that this type of continuity is generally incompatible with strategies in terms of numbers of options. We shall discuss model alternatives, where special attention will be given to their tractability for applications. A continuous time model (Bruss [1]) seems to be a suitable compromise. It will be extended to general pay-off functions and leads to interesting comparisons with other models.

References

[1] F.T. Bruss, A unified approach to a class of best choice problems with an unknown number of options, to appear in Ann. Probab.

[2] P.R. Freeman, The secretary problem and its extensions, Intern. Stat. Rev. 51 (1983) 189–206.

On Existence of Stationary Optimal Plans of Markovian Processes with Compact Action Space

Kwo-Jean Farn, *Industrial Technology Research Institute, Taiwan*
Wen-Tao Huang, *Institute of Mathematics, Academia Sinica, Taiwan*

We consider Markovian decision processes with compact action spaces. Some sufficient conditions for the existence of a stationary optimal plan of a Markovian decision process with finite state space and compact action space are proposed. The optimal properties are studied, respectively, in the sense of Derman and a new proposed criterion. Some examples are also given to illustrate that an optimal plan exists in a certain sense and does not exist in another sense for various kinds of senses of Derman, Blackwell and the proposed new criterion.

An Optimal Betting Strategy for Repeated Games

Gary Gottlieb, *New York University, NY, USA*

A gambler is faced with an infinite sequence of identical wagers whose mean is small compared to their variance. We propose an optimality criterion to minimize the expected first exit time of his wealth from a specified interval subject to the condition that the probability that the upper boundary is reached first is at least p. The optimal strategy is derived for a diffusion approximation to the original process.

A Discounted Uniform Two-Armed Bandit Problem with One Arm Known

Toshio Hamada, *Himeji College, Himeji, Japan*

A sequential experimental design procedure to select and perform one of two experiments e_0 and e_1 sequentially for n times to maximize the total expected discounted reward is analyzed. By performing e_0 and e_1, an observation is obtained as a reward from the uniform distribution on the interval $[0, u]$ or $[0, v]$, respectively. The value of v is known, but the true value of u is unknown, and there is the prior knowledge that u has a Pareto distribution as a prior distribution. The present value of reward r obtained at the next stage is βr, where $0<\beta<1$. This problem is formulated by dynamic programming and the optimal strategy is derived. An extension to the case of infinite time horizon is considered, and as a result, the optimal strategy to the discounted multi-armed bandit problem is discussed.

Product Price Risk and the Investment Intensity in a Two Stage Decision Process

Peter Lohmander, *Swedish University of Agricultural Sciences, Umeå, Sweden*

Most production needs several production factors. The decisions concerning the intensity of the factors are normally taken successively. As time passes, the precision in the prediction of the product price increases.

The effect of increasing price risk on the optimal intensity of the first factor is analysed. It is shown that unambiguous qualitative statements can be made if certain production function criteria are satisfied.

Stochastic Control of Two-Parameter Processes

G. Mazziotto, *CNET, Issy les Moulineaux, France*
A. Millet, *Université d'Angers, Angers, France*

This paper studies a classical control problem, in a two-parameter situation where the filtration $(\boldsymbol{F}_z\,;\, z \in \mathbb{R}_+^2 \cup \{\infty\})$ satisfies the usual conditions $(F1)$–$(F4)$. Let $\boldsymbol{Z}$ denote the set of optional increasing paths (o.i.p.) $Z = (Z_u\,;\, u \in \mathbb{R}_+)$. Given a stopping point T, let $\boldsymbol{Z}(T)$ denote the set of o.i.p. Z passing through T, i.e., such that $T = Z_{|T|}$ for some one-parameter stopping time $|T|$ for the filtration $(\boldsymbol{F}_{Z_u}\,;\, u \in \mathbb{R}_+)$. A strategy S is a pair (Z, T) such that T is a stopping point and Z an o.i.p. of $\boldsymbol{Z}(T)$. Let $\boldsymbol{S}$ (resp. $\boldsymbol{T}$) denote the set of strategies (resp. stopping points). Given optional processes X and Y such that $E(\sup |X_z|) + E\,(\sup |Y_z|) < \infty$, and given $\alpha > 0$, for any strategy S let

$$G(S) = \int_0^T \mathrm{e}^{-\alpha u} X_{Z_u}\,\mathrm{d}u + Y_T$$

be the reward obtained when using the strategy S. We study optimal strategies S^*, i.e., such that $E(G(S^*)) = \sup\{E(G(S))\,;\, S \in \boldsymbol{S}\}$.

We use a generalization of the Snell envelop J in order to characterize optimal strategies, and show that maximal strategies are optimal, under regularity assumptions on J. When X and Y are continuous we prove the existence of the value process W defined by

$$W_T = \operatorname{esssup}\left\{ E\left(\int_T^\tau \mathrm{e}^{-u} X_{Z_u}\,\mathrm{d}u + Y_{Z_\tau} \Big/ \boldsymbol{F}_T \right);\, (Z, \tau) \in \boldsymbol{S}\colon Z \in \boldsymbol{Z}(T),\, \tau > |T| \right\} \quad \forall T \in \boldsymbol{T}.$$

and obtain an analog of Bellman's dynamic programming equation. Given a continuous bi-Markov flow $((\xi_z^x\,;\, z \in \mathbb{R}_+^2)\,;\, x \in E)$, bounded Borel functions f and g on E, for $X_z^x = f(\xi_z^x)$ and $Y_z^x = g(\xi_z^x)$, we obtain the existence of a function q on E such that the value processes W^x satisfy $W_T^x = \mathrm{e}^{-\alpha T}\, q(\xi_T^x)\ \forall T \in \boldsymbol{T}$ and $\forall x \in E$, and give sufficient conditions for the continuity of q.

Given a stopping line L, for any o.i.p. Z let $Z(L)$ denote one stopping point of $Z \cap L$, and set $G_L(Z) = G(Z, Z(L))$. We show that the map $Z \to E(G_L(Z))$ can be extended to a continuous linear map on the convex compact set of randomized o.i.p. when X and Y are continuous and $X \geqslant 0$. This proves the existence of an optimal o.i.p. Z^*, i.e., such that $E(G_L(Z^*)) = \sup\{E(G_L(Z)\,;\, Z \in \boldsymbol{Z}\}$.

Finally, by using this result, we prove the existence of an optimal strategy for the general problem stated above.

Discretization of the Time Parameter for Controlled Jump Processes

Hans-Joachim Plum, *University of Bonn, FR Germany*

The paper considers generalized controlled jump processes. On the one hand jumps are generated by a controllable jump-intensity. On the other hand impulsive controls are admitted, causing instantaneous jumps. Such models have been studied recently by van der Duyn Schouten, Hordijk and Yushkevich.

Policies and their total reward are defined. The problem of finding optimal or good policies is attacked by discretizing the time parameter. A family of discrete time models is constructed, converging to the given continuous time process (in an appropriate sense), as the distance between the decision epochs tends to zero. The idea is used to carry over results known from the discrete time theory to the continuous time model.

2.6. Point processes and random measures

The Linear Spatial Birth-and-Death-Process as a Tool for the Construction of Point-Processes with Aggregation

L.G. Barendregt, *Institute TNO for Mathematics, Information Processing and Statistics (IWIS-TNO), The Hague, The Netherlands*

The linear spatial birth-and-death process, as it is considered in this lecture, is a branching process with immigration, where the points are situated in a subset of a Euclidean space, and where a dying point can give birth to at most two new points. Emphasis is put on the subcritical case with invariance under translation.

Then an equilibrium distribution exists, with very simple expressions for its intensity and its covariance density.

If the process acts on a finite rectangle rather than on the whole Euclidean space, invariance under translation is no longer possible. But by allowing the particles to bounce against the walls, one can construct a process with the same mathematical tractability as the translation-invariant-process.

The total number of points is now negative-binomially distributed under the equilibrium distribution. Expressions can be derived for the covariance density, conditional on the total number of points.

Random Coverings in Several Dimensions

Svante Janson, *Uppsala University, Sweden*

Small balls, cubes or other convex sets are placed at random positions, independently of each other and with the same uniform distribution, until a given d-dimensional set is completely covered. The distribution of the required number of small sets is investigated, and the asymptotic distribution as the small sets are shrunk is determined. The case of covering the surface of a sphere by spherical caps is included. The small sets are not necessarily congruent; they may have random size and shape, according to a common distribution.

The asymptotic distribution depends on the dimension in a simple way; the shape of the small sets enters in a rather complicated way in a low-order term.

The method used for this problem is based on a study of the properties of a random, Poisson distributed, number of small sets, which is regarded as a Poisson process of sets.

Changing Time for Spatial Point Processes

Ely Merzbach, *Bar-Ilan University, Ramat-Gan, Israel 52100*

This paper studies the problem of transforming a two-parameter point process into a two-parameter Poisson process by means of a time change. In the one-parameter case, this was done by P.A. Meyer. In the two-parameter case, D. Nualart and M. Sanz gave conditions in order to transform a square integrable strong martingale into a Wiener process. Here, we do the same for the Poisson process by a closely related method.

A Note on One-Dimensional Distances between Two Counting Processes

Martti Nikunen, *University of Helsinki, Finland*

Kabanov, Liptser and Shiryayev have in [1] derived conditions for weak convergence of counting processes. These conditions are given in terms of the compensators of the processes. Hence it is natural to require that the rate of convergence could be expressed in terms of compensators.

Let d denote the total variation metric and d_0 the uniform metric between random variables.

Theorem 1. *Suppose that N is a counting process having a deterministic compensator A and let M be another counting process on the same probability space with compensator B. Then for any $t>0$*

$$\mathrm{d}(N_t, M_t) \leqslant \varepsilon_t(2A)\left\{\int_0^t (\varepsilon_s(2A))^{-1}\,\mathrm{d}U_s + 2E\int_0^t (\varepsilon_s(2A))^{-1}|A_s - B_s|\,\mathrm{d}B_s\right\}$$

and

$$\mathrm{d}_0(N_t, M_t) \leqslant \varepsilon_t(2A)\min\left\{2\int_0^t (\varepsilon_s(2A))^{-1}\,\mathrm{d}V_s, \int_0^t (\varepsilon_s(2A))^{-1}\,\mathrm{d}U_s\right.$$

$$\left. + E\int_0^t (\varepsilon_s(2A))^{-1}|A_s - B_s|\,\mathrm{d}B_s\right\},$$

where $U_t = \sup_{s \leq t} E|A_s - B_s|$, $V_t = E \sup_{s \leq t} |A_s - B_s|$ *and* $\varepsilon_t(2A)$ *is the exponential of Doleans-Dade.*

Theorem 1 improves and extends the results of [2].

References

[1] Yu.M. Kabanov, R.S. Liptser and A.N. Shiryayev, Some limit theorems for simple point processes (A martingale approach), Stochastics 3 (1980) 203-216.
[2] E. Valkeila, A general Poisson approximation theorem, Stochastics 7 (1982) 159-171.

On Distributions of Random Capacities

Tommy Norberg, *University of Göteborg, Sweden*

The purpose with this talk is to present a general theory for distributions of random capacities, which unifies some of the results for random measures, random semicontinuous functions and random sets.

