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Experimental and numerical investigation of fracture in fillet welds by cross joint specimens

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Abstract

The load capacity of fillet welds is investigated experimentally and numerically in cross joint specimens with the objective of calibrating multiple failure criteria for the ductile fracture of weld metal. The cross joint specimens are composed of mild shipbuilding steel with plate thicknesses of 20 mm and 30 mm respectively. The fillet welds are manually produced by flux-cored arc welding, using the filler metal Elgacore MXX100. To achieve different stress states in the weld joints, forces in two different directions and moments about two different axes are applied separately on various specimens. A uniquely designed specimen is employed for each load scenario. For the numerical investigations, an appropriate discretization of the weld joints is established by taking into account the distribution of the metallographic structure with the weld metal and the heat-affected zone. The distribution of the different materials is determined by a macrosection of a weld joint. The material behavior, in terms of true stress-strain curves for the weld metal and the material of the heat-affected zone, is identified by hardness measurements. With the established discretization of the weld joints and true stress-strain curves of the different materials, the experimentally determined force-displacement curves are reproduced in finite element analyses. Furthermore, the Rice and Tracey (1969) failure criterion and the Gurson (1977) damage model are successfully calibrated for the weld metal.

1. Introduction

Collisions between ships are a danger to human life and may lead to environmental pollution through the loss of cargo and the spillage of oil. Active measures such as electronic navigation aids help improve maritime traffic.
safety. However, collisions still occur in frequently used waters and coastal areas, due to either human error or technical failure. Passive measures are necessary in order to increase the collision resistance of double hull structures and to ensure the structural integrity of ships. The collision resistance can be increased through alternative stiffening systems (e.g. Schöttelndreyer et al. (2013), Ringsberg and Hogström (2013) and Naar et al. (2002)). Plate-strengthened stiffeners, developed by Röhr and Heyer (2007), are one such alternative stiffening system. Hereinafter, the German acronym PVPS, which stands for plattenverstärkte Profilsteifen, is used to refer to the alternative stiffening system. It is characterized by plating with a trapezoidal cross section on the bulb profiles of the outer shell (Fig. 1). The plating or the PVPS are connected to the bulb profiles by weld joints. The load capacity of these weld joints is important for the mode of operation of the alternative stiffening system in the case of a collision. Only if the load capacity of these weld joints is maintained at large plastic deformations of the double hull structure and at large penetration depth of the striking ship the alternative stiffening system PVPS will lead to a higher collision resistance compared to a conventional double hull. The fracture behavior of the weld joints between PVPS and bulb profiles under collision load has been numerically analyzed using the Rice and Tracey (1969) failure criterion as well as the Gurson (1977) damage model and the results of these analyses will be published soon. The present paper describes solely the experimental and numerical investigations of the calibration procedure for the weld metal through the Rice and Tracey failure criterion and the Gurson damage model.

2. Experimental setup and specimens

The ductile fracture of fillet weld joints is investigated by applying various load types onto four different cross joint specimens until failure occurs. Tensile forces in two different directions and bending moments about two different axes in relation to the fillet welds are applied on the specimens. The specimens are made of Grade A shipbuilding steel with 20 mm and 30 mm plate thickness. The weld joints are manually manufactured through gas metal arc welding with the filler metal Elgacore MXX100. The four different experiments are labeled as K1, K2, K3, and K4 respectively and are carried out on specifically designed specimens. The experiments K2 and K4 are performed three times, while K1 is done twice and K3 only once.

Fig. 1. Section of a double hull structure with outer shell, bulb profiles, stringer decks and the alternative stiffening system PVPS
In the experiment K1, the specimen is loaded with a tensile force crosswise to the weld joints (Fig. 2a). The width of the specimen, which is equal to the length of the fillet welds, is 58 mm. The horizontal plates are 30 mm thick while the vertical ones are 20 mm thick.

