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Numerical solution of boundary layer flow equation with viscous dissipation effect along a flat plate with variable temperature

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Abstract

The present paper considers the classical problem of hydrodynamic and thermal boundary layers over a flat plate in a uniform stream of fluid with viscous dissipation effect and variable plate temperature. Using a similarity variable, the governing nonlinear partial differential equations have been transformed into a set of coupled nonlinear ordinary differential equations and solved numerically by the finite difference method along with Newton's linearization approximation namely Keller box method. Due to viscous dissipation, conversion of mechanical energy to thermal energy results in temperature variation in the fluid and variation of fluid properties. A discussion is provided for the effects of Eckert number (Ec), Prandtl number (Pr) and temperature power coefficient n on two-dimensional flow. It is observed that as Ec increases the temperature distribution increases whereas Pr increases the temperature distribution decreases for variable temperature. Detailed analysis of the velocity profile, temperature distribution and rate of heat transfer are tabulated and presented graphically.

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Keywords: boundary layer flow, heat transfer, viscous dissipation, Prandtl number, Eckert number

Nomenclature

U	Fluid velocity
u, v	Velocity components
ν	Kinematic viscosity
α	Thermal diffusivity
Pr	Prandtl number
Ec	Eckert Number
θ	Dimensionless temperature

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1. Introduction

When the boundary layer flow is considered with the influence of heat transfer usually the velocities and velocity gradients are assumed sufficiently small as a result of which the effects of kinetic energy and viscous energy dissipation is neglected. In the present paper the boundary layer flow is considered with viscous dissipation effect and convection of heat transfer along a flat plate. Hence, the velocity is to be considered very high. The high velocity effect on the heat transfer plays an important role in many technical applications like heat transfer at high velocities around gas turbine blades or in rocket engines. High velocity convection involves essentially two different phenomena viz. conversion of mechanical energy to thermal energy, resulting in temperature variations in the fluid and variation of the fluid property as a result of temperature variation. The well-known dimensionless Eckert number [1] expresses the ratio of a flow's kinetic energy to the boundary layer enthalpy difference and is used to characterize viscous thermal dissipation of convection, especially for force convection. If the viscous thermal dissipation is ignored then the Eckert number is regarded as zero.

Due to the considered viscous dissipation effect and convection of heat transfer along a flat plate, the energy equation is modified by including a term representing the viscous dissipation effect [2]. Numerous investigations have been done by the various authors to study effect of viscous thermal dissipation on the fluid flows and heat transfer. Studies reveals that in the fluid flows with high Eckert number, generated heat due to the viscous thermal dissipation dominates the fluid temperature and the Eckert number cannot be zero in the investigation of convection heat transfer. The work of Brinkman [3] appears to be the first theoretical work dealing with viscous dissipation. Tyagi [4] performed a wide study on the effect of viscous dissipation on the fully developed laminar forced convection in cylindrical tubes with an arbitrary cross-section and uniform wall temperature. Basu et al. [5] analysed the Graetz problem considering viscous dissipation but neglecting the effect of axial conduction. Aydin et al. [6] discussed mixed convection of viscous dissipating fluid about a vertical plate. They considered four different flow situations according to the direction of the free stream flow and thermal boundary condition applied at the wall. Pantokratoras [7] studied, the steady laminar boundary layer flow along a vertical stationary heated plate taking into account the viscous dissipation of the fluid. While Mamun et al. [8] consider the effects of magnetic field, viscous dissipation and heat generation on natural convection flow of an incompressible, viscous and electrically conducting fluid along a vertical flat plate in the presence of conduction. The problem of steady laminar magnetohydrodynamic (MHD) mixed convection heat transfer about a vertical plate is studied numerically considering the effects of Ohmic heating and viscous dissipation by Aydın et al. [9]. Abo-Eldahab et al. [10] studied the effects of viscous dissipation and Joule heating on MHD-free convection flow past a semi-infinite vertical flat plate in the presence of the combined effect of Hall and ion-slip currents for the case of power-law variation of the wall temperature.

2. Governing Equations

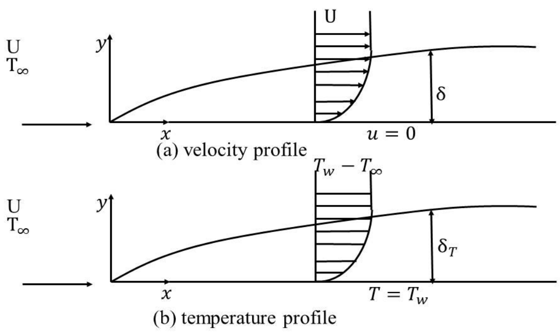

Fig. 1. Convective boundary layer heat transfer

Consider the flow of a fluid velocity of U over a flat plate whose entire surface is held at uniform temperature T_w which is different from that of the fluid ahead of the body which is T_∞. The flow situation is shown in Fig. 1. It is assumed that the Reynolds number is large enough for the boundary layer assumption to be applicable. It is further assumed that the flow is two-dimensional which means that the plate is assumed to be wide compared to its longitudinal dimension. As a result of these assumptions, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Under a boundary layer assumption, the energy transport equation is also simplified.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are the x (along the plate) and the y (normal to the plate) components of the velocities respectively, T is the temperature, ν is the kinematics viscosity of the fluid and α is the thermal diffusivity of the fluid. The last term in Eqs. (3), is the viscous dissipation term. Dissipation has no effect on the continuity and momentum equations because the fluid properties are being assumed constant. The velocity and temperature boundary conditions can be expressed as