By a capacity on a topological space S we mean an extended real-valued, increasing and outer continuous function on the class of compact sets in S that maps the empty set on zero. A mapping ξ of some probability space into the class U_1 of capacities on S is called a random capacity on S if $\xi(K)$ is a random variable for all compact $K \subset S$.

Any random measure is, of course, a random capacity. Let $\mathscr{F}_0$ be the class of upper semicontinuous functions on S. There exists a natural one-to-one mapping of $\mathscr{F}_0$ into U_1. Clearly the class of closed sets in S may be regarded as a subclass of $\mathscr{F}_0$. Thus random semicontinuous functions and random sets may be regarded as random capacities.

We shall see that there exist existence and convergence theorems for distributions of random capacities, which generalize or are similar to those known for random measures, random semicontinuous functions and random sets.

On Convex Orderings of Point Processes

Tomasz Rolski, *Wrocław University, Poland*

Let $\bar{\mathrm{f}}$ be a class of Borel functions $f: \mathbb{R}^k \to \mathbb{R}$ $(k = 1, 2, \ldots)$. For two random measures Φ and Ψ on $\mathbb{R}$, we define

$$\Phi <_f \Psi$$

if for any $f \in \bar{\mathrm{f}}$ and disjoint bounded sets $B_1, \ldots, B_k$,

$$Ef(\Phi(B_1), \ldots, \Phi(B_k)) \leq Ef(\Psi(B_1), \ldots, \Psi(B_k)).$$

For example the class of nondecreasing, L-superadditive functions which are convex with respect to each variable is useful for deriving some bounds for characteristics in the DSP/GI/1 queueing system. These are single server systems at which customers arrive according to a stationary and ergodic doubly stochastic Poisson process.

It is proven that the mean stationary work-load (also the queue size and the delay) is bounded above by the mixture of this characteristic in the standard M/GI/1 queue with respect to the arrival rate.

A Methodology for Solving a General Class of Visibility Problems under Poisson Shadowing Processes

M. Yadin, *Technion, Haifa, Israel*
S. Zacks, *SUNY, Binghamton, USA*

The paper discusses a general methodology for studying a class of visibility problems in the framework of Poisson shadowing processes. Consider a star shaped curve, $\mathscr{C}$ in the plane or in a 3-dimensional space.

An observer (source of light) is located at the origin, O. A random number of objects (disks or spheres) are randomly distributed in the space. The objects have random size and the number of objects whose centers are located in any specified Borel set, B, has a Poisson distribution with mean depending on B. The objects in this Poisson random field cast shadows on the curve $\mathscr{C}$. Portion of the curve is invisible from O. The problems solvable by the proposed methodology are:

(1) Determination of moments of all orders of the random measure of the total portion of $\mathscr{C}$ which is visible.

(2) Approximation to the distribution of the random total visibility measure along $\mathscr{C}$.

(3) Approximation to the distribution of the number of trials that can be performed along $\mathscr{C}$, when each trial requires a segment of $\mathscr{C}$, of length L, which is completely visible.

(4) Approximating the probability of at least one success in a (random number) of Bernoulli trials performed along the visible segments of $\mathscr{C}$, of length L.

Other related problems can be studied by applying the approach described in the paper. Applications of this theoretical methodology are also discussed.

2.7. Probabilistic measure theory

Weak Limits of Probability Measures on Metric Schauder Spaces

Harald Bergström, *Chalmers University of Technology, Göteborg, Sweden*

In my book "Weak convergence of measures", I have presented a reduction procedure for weak convergence of measures on normal spaces. For metric spaces this method is submitted by Lemma III.6.1. in this book. Here I shall consider this lemma for metric Schauder spaces which I am going to define. I shall then give complements to the proofs of some theorems in Chapter V and Chapter VI of the book.

Multilinear Forms and Measures of Dependence Between Random Variables

Richard C. Bradley, *Indiana University, USA*
Włodzimierz Bryc, *Politechnika Warszawska, Poland*

Various moment inequalities for families of dependent random variables are derived by applying the techniques of interpolation theory, in particular the Riesz–Thorin and Marcinkiewicz interpolation theorems and the techniques of Stein and Weiss for handling indicator functions. The relationship of some of these moment inequalities to central limit theory under strong mixing conditions is discussed.

On the Linearity of Regression and Homoscedatity

W. Bryc, *Politechnika Warszawska, Warszawa, Poland*

Consider these two properties of multidimensional normal distributions: (a) linearity of regression, and (b) homoscedatity i.e. nonrandomness of the conditional variance. In the case when all the random variables are linear combinations of a fixed sequence of independent r.v.'s, (a) and (b) are known to imply the normal law (see e.g. Kagan, Linnik, Rao, Characterization problems in math. statistics, Moscow 1972).

Surprisingly (a) and (b) when properly formalized provide more information, then one might expect in the general case. Here is the simplest example:

Theorem. *If* X_1, X_2 *are square-integrable r.v.'s such that* $0<|\mathrm{corr}(X_1, X_2)|<1$, *then the conditions*

(a) $E(X_i\,|\,X_j)$ *is a linear function of* X_j $\forall i, j$,

(b) $\mathrm{var}(X_i\,|\,X_j)$ *is nonrandom* $\forall i, j$,

imply that X_i $(i=1,2)$ *have moments of all orders.*

A Lévy–Khintchin Formula for SU(1; 1)

B.-J. Falkowski, *Hochschule der Bundeswehr München, FR Germany*

In [2] so-called Lévy-Schoenberg kernels on certain homogeneous spaces and their relation to Brownian Motion of several parameters are discussed.

Here we describe the continuous 1-cohomology of $SU(1;1)$ (or equivalently $SL(2;\mathbb{R})$, the group of 2×2 matrices with real entries and determinant 1). This cohomology group is used to derive an abstract Lévy–Khintchin formula. The concrete realization of this formula allows us to rediscover some of the results in [2] thus shedding some new light on the structure of infinitely divisible positive definite functions on SU(1; 1) by exhibiting their relationship to the 1-cohomology. In view of [3] these results should have some interesting consequences in Mathematical Physics: they give a complete overview of irreducible representations of the 'current group' of SU(1; 1) which can be obtained by a standard construction, cf. [1].

References

[1] J. Erven and B.J. Falkowski, Low order cohomology and applications, Lecture Notes in Mathematics 877 (Springer, 1981).
[2] R. Gangolli, Positive definite kernels on homogeneous spaces and certain stochastic processes related to Lévy's Brownian Motion of Several Parameters, Ann. Inst. Poincaré, III (2) (1967).
[3] C.M. Newman, Ultralocal quantum field theory in terms of currents, Comm. in Math. Phys. 26 (1972).

Supremum-Self-Decomposability in $\mathbb{R}^d$

Gerard Gerritse, *Katholieke Universiteit Nijmegen, The Netherlands*

An $\mathbb{R}^d$-valued random variable (rv) X and its distribution function F are called $\vee$-self-decomposable if for every $t>0$ there exists a rv X_t such that X has the same distribution as $(X-t\cdot\mathbf{1})\vee X_t$, where X and X_t are independent and $\mathbf{1}:=(1,1,\ldots,1)\in\mathbb{R}^d$.

The main results are:

1. $\vee$-self-decomposability in $\mathbb{R}^1$ can be expressed in purely analytical terms (this result is due to Mejzler, see Galambos [2]).

2. $\vee$-self-decomposable distributions are limit distributions of normalized partial maxima of sequences of independent rv's.

3. The $\vee$-self-decomposable rv's are a subclass of the $\vee$-infinitely divisible rv's, which are studied by Balkema and Resnick [1].

4. $\vee$-self-decomposable rv's can be represented as functions of Poisson processes. Results 2, 3 and 4 are analogues of known results for the classical additive case (+ instead of $\vee$, $e^{-t}X$ instead of $X-t\cdot\mathbf{1}$). The contents of the present talk can be found in Gerritse [3].

References

[1] Balkema and Resnick, Max-infinite divisibility, J. Appl. Prob. 14 (1977) 309–319.
[2] Galambos, The asymptotic theory of extreme order statistics (John Wiley, New York, 1978).
[3] Gerritse, Supremum-self-decomposability in $\mathbb{R}^d$, Report 8341, Dept. of Mathematics, Katholieke Universiteit, Nijmegen, The Netherlands, 1983.

Infinite Divisibility, Branching Processes, Renewal Theory: 'Next Term' Results Using Banach Algebra Methods

Rudolf Grübel, *Universität Essen, FR of Germany*

In several different areas of probability theory one is lead to problems where the Fourier transform of a measure μ_1 is a known function of the Fourier transform of a second measure μ_2. Banach algebra techniques have proved useful in relating the tail behaviours of μ_1 and μ_2. We present a new variant of this method; the following theorem was obtained using such techniques. It is a refinement of some known results and may be interpreted as a 'next term' result:

Theorem. *If the Lévy-Khintchin measure ν of an infinitely divisible distribution P has finite second moment and a density f which is monotone outside some compact interval with*

$$\sup_{y \geq x/2} f(y) = O(f(x)), \quad x \to f(\log x) \textit{ slowly varying},$$

then

$$\lim_{x\to\infty} \frac{P((x,\infty)) - \nu((x,\infty))}{f(x)} = \kappa,$$

where κ denotes the first moment of P.

Similar results are obtained for the expected population size of an age-dependent branching process and in renewal theory.

Two Random Systems with Complete Connections Associated to the Brodén–Borel–Lévy Type Relations

Sofia Kalpazidou, *University of Thessaloniki, Greece*

Let Y be the set of irrationals in $[-\frac{1}{2}, \frac{1}{2}]$ and $X = (N\backslash\{0, 1\}) \times \{-1, 1\}$. We associate with the sequence $(a_1, \varepsilon_1), (a_2, \varepsilon_2) \cdots$ where $(a_n, \varepsilon_n) \in X$, $n \in N^*$, a $y \in Y$ by means of the *continued fraction to the nearer integer* expansion. Next, if λ is the Lebesgue measure in $[-\frac{1}{2}, \frac{1}{2}]$ and

$$E_{i_1 \cdots i_n}^{j_1 \cdots j_n} = \{y \in Y \colon (a_k, \varepsilon_k) = (i_k, j_k),\ 1 \leq k \leq n\},$$

then we obtain that

$$\lambda(a_{n+1} = k, \varepsilon_{n+1} = 1 | E_{i_1 \cdots i_n}^{j_1 \cdots j_n})$$
$$= \begin{cases} (5+2s_n)^{-1} & \text{if } k=2,\ i_n=2, \\ 4^{-1}(2-s_n)(5+2s_n)^{-1} & \text{if } k=2,\ i_n>2, \\ 4(2+s_n)(2k+2s_n+1)^{-1}(2k+2s_n-1)^{-1} & \text{if } k \geq 3,\ i_n=2, \\ (4-s_n^2)(2k+2s_n+1)^{-1}(2k+2s_n-1)^{-1} & \text{if } k \geq 3,\ i_n>2, \end{cases}$$

$$\lambda(a_{n+1}=k,\ \varepsilon_{n+1}=-1\,|\,E^{j_1\cdots j_n}_{i_1\cdots i_n}) = \begin{cases} 0 & \text{if } k\geq 2,\ i_n=2,\\ 4^{-1}(2+s_n)(5-2s_n)^{-1} & \text{if } k=2,\ i_n>2,\\ (4-s_n^2)(2k-2s_n+1)(2k-2s_n-1) & \text{if } k\geq 3,\ i_n>2. \end{cases}$$

We name the above relations *Brodén-Borel-Lévy type* relations and by making use of them we construct two random systems with complete connections and study their ergodic behaviour. We mention that in the case of [0, 1] this study was made by M. Iosifescu in "Dependence with complete connections and applications" (in Romanian), Ed., St. Encicl. Bucharest (1982).

