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Singular stress field near the edge of interface of bonded dissimilar materials with an interlayer

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Abstract

For bonded dissimilar materials, the free-edge stress singularity usually prevails near the intersection of the free-surface and the interface. When two materials are bonded by using an adhesive, an interlayer develops between the two bonded materials. When a ceramic and a metal are bonded, the residual stress develops because of difference in the coefficient of thermal expansion. An interlayer may be inserted between the two materials to defuse the residual stress. Stress field near the intersection of the interface and free-surface in the presence of the interlayer is then very important for evaluating the strength of bonded dissimilar materials.

In this study, stress distributions on the interface of bonded dissimilar materials with an interlayer were calculated by using the boundary element method to investigate the effect of the interlayer on the stress distribution. The relation between the free-edge singular stress fields of bonded dissimilar materials with and without an interlayer was investigated numerically. It was found that the influence of the interlayer on the stress distributions was confined within a small area of the order of interlayer thickness around the intersection of the interface and the free-surface when the interlayer was very thin. The stress distribution near the intersection of the interface and the free-surface was controlled by the free-edge stress singularity of the bonded dissimilar materials without the interlayer. In this case, the interlayer can be called free-edge singularity-controlled interlayer. If a stress distribution on the interface is known for one thickness of an interlayer h, stress distributions on the interface for other values of h can be estimated.

Keywords: Bonded dissimilar materials; Interface; Free-edge stress singularity; Boundary element method

1. Introduction

In recent years, bonded dissimilar materials and composite material have been widely used in many engineering structures. With an increase in use of bonded dissimilar materials or composite materials, demands for strength evaluation of bonded dissimilar materials increase (Toyoda, 1991; Yuuki et al., 1993).

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When two materials are bonded, the free-edge stress singularity usually develops at the intersection of the interface and the free-surface. In the free-edge singular stress field, the stress distribution is expressed by the following equation:

$$\sigma = Kr^{-p}. \tag{1}$$

Here, p is the exponent of the free-edge stress singularity, K is the intensity of the free-edge singular stress field, and r is the distance from the intersection of the interface and the free-surface. The existence of free-edge stress singularity is very important for the evaluation of the strength of the bonded dissimilar materials. Many researchers have studied the free-edge stress singularity (Bogy, 1968, 1970; Chen, 1996; Dundurs, 1969). It was pointed out that the free-edge stress singularity disappeared for certain combinations of material properties and wedge angles (Kubo and Ohji, 1991; Ohji et al., 1992).

When two dissimilar materials are bonded by using an adhesive, an interlayer develops between two bonded materials. When a ceramic is bonded to a metal, the residual stress develops because of the difference in the coefficient of the thermal expansion. An interlayer may be inserted between a ceramic and a metal to defuse the residual stress. The stress field near the intersection of the interface and the free-surface in the presence of an interlayer is then very important for evaluating the strength of bonded dissimilar materials. Munz et al. (1995) studied the effect of the interlayer under thermal stress loadings. They proposed a relation between the thickness of the interlayer and the intensity of free-edge stress singularity.

In this study, stress distributions on the interface of bonded dissimilar materials with an interlayer subjected to a remote mechanical loading are calculated by using the boundary element method (BEM). The effects of interlayer on the stress distribution on the interface or the free-edge stress singularity are discussed. The relation between the free-edge stress singularity of bonded dissimilar materials with and without an interlayer is investigated numerically and theoretically.

2. Model of plate used for BEM analyses

Fig. 1 shows a model of a plate used for boundary element analyses. This plate consists of three materials, and material 3 is inserted between materials 1 and 2. The widths of the materials 1 and 2 are set to be $2.0W$ and the heights of each material are set to be $1.5W$. The thickness of material 3 is denoted by h. Boundary element analyses were made with changing the value of h. It was assumed that the displacement in the y-direction on the bottom surface was set to be 0, and the uniform tensile stress σ_0 in the y-direction was applied on the top surface of plate. All analyses were made under the plane strain condition.

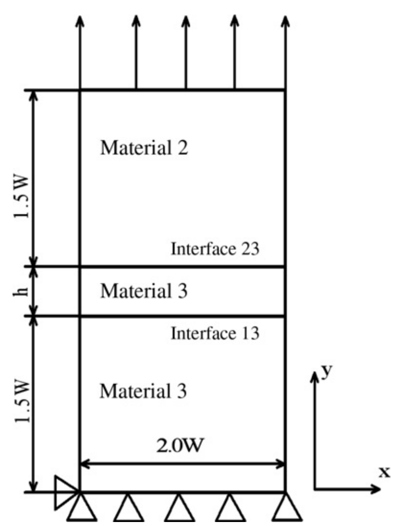

Fig. 1. Model used for BEM analyses.