The specimens used in the experiment K2 are composed of two vertical plates, each with a thickness of 20 mm. These two plates are clamped into the servohydraulic testing machine and connected by two additional plates with a thickness of 30 mm each (Fig. 2b). The four plates of the specimen are welded with eight fillet welds, each 80 mm long. In the experiments K1 and K2, the displacement is measured by the servohydraulic testing machine as well as by an extensometer. The extensometer has a gauge length of 120 mm and is centered vertically on the specimen (Fig 3a). Both experiments are carried out with a load velocity of 0.1 mm/s.

In experiment K3, the load of the specimen is applied through a four-point bending fixture, whereby the specimen has a horizontal position (Fig. 2c and 3b). The specimen consists of three plates, each with a thickness of 30 mm and an overall length of 980 mm.
To achieve failure in the weld joints through the applied bending moment, the fillet welds need to be shortened from their original length of 85 mm to 55 mm. All four weld joints are shortened about 15 mm by milling both sides (Fig. 3c).

The experiment K4 is carried out with specimens similar to those of experiment K3. The fillet welds run across the entire breadth of the specimens, i.e. 85 mm, and have no shortened ends. Furthermore, unlike experiment K3, the specimens in experiment K4 have an upright position in the four-point bending fixture and are turned about 90° (Fig. 2d).

The moment in the four-point bending fixture is applied to the specimens through cylindrical bearings each with a diameter of 38 mm, as shown in Fig. 3b. The bearings are characterized by a smooth surface that keeps the friction between the bending fixture and the specimen small. The central bearings are separated by a distance of 250 mm and the outer bearings by a distance of 700 mm. The four-point bending fixture is built into a stiff frame. Due to the stiffness, the deformation of the frame during an experiment is assumed to be negligible. The displacement of the bending fixture is generated by a servohydraulic testing cylinder and is applied to the specimens through the central bearings with a load velocity of 0.1 mm/s. The displacement of the central bearings is measured by four displacement transducers.

3. Experimental results

The force-displacement curves of the four experiments are crucial results of the investigations. In addition, the hardness of the weld joints of the cross joint specimens is measured multiple times to obtain the material behavior in terms of true stress-strain relations of the weld metal and the heat-affected zone.

3.1. Force-displacement curves of the cross joint specimens

Both of the force-displacement curves resulting from the experiment K1 have a nearly linear slope of force, going up to 300 kN and then turning into a plateau after a displacement of 0.5 mm (Fig. 4a). The test K1 c yields a maximum reaction force of 345 kN and the fillet welds lose their entire load capacity and fail at a displacement of 1.6 mm. The course of the curve resulting from K1 a is similar to the force-displacement curve of K1 c.

The three force-displacement curves resulting from the experiment K2 are nearly identical and have a maximum reaction force of 460 kN (Fig. 4b). The weld joints fail at displacements ranging from 2.37 mm to 2.55 mm. The displacements during the experiments K1 and K2 shown in Fig. 4 are acquired by the extensometer.

Fig. 4. (a) Experimentally determined force-displacement curves along with a force-displacement curve of a finite element simulation applying the Rice and Tracey failure criterion of experiment (a) K1 and (b) K2
The force-displacement curve resulting from the experiment K3 is characterized by a nearly linear course going up to 5 mm and from 9 mm to 16 mm (Fig. 5a). The maximum reaction force of 43 kN is reached at 16.8 mm displacement and failure subsequently occurs in one of the weld joints.

Compared to the experiment K3, the force-displacement curves resulting from the experiment K4 have substantially greater reaction forces of 140 kN (Fig. 5b). The course of the force-displacement curve in K4 c is similar to those in K4 a and K4 b, though it differs in terms of the maximum reaction force and the displacement at the fracture of the weld joint. The force-displacement curves of K4 a and K4 b are nearly identical and have a maximum reaction force of 130 kN. These differences are assumed to exist due to a slightly greater throat thickness of the specimens weld joints. Ductile fracture occurs in all three tests at displacements between 5.0 mm and 5.5 mm.

