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Basic oscillation measurables in the neutrino pair beam

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ABSTRACT

It was recently shown that the neutrino-pair emission may occur with large rates, their energy being extended to GeV region, if appropriate heavy ions are circulated in a quantum state of mixture. In the present work it is further demonstrated that the vector current contribution of neutrino interaction with electrons in ion, not necessarily suppressed in high atomic number ions, gives rise to the oscillating component, even when a single neutrino is detected alone. On the other hand, the single neutrino detection in Z-boson decay does not show the oscillating component, as known for some time. CP violation measurements in the neutrino pair beam may become a possibility, along with determination of mass hierarchical patterns.

1. Introduction

A new strong source of neutrinos consisting of all flavor pairs of ν_a and $\bar{\nu}_a$ ($a = e, \mu, \tau$) was recently proposed [1]. The proposal uses, as the emitting source, circulating heavy ions in a quantum state of two-state mixture, $\sin\theta_c e^{-i\epsilon_{eg}t/\gamma}|e\rangle + \cos\theta_c|g\rangle$ ($|e\rangle$ is a suitable excited state, while $|g\rangle$ is the ground state of ion) with a mixing angle θ_c. A high coherence quantified by a large value of $\sin(2\theta_c)/2$ produces high energy neutrino-pair with its energy sum extending up to $2\gamma\epsilon_{eg}$ where $\gamma = 1/\sqrt{1-\beta^2}$ is the boost factor of circulating ions and ϵ_{eg} is the level energy difference of the two states. Produced pair-beam is well collimated to the angular region $1/\gamma$ from the tangential direction of circulating ion. Moreover, produced neutrinos in the pair are in a highly entangled quantum state. These features of the neutrino-pair beam make it worthwhile to investigate the new possibility of neutrino oscillation experiments.

It was however shown in [2] that the neutrino oscillation can be measured only when both neutrinos in the pair are detected, which is experimentally difficult due to the smallness of the double detection rate. This disappearance of oscillation for a single neutrino detection is based on (i) the unitarity of the 3×3 neutrino mixing matrix and (ii) the equality of pair emission amplitude squared that holds for the dominant axial vector contribution of light ions.

In the present work we show that the second condition, the equality of pair emission amplitude squared, does not hold in the vector current contribution (sub-dominant, when ionic electrons move with non-relativistic velocities, but may be comparable to the axial vector contribution in heavy ions) of pair emission amplitude, hence the emergence of oscillation patterns occurs from the vector contribution. When neutrino oscillation is made possible this way, the CP violating (CPV) parameter determination (the CPV phase δ common to both Dirac and Majorana cases) becomes possible.

We derive basic formulas for the three-flavor neutrino scheme including the earth matter effect and present numerical outputs of quantities for new experiments using the neutrino-pair beam. It is found that oscillation patterns appear in all $\nu_a, \bar{\nu}_a$, $a = e, \mu, \tau$, but determination of CPV parameter is possible only by detection of $\nu_\mu, \bar{\nu}_\mu$ and tau-neutrinos. Electron neutrinos do not allow CPV determination.

One of the most important advantages of the neutrino-pair beam is that it is a CP even beam consisting of equal mixture of neutrinos and anti-neutrinos. This is the reason why the direct measurement of CP-odd quantity, showing genuine CP violation effects, is possible in our neutrino-pair beam. CPV parameter determination is also possible by measuring CP-even observables alone. Simultaneous measurements of CP-odd and CP-even observables should be of great help for determination of CPV parameter.

Moreover, we would like to show that the neutrino pair beam offers experiments for other interesting neutrino physics. One example is the determination of the mass hierarchy of neutrinos by using the oscillation patterns in the ν_e ($\bar{\nu}_e$) and ν_μ ($\bar{\nu}_\mu$) ap-

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pearance probabilities. Another interesting topic is a possibility to probe the deep interior of earth by exploiting a large matter effect for sufficiently long baseline experiments of length $\gg 1000$ km.

Our estimate of the feasibility of high energy neutrino-pair is admittedly crude, but results thus obtained appear promising for further studies.

In the rest of paper we first explain under what conditions the disappearance of oscillation pattern occurs when a single neutrino is detected and how this can be evaded. We shall then proceed to calculation of oscillation effects.

Throughout this work we use the natural unit of $\hbar = c = 1$.