Table 1
Material properties used for BEM analyses

Group		Material 1	Material 2	Material 3
1	Young's modulus (GPa)	206.0	70.3	4.93
	Poisson's ratio	0.30	0.345	0.33
2	Young's modulus (GPa)	100.0	1.0	20.0
	Poisson's ratio	0.30	0.30	0.30

Table 2
Theoretical values of exponent of free-edge stress singularity

Group	p_{12}	p_{13}	p_{23}
1	0.09336	0.28226	0.23925
2	0.26770	0.05978	0.23915

Young's modulus and Poisson's ratio used for BEM analyses are shown in Table 1. Exponent of the free-edge stress singularity for the combination of material i and j is denoted by p_{ij}. Theoretical values of p_{ij} calculated using the characteristic equation deduced in terms of the Airy stress function (Kubo and Ohji, 1991; Ohji et al., 1992) are shown in Table 2.

In this paper, the interface between the materials 1 and 3 is denoted by interface 13, and the interface between the materials 2 and 3 is denoted by interface 23. As is seen from Table 2, we compared the stress distributions on the interface for two groups of materials. In the first group (Group 1) the adhesive has the effect of increasing the order of the singularity, i.e. $p_{12} < p_{13}, p_{23}$, while in the second group (Group 2) the adhesive has the effect of decreasing the order of the singularity, i.e. $p_{12} > p_{13}, p_{23}$.

3. Stress distributions on interface

3.1. Case of $0 < p_{12} < p_{13}$ and $0 < p_{12} < p_{23}$

For the Group 1 in Table 1, the distributions of normal stress σ_{yy} on the interface 13 and the interface 23 are shown in Figs. 2(a) and (b), respectively. The abscissas of the figures show the distance r from the intersection of the interface and the free-surface normalized by W. The ordinates of the figures show the normal stress σ_{yy} on the interface normalized by σ_0. Thickness of an interlayer h is set to be $0.002W$, $0.005W$ and

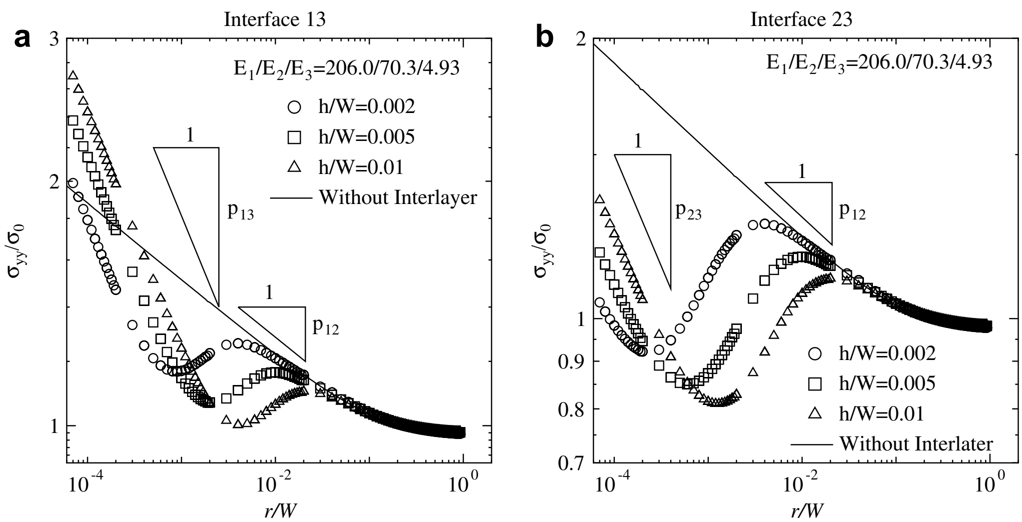

Fig. 2. Stress distributions on interfaces 13 and 23 for pair 1. (a) Distribution on interface 13. (b) Distribution on interface 23.

$0.01\,W$. The solid lines in the figures show the stress distribution on the interface of bonded dissimilar materials without the interlayer.