3.2. Hardness measurements of the weld joints

To investigate the hardness and the shape of the cross section of the fillet welds, a sample of a macrosection is taken from one cross joint specimen, as shown in Fig. 6a. The heat-affected zone in the macrosection, which surrounds the weld metal, can be seen through a comparatively darker metallographic structure. In addition, the path of the hardness measurement is visible through imprints. The Vickers hardness is measured by using a loading force of 98.1 N and a 10-second time duration. A maximum hardness of 330 HV is obtained in the heat-affected zone, while in the weld metal the hardness is approximately 255 HV. Since the maximum value of 330 HV appears only of the hardness measurement is visible through imprints. The Vickers hardness is measured by using a loading force up to 5 mm and from 9 mm to 16 mm (Fig. 5a). The maximum reaction force of 43 kN is reached at 16.8 mm displacement and failure subsequently occurs in one of the weld joints.

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3.2.1. Hardness measurements

The stress-strain relation for the weld metal is determined through experimental investigations of round tensile specimens from a butt weld and the specimens are taken from this weld joint. The true stress-strain relation is developed in the manner mentioned above, i.e. by using the modified version of the weighted average method by Ling (1996).

The base material is a mild Grade A shipbuilding steel. For the numerical investigations, a true stress-strain relation is applied, determined by tensile tension tests with flat tensile specimens manufactured from a batch of Grade A shipbuilding steel sheets, of 10 mm thickness. The true stress-strain relation is described by using a modified version of the weighted average method by Ling (1996), using a weighting factor of 0.46. It is shown as a black curve in Fig. 7a. In the experimental investigation of the round tensile specimens of the Grade A shipbuilding steel, an ultimate strength value of 820 MPa is determined. Using the revaluation table according to DIN 50150 in the range of uniform strain, the true stress-strain curve of the weld metal is modified. The modified stress-strain relation is described through the Hollomon equation in the range of uniform strain. The yield stress of 530 MPa and the hardening exponent through the Hollomon equation in the range of uniform strain.
of 98.1 N and a 10-second time duration. A maximum hardness of 330 HV is obtained in the heat-affected zone, while in the weld metal the hardness is approximately 255 HV. Since the maximum value of 330 HV appears only on one side of the weld joint, the hardness of the heat-affected zone is determined by using a load force of 9.81 N on two additional paths (Fig. 6a and Fig. 6b). These paths start in the base material and continue across the heat-affected zone into the weld metal. In the heat-affected zone, maximum hardness levels of 330 HV and 360 HV are obtained. Both paths confirm the typical behavior of higher hardness levels in the heat-affected zone compared to the weld metal and the base material.

3.3. True stress-strain relations

The base material is a mild Grade A shipbuilding steel. For the numerical investigations, a true stress-strain relation is applied, determined by tensile tension tests with flat tensile specimens manufactured from a batch of Grade A shipbuilding steel sheets, of 10 mm thickness. The true stress-strain relation is described by using a modified version of the weighted average method by Ling (1996). In this method, true stress-strain curves are simply described analytically up to the point of necking. In the post-necking range, a weighted average is defined between the Hollomon equation ($\sigma = K_0 \varepsilon^n$) and a linear function with the slope of the true stress-strain curve at necking. A detailed description of the approach can be found in the works of Ling (1996) and Werner et al. (2015). The stress-strain curve of the Grade A shipbuilding steel is depicted in Fig. 7a.

The stress-strain relation for the weld metal is determined through experimental investigations of round tensile specimens. To manufacture the round tensile specimens, two plates of Grade A shipbuilding steel are joined through a V-shaped butt weld and the specimens are taken from this weld joint. The true stress-strain curve is developed in the manner mentioned above, i.e. by using the modified version of the weighted average method by Ling (1996). This curve is characterized by a yield stress of 530 MPa, a hardening exponent of $n = 0.11$, and a weighting factor of $w = 0.46$. It is shown as a black curve in Fig. 7a. In the experimental investigation of the round tensile specimens of the weld metal, an ultimate strength of 635 MPa is determined. Using the reevaluation table according to DIN 50150 (1976), the measured hardness value of 255 HV can be transformed into an ultimate strength value of 820 MPa. In order to take the difference of the material behavior in the numerical analysis of the cross joint specimens into account, the true stress-strain curve of the weld metal is modified. The modified stress-strain relation is described through the Hollomon equation in the range of uniform strain. The yield stress of 530 MPa and the hardening exponent $n = 0.11$ remain unchanged and an ultimate strength value of 820 MPa is used. The post-necking range is determined by the weighted average method by Ling (1996), using a weighting factor $w = 0.46$, i.e. the same as the original true stress-strain curve of the weld metal. The resulting stress-strain curve is plotted in green in Fig. 7a.