2. How oscillation pattern appears in single neutrino detection

We shall recapitulate how neutrino oscillation may arise in the case of pair-beam following notations of [2]. The probability amplitude of the entire process consists of three parts: the production, the propagation, and the detection due to charged current interaction (neutral current interaction is much smaller, hence not considered here), each to be multiplied at the amplitude level. Thus, one may write the probability for the ν_a neutrino quasi-elastic scattering (with J^α the nucleon weak current) as

$$\sum_c \left(\frac{G_F}{\sqrt{2}}\right)^2 \bar{\nu}_a\gamma_\alpha(1-\gamma_5)l_a J^\alpha \bar{l}_a\gamma_\beta(1-\gamma_5)\nu_a(J^\beta)^\dagger \times \left|\sum_b \langle\bar{c}|e^{-i\bar{H}\bar{L}}|\bar{b}\rangle\langle a|e^{-iHL}|b\rangle\mathcal{M}_{\bar{b}b}(1,2)\right|^2, \tag{1}$$

where H ($\bar{H}$) is the hamiltonian for propagation of neutrino (anti-neutrino) including earth-induced matter effect [3–5], which is in the flavor basis

$$H = U^* \begin{pmatrix} \frac{m_1^2}{2E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} \end{pmatrix} U^T + \sqrt{2}G_F n_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2}$$

where U_{ai} is the neutrino mixing matrix with $|a\rangle = \sum_i U_{ai}^*|i\rangle$, $a = e, \mu, \tau$, $i = 1, 2, 3$, and n_e is the number density of electrons in the earth. $\bar{H}$ can be obtained by replacing $U \to U^*$ and changing the sign in the second term $\propto G_F$.

We shall denote three eigenvalues by λ_i for neutrinos, and $\bar{\lambda}_i$ for anti-neutrinos. Let $V(\sim U)$ and $\bar{V}$ are unitary 3×3 matrices that diagonalize the hamiltonian H for neutrino and $\bar{H}$ for anti-neutrino, including the earth matter effect. The propagation amplitude is then

$$\langle a|e^{-iHL}|b\rangle = \sum_i V_{ai}V_{bi}^* e^{-i\lambda_i L},$$

$$\langle\bar{c}|e^{-i\bar{H}\bar{L}}|\bar{b}\rangle = \sum_i \bar{V}_{ci}^*\bar{V}_{bi}e^{-i\bar{\lambda}_i\bar{L}}, \tag{3}$$

$$\sum_b \langle\bar{c}|e^{-i\bar{H}L}|\bar{b}\rangle\langle a|e^{-iHL}|b\rangle c_b = \sum_{ij} V_{ai}\bar{V}_{cj}^*\xi_{ij}\exp[-i(\lambda_i L + \bar{\lambda}_j\bar{L})],$$

$$\xi_{ij} = \sum_b c_b V_{bi}^*\bar{V}_{bj}. \tag{4}$$

The factor c_b arises from the production amplitude $\mathcal{M}_{\bar{b}b}(1,2)$ and it is $(c_b^A) = \frac{1}{2}(1, -1, -1)$ for the axial vector contribution and for the vector contribution,

$$(c_b^V) = \left(\frac{1}{2}(1 + 4\sin^2\theta_w), -\frac{1}{2}(1 - 4\sin^2\theta_w), -\frac{1}{2}(1 - 4\sin^2\theta_w)\right), \tag{5}$$

with the weak mixing angle θ_w. The precise relation between neutrino and anti-neutrino eigenvalue problem is given by

$$\bar{\lambda}(G_F) = \lambda(-G_F), \quad \bar{V}_{ai}(G_F) = V_{ai}(-G_F). \tag{6}$$

The rate of neutrino ν_a detected and $\bar{\nu}_c$ undetected contains the squared propagation factor,

$$\begin{aligned} &\sum_c \left|\sum_{ij} V_{ai}\bar{V}_{cj}^*\xi_{ij}\exp[-i(\lambda_i L + \bar{\lambda}_j\bar{L})]\right|^2 \\ &= \sum_{ijkl}\sum_c V_{ai}V_{ak}^*\bar{V}_{cj}^*\bar{V}_{cl}\xi_{ij}\xi_{kl}^*\exp[-i(\lambda_i - \lambda_k)L]\exp[-i(\bar{\lambda}_j - \bar{\lambda}_l)\bar{L}] \\ &= \sum_{ik} V_{ai}V_{ak}^* p_{ik}\exp[-i(\lambda_i - \lambda_k)L], \quad p_{ik} = \sum_j \xi_{ij}\xi_{kj}^*. \end{aligned} \tag{7}$$

When $(|c_b^A|^2) = (1, 1, 1)/4 \propto 1$ for the axial vector contribution, $p_{jl} = \delta_{jl}/4$ and the detection probability becomes 1/4, hence no oscillation pattern exits.