It is found from Fig. 2 that the stress on the interface of the bonded materials without the interlayer increases linearly on a log–log diagram with decreasing r in the region where r is smaller than $0.01\,W$. It is therefore seen that stress distributions on the interface without the interlayer have the r^{-p}-type singularity in the vicinity of the intersection of the interface and the free-surface. In Fig. 2(a), when material 3 is inserted, the stress on the interface 13 increases linearly with decreasing r in the region where r is smaller than $0.001\,W$, and the r^{-p}-type singularity prevails. The exponents of free-edge stress singularity, which are determined from the stress distributions on the interface in the vicinity of the intersection of the interface and the free-surface using the least squares method, agree well with the theoretical value p_{13}. The region, where the free-edge stress singularity is predominant, becomes smaller as the interlayer thickness h becomes smaller. On the other hand, the stress distributions in the region, where r is larger than h, agree well with the stress distribution without an interlayer. From this result, it is seen that the influence of the interlayer on the stress distribution is confined in the region where r is smaller than h. In Fig. 2(b), stress distributions on the interface 23 also have the r^{-p}-type singularity in the vicinity of the intersection of the interface and the free-surface. The exponents of the singularity of stress distributions on interface 23 agree well with the theoretical value p_{23}.

When the thickness of the interlayer is small, the stress distribution in the vicinity of the intersection of the interface and the free-surface is controlled by the free-edge stress singularity of the bonded dissimilar materials without an interlayer. In this case, the interlayer can be called free-edge singularity-controlled interlayer. In the followings, the normalization of the stress distributions on the interface with an interlayer by the free-edge singular stress field without an interlayer is discussed. The stress distribution on the interface of the bonded materials without an interlayer is expressed by

$$\sigma_{yy} = K_{y0} r^{-p_{12}}. \tag{2}$$

Therefore, the value of the stress is equal to $K_{y0}h^{-p_{12}}$ at the location $r = h$. Stress distributions on the interface with an interlayer normalized by $K_{y0}h^{-p_{12}}$ are plotted against r/h in Fig. 3. It is seen that stress distribution normalized by $K_{y0}h^{-p_{12}}$ is independent of h in the region where $r/h < 1.0$. When the interlayer is very thin, the normalized stress $\sigma_{yy}/(K_{y0}h^{-p_{12}})$ can be expressed by the following equation.

$$\frac{\sigma_{yy}}{K_{y0}h^{-p_{12}}} = f_{13}\left(\frac{r}{h}\right). \tag{3}$$

Here $f_{13}(r/h)$ denotes a function of r/h.

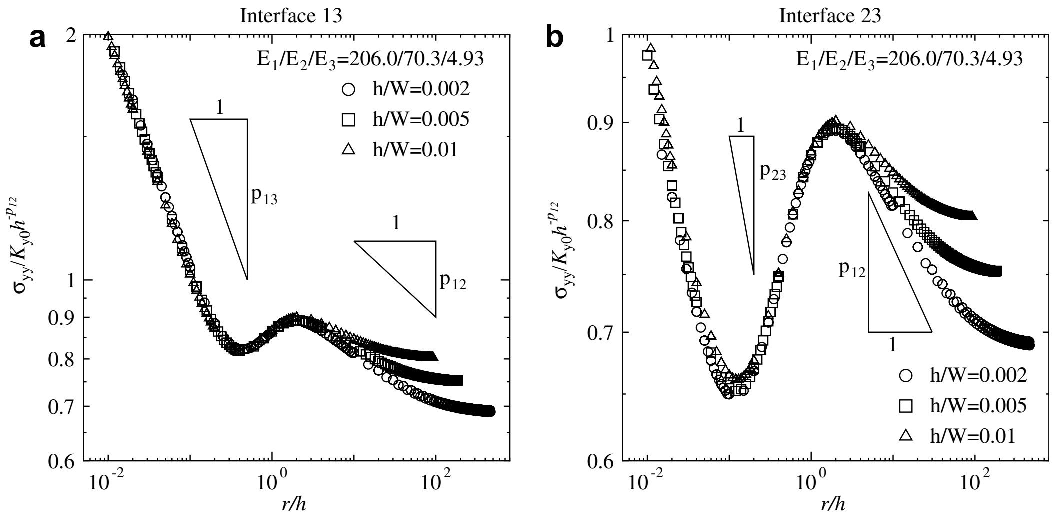

Fig. 3. Normalized stress distributions on interfaces 13 and 23 for pair 1. (a) Distribution on interface 13. (b) Distribution on interface 23.

