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Reliability analysis of slope Stability using stochastic finite element method

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Abstract

This paper deals with the reliability analysis of slope stability with spatial variability of soil properties. Supposing an elastic soil behavior until yield criteria, a stresses mobilized along slip surface are evaluated to formulate the performance function. The results of SFEM (Stochastic Finite Element Method) and the limit equilibrium method, such as Bishops simplified method are analyzed to check their efficiencies and accuracies. To look for critical slip surface, an optimization strategy is performed, also a sensitivity studies are carried out in order to investigate the influence of random field parameters used to model the spatial variability of soil properties.

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Keywords: Slope stability; stochastic finite element method; limit equilibrium method

1. Introduction

The uncertainty in the calculation of slope stability results from a number of sources in particular; the geotechnical properties of soils have several components of randomness. The values of parameter at any point are uncertainly due to the spatial variability of soil; in addition they are estimated using a finite number of trials. The analytical models adopted for slope stability problems are simplifications of more complex mathematic problems. Recently, many studies of probabilistic slope stability have been presented, which takes into account the uncertainties and randomness in soil properties. (Yang[1], SU Yong-huo and al[2];Tan[3]; Suchomel [4];Tan and Wang[5]).

The objective of a deterministic slope stability analysis is to estimate the minimum safety factor which corresponds to a critical slip surface. For simplicity we generally opt for computing a circular surface. However, the slip surface can have a more complicated form for a slope with a heterogeneous soil l.

The search procedure of the slip surface uses an algorithm with a limited finite number of slip surfaces.

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In practice, to find a circular critical slip surface we precede as follows, from a given centre, we give a different radius and are looking for the one who gives the minimum safety factor, repeat the same operation for other centres according to a predetermined grid. Since the limit equilibrium methods use different approximations, the location of critical slip surface may change slightly. With the evolution of computation tools , the optimization techniques for finding the critical slip surface were developed as dynamic programming (Baker[6]), Unconstrained minimization technique sequential (Basudhar [7]), simplex reflection technique (Nguyen[8]) and more recently, Monte Carlo technique

In all the algorithms listed above, the identification of deterministic surface associated with a minimum safety factor is formulated as an optimization problem. By optimizing the various geometric variables of the slip surface, the critical slip surface can be obtained. The determination of the probabilistic critical surface may be driving by the same way, with the consideration of the random character of soil properties.

In this work the probabilistic approach proposed considers the spatial variability of soil mechanical parameters and the Yong's modulus. By applying the stochastic finite element method SSFEM, we determine the stresses along the slip surface necessary for the formulation of limit state function. The mechanical model of Bishop and Spenser were adopted for evaluating the reliability index and the determination of the critical probabilistic slip surface. An optimization procedure for finding critical probabilistic surface is proposed and is evaluated by numerical examples. It uses the technique developed by (Celestino and Duncan [9]), which is an research of the slip surface associated with a minimum safety factor. This procedure is modified to evaluate a maximum probability of ruin. A comparison between the different approaches used to this reliability analysis is presented. The probabilistic calculation was performed using the program FERUM which was developed by (Haukaas and Der Kiuregian [10]) and with a direct coupling through a programation of the proposed algorithm written in Matlab to analyze the stability of a slope

2. Probabilistic approaches

The probabilistic approach allowed measuring the performance of a slope stability failure probability or reliability index. It considers that soil properties are modelled by variables or random fields. By choosing as failure criterion the safety margin, the performance function is defined by the following expression:

$$G = F_S - 1 \qquad (1)$$

The safety factor can be evaluated by limit equilibrium methods or by applying the finite element method is a function of random variable

2.1. Limit equilibrium method

The limit equilibrium method uses different assumptions and differing in the number of the equations satisfying equilibrium, among used in this work: The simplified Bishop method was developed by Bishop. This procedure uses the method of slices to find the factor of safety for the soil mass. It is based on the assumptions that the forces on the sides of the slice are horizontal and the failure occurred by rotation of soil mass on a circular slip surface. Also, the normal force is assumed to act at the centre of the base of slice. By using the Mohr Coulomb criteria and the definition of the factor of safety. The Spencer‘s Method based on the assumption that the interslice forces are parallel. It presented originally for the analysis of circular slip surfaces. By considering overall force equilibrium and overall moment, two values of factor of safety respectively Ff and Fm are obtained

Note that the term of Fs is present in both sides of equilibrium equation. Hence, the algorithm proposed by (SU Yong –hua and al [2]) based on checking point and iteration method is used to evaluate the probability of failure.

The expression of the factor of safety obtained by Bishop‘s simplified method and Spencer‘s Method is provided in the literature.

2.2. Finite element method

The finite element method provides a distribution of stresses and displacements. It represents available tools that satisfy all condition necessary for the evaluation of the solution of slope stability problem. Zheng [11]have used this procedure.