De Finetti-Type Theorems: An Analytical Approach

Paul Ressel, *Katholische Universität Eichstätt, FR Germany*

A famous theorem of de Finetti (1931) shows that an exchangeable sequence of $\{0, 1\}$-valued random variables is a unique mixture of coin tossing processes. Many generalizations of this result have been found; Hewitt and Savage (1955) for example extended de Finetti's theorem to arbitrary compact state spaces (instead of just $\{0, 1\}$).

Another type of question arises naturally in this context. How can mixtures of independent and identically distributed random sequences with certain specified (say normal, Poisson, or exponential) distributions be characterized among all exchangeable sequences?

We present a general theorem from which the 'abstract' theorem of Hewitt and Savage as well as many 'concrete' results - as just mentioned - can be easily deduced. Our main tools are some rather recent results from harmonic analysis on abelian semigroups.

On the Distributions of Sums of Symmetric Random Variables and Vectors

Thomas Sellke, *Purdue University, USA*

Let F be a probability distribution on $\mathbb{R}^n$. Then there exist (possibly dependent) spherically symmetric random vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ whose sum $\boldsymbol{X}+\boldsymbol{Y}$ has distribution F if and only if all the one-dimensional distributions obtained by projecting F onto lines through the origin either have mean zero or no mean, finite or infinite. An easy corollary is that any distribution on $\mathbb{R}^n$ can be attained by a sum of three spherically symmetric random vectors.

An Integral Representation for the Hellinger Distance

Esko Valkeila, *University of Helsinki, Finland*
Ljudmilla Vostrikova, *Eötvös Loránd University, Budapest, Hungary*

The total variation metric and the Hellinger metric between two probability measures define the same topology in the space of probability measures on a probability space. The convergence with respect to this topology is called strong convergence. If we have a filtration on a probability space then we can define the Hellinger distance between two probability measures locally. We give an integral representation for the Hellinger distance between two probability measures. This representation uses the Hellinger process, which depends on the two measures and the corresponding filtration. Using this representation we give upper and lower bounds for the Hellinger distance in terms of the Hellinger process. We give examples on how to apply these results to the strong convergence of stochastic processes.

Reference

R. Lipster and A. Shiryayev, On the problem of 'predictable' criteria of contiguity, IV Japan USSR symposium on probability theory, Lecture Notes in Mathematics 1021 (1983) 386-418.

Lower and Upper Bounds for the Hellinger Integral

L. Vostrikova, *Eötvös Loránd University, Hungary*

In studying properties of statistical estimators the Hellinger integral $H(P, \tilde{P})$ is often used as characteristic of distinctions between two probability measures P, $\tilde{P}$. If a measurable space is equipped with a filtration, then the Hellinger integral may be expressed in terms of local predictable characteristics of P, $\tilde{P}$ or, more precisely, in terms of the corresponding Hellinger process [1].

We give lower and upper bounds for the Hellinger integral $H(P, \tilde{P})$ using the Hellinger process. These bounds are useful when $H(P, \tilde{P})$ is near to zero.

The bounds are applied to obtain 'predictable' criteria for quick consistency of estimators and 'predictable' criteria for entire separation of sequences of probability measures corresponding to stochastic processes.

Reference

[1] R. Liptser and A. Shiryayev, On the problem of 'predictable' criteria of contiguity, Theory Probab. and Math. Stat. IV. JAPAN-USSR Congress. Proceedings. Lecture Notes in Math. 1021 (1983) 386-418.

Korovkin Systems of Stochastic Processes

Michael Weba, *TH Darmstadt, FR Germany*

Let $X(t)$ be an L^1-continuous stochastic process with compact parameter set P which is approximated by a stochastic process $Y(t)$ where $Y(t)$ depends upon $X(t)$ by means of n observations $X(t_1), \ldots, X(t_n)$ at times $t_1, \ldots, t_n \in P$. In general, this approximation will cause a positive error. The question arises whether this error can be made arbitrarily small, e.g. by increasing the number n of observations. In other words: If a sequence $Y_1(t)$, $Y_2(t), \ldots$ of processes is given, can we find criteria in order to decide whether the associated sequence of errors tends to zero? Since $Y_n(t)$ emerges from $X(t)$ by means of a transformation, say T_n, a reformulation of the above question is: which conditions must be imposed on $X(t)$ and a sequence of transformations T_1, $T_2, \ldots$ in order to guarantee $\lim(T_nX)(t) = X(t)$?

For the moment consider the space $C[a, b]$ of real-valued continuous functions $f(t)$ defined on a compact interval of reals. A celebrated theorem of Korovkin states that $\lim(T_nf) = f(t)$ (in the sup-norm) is true for every $f(t) \in C[a, b]$ if the transformations $T_n : C[a, b] \to C[a, b]$ are monotone and if $\lim(T_nf) = f(t)$ is true for the three 'easy' functions $f_1(t) = 1$, $f_2(t) = t$, $f_3(t) = t^2$.

It is the purpose of this paper to utilize this idea for the approximation of stochastic processes. A theorem is established that guarantees asymptotic accuracy of a given approximating procedure for every L^1-continuous stochastic process provided the procedure works for all elements of a prescribed finite set of 'easy' processes. This theorem can be used to prove stochastic versions of Weierstrass' and Fejér's theorems and to derive a result about numerical integration of random functions.

2.8. Queuing, storage and renewal processes

On the Time Intervals between the Registration Moments of Registered Particles for a Modified Counter with Prolonged Dead Time

Anatolij Dvurečenskij, *JINR Dubna, USSR*

We study the modified counter with prolonged dead time (type II counter, too) in that any registered particle has a distribution function of an impulse length different, in the general case, from distribution functions of the lengths of impulses for non-registered particles (which are assumed to be equal). Particles arrive at the counter according to a recurrent process.

We derive the explicit form of the Laplace transform for the length of the time interval between the registration moments, and some of its asymptotic properties, such as an exponential law, are proved.

Similar processes also appear in the film or filmless measurements of track ionization in the so-called bubble chambers in high energy physics.

References

[1] A. Dvurečenskij and G.A. Ososkov, Note on one type II counter problem, E5-83-255, JINR Dubna, 1983 (Sent to Aplikace Matem.).

[2] A. Dvurečenskij and G.A. Ososkov, On limit properties of a modified counter with prolonging dead time, P5-83-873, JINR Dubna, 1983 (Sent to Izv. AN USSR, Techk. Kiber.).

[3] A. Dvurečenskij and G.A. Ososkov, On a modified counter with prolonging dead time, E5-83-882, JINR Dubna, 1983 (Sent to J. Appl. Prob.).

Control Strategy for some Queues

Asaf Gadziev, *Azerbaijan Academy of Sciences, Baku, USSR*

Mathematical models which describe the behaviour of transport systems, networks of computers, and control strategy for such models are considered. The control strategy includes delays of the beginning of services. The class of systems for which it is advisable to include delays (decreasing waiting time before service) is described. The optimal function minimizing the expected waiting time before service is derived. It is shown that delays diminish the variance of the waiting time before service. A numerical example is given.

A Tandem Storage System and its Diffusion Limit

J. Michael Harrison, *Stanford University, Stanford, CA, USA*
L.A. Shepp, *Bell Laboratories, Murray Hill, NJ, USA*

We consider a two-dimensional diffusion process $Z(t)=[Z_1(t), Z_2(t)]$ that lives in the half strip $\{0 \leq Z_1 \leq 1, 0 \leq Z_2 < \infty\}$. On the interior of this state space, Z behaves like a standard Brownian motion (independent components with zero drift and unit variance), and there is instantaneous reflection at the boundary. The reflection is in a direction normal to the boundary at $Z_1 = 1$ and $Z_2 = 0$, but at $Z_1 = 0$ the reflection is at an angle θ below the normal ($0 < \theta < \frac{1}{2}\pi$). This process Z is shown to arise as the diffusion limit of a certain tandem storage or queuing system. It is shown that $Z(t)$ has a non-defective limit distribution F as $t \to \infty$, and the marginal distributions of F are computed explicitly. The marginal limit distribution for Z_1 is uniform (this result is essentially trivial), but that for Z_2 is much more complicated.

A Multiple Queue with Coupling

G. Hooghiemstra, M.S. Keane and S.v.d. Ree, *Delft University of Technology, Delft, The Netherlands*

An exponential queuing system with coupled processor (server) is considered. For the equilibrium situation we show that the stationary probabilities of the imbedded Markov chain, representing the number of customers in the various queues, can be expressed as a power series in the traffic intensity parameter. The coefficients of this power series can be obtained from an iterative scheme, whose implementation is very easy. The method renders excellent results.

In the two-dimensional symmetric case (cf. [1]) complete proofs are available. For the general case (dimension ≥ 3 and/or the non-symmetric case) some pieces of the proof of the validity of this method are not yet forehand.

Reference

[1] A.G. Konheim, I. Meilijson and A. Melkman, Processor sharing of two parallel lines, J. Appl. Probability, 18 (1981) 952–956.

On Coupling of Renewal Processes with use of Failure Rates

Torgny Lindvall, *University of Göteborg, Sweden*

During the latest decade, much attention has been paid to the so called coupling method; for a survey, see Griffeath [3], for its use in the study of the asymptotics of renewal processes, see [1, 4, 5, 6 and 7].

The prupose of this contribution is to present a new coupling, suitable for the analysis of renewal processes when the life-length distribution is of IFR or, in particular, DFR type. For the latter case, we extend results due to Mark Brown on monotonicity properties and inequalities for the renewal measure and point process associated with a renewal process.

References

[1] E. Arjas, E. Nummelin and R.L. Tweedie, Uniform limit theorems for non-singular renewal and Markov renewal processes, J. Appl. Prob. 15 (1978) 112–125.
[2] M. Brown, Bounds, inequalities and monotonicity properties for some specialized renewal processes, Ann. Prob. 8 (1980) 227–240.
[3] D. Griffeath, Coupling methods for Markov processes, in: Advances in Mathematics Supplementary Studies, vol. 2: Studies in Probability and Eegodic Theory (1978) 1–43.
[4] T. Lindvall, A probabilistic proof of Blackwell's renewal theorem, Ann. Prob. 5 (1977) 482–485.
[5] T. Lindvall, On coupling of discrete renewal processes, Z. Wahrsch. 48 (1979) 57–70.
[6] T. Lindvall, On coupling of continuous time renewal processes, J. Appl. Prob. 19 (1982) 82–89.
[7] P. Ney, A refinement of the coupling method in renewal theory, Stoch. Proc. Appl. 11 (1981) 11–26.