In the region where $r/h < 0.1$, the stress distribution is expressed by

$$\frac{\sigma_{yy}}{K_{y0}h^{-p_{12}}} = C_{y13}\left(\frac{r}{h}\right)^{-p_{13}}. \tag{4}$$

On the other hand, the free-edge singular stress field is expressed by the following equation.

$$\sigma_{yy} = K_{y13}r^{-p_{13}}. \tag{5}$$

Here, K_{y13} is the intensity of singular stress field on the interface 13 when an interlayer is inserted. On the interface 23, similar equations are obtained by replacing the subscript 13 by 23.

By combining Eqs. (4) and (5), K_{y13} is expressed by the following equation.

$$K_{y13} = K_{y0}C_{y13}h^{p_{13}-p_{12}}. \tag{6}$$

From this equation, the intensity of the free-edge singular stress field K_{y13} becomes smaller as the thickness of the interlayer h becomes smaller when $p_{13} > p_{12}$.

The intensity of the singular stress field K_{y23} can be derived in the same way, and is given as,

$$K_{y23} = K_{y0}C_{y23}h^{p_{23}-p_{12}}. \tag{7}$$

Munz et al. (1995) proposed a relation between the intensity of the singularity and the thickness of the interlayer written as follows:

$$\frac{K_{yi3}}{K_{y12}} = A + B\log\left(\frac{h}{W}\right) \qquad \left(\frac{h}{L} \leqslant 0.01\right) \tag{8}$$

$$= C + D\log\left(\frac{h}{W}\right) + E\left[\log\left(\frac{h}{W}\right)\right]^2 + F\left[\log\left(\frac{h}{W}\right)\right]^3 + G\left[\log\left(\frac{h}{W}\right)\right]^4 \qquad \left(0.01 \leqslant \frac{h}{L} \leqslant 2\right) \tag{9}$$

$$= 1 \qquad \left(2 \leqslant \frac{h}{L}\right). \tag{10}$$

Parameters A, B, C, D, E, F and G dependent on the material properties were determined from finite element analyses. The results obtained by Munz et al. are different from those of this paper. It is due to the reason that the work of Munz et al. treated the case under the thermal stress loadings where the constant stress term was not negligible.

3.2. Case of $0 < p_{13} < p_{12}$ and $0 < p_{23} < p_{12}$

For the Group 2 in Table 1, the stress distributions on the interface 13 and the interface 23 are shown in Figs. 4(a) and (b), respectively. The abscissas of the figures show the distance r from the intersection of the interface and the free-surface normalized by W. The ordinates of the figures show the normal stress σ_{yy} on the interface normalized by σ_0. The thickness of an interlayer h is set to be $0.002W$, $0.005W$ and $0.01W$. The solid lines in the figures show the stress distribution on the interface of bonded dissimilar materials without the interlayer.

It is seen in Fig. 4(a) that the stress on the interface 13 increases linearly with decreasing r in the region where r is smaller than $0.001W$ on a log–log diagram, and the r^{-p}-type singularity prevails. The slope of the stress distribution is smaller than that of the stress distribution without an interlayer; thus the exponent of the free-edge stress singularity p_{13} is smaller than p_{12}, as predicted theoretically. There is a region where the stresses are higher than that of the no-interlayer case. This can be seen for the pair of material properties like the Group 2.

On the other hand, the stress distributions on the interface 23 are similar to the stress distribution without an interlayer shown as a solid line in Fig. 4(b), because the exponent of the singularity p_{23} is 0.23915 and nearly equal to p_{12}.