The relevant weak amplitude for the vector part gives oscillating components. Candidate ions for circulation that contribute to the vector current interaction are Be-like heavy ions of $2p2s^3P_1^-$ and Ne-like heavy ions of $2p^+3s^3P_1^-$ (electron–hole system).

3. Basic measurable quantities in neutrino pair beam

We first note

$$\begin{aligned} p_{ik} &= \sum_j \xi_{ij}\xi_{kj}^* = \sum_b |c_b^V|^2 V_{bi}^* V_{bk} \\ &= \frac{1}{4}(1 + 4\sin^2\theta_w)^2\, V_{ei}^* V_{ek} \\ &\quad + \frac{1}{4}(1 - 4\sin^2\theta_w)^2\,(V_{\mu i}^* V_{\mu k} + V_{\tau i}^* V_{\tau k}) \\ &= \frac{1}{4}(1 - 4\sin^2\theta_w)^2\delta_{ik} + 4\sin^2\theta_W\, V_{ei}^* V_{ek}. \end{aligned} \tag{8}$$

The detection probability of ν_a (when the other neutrino of the pair is undetected) is given by the oscillation formula based on the vector part of weak current,

$$P_a(E, L; m_i, \delta) \equiv \frac{1}{3(1 - 4\sin^2\theta_w)^2/4 + 4\sin^2\theta_w} \times \left(\frac{1}{4}(1 - 4\sin^2\theta_w)^2 + 4\sin^2\theta_w\left|\sum_j U_{ej}^* U_{aj}\exp\left(-i\frac{m_j^2 L}{2E}\right)\right|^2\right), \tag{9}$$

with $\sin^2\theta_w \sim 0.231$. The formula (9) is valid when the earth matter effect is neglected. When the earth matter effect is included, one replaces $U \to V$, $m_j^2/2E \to \lambda_j$. The quantity $P_a(E, L; m_i, \delta)$ is the normalized probability: $\sum_a P_a(E, L; m_i, \delta) = 1$. The oscillating component in eq. (9) is equivalent to the $\nu_e \to \nu_a$, $a = \mu, \tau$ appearance probability multiplied by

$$\frac{4\sin^2\theta_w}{3(1 - 4\sin^2\theta_w)^2/4 + 4\sin^2\theta_w} \sim 0.995. \tag{10}$$

Thus, the constant off-set term $\propto (1 - 4\sin^2\theta_w)^2$ in eq. (9) is very small. In the limit of $\sin^2\theta_w = 1/4$ there is no contribution to the

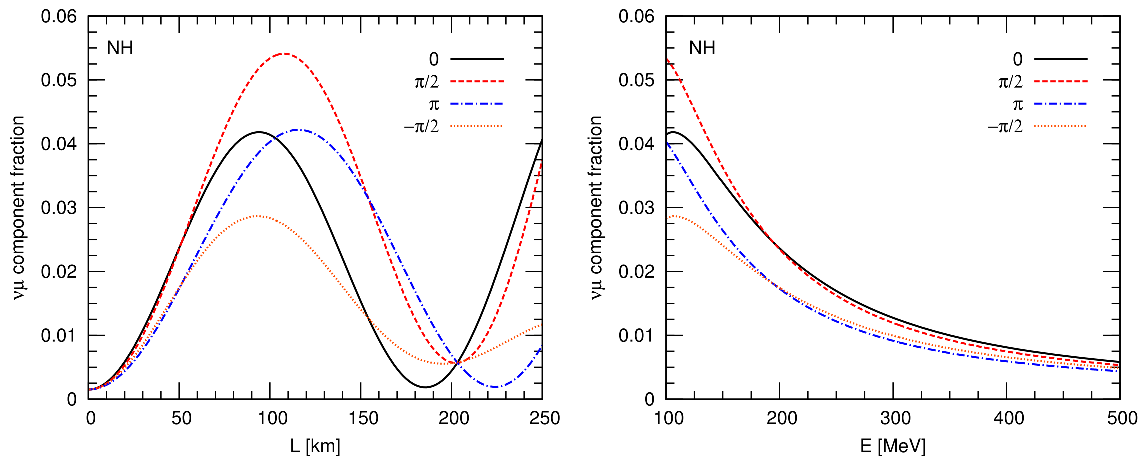

Fig. 1. ν_μ component fraction of the pair beam, calculated by using eq. (9). A few choices of CPV parameter δ are taken: 0 in solid black, $\pi/2$ in dashed red, π in dash-dotted blue, and $-\pi/2$ in dotted orange. In the left panel the neutrino energy is fixed at 200 MeV. In the right panel the distance is 50 km away from the ring and the lowest ν_μ energy should be set at $\sim$200 MeV to avoid $\nu_\mu \to \mu$ threshold effect. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