A Relation for the GI/G/1 Queuing System

Teunis J. Ott, *Bell Communications Research, Incorporated, Holmdel, NJ, USA*

Consider the stable GI/G/1 queue where interarrivals have distribution function A, and where service times have distribution function F and moments $\mu_1, \mu_2, \ldots$.

$V(t)$ denotes the virtual waiting time process of this system. At time $t = 0^-$, let $V(0^-)$ have the customer arrival stationary distribution. At time $t = 0$ a customer arrives and after that customer no more customers are allowed into the system. The resulting virtual waiting time process $(t \geq 0)$ is denoted by $V^*(t)$.

The stationarity condition now reads:

$$P\{V(0^-) \leq x\} = \int_{0^-}^{\infty} P\{V^*(t) \leq x\}\, \mathrm{d}A(x) = \int_{0^-}^{\infty} P\{V^*(0) \leq x + t\}\, \mathrm{d}A(t) \quad (1)$$

It is shown that for $k \geq 1$:

$$\int_0^\infty P\{V^*(t)>0\}\left(\frac{t^{k-1}}{(k-1)!}-\sum_{j=1}^{k}\frac{\mu_{k-j}}{(k-j)!}\int_{0^-}^{t}\frac{(t-x)^{j-1}}{(j-1)!}\,\mathrm{d}A(x)\right)\mathrm{d}t=\frac{\mu_k}{k!} \qquad (2)$$

With an appropriate system of orthogonal functions this can be used to compute $P\{V^*(t)>0\}$. Result (2) can also be used to derive inequalities for queues. In the case of the D/G/1 queue, which is of importance in software scheduling in clocked operating systems, this is a convenient mechanism to translate information on $F(\cdot)$ into bounds for $E[V(0^-)]$ etc. 'Minimal' information on F already reproduces the Kingman upper bound and the Marshall lower bound, and a corollary to the derivation shows that the Marshall lower bound is tight.

Reference

[1] K.T. Marshall, Some Inequalities in Queuing, *OR* 16 (1968) 651-665.

Analysis of a Storage Model with Markov Additive Inputs

Dr. Süleyman Özekici, *Boğaziçi University Bebek, Istanbul, Turkey*

In this paper a storage model with Markov additive inputs is analysed. It is assumed that the input process is a Markov additive process with an increasing second component which has a finite jump rate; furthermore the first component which describes the random changes in the environment is assumed to be a pure jump process. Admissibility requirements on the release rules which necessarily depend on environmental conditions are discussed and the storage process is constructed. It is shown that the storage process is a strong Markov process and an expression for its infinitesimal generator together with characterizations of its range and domain are obtained. Furthermore, the case where the store has finite physical capacity is discussed.

The Queue M/G/1 with Markov Modulated Arrivals and Services

G.J.K. Regterschot and J.H.A. de Smit, *Twente University of Technology, The Netherlands*

We discuss the M/G/1 queue with Markov modulated arrivals and services. Let $(X_t, t \geq 0)$ be a Markov chain with state space $\{1, 2, \ldots, N\}$ and indecomposable generator Q. T_n is the arrival epoch of the n-th customer and S_n is his service time. If $X_t = i$ customers arrive according to a Poisson process with intensity λ_i and an arriving customer has service time distribution G_i. Let $Y_n = X_{T_n}$.

We study the process$\{(W_n, T_n, Y_n), n = 1, 2, \ldots\}$ and obtain a system of Wiener-Hopf-type equations for the joint distribution of W_n, T_n and Y_n. This system is solved by factorizing its symbol. Thus we obtain explicit results for the steady state distributions of the actual waiting time, the virtual waiting time and the queue length both at arrival epochs and in continuous time. Finally we give some numerical examples.

Queuing Models in Application to the Design of Communication Networks

Diane D. Sheng, *AT & T Bell Laboratories, Holmdel, New Jersey, USA*

For a communications network, as the traffic load increases, the network becomes 'congested' and traffic takes longer to be communicated across the network, is communicated less clearly, and/or is altogether rejected by the network. A general goal in designing a communication network is to design a minimum cost network satisfying specific performance objectives. A common approach is to assign local performance objectives to individual components in the network, optimally design the network so as to satisfy the local objectives, check the overall performance of the network in light of the original global objectives, and to then redesign the network so as to alleviate performance bottlenecks.

We discuss here queuing models that are currently being used in the performance analysis of communication networks for which the fundamental measures of performance are the frequency of blocking traffic into the network and the transport time of traffic across the network. These queuing models were chosen so as to take into account the stochastic nature by which traffic arrives to the network, and to allow for the tractable characterization of congestion throughout the network. We also outline several open problems relevant to the performance analysis and optimal design of communication networks.

2.9. Random walks and i.i.d. *random variables*

Rates of Convergence and the Berry–Esséen Theorem

A.D. Barbour, *University of Zürich, Switzerland*

The Berry-Esséen theorem, applied to independent and identically distributed random variables $(X_j)_{j\geq 1}$ with zero mean, unit variance and finite third absolute moment γ, states that

$$\sup_x \left| P\left[n^{-1/2} \sum_{j=1}^{n} X_j \leq x \right] - P[N(0,1) \leq x] \right| \leq C\gamma n^{-1/2},$$

for a universal constant C. Because the distribution of the X_j's may be lattice valued, the right hand side cannot be directly improved: yet, for many distributions, the rate of convergence is much faster. In this paper, which describes work done jointly with P. Hall, the discrepancy between the distribution of the partial sums and the normal distribution is measured by a metric based on the expectations of smooth functions of random variables. In this framework, a precise rate of convergence can be determined, to within a tolerance of order n^{-1}.

From Discrete to Continuous Time in a Random Walk Model

H.F.P. van den Boogaard and P.I.M. Johannesma, *University of Nijmegen, The Netherlands*

In discrete time $\{t_k \mid t_k = k\,\Delta t,\ k \in \mathbb{N}\}$ a random walk $\{U_k\}$ is considered: if at time t_k $U_k = u$ then at t_{k+1} $U_{k+1} = u + \Delta U_k$ where the random variable $\Delta U_k = -bu\,\Delta t + wQ_k\,\Delta t^{\alpha}$, $Q_k \in \{-1, 0, +1\}$ with distribution $P[Q_k = q \mid U_k = u] = \psi(q|\Delta t|u)$. So Q_k gives the sign of the stochastic displacement $w\,\Delta t^{\alpha}$ and $-bu\,\Delta t$ is a deterministic movement towards the origin. In the following $b = 0$. This Markov chain can be scaled with respect to Δt such that $\lim_{\Delta t \downarrow 0}$ yields a 'decent' process $U(\cdot)$ in continuous time. For that it is chosen:

$$\psi(+|\Delta t|u) - \psi(-|\Delta t|u) = g_1(u)\Delta t^{\beta},$$

$$\psi(+|\Delta t|u) + \psi(-|\Delta t|u) = g_2(u)\Delta t^{\gamma} \qquad (0 \leqslant \gamma \leqslant \beta)$$

and a partial differential equation for the second order transition probability density of $U(\cdot)$ (physics: master equation) is derived. It is found that:

1. if $\alpha + \beta < 1$ or $2\alpha + \gamma < 1$ it is not allowed to take $\lim_{\Delta t \downarrow 0}$.
2. if $\alpha + \beta \geqslant 1$, $2\alpha + \gamma \geqslant 1$ the master equation is:
   a. trivial if $\alpha + \beta > 1$, $2\alpha + \gamma > 1$.
   b. an ordinary first order partial differential equation if $\alpha + \beta = 1$, $2\alpha + \gamma > 1$. $U(\cdot)$ represents a deterministic motion.
   c. a diffusion (or Fokker-Planck) equation if $2\alpha + \gamma = 1$, $\alpha > 0$.
   d. a partial differential equation with non-local u arguments if $2\alpha + \gamma = 1$, $\alpha = 0$. $U(\cdot)$ performs jumps.

This abstract generalizes known ($\alpha = \frac{1}{2}$) methods to obtain a diffusion equation and shows how deterministic, diffusion and jump processes are linked. The foregoing resulted from the authors' theoretical investigation of modelling interaction of nerve cells.

A Conditional Limit Theorem for Asymptotically Stable Random Walk

R.A. Doney, *University of Manchester, UK*

Earlier results on limit theorems for random walk with zero mean and finite variance, conditioned in the first n values being non-negative, were unified in a functional conditional central limit theorem, due to Bolthausen [1]. If by an excursion of a random walk we understand the section of a random walk between successive downgoing ladder epochs, the conditioning event may be reformulated as 'the length of the first excursion exceeds n'. Shimura [3] has recently shown that Bolthausen's result, and his method of proof, may be extended to the case where the conditioning event is one of a large class of events definable in terms of the first excursion of the random walk. Similar results, derived in a quite different way, are contained in [2]. In this paper we establish a similar result for a recurrent random walk in the domain of attraction of a suitable stable law. We apply it to deduce the asymptotic behaviour of various functionals of the first excursion of the random walk.

References

[1] E. Bolthausen, On a functional central limit theorem for random walk conditioned to stay positive, Ann. Probability 4 (1976) 480–485.
[2] P.E. Greenwood and E. Perkins, Limit theorems for excursions from a moving boundary. To appear in Theory of Probability and its Applications, 1984.
[3] M. Shimura, A class of conditional limit theorems related to ruin problems, Ann. Prob. 11 (1983) 40–45.

Weak Limits of Elementary Symmetric Polynomials

A.J. van Es, *Centre for Mathematics and Computer Science, Amsterdam, The Netherlands*

Let the k-th order elementary symmetric polynomial of i.i.d. random variables $X_1, \ldots, X_n$ be defined by

$$S^{(k)}(X_1, \ldots, X_n) = \sum_{1 \leq i_1 < \cdots < i_k \leq n} X_{i_1} \cdots X_{i_k}, \quad 1 \leq k \leq n.$$

Our concern is the weak limits of $S^{(k)}(X_1, \ldots, X_n)$ if $k \to \infty$, $n \to \infty$ and $k/n \to c$ $(0 \leq c \leq 1)$.

Recent results of G.J. Székely and others treat the limit behaviour for special classes of X_j,

(a) for strictly positive X_j,

(b) for X_j with distribution $P(X_j = -1) = P(X_j = 1) = \frac{1}{2}$.

A detailed treatment of $S^{(k)}(X_1, \ldots, X_n)$ for zero-one X_j and the observation

$$S^{(k)}(X_1, \ldots, X_n) \stackrel{\mathrm{d}}{=} S^{(k)}(Y_1, \ldots, Y_{E_n}),$$

where $X_j = Z_j Y_j$, with $Z_1, \ldots, Z_n$ i.i.d. zero-one random variables, independent of $Y_1, \ldots, Y_n$, i.i.d. random variables with $P(Y_j = 0) = 0$, and $E_n = Z_1 + \cdots + Z_n$, provide us with a means to derive the weak limits for

(a′) non-negative X_j,

(b′) three valued symmetric X_j, $P(X_j = -1) = P(X_j = 1) = \frac{1}{2}P(X_j \neq 0)$ from the results in cases (a) and (b).