![Fig. 7. (a) True stress-strain curves of the base material (shipbuilding steel grade A), the heat affected zone and the weld metal; (b) equivalent plastic strain over stress triaxiality of the elements at the location of crack initiation in the finite element simulation with the failure criterion according to Rice and Tracey](image-url)
The stress-strain curve of the heat-affected zone is determined by the hardness measurement shown in Fig. 6. According to DIN 50150 (1976), the hardness value of 330 HV corresponds to an ultimate strength value of 1060 MPa. Furthermore, a yield stress is calculated by

\[
HV = 4.08 \left( 1 + 1755 \left( \frac{\sigma_y}{E} \right)^2 - 44.1 \left( \frac{\sigma_y}{E} \right) \right)
\]

(1)

according to He et al. (2007), whereby the yield stress is determined from the hardness and the Young’s modulus. In the same manner as the modified true stress-strain curve of the weld metal, the post-necking range of the true stress-strain relation is described by the weighted average method by Ling (1996), using an assumed weighting factor \( w = 0.5 \) (Fig. 7a).

4. Numerical investigations of the welded cross joint specimens

The cross joint specimens are numerically investigated using LS-Dyna. For the investigation of the experiments K1 and K2, symmetry in three directions is used for the finite element models, so that only one-eighth of the whole specimen is needed (Fig. 8a and Fig. 8b). For the numerical investigation of the experiments K3 and K4, the finite element models are reduced to a quarter of the specimen through symmetry conditions (Fig. 8c and Fig. 8d). In the numerical models, the cylindrical bearings of the four-point bending fixture are represented through rigid cylindrical bodies. For the contact condition between the specimen and the bearing the Coulomb friction with a friction coefficient of \( \mu = 0.15 \) is assumed. In the finite element investigations, the rigid body motions are suppressed by means of contact and symmetry conditions. The displacement is applied with a load velocity of 100 mm/s onto the nodes of the upper cross sections of the finite element models for the experiments K1 and K2 as well as onto the central bearings in the models for K3 and K4.

Since the hardness tests of the weld joints indicate a different material behavior of the heat-affected zone compared to the weld metal and the base material, the heat-affected zone is taken into account in preliminary numerical investigations. In these numerical investigations, an influence of the heat-affected zone on the force-displacement curves is apparent compared to numerical analyses that do not take the heat-affected zone into consideration. Therefore, it is necessary to consider the heat-affected zone to reproduce the experimentally determined force-displacement curves in the most accurate manner. An initial shape of the weld joint cross section (weld metal and heat-affected zone) for the numerical investigations is taken from the macrosection (Fig 9c). To
4. When the strain relation is described by the weighted average method by Ling (1996), using an assumed weighting factor, the same manner as the modified true stress-strain curve of the weld metal, the post-necking range of the true stress is determined according to He et al. (2007), whereby the yield stress is determined from the hardness and the Young’s modulus. In the same manner, a yield stress is calculated by the hardness value of 330 HV corresponding to an ultimate strength value of 1060 MPa. Furthermore, a yield stress is calculated by the hardness measurement shown in Fig. 6.

According to DIN 50150 (1976), the hardness value of 330 HV corresponds to an ultimate strength value of 1060 MPa. Furthermore, a yield stress is calculated by the hardness measurement shown in Fig. 6.

4.08 1 1755 44.1

The stress-strain curve of the heat-affected zone is determined by the hardness measurement shown in Fig. 6.