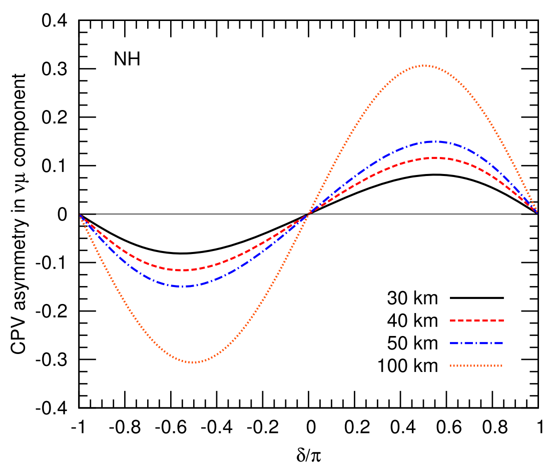

Fig. 2. ν_μ CPV asymmetry plotted against CPV parameter δ given by eq. (11). Assumed parameters are the neutrino energy 200 MeV, the distance = 30 km in solid black, 40 km in dashed red, 50 km in dash-dotted blue, and 100 km in dotted orange. The earth matter effect is negligible in these distances as shown in Fig. 5. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

vector part from Z-boson exchange. Due to the dominance of $\nu_e \to \nu_a$, $a = \mu, \tau$ in the oscillating term, oscillation patterns in the pair beam have similarities to the β [6] and $\beta^\pm$ beam [7].

The most striking feature of the neutrino pair beam is that circulating quantum ions produce coherent pairs of all flavors, $\nu_a \bar{\nu}_a$, $a = e, \nu, \tau$. When these pairs propagate, all mass eigen-states get involved, and relevant oscillation extrema at $L/E = 2\pi/\delta m_{ij}^2$, $(ij) = (12), (23), (13)$ may become relevant, making short baseline experiments a feasible approach.

We illustrate numerical results of oscillation patterns and CPV asymmetry in Figs. 1 and 2, respectively. We used neutrino data as determined from neutrino oscillation experiments [8]. CPV asymmetry here is defined by

$$A_a(E, L; m_i, \delta) = \frac{P_a(E, L; m_i, \delta) - P_a(E, L; m_i, -\delta)}{P_a(E, L; m_i, \delta) + P_a(E, L; m_i, -\delta)}. \tag{11}$$

Experimentally, this quantity may be derived from measurements of both ν_a and $\bar{\nu}_a$ events.

We note a few important results: (i) CPV δ measurement is impossible for $\nu_e, \bar{\nu}_e$ events, because the oscillation probability appearing in eq. (9) depends on the quantity $|U_{ej}|^2$, hence is insensitive to δ. (ii) CPV asymmetry measured by detection of $\nu_\mu, \bar{\nu}_\mu$ is small at the ion ring site due to the unitarity relation $\sum_j U_{ej}^* U_{\mu j} = 0$ valid at small L. (iii) Interestingly, as shown in Fig. 2, CPV asymmetry of $\mathcal{O}(0.1)$ can be obtained even if the distance is $L \sim 40$ km for $E = 200$ MeV. (iv) The determination of the mass hierarchical pattern, normal or inverted, namely NH/IH distinction is possible in the ν_e and ν_μ components as seen in Fig. 3. The advantage of ν_e component was also pointed out in the reactor neutrino experiment [9].

Numerical results of Fig. 1 and Fig. 2 suggest that an ideal CPV parameter determination is possible for detector located at $\sim$100 km away from the heavy ion synchrotron. Events that confuse CPV parameter determination are generated by the earth matter effect, which is minimal at these short baseline experiments.

Comparison with Z-decay pair may be of interest. In this case $(c_b^V) = -\frac{1}{2}(1 - 4\sin^2\theta_W)(1, 1, 1)$. Hence, the neutrino-pair beam from Z-boson decay does not show the oscillation pattern when only one neutrino is detected [10].