It turns out that the limit behaviour in case (a) doesn't change when we allow zeros while it changes drastically in case (b).

Fluctuation Behavior of Poisson Triangular Arrays

Lutz Edler, *German Cancer Research Center, D-6900 Heidelberg, F.R. of Germany*

Let $(X_{nk}, (n, k) \in N^*)$ denote a triangular array (t.a.) of random variables (r.v.) $N^* = \{(n, k), k = 1, \ldots, n \text{ and } n = 1, 2, \ldots\}$. The fluctuation behavior of the sequence of the row sum $S_n = \sum_{k=1}^{n} X_{nk}$ was investigated by analogues of the law of the iterated logarithm (LIL) for t.a. of Bernoulli variables. Baxter (1955) studied two classes of t.a.: the completely independent array (I.A.) where X_{nk}, $k = 1, \ldots, n$ are i.i.d. and the rows independent, and the column independent and row ordered array (CI.RO.A.) where the σ-algebras $\mathfrak{A}_k = \mathfrak{A}(X_{mk}, m \geq k)$ generated by the r.v. in the k-th column are independent for $k = 1, 2, \ldots,$ and the r.v. within each row are almost surely ordered as $X_{kk} \geq X_{k+1,k} \geq \cdots$. If $p_n = \lambda/n, 0 < \lambda < n$, he obtained the LIL $P(\limsup(S_n - ES_n)/\varphi_n = 1) = 1$ for I.A. with $\varphi_n = \log n/\log_2 n$ (Baxter, Theorem 1) and for CI.RO.A. with $\varphi_n = \log_2 n/\log_3 n$ (Baxter, Theorem 2). Recently, Rosalsky (1983) modified Baxter's proof and obtained a LIL for a sequence of i.i.d. r.v. $\{Y_n\}$ having a Poisson distribution $P(\lambda)$. Using the infinite divisibility of $P(\lambda)$ Rosalsky's fluctuation theorem for i.i.d. Poisson r.v. $\{Y_n\}$ obtained from Baxter, Theorem 1 follows from a LIL for an I.A. $\{Y_{nk}, (n, k) \in N^*\}$ of Poisson r.v. obtained by Edler (1970). Furthermore, a fluctuation theorem for a sequence of identically distributed but dependent Poisson r.v. $\{Y_n\}$ can be obtained from a LIL of CI.RO.A. of Poisson r.v. $\{Y_{nk}, (n, k) \in N^*\}$ analogously to Baxter, Theorem 2, for CI.RO.A., The dependency lets the norming sequence increase slower than in the independent case.

References

G. Baxter, An analogue of the law of the iterated logarithm, Proc. Amer. Math. Soc. 5 (1955) 177–181.

L. Edler, Verallgemeinerungen des Gesetzes vom iterierten Logarithmus auf Doppelfolgen von zufälligen Veränderlichen. Diplomarbeit, Universität Freiburg (1970).

A. Rosalsky, A remark on the fluctuation behavior of i.i.d. Poisson random variables, Statist. Prob. Letters 1 (1983) 181–182.

On the Waiting Time in a Generalized Roulette Game

Allan Gut and Lars Holst, *Uppsala University, Sweden*

Consider a multinomial experiment with outcomes $E_0, E_1, \ldots, E_{N+1}$ having probabilities $p_0, p_1, \ldots, p_{N+1}$ with $p_i > 0, 0 \leq i \leq N, p_{N+1} \geq 0$. Independent repetitions are performed and the following game is played: The game ends as soon as $E_1, \ldots, E_N$ all have appeared without any E_0 in between (E_{N+1} may or may not have occurred). If E_0 appears too early the game starts again from the beginning. What is the duration of the game? Special cases are the waiting times in a roulette game and in some dice problems sought for earlier. The main ingredients are the theory of stopped sums and an alternative formulation using independent Poisson processes.

The Rate of Growth of Sample Maxima with Multidimensional Indices

Allan Gut, *Uppsala University, Sweden*
Jürg Hüsler, *Bern University, Switzerland*

We extend two results due to de Haan and Hordijk (1972) concerning asymptotics for the partial maxima of i.i.d. random variables to the cases where the index sets are the positive integer d-dimensional lattice points and the sector.

On the Rate of Escape of a Random Walk

Joop Mijnheer, *University of Leiden, The Netherlands*

Let $X_1, X_2, \dots$ be i.i.d. symmetric random variables with common distribution function F, satisfying

$$1 - F(x) = x^{-1}L(x) \quad \text{for } x > x_0,$$

where L is slowly varying at infinity. $S_n = X_1 + \cdots + X_n$.

In [1] Griffin gives examples such that, for each $\delta \in (0, 1)$, we have a function L and

$$\liminf n^{-\alpha}|S_n| = \begin{cases} \infty & \text{a.s.} \quad \text{if } \alpha < \delta, \\ 0 & \text{a.s.} \quad \text{if } \alpha > \delta. \end{cases}$$

We shall give a new simple proof for this result and we also consider the case $\alpha = \delta$. Finally we shall prove the following integral test. Write

$$L(x) = \exp\left\{\int_{x_0}^{x} y^{-1} (\log y)^{-1} \psi(y)\, \mathrm{d}y\right\},$$

where ψ is a non-decreasing slowly varying function.

We define

$$I(\psi) = \int^{\infty} x^{-1}(\log x)^{-1}\psi(x) \exp\{-(\log \alpha^{-1})\psi(x)\}\, \mathrm{d}x.$$

Theorem. *Let S_n and ψ be defined as above. Then we have*

$$\liminf n^{-\alpha}|S_n| = \begin{cases} 0 & a.s. \\ \infty & a.s. \end{cases} \quad \textit{according as } I(\psi) \begin{cases} = \infty, \\ < \infty. \end{cases}$$

We shall discuss the further assumptions on ψ.

Reference

[1] P.S. Griffin, Rates of escape of random walks, Ann. of Prob. 10 (1982) 869-871.

A Limit Theorem for Two-Dimensional Conditioned Random Walk

M. Shimura, *University of Tsukuba, Japan*

Let $\boldsymbol{X}_n$, $n = 0, 1, 2, \ldots$, be a two-dimensional random walk with stationary independent increments such that $\boldsymbol{X}_0 = \boldsymbol{0}$. We assume

$$E(\boldsymbol{X}_1) = \boldsymbol{0} \quad \text{and} \quad \operatorname{cov}(\boldsymbol{X}_1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Let $\boldsymbol{X}^{(n)}(t)$, $t \geq 0$, denote the normalized random walk $n^{-1/2}\boldsymbol{X}_{[nt]}$, where $[a]$ is the integral part of a.

In this paper we will show the following result: Let F be a closed domain in the upper half-plane such that $\boldsymbol{0} \in \partial F$ and such that $\boldsymbol{0}$ is an *irregular point* to the set F^c for the *upper half-plane excursions* of the two-dimensional Brownian motion. Then *a sequence of conditional probabilities*

$$P(\boldsymbol{X}^{(n)}(\cdot) \in * \,|\, \sigma_F(X^{(n)}) > 1), \quad n = 1, 2, \ldots,$$

converges weakly in $\boldsymbol{D}([0, 1] \to \boldsymbol{R}^2)$ *to a conditioned Brownian motion*, where $\sigma_F(X^{(n)}) = \inf\{t : X^{(n)}(t) \notin F\}$.

Reference

M. Shimura, A limit theorem for two-dimensional conditioned random walk, to appear in Nagoya Math. J. 95 (1984).

Run Probabilities and The Motion of a Particle on a given Path

A. Reza Soltani, *Shiraz University, Iran*

Let $\{X_n\}$ be a sequence of independent (or Markov dependent) trials taking values in a given set S. Let R be a given path of length k in S, i.e. R is a run of length k whose elements come from S. $\{X_n\}$ may indicate the motion of a particle on S. We consider the problem of finding the probability that at trial m, the particle has for the first time moved length $l \leq k$ on R which is equivalent to finding the probability of the first occurrence of any subrun of length $l \leq k$ of R. This in the case of $l = k$ gives the result of Steven J. Schwager (Run probabilities in a sequence of Markov-dependent trials, JASA 78 (1983) 168–175.).

Poisson Processes and Bessel Functions

F.W. Steutel, *Eindhoven University of Technology, The Netherlands*

Several simple models in applied probability can be translated into the following game: two persons, X and Y, take steps of random length, one by one, alternately. All steps have independent, exponentially distributed lengths with expectation one.

X starts, and the question of interest is: what is the probability that X covers a distance x before Y has gone a distance y? The answer is: $J(x, y)$, where J is a well-known Bessel function integral (cf. [2, p. 217, ft.])

Many properties of J as given in [2], and some others, follow directly from the interpretation above. Especially, the central limit theorem yields a simple approximation for $J(x, y)$ if x and y are large.

The integrated random telegraph model considered by Kac [1] can be made a special case of our model by a simple transformation, and properties of the process follow.

References

[1] M. Kac, A stochastic model related to the telegrapher's equation, Rocky Mountain J. of Math. 4 (3) (1974) 497–509.
[2] Y.L. Luke, Integrals of Bessel functions (McGraw-Hill, New York, 1962).

Some Almost Sure Results for the Maxima of Trigonometric Polynomials with Random Coefficients

K.F. Turkman, *Faculdade de Ciências da Universidade de Lisboa, Portugal*

Let $\{X_t,\ t = 1, 2, \ldots\}$ be i.i.d. random variables with standard normal distribution. Let

$$M_{n,X} = \max_{\lambda \in [0,\pi]} \left[\sqrt{\frac{2}{n}} \sum_{t=1}^{n} X_t \cos \lambda t \right]$$

and

$$M_{n,I} = \max_{\lambda \in [0,\pi]} \left[\frac{2}{\pi} \left| \sum_{t=1}^{n} X_t \, \mathrm{e}^{\mathrm{i}\lambda t} \right|^2 \right].$$

It is shown that almost surely

$$\lim_{n\to\infty} [M_{n,X} - (2 \log n)^{1/2}] = 0,$$

$$\lim_{n\to\infty} \frac{M_{n,I}}{2 \log n} = 1.$$

Also almost sure upper bounds for $M_{n,X}$ and $M_{n,I}$ are established.

2.10. Reliability and risk theory

A Simple Model for Reliability Growth in Hardware

E. Arjas and H. Laaksamo, *University of Oulu, Finland*

Reliability growth, an increase in the reliability of a system, is a common phenomenon in both hardware and software. It occurs mainly at the beginning of the operation. There are a number of widely used models for reliability growth, such as the AMSAA and EDRIC-models.

In the case of hardware it can be argued that, apart from active improvement of systematic weaknesses (such as design errors), reliability growth results from the replacement (or repair) of the system's 'weak' components that 'have proven their weakness by failing'. The present model simply postulates that the considered system consists of independent components, each component having a lifetime distribution which is a mixture of two exponential distributions (representing the populations of 'weak' and 'normal' components). The elementary properties of this model are discussed.