The mean factor of safety is defined as a weighted average of shear strength over shear stress

$$F_S = \frac{\sum_1^n (c+\sigma_n \tan(\varphi))\Delta l_i}{\sum_1^n \tau_n \Delta l_i} \quad (2)$$

Where:

n:the number of elements on a slip surface, c: cohesion,φ is the friction angle, σ_n is the normal stress andτ_n is the shear stress,Δl_i is length of the slip surface of the ith element. For an Δl_i inclined at an angle α to the horizontal, the stresses are calculated from stress components ($\sigma_x, \sigma_y, \sigma_{xy}$)by the following equations:

$$\tau = 0.5(\sigma_y - \sigma_x)\sin 2\alpha + \sigma_{xy}\cos 2\alpha \quad (3)$$

$$\sigma_n = \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha - \sigma_{xy}\sin 2\alpha \quad (4)$$

In the case of the stochastic finite element method SSFEM (Ganem and Spanos[12]based on a decomposition of random fields to an order M using a discretization of Karhunem-Loeve and another decomposition of vector nodal displacement solution of the problem based on polynomial chaos. This vector is truncated by a P term. P is given by the equation:

$$P = \frac{(M+p)!}{M!p!} \quad (5)$$

The stresses σ are obtained from stochastic nodal displacement.

In considering the truncation of the decomposition of the Karhunem-Loève and the nodal displacement vector, the convergence of the SSFEM method depends of the orders of expansion M and p
To determine the probability of failure several approaches can be adopted as First Order relibility Method FORM, Second Order Reliability Methods SORM and Monte Carlo.
The method MVFOSM (mean value first order second moment reliability index) is a convenient to estimate the reliability index. The reliability index is given by:

$$\beta_{MVFOSM} = \frac{G(\mu_X)}{\sqrt{(\nabla_X G)^T_{X=\mu_X} \Sigma_{XX} (\nabla_X G)^T_{X=\mu_X}}} \quad (6)$$

$\nabla_X G$ and $\sum_{XX}$ are respectively the gradient of the performance function and covariance matrix

The probability of failure is estimated by the following equation:

$$P_f = \Phi(-\beta_{MVFOSM}) \quad (7)$$

Where Φ is standard normal cumulative function.

3. Optimization procedure for evaluating probability of failure

The Optimization of the research algorithm of critical surface associated with the lowest index of reliability (or maximum probability of failure) can be determined by the following steps:

a. From the displacement and contour of stress, we can identify an initial critical surface of failure. A deterministic program of slope stability analysis for the evaluation of the safety factor is modified in order to determine the reliability index or probability of failure.

b. Determination of a critical circular surface by using the grid method. It represents an initial slip surface for the search for a non-circular slip surface in reliability context. The technique developed by (Celestino and Duncan, [9]) to looking for critical slip surface is embedded in an algorithm for calculating the probability of failure or reliability index

4. Evaluation of the proposed procedure

In order to verify the effectiveness of the procedure adopted and to find the effect of the variability of soil properties considered as random, numerical examples and a comparative study are presented.
Consider a homogeneous slope, the Young's modulus, cohesion and friction angle are modelled by a Gaussian random fields. The parameters values are listed in Table 1

Table.1 Input soil properties

Soil Properties	c (kPa)	$\varphi(°)$	γ(KN/m^3)	E(MPa)	ν
Mean (μ)	7	20	19	20	0.3
Coefficient of variation (cov_X)	0.10	0.10	-	0.10	-
Correlation length (m) [l_x,l_y]	[60,6]	[60,6]	-	[60,6]	-

c: Cohesion. φ : Friction angle. γ : Unit weight. E: Young's modulus. ν : Poisson's ratio

For this slope stability analysis using SSFEM method, the finite element model considered a mesh composed of 1081nœud. The elements used are linear quadrilaterals QUAD4. The boundary conditions are: the bottom of the slop is fixed also the horizontal displacement on the left and the right sides are fixed.

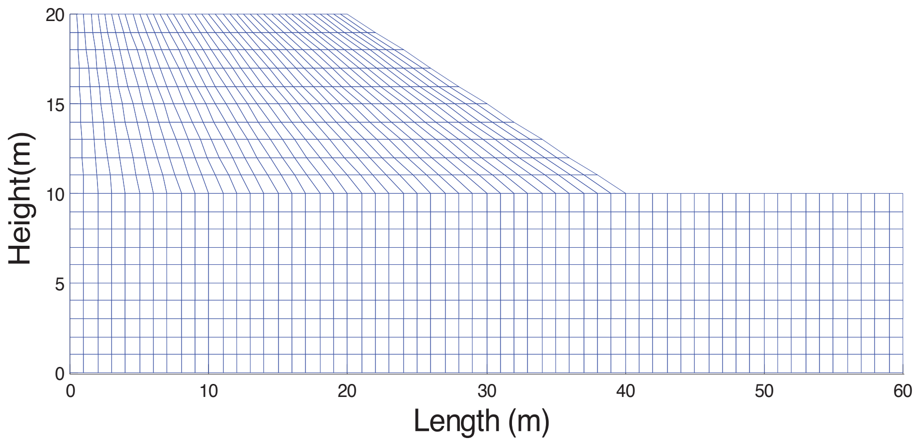

Fig.1. Meshing of the slope

In this example, we consider that the random fields have an exponential autocorrelation function. The soil behaviour is assumed linear. The shear stress is calculated using the criterion of Mohr-Coulomb. The autocorrelation function is given by the following expression:

$$\rho(\underline{x},\underline{\acute{x}}) = \exp\left(-\frac{|x1-x2|}{l_x} - \frac{|y1-y2|}{l_y}\right) \qquad (8)$$