So far we focused on short baseline experiments. For long baseline experiments the earth matter effect may become important. To incorporate the earth matter effect, we numerically diagonalize the effective hamiltonian (2) [11]. The oscillation patterns including the earth matter effect are illustrated in Figs. 4 and 5. We took a pure SiO_2 model with density 2.8 g/cm^3 for the earth matter. It is seen that the neutrino pair beam offers also a strong experiment to test the earth matter effect if the baseline length is sufficiently large as $L \gg 1000$ km.

4. Rates of neutrino-pair production from quantum ion beam

Calculations in [2] are for the axial-vector contribution giving the spin matrix element of ion. We now repeat calculations based on the vector contribution, following [12] of the spin current contribution derived for any ion velocity. Two contributions, the vector and axial-vector parts, never interfere since they have different parity relations between two specified states, $|e\rangle, |g\rangle$. In terms of two-component spinors, the axial vector operators of electrons in ions have the form, $(0, 2\vec{S}_e)$ in the 4-vector notation, while the vector operators have $(1, \epsilon_{eg}\vec{r}_{eg})$ with $\vec{r}_{eg}$ the transition dipole moment $\vec{d}_{eg}$ divided by the electric charge, although the time component 1 is usually small due to the orthogonality of wave functions. Calculation of the vector current matrix element is identical to

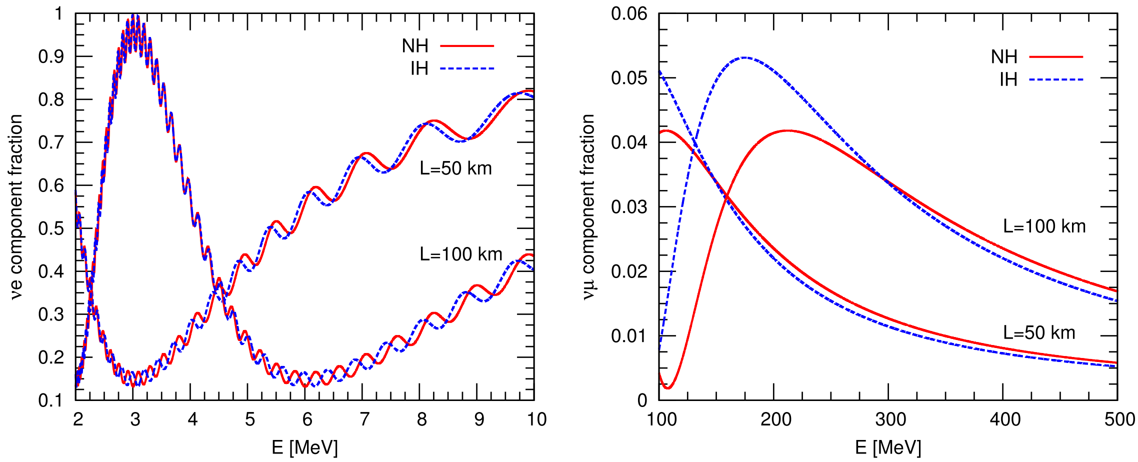

Fig. 3. ν_e (left) and ν_μ (right) component fractions of the pair beam, calculated by using eq. (9) for NH (solid red) and IH (dash-dotted blue). The distances are 50 km and 100 km away from the ring. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

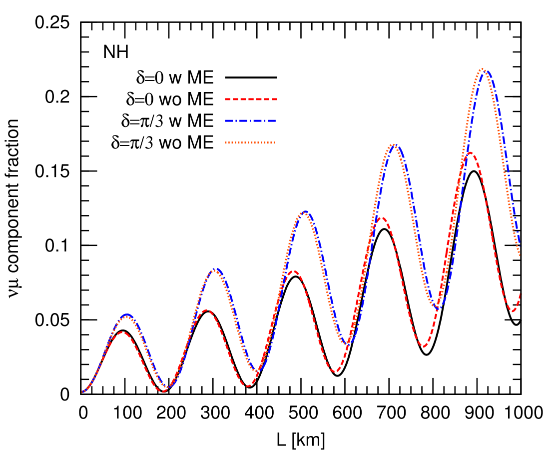

Fig. 4. ν_μ oscillation pattern with and without the earth matter effect (ME). The neutrino energy is fixed at 200 MeV. $\delta = 0$ with ME in solid black, without ME in dashed red, $\delta = \pi/3$ with ME in dash-dotted blue, and without ME in dotted orange. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