A Planned Maintenance Model with Lead Time and Minimal Repair

Naoto Kaio and Shunji Osaki, *Hiroshima Shudo University and Hiroshima University, Japan*

A planned maintenance model is discussed taking account of lead time and minimal repair. Consider a one-unit system in which each spare is only provided by an order with a constant lead time, where the lifetime obeys an arbitrary distribution. Minimal repair is made whenever system failure takes place. If the system does not fail up to a prespecified time t_0, a regular order is made at time t_0. When the spare is delivered, (1) the system is replaced if it has experienced one or more failures, or (2) the spare is put into inventory until the first system failure. Otherwise, if the system fails up to time t_0, an expedited order is made at the failure time and the system is replaced as soon as the spare is delivered. The same cycle repeats itself again and again.

This maintenance model can be analyzed by noting a regeneration point. The long-run expected total discounted cost can be derived by introducing several costs, lead time, discount factor, etc. Our interest is to obtain the optimum regular ordering time which minimizes the long-run expected total discounted cost. It is shown that, under certain conditions, there exists a finite and unique optimum regular ordering time.

Entropy Estimates for First Passage Times

Anders Martin-Löf, *The Folksam Group, Stockholm, Sweden*

In risk theory one considers the first passage time $N(c)$ of e.g. a random walk with negative drift to a high level $c > 0$. As $c \to \infty$ the well known asymptotic formulas of Lundberg, Cramér, Segerdahl etc. tell us that $P(N(c) < \infty) \approx e^{-Rc}$, and that the conditional distribution of $N(c)$ given that $N(c) < \infty$ is approximatively normal. These estimates were derived using Wiener–Hopf methods and later by Feller using renewal theory. In this paper we show that they can be derived directly using ideas from the theory of large deviation estimates in path space. One considers the scaled process

$$X_c(t) = S_{[ct]}/c.$$

The probability of a path $x(t)$, $0 \leq t \leq T$, is approximatively $\exp c \int_0^T h(x(t))\,dt$, where $h(x')$ is the entropy function. $-R$ is then obtained as the maximum entropy of all paths reaching the level 1, and the central limit theorem is obtained by linearization around the path of maximal entropy.

These ideas allow generalizations to time dependent barriers and more general Markov processes to be derived straightforwardly.

Systems Weakened by Failures

Ilkka Norros, *University of Helsinki, Finland*

Consider a technical system consisting of k components with respective life lengths S_i, $i = 1, \ldots, k$. The system $\boldsymbol{S} = (S_1, \ldots, S_k)$ is called *weakened by failures*, if the conditional distribution of $\boldsymbol{S}$ given the failure history up to time t jumps downwards at each failure time $t = S_i$, in the sense of stochastic order.

'Weakened by failures' is a new notion of multivariate positive dependence. It is based on the 'dynamic' approach to multivariate reliability systems, introduced in Arjas (1981).

In the present paper, some properties of systems weakened by failures are discussed, i.e. the following result:

Theorem. *If $\boldsymbol{S}$ is weakened by failures, then it is associated, that is,* $\operatorname{cov}(f(\boldsymbol{S}), g(\boldsymbol{S})) \geq 0$ *for increasing bounded real-valued functions f and g.*

References

[1] E. Arjas, A stochastic process approach to multivariate reliability systems: Notions based on conditional stochastic order, Math. Oper. Res. 6 (1981) 263–276.

[2] E. Arjas and I. Norros, Life lengths and association: a dynamic approach, To appear in Math. Oper. Res. (1984).

Performance/Reliability Evaluation for Multi-Processor Systems with Computational Demands

Shunji Osaki, *Hiroshima University, Higashi-Hiroshima 724, Japan*
Toshio Nakagawa, *Meijo University, Nagoya 468, Japan*

A multi-processor system can attain high reliability and performance simultaneously. Though several measures featuring the reliability or performance have been proposed independently, it is not enough to seek the measures featuring the reliability and performance independently. We propose a few measures featuring the reliability and performance simultaneously such as the expected numbers of lost jobs by the failure and by the cancellation. We discuss a multi-processor system with a buffer, in which each processor and buffer can fail, and be repaired independently. Assuming that the failure time is exponentially distributed and the repair time is arbitrarily distributed, and introducing the coverage, we analyze such a generalized model by applying Markov renewal processes and queuing theory, and derive some important measures numerically. Numerical examples show several measures that are the expected number of lost jobs by the cancellation which should be minimized, and the expected system throughput which should be maximized. Such measures give an optimum storage capacity of the buffer balancing the tradeoffs between performance and reliability.

An Adaptive Block Replacement Plan for a Series System

H. Sandoh and S. Fujii, *Kanazawa Institute of Technology, Japan*

This study deals with a block replacement for a series system when failure rates of its component are not exactly known.

A cost function which considers uncertainty of failure rates is formulated, making use of the prior distribution. It is based on the cost function which consists of costs for preventive and corrective replacements as well as adjustment costs. The adjustment cost for each component is considered here to increase according to its age.

In the process of executing a preventive replacement plan, field data such as failure times of the system are available. The prior distribution mentioned above can be improved via Bayes theorem in conjunction with such incoming data.

From this point of view, an adaptive method is presented in which the block replacement interval as well as the posterior distribution are revised after each replacement. The replacement interval obtained by this method has the property of converging to the optimal replacement interval for the true failure rates of components.

Some Inequalities in Risk Theory

J.L. Teugels, *Katholieke Universiteit, Leuven, Belgium*

Consider the classical case of a compound Poisson process $X(t)=\sum_{i=1}^{N(t)} Y_i$ where $\{N(t);\, t\geq 0\}$ is the counting process of the claim arrivals with intensity λ, while $\{Y_i;\, i\in\mathbb{N}\}$ is the sequence of successive claims with distribution F.

If the initial capital is u while the incoming premiums up to time t are given by ct, $c>0$, then the risk reserve at time t is $Y(t)=u+ct-X(t)$. Ruin occurs whenever $Y(t)$ becomes negative.

So let $T_u = \inf\{t > 0: Y(t) \leqslant 0\}$, if finite, be the time of ruin. An easy renewal argument yields that

$$W(t, u) = P\{T_u > t\}$$

satisfies the integral equation

$$W(t, x) = \mathrm{e}^{-\lambda t} + \lambda \int_0^t \mathrm{e}^{-\lambda s}\,\mathrm{d}s \int_0^{x+cs} W(t-s, x+cs-y)F(\mathrm{d}y).$$

We solve the equation and derive a set of inequalities for the ruin probabilities.

2.11. Stationarity, invariance and iterated mappings

A Principle of Subsequences in Probability Theory and Restricted Exchangeability
R. Ahmad, *University of Strathclyde, Glasgow, Scotland*

In investigating a principle of subsequences in probability theory, Chatterji (1974) formulated the general principle as follows. If a certain quantitative asymptotic property (QAP) is valid for any sequence of i.i.d. r.vs. $\{X_n\}$ belonging to some integrability class determined by the finiteness of norm $\|\cdot\|_L$, an analogous property QAP* will be valid for a suitable subsequence $\{h_n\}$ of any sequence $\{H\}$ of functions on any probability space such that $\sup\{\|h\|_L: h \in H\} < \infty$. Moreover, the subsequence can be chosen in such a way that any further subsequence will have the same property QAP*. For example, QAP could be: the Kolmogorov strong law of large numbers, the Marcinkiewicz generalization of Kolmogorov's strong law, the classical central limit theorem, any almost sure limit theorem, any weak limit (convergence in distribution) theorem, etc., under mild restrictive conditions. Other relevant work in this area is Komlós (1967) and Aldous (1977).

Motivated by various practical aspects for several common applications, in this paper by considering arbitrarily dependent sequences of random variables, we give a slightly extended and modified principle of subsequences. The technique used employs exchangeable and partially exchangeable sequences of random variables.

References

R. Ahmad, On the structure and applications of restricted exchangeability, in: G. Koch and F. Spizzichino, eds., Exchangeability in Probability and Statistics, North-Holland (1982) 157-164.

D.J. Aldous, Limit theorems for sequences of arbitrarily-dependent sequences of random variables, Z. Wahrsch. verw. Gebiete 40 (1977) 59-82.

S.D. Chatterji, A principle of subsequences in probability theory: The central limit theorem, Advances in Math. 13 (1974) 31-54.

G. Koch and F. Spizzichino, eds., Exchangeability in Probability and Statistics, A symp. in Honour of Professor B. de Finetti on his 75th Birthday (North-Holland, Amsterdam, 1982).

J. Komlós, A generalization of a problem of Steinhaus, Acta Math. Acad. Sci. Hung. 18 (1967), 217-229.

Stationarity and Periodicity

H.C.P. Berbee, *Centre for Mathematics and Computer Science, Amsterdam, The Netherlands*

We assign a period to an ergodic stationary process. For Markov sequences it corresponds to the familiar periodicity notion. One has

$$\text{period} = 1 \Rightarrow \text{(double) tail field is trivial.}$$

However in this more general context the converse is not valid, as is shown by a nice example related to a random walk. However if one considers absolute regular (or ϕ-mixing) processes the well known picture arises again of p 'cyclic moving subclasses' corresponding to period p. These results generalize renewal theory.

The Stochastic Equation $Z_{n+1} = f(Z_n, X_n)$: Existence of Stationary Solutions and Some Applications

Andreas Brandt, Peter Franken and Bernd Lisek, *Homboldt-Universität, Berlin, DDR*

Some general results concerning the existence and uniqueness of stationary solutions of the stochastic equation

$$Z_{n+1} = f(Z_n, X_n), \quad n \in \Gamma, \tag{1}$$

will be sketched, where (X_n) is a given strictly stationary sequence and the random variables X_n and Z_n take values in Polish spaces. Equations of type (1) describe the temporal behaviour of some queuing models, cf. e.g. [1, 2]. However, they arise in other fields, too, see e.g. [3, 4, 5].

References

[1] A.A. Borovkov, Asymptotic methods in queuing theory (In Russian) (Nauka, Moscow, 1980).

[2] A. Brandt, P. Franken and B. Lisek, Ergodicity and steady state existence. Continuity of stationary distributions of queuing characteristics, Lecture Notes on Information and Control Theory, to appear (1984).

[3] P. Franken, B. Lisek, On Wald's identity for dependent variables, Z. Wahrsch. Verw. Geb. 60 (1982) 143–150.

[4] B. Lisek and M. Lisek, On stationary distributions of Markov chains, submitted to Stochastic Models (1984).

[5] W. Vervaat, On a stochastic difference equation and a representation of non-negative infinitely divisible random variables, Adv. Appl. Prob. 11 (1979) 750–783.

Asymptotic Representation Results for Products of Random Matrices

C.C. Heyde, *University of Melbourne, Australia*

Representations which are suitable for extraction of limit theorems can be obtained for products of stationary non-negative matrices using an approach via subadditive process theory and approximating stationary processes. The results are very useful when applied to the theory of demographic population projections.