Fig. 8. Finite element models for the numerical investigation of the experiments (a) K1, (b) K2, (c) K3 and (d) K4

bodies. For the contact condition between the specimen and the bearing the Coulomb friction with a friction coefficient of \( \mu = 0.15 \) is assumed. In the finite element investigations, the rigid body motions are suppressed by means of contact and symmetry conditions. The displacement is applied with a load velocity of 100 mm/s onto the central bearings in the models for K3 and K4.

Fig. 9. (a) Discretization of the weld joint with approximately 0.5 mm element size; (b) weld joint measurements for the numerical analyses; (c) macrosection of one weld joint of the cross joint specimens

determine the final shape of the weld joint cross section, two different geometries and different positions in regard to the two plates are proven. The shape of the weld joint cross section in Fig. 9a delivers the best results. This weld joint has a throat thickness of 3 mm and is surrounded by a heat-affected zone that is approximately 1 mm in width (Fig. 9b).

The specimens are discretized by using solid elements with an element size of approximately 0.5 mm at the weld joint and the surrounding area and increases up to 4.5 mm at the edges. The element type is characterized through one integration point per element, which tends to zero energy modes and the hourglass effect respectively. This phenomenon is avoided by adding an artificial stiffness to each element by the finite element software.

4.1. Rice and Tracey

Rice and Tracey analytically investigated the behavior of a spherical void in an ideal plastic material in a triaxial stress field. Through their investigations, they determined a relation between the ductility of the material up to the point of failure and the hydrostatic stress, whereby the ductility decreases with increasing hydrostatic stress. Hancock and Mackenzie (1976) and Atkins (1997) express the Rice and Tracey failure criterion as a function of the fracture strain

\[
\varepsilon_f = 1.65 \varepsilon_c \exp \left( -\frac{3}{2} T \right)
\]

(2)

depending on the stress triaxiality \( T \) and assuming a proportional stress state. The failure criterion is implemented through a damage integral

\[
D = \frac{1}{1.65 \varepsilon_c} \int \exp \left( \frac{3}{2} T \right) d\varepsilon
\]

(3)

which describes the accumulated damage of an element. An element gets deleted after \( D \) reaches unity. The failure criterion is calibrated onto the material and adjusted to the element size through the critical strain \( \varepsilon_c \).

In the numerical investigations of the welded cross joint specimens, the critical strain is determined by the experiment K1 with \( \varepsilon_c = 0.34 \). Due to the calibration of the failure criterion with the experiment K1, both force-displacement curves and especially the displacement at failure are well reproduced in the finite element simulations.
The initial crack results in a drop or decline of the reaction force in the numerical investigation at a displacement of 1.4 mm. The crack in the weld metal grows from the welding gap through the weld joint. The crack initiation is located close to the outside of the specimen and is highlighted through white elements in Fig. 10a. With increasing load the crack grows through the weld joint (gray elements in Fig. 10a) in a small range of displacement and loses its load capacity. Fig. 7b shows the path of the plastic strain ε over the stress triaxiality $T$ of the element at the location of crack initiation in the weld joint as well as the limiting curve of the failure criterion. The stress triaxiality increases steadily up to $T = 2$ through the load history and the plastic strain reaches $\varepsilon = 4.2\%$.

By employing the failure criterion according to Rice and Tracey to the experiment K2 with a critical strain of $\varepsilon_{cr} = 0.34$, the numerically determined force-displacement curve reproduces the experimental results with a limited correlation. The numerically determined force-displacement curve has a steadily increasing course of reaction force, whereas the results of the experiment show a maximum reaction force at a displacement of 1.6 mm (Fig. 4b). At a displacement of 1.48 mm, cracks appear at the welding gap from both ends of the fillet welds. The cracks grow slowly with increasing load and at a displacement of 2.9 mm they are still locally restricted to the two ends of the weld joint. The weld joint and one of the plates of the specimen are shortened by about 5 mm after the welding process. In Fig. 8b, the shortened plate, shown in green, is milled flat onto the yellow plate. The milling of the specimen can also be seen in Fig. 3a. One of the initial cracks in the numerical investigation occurs in the flat-milled area at the root of the weld joint and is highlighted by white elements on the right hand side of Fig. 10b. The element at the location of crack initiation is characterized by a stress triaxiality of $T = 0.56$ and a plastic strain of $\varepsilon = 40\%$ (Fig. 7b). At the same displacement (1.48 mm), a crack appears in the fillet weld at the welding gap at the other end of the weld joint (left in Fig. 10b). The stress state differs distinctly compared to the flat-milled area and is characterized by a stress triaxiality of $T = 1.17$ and a plastic strain of $\varepsilon = 12.6\%$ at failure (Fig. 7b).