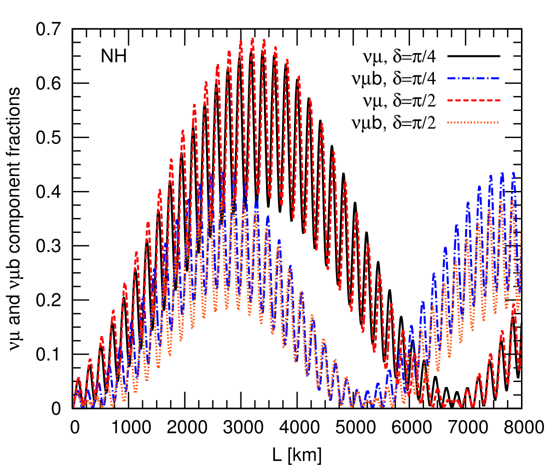

Fig. 5. ν_μ oscillation pattern with the earth matter effect. The neutrino energy is fixed at 200 MeV. $\delta = \pi/4$ neutrino (ν_μ) in solid black, anti-neutrino ($\bar{\nu}_\mu = \nu_{\mu b}$) in dash-dotted blue, $\delta = \pi/2$ neutrino in dashed red, and anti-neutrino in dotted orange. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

photon emission in QED. When the squared amplitude is averaged over directions of dipole vector for probability calculation, the result is given by

$$\frac{4}{3}\epsilon_{eg}^2 \vec{r}_{eg}^{\,2} = \frac{4}{3}\epsilon_{eg}^2 \frac{\vec{d}_{eg}^{\,2}}{4\pi\alpha}. \tag{12}$$

This replaces the previous factor of [2], $4\vec{S}_e^{\,2}\frac{1}{\gamma^2}(1+\frac{2}{3}\beta^2\gamma^2)$. We have replaced the flavor component fraction at production by a simplified result of $\sin^2\theta_w = 1/4$, namely $(c_b^V)^2 = (1, 0, 0)$.

The differential energy spectrum of a single detected neutrino at the forward direction is, in the high energy limit of $\gamma \gg 1$,

$$\frac{d^2\Gamma}{dy d\varphi} = \frac{8}{27\sqrt{\pi}(2\pi)^4} N\sqrt{\rho\epsilon_{eg}}\frac{G_F^2\epsilon_{eg}^5}{\alpha}\gamma^{11/2}\varphi$$
$$\times \int dy_2 y^2 y_2 (y+y_2)^{-1/4}\left(4\gamma^2 - y - y_2 - \gamma^2 y\varphi^2\right)^{3/4}, \tag{13}$$
$$\frac{8}{27\sqrt{\pi}(2\pi)^4}\sqrt{\rho\epsilon_{eg}}\frac{G_F^2\epsilon_{eg}^5}{\alpha}\gamma^{11/2}$$
$$\sim 7.1\times 10^{10}\ \mathrm{Hz}\left(\frac{\rho\epsilon_{eg}}{10^{14}}\right)^{1/2}\left(\frac{\epsilon_{eg}}{10\ \mathrm{keV}}\right)^5\left(\frac{\gamma}{10^3}\right)^{11/2},$$
$$y = \frac{E}{\epsilon_{eg}}\sqrt{\frac{1-\beta}{1+\beta}}. \tag{14}$$

Here N is the available number of ions, ρ is the radius of the ring. The angle φ is defined from the usual angular variables (θ, ψ) by $\varphi = \sqrt{\theta^2+\psi^2}$ in which θ is the angle within the orbit plane.

The angular distribution is readily calculable, and is plotted in Fig. 6. Here we take the radius of the ion ring as $\rho = 10^{14}/\epsilon_{eg} \sim 2$ km $(10\ \mathrm{keV}/\epsilon_{eg})$ and the Lorentz boost factor $\gamma = 2000$ for illustration. The forward production rates are the most relevant to neutrino oscillation experiments away from the ring. The forward rate is estimated by taking the angular area π/γ^2 times the right hand side of eq. (13). The following figure, Fig. 7, illustrates these rates. The forward rates scale with ion parameters $\propto A_{eg}\epsilon_{eg}^{5.5}$, with A_{eg} the decay rate (Einstein's A-coefficient), ϵ_{eg} the level spacing, and scale with the boost factor $\propto \gamma^{3.5}$. In order to detect ν_μ events, neutrino energies larger than 200 MeV are desired, which gives a constraint on $2\gamma\epsilon_{eg}$.