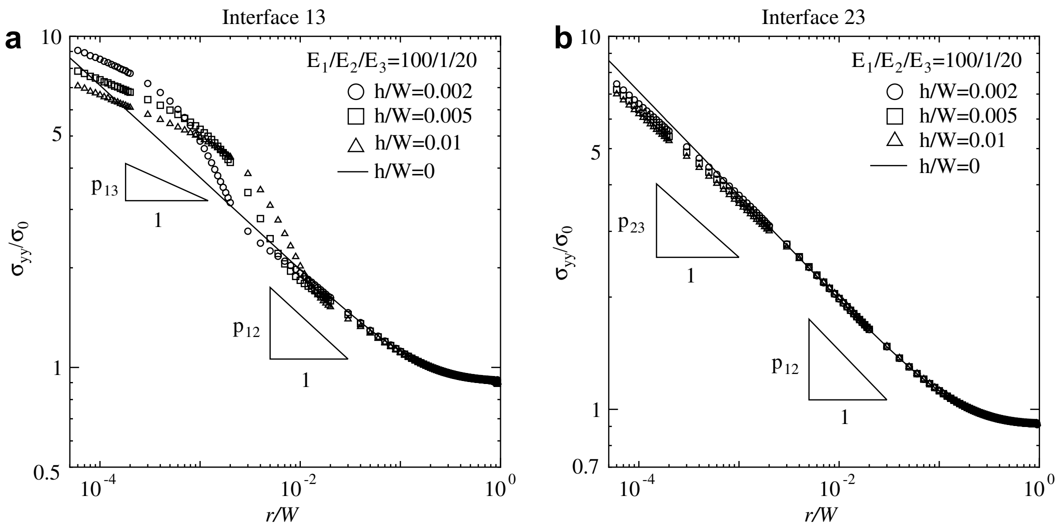

Fig. 4. Stress distributions on interfaces 13 and 23 for pair 2. (a) Distribution on interface 13. (b) Distribution on interface 23.

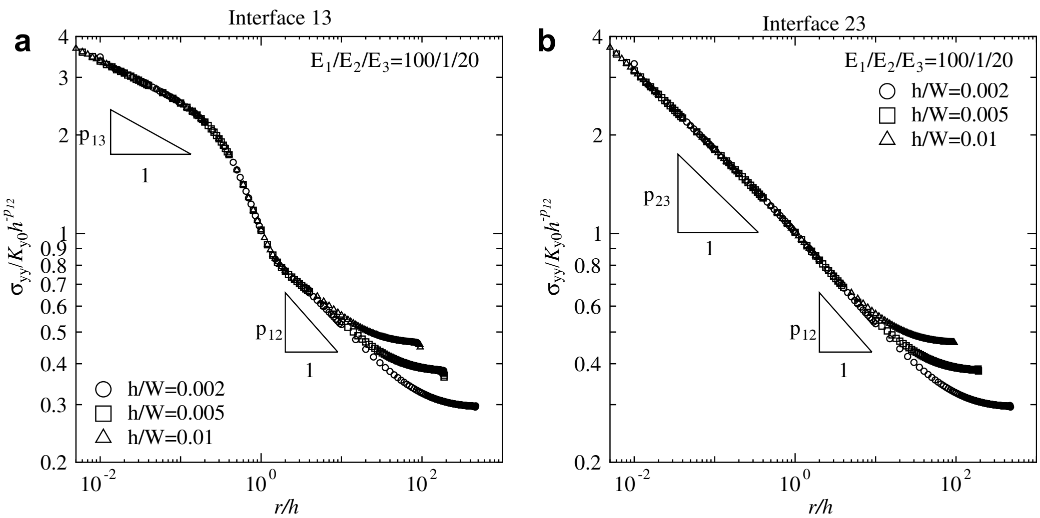

Fig. 5. Normalized stress distributions on interfaces 13 and 23 for pair 2. (a) Distribution on interface 13. (b) Distribution on interface 23.

The stress distributions normalized by $K_{y0}h^{-p_{12}}$ are plotted against r/h in Fig. 5. Figs. 5(a) and (b) show the distributions on the interface 13 and the interface 23, respectively. From these figures, the distribution of normalized stress $\sigma_{yy}/(K_{y0}h^{-p_{12}})$ is expressed by a function of r/h as mentioned in the previous section.

If the stress distribution on the interface is known for a value h of the interlayer thickness, the stress distribution on the interface for the other value of h can be estimated by using Eq. (6) or Eq. (7). In the region where $r \gg h$, stress distribution with an interlayer is similar to that without an interlayer.

4. Conclusions

The stress distributions in bonded dissimilar materials with an interlayer were investigated using the boundary element method with special emphasis on the effect of the interlayer on the stress distribution on the interface. The following conclusions are obtained.

The stress distribution in the region where r is larger than the interlayer thickness h is similar to the stress distribution without the interlayer. The influence of the interlayer is confined in the region of the order of the interlayer thickness around the intersection of the interface and the free-surface.

The stress distribution near the intersection of the interface and the free-surface is controlled by the free-edge stress singularity without the interlayer. If the stress distribution on the interface is known for one thickness of an interlayer h, the stress distribution on the interface for the other values of h can be estimated.

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