Applying the Rice and Tracey failure criterion to the numerical analysis of experiment K3, fracture of the weld joint is predicted at a displacement of 13.7 mm and loses the entire load capacity at 14.5 mm (Fig. 5a). Failure of the weld joint occurs in the finite element simulation at a smaller displacement compared to the experimental results with a displacement of 17.7 mm at fracture. In the displacement range of 3.8 mm to 11 mm, slight differences are visible between the experimentally and numerically determined force-displacement curves. The initial crack grows from the welding gap (white elements in Fig. 11a) into the weld joint. With increasing displacement of the cylindrical central bearings, the crack grows uniformly over the entire width of the specimen through the weld joint (gray elements in Fig. 11a). The stress triaxiality of the element at the location of crack initiation increases steadily before failing at $T = 1.63$ and a plastic strain of $\varepsilon = 7.5\%$ (Fig. 7b).

The force-displacement curve of the numerical investigation of experiment K4 is characterized by a high correlation with the experimentally determined force-displacement curves in experiments K4a and K4b (Fig. 5b). The onset of failure in the weld joint at 3.2 mm displacement causes oscillations in the force-displacement curve. Final failure takes place at 4.4 mm displacement. Due to the position of the specimen in the four-point bending

![Fig. 10. (a) Location of crack initiation (white) at the weld joint in the finite element simulation of experiment K1 and the following crack path through the fillet weld (gray); (b) Two locations of crack initiation at the weld joint in the finite element simulation of experiment K2 highlighted through white elements](image-url)
fixture, the crack grows in longitudinal direction of the weld joint. The initial crack appears on the opposite side of the central bearings (blue in Fig. 8d). The location of the initial crack is highlighted using white elements in Fig. 11b. Due to the load direction of the specimen, the crack grows from the location of initiation to the other side of the specimen. It is illustrated in an initial state using gray elements in Fig. 11b. The stress state at the location of crack initiation is almost identical to the results of the numerical investigation of the experiment K1 (Fig. 7b). The stress triaxiality increases up to $T = 2.17$, while the plastic strain reaches $e = 3.6\%$ at failure.

4.2. Gurson

The damage model according to Gurson (1977) is used often and in various ways to predict failure in porous materials through nucleation and growth of voids (Dunand and Mohr (2011), Xue et al. (2010), Xue et al. (2013), Nègre et al. (2004), Nielsen and Tvergaard (2010) and Zhou et al. (2014)). The porosity of a material is described by an approach based on continuum mechanics through the effective void volume fraction. Whereby the yield condition

$$
\Phi = \frac{\sigma_{\text{vol}}}{\sigma_y} + 2q_1f^+ \cosh \left( \frac{3q_2\sigma_y}{2\sigma_y} \right) - 1 - q_3f^+^2
$$

(4)

of the Gurson damage model depends on $f^+$, the von Mises equivalent stress $\sigma_{\text{vol}}$, and the hydrostatic stress state $\sigma_y$. Tvergaard (1981) introduced the parameters $q_1$, $q_2$, and $q_3$ into the yield condition and showed that, in numerical investigations, the values of $q_1 = 1.5$, $q_2 = 1$, and $q_3 = q_1^2$ reproduce the hardening behavior more accurately than the original yield condition by Gurson, wherein $q_1 = q_2 = q_3 = 1$. The function of the effective void volume fraction

$$
f^+ = \begin{cases} 
 f & \text{for } f \leq f_c \\
 f_c + \frac{q_1 - f_c}{f_f - f_c} (f - f_c) & \text{for } f > f_c
\end{cases}
$$