5. Detection rates of neutrino events away from ion ring

We next estimate single neutrino event rates by a detector placed at 50 km away from the ion ring. Magnetized detector to

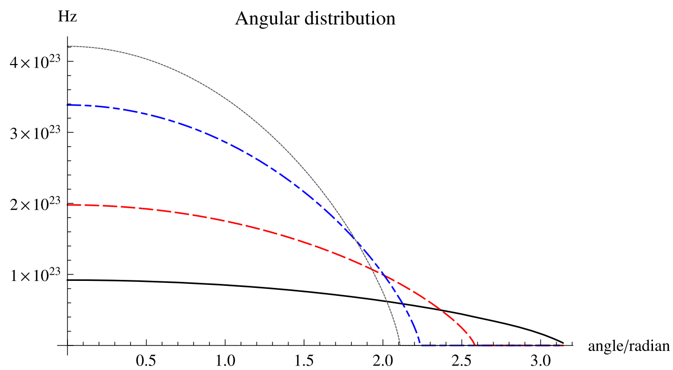

Fig. 6. Angular distribution of single neutrino for $\gamma = 5000$, $\rho\epsilon_{eg} = 10^{14}$, $N = 10^8$, $\epsilon_{eg} = 50$ keV: neutrino energy 200 MeV in solid black, 300 MeV in dashed red, 400 MeV in dash-dotted blue, and 450 MeV in dotted black. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

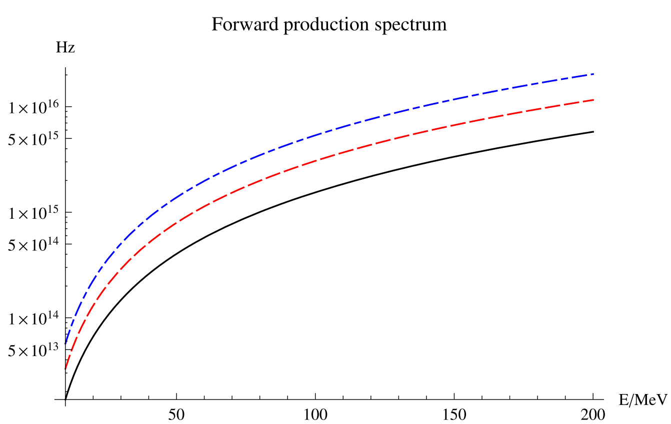

Fig. 7. Neutrino energy spectrum rate at the forward direction of solid angle area π/γ^2. Assumed parameters are $\rho\epsilon_{eg} = 10^{14}$, $N = 10^8$ and $\epsilon_{eg} = 50$ keV, $\gamma = 4000$ in solid black, 5000 in dashed red, 6000 in dash-dotted blue. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

distinguish produced lepton or anti-lepton from either neutrino or anti-neutrino should be considered to directly measure the difference of rates, namely, CP-odd asymmetry. 10 kt class of ${}^{56}_{26}$Fe target is considered. We shall use PDG compilation [13] of neutrino quasi-elastic cross section extrapolated to lower neutrino energy region. Event rates for an ideal detector of the active target volume covered fully by the neutrino pair beam (for example, the cylindrical detector located at the distance D_d with the cross section area $S = \pi D_d^2/\gamma^2$, the length L_d and the total volume $SL_d = T$), are

$$\nu_\mu(\text{Fe}); \quad 6.2 \times 10^{-11} P_\mu \left(\frac{E}{1\ \text{GeV}}\right)\left(\frac{T}{10\ \text{kt}}\right)\frac{d\Gamma}{dE}\Delta E\ \text{Hz}, \tag{15}$$

$$\bar\nu_\mu(\text{Fe}); \quad 1.8 \times 10^{-11} P_{\bar\mu} \left(\frac{E}{1\ \text{GeV}}\right)\left(\frac{T}{10\ \text{kt}}\right)\frac{d\Gamma}{dE}\Delta E\ \text{Hz}, \tag{16}$$

where ΔE is the width of the energy bin. Neutrino event rates scale with $N(\sin\theta_c\cos\theta_c)^2$ where N is the circulating ion number and $\sin\theta_c\cos\theta_c$ is the coherence factor. We took for this product 10^8. The precise number to be taken depends on accelerator R and D works and is difficult to pin down at the moment. But one can readily obtain neutrino event rates by this scaling.