Where, l_x , l_y:is the correlations lengths of a 2D domain, $\underline{x}$, $\underline{\acute{x}}$ and the location of the soil mass

4.1. Determination of the reliability index and the critical probabilistic surface

The results of calculation the reliability index and safety factors obtained by different approaches are listed in Table.1

Table1: reliability indexes and safety factors for both probabilistic and deterministic slip surfaces evaluated by the different methods

	Probabilistic slip surface				Deterministic slip surface			
	SSFEM (p=2, M=4)	SSFEM (p=3, M=4)	Bishop	Spenser	SSFEM (p=2, M=4)	SSFEM (p=3, M=4)	Bishop	Spenser
β	3.783	3.785	4.312	4.489	2.759	2.758	2.768	2.771
F	1.767	1.765	1.658	1.651	1.871	1.869	1.874	1.881

Where β the reliability index and F the factor of safety
The results of calculating the reliability index and safety factors obtained by different approaches are listed in Table 1.
From table1, It can be seen that the values obtained by the Bishop's and Spenser's model are comparable whereas compared to the values obtained by SSFEM, a remarkable difference is observed

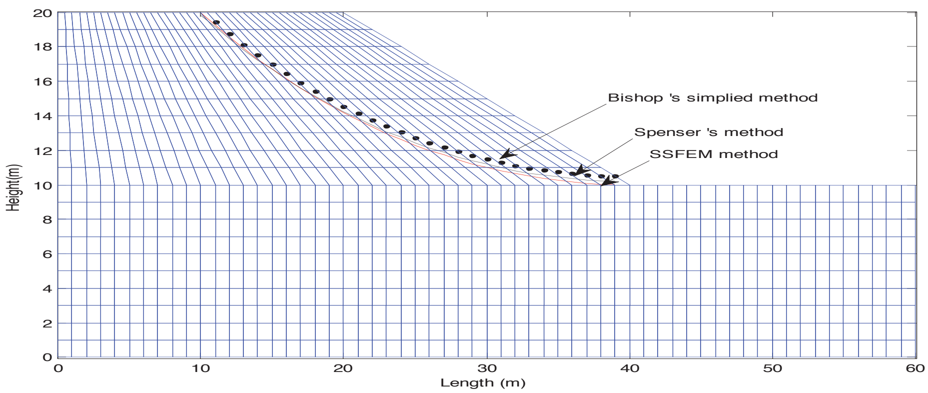

Fig1: Critical probabilistic slip surface obtained by the different methods

The corresponding critical probabilistic slip surface evaluated by the different approaches are presented in Fig.1.The small difference between the values of the minimum reliability index presented in table1 is confirmed by the positions of the critical probabilistic slip surface. This result demonstrate the effectiveness of the developed method

4.2. Effect of the input random field

4.2. 1.Effect of the coefficient of variation

The minimum reliability indexes are evaluated by the proposed approaches for different coefficient of variation of the random fields. The results are reported in Table 2

Table2: The minimum reliability indexes for different coefficient of variation and correlation length=[60,6]

	0.1	0.20	0.25	0.3
SSFEM(p=3, M=4)	3.848	3.756	3.679	3.586
Bishop	4.227	4.205	4.121	4.023
Spenser	4.223	4.167	4.107	4.015

4.2. 2.Effect of the correlation length

The reliability indexes are evaluated by the proposed approaches for different correlation length of the random fields. The results are reported in Table 3

Table3: The minimum reliability indexes for different correlation length and coefficient of variation=0.1

	Correlation length			
	[120,12]	[240,24]	[480,48]	[960,96]
SSFEM (p=3, M=4)	3.945	3.845	3.764.	3.734
Bishop	4.345.	4.231	4.125	3.946
Spenser	4.325	3.221	4.113	3.926

From Table2 and, Table3 we can find that the results of the three methods are similar and the reliability index decrease when the coefficient of variation or the correlation length increase. These results demonstrate the accuracy and feasibility of the adopted procedure

5. Conclusions

In this work, a solution of the procedure for the analysis of slope stability problem is presented, in which it was integrated an optimization technique to identify a critical probabilistic slip surface. We considered the spatial variability of soil strength parameters and Young's modulus necessary for the determination of stress field mobilized along the slip surface. These soil properties are modelled using the random field's theory. In this reliability study by the stochastic finite element, soil mass is assumed elastic until the verification of Mohr-Coulomb criterion. A comparison between the values of the reliability index and the position of critical surface obtained by the probabilistic methods using limit equilibrium: Spenser and Bishop and SSFEM are presented. The results of the proposed approaches showed a small difference between the values of reliability index and the position of critical probabilistic slip surface.

The procedure established to search the probabilistic slip surface can be generalized to study the stability of a heterogeneous slope and taking into account the effects of water and loading.

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