(5)

is subdivided into the range for values less than or equal to the critical void volume fraction $f_c$ and for the range of values $f > f_c$ modified by Tvergaard and Needleman (1984). Apart from $f_c$, the yield condition parameter $q_1$ and the void volume fraction at fracture $f_f$ are included. On reaching the critical void volume fraction $f_c$, the load capacity of the material declines, till it vanishes entirely when $f^+ = f_c$. The increment of the void volume


\[ \dot{f} = \dot{f}_g + \dot{f}_n \]  

comprises an increment of the growing process of existing voids as well as an increment of nucleation. The incremental character of the values is represented by the time derivative. The incremental growth of the existing voids

\[ \dot{f}_g = (1-f) \dot{\varepsilon}_{\text{vol}} \]  

is calculated from the first invariant of the plastic strain \( \dot{\varepsilon}_{\text{vol}} \) which corresponds with the volume dilatation. The increment of nucleation

\[ \dot{f}_n = \frac{f_n}{s_n \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\varepsilon - \varepsilon_n}{s_n} \right) \right] \dot{\varepsilon} \]  

is calculated from the plastic strain rate \( \dot{\varepsilon} \) and the existing void volume fraction through nucleation \( f_n \). Furthermore, the equation includes the normal distribution specified by Gauss with the standard deviation \( s_n \) and the average value \( \varepsilon_n \).

Nahshon and Hutchinson (2008) extended the Gurson-Tvergaard-Needleman damage model for softening under shear stress. Softening under shear load occurs solely through the deformation and rotation of voids and is therefore different from the process of softening under tensile load with growth and nucleation of voids. Nahshon and Hutchinson (2008) defined the incremental growth of voids

\[ \dot{f}_g = (1-f) \dot{\varepsilon}_{\text{vol}} + k_\omega \cdot f \cdot \omega(\sigma) \frac{s_n \dot{\varepsilon}}{\sigma_{\text{eff}}} \]  

as an effective damage value and extended \( \dot{f}_g \) by a second mathematical term describing the damage due to shear load. In equation (9) controls the material parameter \( k_\omega \) the magnitude of damage. The equation also includes the void volume fraction \( f \), the function \( \omega(\sigma) \), the deviatoric stress \( s_{ij} \), the plastic strain rate \( \dot{\varepsilon} \), as well as the equivalent von Mises stress \( \sigma_{\text{eff}} \).

\[ f_n = \frac{f_n}{s_n \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\varepsilon - \varepsilon_n}{s_n} \right) \right] \dot{\varepsilon} \]  

\( s_{ij} \), \( \sigma_{\text{eff}} \), \( \dot{\varepsilon} \). The function

![Fig. 12. Experimentally and numerically determined force-displacement curves using the Gurson damage model of experiment (a) K1 and (b) K2](image-url)
Hutchinson (2008) defined the incremental growth of voids different from the process of softening under tensile load with growth and nucleation of voids. Nahshon and shear stress. Softening under shear load occurs solely through the deformation and rotation of voids and is therefore $\epsilon$ von Mises stress.

The incremental character of the values is represented by the time derivative. The incremental growth of the existing comprises an increment of the growing process of existing voids as well as an increment of nucleation. The $\epsilon$ void volume fraction as an effective damage value and extended $\epsilon$ is calculated from the first invariant of the plastic strain $\epsilon$ load. In equation (9) controls the material parameter $f$. The function $f$ by a second mathematical term describing the damage due to shear $\omega$ the magnitude of damage. The equation also includes the $\epsilon$ which corresponds with the volume dilatation. The $\epsilon$ and the existing void volume fraction through nucleation.

The axisymmetric stress state is characterized by $q_2 = 0.9$, following Tvergaard and Needleman (1984) and Nielsen and Tvergaard (2010). The force $\omega = 2$ and $\omega = 3$, the loss of the load capacity of the weld joint is predicted at a displacement.