It is found from Fig. 7 that, when $\gamma = 5000$ and $E = 200$ MeV, $\frac{d\Gamma}{dE}\Delta E \simeq 10^{16}$ Hz (with $\Delta E = 1$ MeV). Then the expected event rate is $\mathcal{O}(10^3)$ per second by using the 10 kt detector for $P_{\mu,\bar\mu} \sim$ a few %, which leads to $\mathcal{O}(10^{10})$ quasi-elastic scattering events per year. This is a large enough event rate which makes possible the measurements of the CP asymmetry, depending on the value of δ as shown in Fig. 2. We stress the advantage of the CP-even neutrino-pair beam: this beam is free from systematic errors in the relative fluxes of neutrinos and anti-neutrinos, which offers a direct test of CP violation, although CP-even observables can also determine CPV parameter δ. The simultaneous measurement of both CP-odd and CP-even observables is a great advantage to enhance the precision of δ measurement. Although the neutrino-pair beam has some similar features to the beta-beam, the simultaneous presence of neutrinos and anti-neutrinos makes a distinctive merit. The more precise estimation of the event rate requires a better understanding of the neutrino scattering cross section in the detector, which seems insufficient near the threshold region.

One might worry about a small branching ratio of the neutrino-pair emission. Actually, a possible smallness of the branching ratio itself is not a problem at all, and what really matters to a realization of the neutrino-pair beam is whether the absolute rate of the neutrino-pair emission is large enough. Multiple photon modes due to quantum electrodynamics, for instance, might have larger rates than that of the neutrino-pair process. Emitted multiple photons in this case are simply dumped somewhere when the neutrino-pair beam is extracted. The situation is similar to ν_μ beam in the pion decay: in this case π^0 backgrounds etc are dumped somewhere, and one does not discuss the branching fraction that goes to the neutrino beam.

The problem of coherence loss is related to the energy consumption of ion in the accelerator ring. After submission of the original manuscript, one of the present authors (MY) worked out the coherence loss problem during the circulation of heavy ions [14]. The coherence loss caused by a strongest photon emission process is described by

$$\frac{d\rho_{eg}}{dt} = -\frac{G}{2}\rho_{eg}^3, \quad \rho_{eg}(t) = \frac{\rho_{eg}(0)}{\sqrt{1 + G\rho_{eg}^2(0)t}},$$
$$\rho_{eg}(0) = \frac{1}{2}\sin(2\theta_c), \tag{17}$$

where the de-coherence coefficient G was calculated by integrating the emitted photon number over all photon energies and angular area π/γ^2. The coefficient G depends on quantum numbers of $|e\rangle$ and $|g\rangle$. The essential point for ion energy recovery is that one can compensate the lost ion energy by acceleration of ions, since the rate of coherence loss given by eq. (17) is much slower than the linear growth of compensation energy given by a constant acceleration gradient. The ratio of lost energy by photon emission to the compensated ion energy was numerically computed for He-like ions, using a high-gradient acceleration of 100 MV/m technically achievable in [14]. It was found that by adjusting the accelerator circumference, the acceleration gradient and appropriate ions, one can compensate the lost energy by ion acceleration within one turn of circulation. If the de-coherence problem is solved this way, a fast radiative decay in the case of neutrino pair emission is not an obstacle, as discussed above. Clearly, the problem needs to be solved along with more detailed design of heavy ion accelerator.

6. Summary and outlook

In summary, we demonstrated that CPV parameter determination is possible in a short baseline experiment using the neutrino pair beam. If the accelerator ring is placed in the underground of depth d, the neutrino pair beam appears on earth at a distance $\sim \sqrt{2dR}$ away from the synchrotron site where R is the radius of the earth. It is found in the present work that the experiment with a short baseline of $50 \sim 100$ km is interesting for the study of CP violation by using the neutrino pair beam. It is then interesting to consider a realization of the neutrino-pair beam by using an updated LHC ring at CERN or new FCC [15] and CEPC [16] rings under discussion. In the LHC case the accelerator is located 100 m underground, and the easiest way is to detect horizontally directed neutrino pair beam right after the beam gets out of the earth. Due to the high intensity beam correlated with the ion's bunch structure it may not be too difficult to eliminate the horizontal component of cosmic ray backgrounds. There may be no need to go underground for detection of neutrinos. The baseline length in this setup becomes 35 km. If it is judged to be better to shield the detector from the cosmic ray backgrounds, the option is to dig a horizontal tunnel in the nearby mountains close to the accelerator site.

The determination of the CPV parameters requires a sufficiently large $N(\sin\theta_c \cos\theta_c)^2$. Clearly, both experimental R and D works of quantum coherent ion circulation and theoretical studies of candidate ions are required for further development of this new project.

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We use the parametrization as given by
$$(U_{ai}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P,$$
$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}, \; a = e, \mu, \tau, \; i = 1, 2, 3,$$
where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. The lightest neutrino mass is taken as 5 meV, on which the results in this article are independent.
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