The Asymptotic Behavior of Extreme Values of Stationary Sequences

George L. O'Brien, *York University, Canada*

Let (X_n) be a strictly stationary sequence of random variables with marginal distribution function F. Let $M_{i,j} = \max(X_{i+1}, X_{i+2}, \ldots, X_j)$ and let $M_n = M_{0,n}$. Let (c_n) be a sequence of real numbers. If (X_n) satisfies a certain asymptotic independence condition, weaker than strong mixing, and if $(p) = (p_n)$ is a sequence of positive integers satisfying a certain growth rate condition, then

$$P[M_n \leqslant c_n] - (F(c_n))^{nP[M_{1,p} \leqslant c_n | X_1 > c_n]} \to 0$$

as $n \to \infty$, provided

$$\liminf\{(F(c_n))^n + P[M_{1,p} \leqslant c_n \mid X_1 > c_n]\} > 0.$$

Most earlier results about the asymptotic nature of $P[M_n \leqslant c_n]$ can be deduced from this result, often in a strengthened form. Some theorems about the simultaneous limiting behavior of $P[M_n \leqslant c_n(x)]$ for x in some index set T are also obtained. The results are applied to functions of positive Harris Markov sequences.

A Spectral Theory for Certain Non-Stationary Stochastic Processes

Bengt Ringnér, *Lund University, Sweden*

Many time-series ocurring in practice such as data from economy, industrial processes, etc. cannot be modelled directly by stationary stochastic processes, since the series contain trends and diurnal or seasonal variations. (Properly speaking these could be considered part of long term stationary variations, but that approach would lead to difficulties when identifying the model.) This type of instationarity can be avoided by taking a suitable number of differences until the resulting process becomes stationary. The differences can have lag one to remove the trend and longer lags when dealing with nearly periodical variations. This procedure is also applicable in the special case when the process is the sum of a linear or polynomial trend, a periodic function, and a stationary process.

The purpose of this research is to use some functions with infinite integrals as spectral densities for processes that can be made weakly stationary after suitable differencing. Such a theory has been made by Kolmogorov; the novelty here is that filtering is treated. The main results are:

(1) A class of processes admitting spectral densities and closed under linear filtering is established.

(2) The class of frequency response functions for linear filters is characterized.

(3) The counterpart of the Wiener filter to extract an unknown signal from noise is studied. Similarities to and differences from the stationary case are mapped out.

An Almost Subadditive Superstationary Ergodic Theorem

Klaus Schürger, *University of Bonn, FR Germany*

A recent ergodic theorem of Derriennic [1] extends Kingman's [3] subadditive ergodic theorem to stationary stochastic processes which are almost subadditive. By using a different proof technique based on a result of Komlós [4], we show that the almost sure part of Derriennic's result holds under a weaker moment condition. Furthermore, we extend the almost subadditive ergodic theorem to stochastic processes which are superstationary in the sense of Kamae, Krengel and O'Brien [2]. As an application, an ergodic theorem for very general classes of random sets is obtained which extends results of Schürger [5].

References

[1] Y. Derriennic, Un théorème ergodique presque sous-additif, Ann. Prob. 11 (1983) 669-677.

[2] T. Kamae, U. Krengel and G.L. O'Brien, Stochastic inequalities on partially ordered spaces, Ann. Prob. 5 (1977) 899-912.

[3] J.F.C. Kingman, Subadditive ergodic theory, Ann. Prob. 1 (1973) 883-909.

[4] J. Komlós, A generalization of a problem of Steinhaus, Acta Math. Acad. Sci. Hungar. 18 (1967) 217-229.

[5] K. Schürger, Ergodic theorems for subadditive superstationary families of convex compact random sets, Z. Wahrsch. Verw. Gebiete 62 (1983) 125-135.

Mappings which Preserve the Pointwise Ergodic Theorem

Štefan Šujan, *Joint Institute for Nuclear Research, Moscow, USSR*

According to Gray and Kieffer, a measurable transformation T of a probability space $(\Omega, \mathscr{F}, m)$ obeys the pointwise ergodic theorem for each bounded measurable function iff m is asymptotically mean stationary (a.m.s.) with respect to T. Here, measurable mappings $\varphi: (\Omega, \mathscr{F}) \to (\Lambda, \mathscr{L})$ are characterized such that if m is a.m.s. relative to T, then $m\varphi^{-1}$ is a.m.s. relative to a measurable transformation U acting on $(\Lambda, \mathscr{L})$. The problems of preservation of ergodicity, exactness in Rokhlin's sense, and of other ergodic properties are discussed. With each φ there is assigned a deterministic channel, and various anticipation and memory constraints upon the channels are shown to yield interesting classes of such mappings. In particular, channel ideas are employed in order to deal with invertibility of the mappings φ.

Quasi-Stationary and Quasi-Isotropic Random Fields

Eugeniusz Szczepankiewicz, *Wrocław, Poland*

The theory of certain random fields is presented in this paper. Let (E, A, P) be a probabilistic space. Each function $f: \mathbb{R}^n \times E \to \mathbb{R}^1$ such that for each $p \in \mathbb{R}^n$ the function $e \mapsto f(p, e)$ is a random variable in E, we call a random field. Assume that in a certain set $D \subset \mathbb{R}^n$ the function $f(p)$ and the coordinate system $0x_1, \ldots, x_n$ are given. In this set, let $C(e)$ be a random coordinate system, i.e., for each point $e \in E$ a system $C(e)$ is given. Each point $p \in D$ can be represented in this system as $p = p(e)$, and a function $f(p, e) = f(p(e))$ is a random field, if f is enough regular, and $e \mapsto f(p(e))$ is a random variable in E. Let us denote by $r(f(p, e)), f(q, e)) = r(p, q)$ the correlation function of the field $f(p, e)$. If $r(p, q) = r(|\overrightarrow{pq}|)$, where $|\overrightarrow{pq}|$ is the length of the vector $\overrightarrow{pq}$, the random field $f(p, e)$ is called isotropic and stationary in the wide sense. If $r(p, q) = r(\|\overrightarrow{pq}\|)$, where $\|\overrightarrow{pq}\|$ is a norm in $D \in \mathbb{R}^n$, the random field is called stationary in wide sense and quasi-isotropic. If $E[f_t(p, e)] = m(t)$ and $D^2[f_t(p, e)] = \sigma^2(t)$ and also $r(p, q) = r(t, \|\overrightarrow{pq}\|)$, where f_t is a function given in $D \times \mathbb{R}^1$, the random field is called nearly stationary in wide sense and quasi-isotropic.

Theorems concerning the magnitude of the class of presented random fields are proved, and applications to the theory of material durability, the theory of filtration, distribution of the air and water pollutions etc., are also given.

Upper Differential Quotients of Self-Similar Processes

Wim Vervaat, *Katholieke Universiteit, Nijmegen, The Netherlands*

Let X be an H self-similar process with stationary increments, i.e.,

$$X(a\,\cdot\,) \stackrel{\mathrm{d}}{=} a^H X(\,\cdot\,) \quad \text{for } a > 0, \tag{H-ss}$$

$$X(b+\cdot\,) - X(b) \stackrel{\mathrm{d}}{=} X(\,\cdot\,) - X(0) \quad \text{for } b > 0, \tag{si}$$

where $\stackrel{\mathrm{d}}{=}$ denotes equality of finite-dimensional distributions. Then $H > 0$ and $X(0) = 0$ wp 1, except in trivial cases. We investigate the process of upper differential quotients

$$D_\beta(t) := \limsup_{u \downarrow 0} u^{-\beta}(X(t+u) - X(t)).$$

It turns out that $D_\beta(t) = D_\beta(0)$ wp 1 for each t separately. Results and open problems are indicated about possible values of $D_\beta(0)$ for various combinations of β and H. More specifically, there is a functional relation (depending on β-H) between $D_\beta(0)$ and $\sup_{0<t\le 1} t^{-\beta}X(t)$. If X is ergodic with respect to (H-ss), then $D_H(0)$ equals wp 1 the right endpoint of the marginal distribution of $X(1)$. There are only partial results as to which right endpoints actually can occur.

2.12. Statistical inference

Signal Detection for Same-Shape Families of Non-Homogeneous Poisson Processes

C.B. Bell, *San Diego, CA, USA*

Each such family of NHPP's is characterized by the property that any two mean functions have a constant ratio. Five inference problems-goodness-of-fit with and without nuisance parameters, the 2- and c-sample problem, and a modified 2-sample problem are treated. Each problem is considered from the point of view of Type I and Type II censoring as well as Regular Sampling, equal-distance and same-shape sampling. Applications to epileptic seizures are indicated.

Estimating Failure Rate Parameters from Contaminated Data

C.A. Clarotti, *CRE-Casaccia, ENEA, Rome, Italy*
G. Koch, *University of Rome, Italy*
F. Spizzichino, *University of Rome, Italy*

When failure data are contaminated, modeling and estimation problems in reliability theory are conveniently dealt with by a dynamical approach, which stresses semimartingales, counting processes, intensity processes.

In this work, maintenance is considered, which periodically 'refreshes' the age of the observed item(s) by a random quantity. A simple model is proposed. Then, the conditional distribution of age and unknown failure rate parameters given the observed item(s) history is obtained by iterative solution of the filter equation.

This distribution undergoes appropriate separate updatings during nonmaintenance intervals, at maintenance times, at failure times. These updatings enjoy a clear bayesian interpretation.

Serial or parallel observation schemes for multiple items are shown to be equivalent.

Weak Convergence of Weighted and Split Multidimensional Rank Processes with Truncation

Michel Harel, *Institut Universitaire de Technologie, Limoges, France*

Among all the suggested methods of establishing convergence of rank statistics, one would be to write these statistics in the form $T_n = \int L_n \cdot r \, \mathrm{d}\mu_n$ (where μ_n is a signed measure, r a positive or null continuous function) and to verify on the one hand the weak convergence of the measure $r \, \mathrm{d}\mu_n$ and on the other hand the convergence with respect to the uniform topology of the process $L_n \cdot (1/r)$. Here, we are only giving necessary conditions for the convergence of a modification of the process $L_n \cdot (1/r)$, which is called split process and written $L_n^* \cdot (1/r)$, with respect to a sequence of non-stationary φ-mixing $\mathbb{R}^k$-valued observations.

Maximum Likelihood Estimation in the Multiplicative Intensity Model

Alan F. Karr, *The John Hopkins University, USA, and The University of North Carolina at Chapel Hill, USA*

For point processes comprising i.i.d. copies of a multiplicative intensity process it is shown that even though log-likelihood functions are unbounded, consistent maximum likelihood estimators of the unknown function in the stochastic intensity can be constructed using the method of sieves. Conditions are given for existence and strong and weak consistency, in the L^1-norm, of suitably defined maximum likelihood estimators. Strong consistency and asymptotic normality are demonstrated for maximum likelihood estimators in submodels parameterized by Euclidean space. A general contiguity theorem for differences of log-likelihood functions is established. Martingale limit theorems are a principal tool throughout.

The Influence Function and an Asymptotic Theory for Estimation in Markov Processes

H. Künsch, *ETH-Zentrum, Zürich, Switzerland*

Recently the author has introduced an influence function IC for estimators (T_n) in autoregressions. From the point of view of asymptotic expansions, IC is characterized by

$$T_n = \theta + \frac{1}{n}\sum_{i=1}^{n-1} \mathrm{IC}(X_i, X_{i+1}, \theta) + o_p(n^{-1/2}) \quad \text{and} \quad E_\theta[\mathrm{IC}(X_i, X_{i+1}, \theta) | X_i] = 0.$$

At the same time IC describes also the bias caused by small contaminations of the model. We extend the notion of the influence function to general Markov processes and consider higher-order terms in the expansion. As an application we investigate the jackknife estimate of variance.