The location of crack initiation in the numerical analysis of the experiment K1 is identical to the results of the simulation using the Rice and Tracey failure criterion. The crack continues to grow through the weld metal as load is increased (Fig. 10a).

Similar to the numerical analysis of the experiment K2 with the Rice and Tracey failure criterion, the experimentally determined force-displacement curves are reproduced with a limiting correlation by using the Gurson damage model (Fig. 12b). With increasing shear damage parameter $k_{\omega}$, the weld joint in the numerical analysis of K2 tends to fail at smaller displacements. With values of $k_{\omega} = 0$ and $k_{\omega} = 1$, a crack initiation appears in the weld joint but grows slowly with the increasing load, resulting in a steadily increasing reaction force in both numerical analyses. At $k_{\omega} = 2$ and $k_{\omega} = 3$, the loss of the load capacity of the weld joint is predicted at a displacement.

$$\omega(\sigma) = 1 - \left( \frac{27J_3}{2\sigma_{\text{vM}}} \right)^2$$  \hspace{1cm} (10)

Fig. 13. Experimentally and numerically determined force-displacement curves using the Gurson damage model of experiment (a) K3 and (b) K4.
of 2.7 mm and 2.4 mm respectively. The numerical analysis with \( k_\omega = 3 \) and a displacement of 2.4 mm at failure is in the range of the experimentally determined force-displacement curves, whereby the simulation with \( k_\omega = 2 \) reproduces the loss of load capacity at a larger displacement. In the numerical analysis with \( k_\omega = 2 \), crack initiation appears on both ends of the weld joint. At a displacement of 1 mm, a crack starts to grow from the welding gap on the upper end of the weld joint (Fig. 8b) and is highlighted using white elements in Fig. 14a. At a displacement of 1.72 mm, a crack starts forming in the flat-milled area mentioned above (Fig. 14b) and leads to a change in the course of the force-displacement curve, from a steadily increasing course to an almost horizontal course.

The influence of the shear parameter \( k_\omega \) on the force-displacement curves of the experiments K3 and K4 can be seen in Fig. 13. The force-displacement curve of the experiment K3 is reproduced with a slight deviation between 4 mm and 10 mm displacement in the finite element simulations. The displacement of 17.7 mm at the point of failure of the weld joint in the experiment is not reached in all four simulations. With an increasing shear damage parameter \( k_\omega \), the displacement at the point of failure of the weld joint decreases. The same tendency can be recognized in the numerically determined force-displacement curves of experiment K4, whereby \( k_\omega \) has a smaller influence on the displacement at the point of failure (Fig. 13b). The force-displacement curves of the finite element simulations are almost identical to the results of K4 a and K4 b up to a displacement of 3.8 mm.

5. Conclusion

Since the critical strain \( \varepsilon_{cr} \) of the Rice and Tracey failure criterion and the parameters \( f_c \) and \( f_i \) of the Gurson damage model are calibrated with the force-displacement curves of the experiment K1, the results of the experiment are properly reproduced in the numerical analyses. Furthermore, the force-displacement curves of the experiment K4 are predicted with good correlation in the finite element simulations, but failure of the weld joint occurs at a smaller displacement compared to the experiment. The experimental results of K2 and K3 are reproduced with obvious differences in the finite element simulations. These differences are attributed to the assumption of identical shapes of the weld joint cross sections in all numerical models, though they might actually differ in the specimens. Furthermore, the true stress-strain relations of the weld metal and the heat-affected zone are determined by hardness measurements and consequently have a degree of uncertainty. The metallographic structure of the weld metal and the heat-affected zone possibly varies in the different specimens due to distinct cooling processes while welding and leads to different material behaviors. The discretization with an element size of 0.5 mm of the weld joint might be too coarse to reproduce the notch geometry of the welding gap. This can have a significant influence on the numerically determined force-displacement curves, since the location of crack initiation of the weld joints is at the welding gap in all finite element simulations.
Despite the challenges of reproducing the experimental results in numerical analyses, the Rice and Tracey failure criterion and the Gurson damage model are both suitable for reproducing failure of the welded cross joint specimens.

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