Nonparametric Estimation of the Integrated Intensity of an Unobservable Transition in a Nonhomogeneous Markov Illness–Death Process

J. Mau, *University of Tübingen, FR Germany*

Consider a nonhomogeneous Markov jump process with transient states 0 and 1 and two absorbing states 2 and 3. Assume: (i) The process starts in 0, (ii) the transition $1 \to 0$ is impossible, (iii) an observer cannot distinguish between 0 and 1, but (iv) at the time the process enters 2 or 3, or observation is discontinued on a random basis, the immediately preceding state becomes known. Though the data may be described in terms of counting processes, properties of the proposed estimator cannot be derived by the usual approach via multiplicative intensities and martingales alone. One can show unbiasedness, though only in a certain asymptotic sense, strong consistency and an approximate variance formula. Animal carcinogenicity experiments with serial sacrifice serve as an example of application.

Estimation of Natural Fertility Parameters

S.N. Singh, *Banaras Hindu University, Varanasi, India*

An attempt has been made to estimate some natural fertility parameters. For this purpose, some probability models have been developed. The models take account of several factors which affect natural fertility. Estimates of the parameters have been obtained from the data taken from (i) Demographic Survey of Varanasi (Rural) 1969–70 and (ii) Evaluation of the Impact of Development Activities and Fertility Regulation Programs on Population growth rate in rural areas (1979–82) conducted by the Centre of Population Studies, Banaras Hindu University, Varanasi, India.

A Recursive Parameter Estimation Algorithm for Counting Process Observations

Peter Spreij, *Centre for Mathematics and Computer Science, Amsterdam, The Netherlands*

Suppose that we are given a counting process n_t, $t \geqslant 0$ defined on some complete probability space (Ω, F, P) and adapted to a filtration $\{F_t\}_{t\geqslant 0}$ satisfying the usual conditions. Let n have an intensity process λ, then n admits the (minimal) Doob–Meyer decomposition with respect to the family of σ-algebras $F_t^n = \sigma\{n_s, 0 \leqslant s \leqslant t\}$

$$\mathrm{d}n_t = \hat{\lambda}_t \, \mathrm{d}t + \mathrm{d}m_t$$

where $\hat{\lambda}_t = E(\lambda_t \mid F_t^n)$ and m is an F_t^n-martingale. We consider the case when $\hat{\lambda}_t = p^{\mathrm{T}} \phi_t$ where ϕ is a stochastic process $\Omega \times [0, \infty) \to \mathbb{R}^d$ adapted to $\{F_t^n\}_{t\geqslant 0}$ and $p \in \mathbb{R}^d$ a parameter. The objective is to estimate p on-line with the observations n_t.

We present a recursive maximum likelihood parameter estimation algorithm and prove its convergence to the true parameter value. The proof is based upon the choice of a suitable stochastic Lyapunov function and uses a stochastic approximation type lemma.

Barrier Estimation Using First Passage Time Data From Brownian Motion

G.A. Whitmore, *McGill University, Montreal, Canada*

First-passage-time models for duration data are encountered occasionally in the statistics literature. The inverse Gaussian model, for example, is the first passage time to a linear barrier in Brownian motion. Often, however, the first passage time to a nonlinear barrier in Brownian motion is an appropriate model. This paper describes such a model and considers the two related problems of estimating (1) the functional form of a barrier in a Brownian-motion process and (2) its associated first-passage-time distribution function based on a sample of first-passage-time data. Estimation procedures based on maximum likelihood and empirical transform methods are presented. Several case applications are given as illustrations of the procedures.

2.13. Stochastic models

Perturbations of the Laplacian; Models of Polymers and Quantum Fields
Sergio Albeverio, Jens Erik Fenstad, Raphael Høegh-Krohn and Tom Lindstrøm, *Norges Tekniska Høgskole, Trondheim, Norway*

We study self-adjoint perturbations of the Laplacian $-\Delta$ in $\mathbb{R}^d$ which can formally be expressed as

$$H_\omega = -\Delta - \int_0^1 \lambda(\omega, b(\omega, t)) \cdot \delta(\cdot - b(\omega, t))\, \mathrm{d}t,$$

where b is a Brownian motion and λ is a measurable function. This is the Hamiltonian of a quantum mechanical particle interacting with a Brownian path through a zero range interaction. When the dimension d is less than or equal to 5, we prove that H_ω exists and is a nontrivial perturbation of $-\Delta$ for proper choices of λ. When $d \leq 3$, any bounded function λ will do, but for $d = 4$ or 5, λ must be chosen infinitesimal. The proof uses methods of nonstandard analysis.

The operators H_ω are closely connected to models of polymers, self-avoiding random walks, and quantum field theory. We discuss this relationship and point to some open questions.

An Application of Feller-Dynkin Diffusions to the Estimation of Stock Market Prices
John Brode, *University of Lowell, Lowell, MA, USA*

The widely used Black & Scholes model for securities markets is based on the familiar Itô Lemma. Although this model has become a classic, it performs badly in certain circumstances. The price increments for some stocks fluctuate in a manner not appropriate to the Brownian motion assumed behind the Itô Lemma. The price of these stocks would be better estimated by a stochastic process generated by a non-Gaussian stable distribution. This paper will present a model of a securities market based on Feller's generalization of the semi-group of solutions to the parabolic Cauchy problem.

On the Variation of Air-Pollutants Concentrations
Jan Grandell, *Royal Institute of Technology, Stockholm, Sweden*

In this talk we shall consider mathematical models for the time variation of air-pollutants. The particles are assumed to be emitted into and removed from the atmosphere with intensities described by stationary stochastic processes.

Non-Negative Matrices, Dynamic Programming and a Harvesting Problem
D.R. Grey, *University of Sheffield, England*

We consider the recursive schemes

$$x_{n+1} = \max_{M \in \mathcal{S}} Mx_n$$

and

$$y_{n+1} = e + \beta \max_{M \in \mathcal{S}} My_n$$

where $\mathcal{S}$ is a suitable class of finite non-negative matrices. Results are obtained on the asymptotic behaviour of x_n or y_n and also of the maximising matrices. These models arise naturally in a dynamic programming problem involving Markov transition matrices. The application made is to the problem of optimally harvesting a randomly reproducing population in a limited environment which cannot sustain it if it is allowed unrestricted growth.

Stochastic Models for the Effects of Radiation on Cells
Imke Janssen, *UC Berkeley, USA*

The stochastic models analyzed in this paper consist of two parts. The first part describes the instantaneous initiation of damage (lesions) caused by radiation, the second one, the time-requiring repair of this damage. The initiation is modeled as a compound Poisson process, the repair as a Markovian transition process. It turns out to be important to allow for some interaction between lesions and for the possibility of misrepair which leads to cell lethality or cell transformation. The modeling of the repair process can also be done in a deterministic manner. The two approaches lead to different conclusions, as is best seen via corresponding hazard functions. Experiments show that the stochastic model leads to more reasonable answers than the deterministic approach.

Application of Random Fields Theory to Description of Gas Flow Through-out Porous Layer
Jan R. Koscianowski, *Opole, Poland*
Eugeniusz Szczepankiewicz, *Wrocław, Poland*

The pressure drop variation during dust filtration process performance is one of the main research problems. It was shown that the change of the gas velocity flow, v, on the length of elementary dust layer can be described by forced vibration equation. The solution of this equation is as follows:

$$v = a \exp(-\varepsilon t) \sin k_1 t \tag{1}$$

where: $\varepsilon^2 - k^2 < 0$ and ε and k are constants depend on the exciting force P and damping force Q, and t is a time. Utilizing the Torricelli equation and dependence (1) we obtain:

$$h = \frac{a^2}{4g^2} e^{-\varepsilon' t} \sin^2 k_1 t \tag{2}$$

where h is energy consumption (proportional to the pressure drop) in the dust filtration process. Variables v and h are given in the point $p \in D$. Treating v and h as random fields we can estimate the mean values of v and h in the set D. The random field is introduced as follows. Let (E, A, P) be a probabilistic space. Put in $\mathbb{R}^2$ the random coordinate system $C(e)$. Each point $p \in D$ can be presented in $C(e)$ as a $p = p(e)$. Functions $v(p) = v(p(e))$ and $h(p) = h(p(e))$ are random fields in D. It is possible to assume that these random fields are stationary in wide sense and isotropic or quasi-isotropic, or that they are quasi-stationary in wide sense and isotropic or quasi-isotropic.

Scaling Sums of Pulses, Fractals and the Modelling of Clouds

B. Mandelbrot, *IBM, Yorktown Heights, NY, USA*

This talk introduces a new class of self similar random functions of time or of several variables. Their increments are interdependent and their isosurfaces are fractals. Analysis and computer generated samples show that the fractals can serve to model several distinct types of clouds. Examples of computer generated fractal mountains will also be shown.

On the Derivation of Hydrodynamics from Molecular Dynamics

Michael Mürmann, *Institut für Angewandte Mathematik, Universität Heidelberg, FR Germany*

Some progress has been made recently in understanding how hydrodynamical behaviour arises from molecular dynamics. A few models of idealized dynamics were studied and could be treated rigorously. They indicate the structure in which the transition from microscopic to macroscopic dynamics takes place. We mention some of these models and use them to explain this structure. We stress especially their probabilistic content.

For the Newtonian dynamics, however, where one expects a microscopic derivation of the classical Euler and Navier–Stokes equations to be possible, a rigorous treatment is too difficult at present. We derived a few exact results in this direction (to appear in J. Math. Phys. (1984)).

Stochastic Modelling of Fluidized Bed Reactors and Packed Beds

S.R.K. Prasad, K. Lakshminarayana and Ahmed Basha, *Coimbatore Institute of Technology, Coimbatore, India*

The two phase mathematical models of a fluidized bed that have been proposed so far are all based on Toomey and Johnstone's (1952) theory of discrete fluidized gas solid mixture. In the present paper a unified two phase stochastic model is developed based on the random behaviour of the system. In the proposed model the fluidized bed reactor is described to have two states corresponding to the bubble phase and the particulate phase, together with a reaction state. The model proposed by Gomez Plata and Shuster is deduced starting from the proposed model.

Also in the present paper a model based on Markov chain analysis has been proposed and analysed for the Residence Time Distribution of the fluid phase in packed beds. The equations describing the random walk of the fluid particle through the bed have been formulated. The Residence Time Distribution of the fluid element in the system is discussed in relation with the random walk probabilities. This model is a generalisation of the model proposed by Raghuraman and Mohan (1975).

Statistical Analysis of the Variation of the Oxygen Concentration in a River by Means of Diffusion Processes

Michael Sørensen, *University of Aarhus, Denmark*

A continuity equation for the concentration of dissolved oxygen in a river is converted to a stochastic differential equation by addition of a white noise term. This diffusion model is used in the statistical analysis of a time series of oxygen concentration, incident light and water temperature. A check of the model indicates that the measurement error is large compared to the system error, and it is discussed how to take the measurement error into account in the statistical analysis.