$$u = 0, \qquad T = T_w(x) \qquad \text{at} \qquad y = 0 \tag{4}$$

$$u = U, \qquad T = T_\infty \qquad \text{at} \qquad y \to \infty \tag{5}$$

We assume that the plate temperature varies as

$$T_w(x) - T_\infty = Ax^n \tag{6}$$

Introducing a similarity variable η and a dimensionless stream function ψ and temperature $\theta(\eta)$ as

$$\eta = y\sqrt{\frac{U}{\nu x}}, \qquad \psi = \sqrt{U\nu x}f(\eta) \qquad \text{and} \qquad \theta(\eta) = \frac{T - T_\infty}{T_w(x) - T_\infty} \tag{7}$$

$$u = \frac{\partial \psi}{\partial y} = Uf' \qquad \text{and} \qquad v = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left(\eta f' - f\right) \tag{8}$$

where prime symbol denotes differentiation with respect to η. Eqs. (1) – (8) reduces to

$$f''' + \frac{1}{2}ff'' = 0 \tag{9}$$

$$\theta'' + \frac{1}{2}f\theta' + EcPr\left(f''\right)^2 - nPrf'\theta = 0 \tag{10}$$

subject to the following boundary conditions

$$f'(0) = 0, f(0) = 0, \theta(0) = 1 \tag{11}$$

$$f'(\infty) = 1, \ \theta'(\infty) = 0 \tag{12}$$

where $\Pr = \frac{\nu}{\alpha}$ is dimensionless Prandtl number and $Ec = \frac{U^2}{C_p(T_w - T_\infty)}$ is dimensionless Eckert number.

The quantity of physical interest $-\theta'(0)$ is the value which represents the heat transfer rate at the surface. Thus, our task is to investigate how the governing parameters Pr, Ec and n influence these quantities.

3. Numerical Solution

The coupled nonlinear Eqs. (9) – (10) with the boundary conditions Eqs. (11) – (12) are solved numerically using implicit finite difference Keller box method [11]. The discretizations of momentum and energy equations are carried out with respect to non-dimensional coordinates x and η to convey the equations in finite difference form by approximating the functions and their derivatives in terms of central differences in both the coordinate directions. Then the required equations are to be linearized by using the Newton's Quasi-linearization method. The linear algebraic equations can be written in a block tri diagonal matrix which forms a coefficient matrix. The whole method namely reduction to first order followed by central difference approximations, Newton's Quasi-linearization method and the block algorithm [12], is well known as Keller-box method.

4. Results and discussion

The present section discusses the behaviour of velocity and temperature distribution profile with the help of numerical values obtained by Keller box method for variable plate temperature. The variable plate temperature is assumed varying with $x-$ coordinate by considering a specific relationship shown in the Eqs (6). From Fig. 2 it is seen that the velocity profile on the flat plate possess a very small curvature at the wall and turns rather abruptly from it in order to reach the asymptotic value. The transverse component of the velocity for the boundary layer flow over a flat plate is shown in Fig. 3. To observe the temperature distribution with the variable plate temperature, various values of n are considered keeping Prandtl number (Pr) and Eckert number (Ec) fixed. It is also observed that for every value of Prandtl number or Eckert number the temperature distribution decreases as η increases. As seen from Fig. 4 the temperature distribution decreases when n increases. A similar observation is made by considering various values of Pr and Ec as shown in Fig. 5. To observe the effect of Eckert number, Prandtl number and n are kept fixed. For Pr=0.7, $n = 1$, the effect of Eckert number is observed for Ec=0.0,0.1,0.4 ,1 and its graphical representation for the temperature distribution is shown in Fig. 6. Thus by increasing Ec, the temperature profile increases. The same is observed for Pr=3, $n = 3$ in Fig. 7. The effect of Prandlt number on the temperature distribution is studied keeping n and Ec fixed. For $n = 3$, Ec=0.0, the numerical results are shown in Fig. 8 for various values of Prandlt number Pr=0.044, 0.7,3. It is observed that by increasing Pr the temperature distribution decreases. The same can be observed in Fig. 9. In Table 1 to Table 3, the numerical values of temperature gradient at wall $-\theta'(0)$ are calculated to find the Nusselt number. In Table 1, the numerical values of $-\theta'(0)$ are calculated for Pr=0.7 for various values of Ec with various values of n. In Table 2, the numerical values of $-\theta'(0)$ are calculated for Ec=0.5 for various values of Pr with various values of n. In Table 3, the numerical values of $-\theta'(0)$ are calculated for $n = 3$ for various values of Pr with various values of Ec. From Table 1, it is seen that the values of $-\theta'(0)$ are decreasing when Ec increases for each n=1, 2, 3, 4 i.e. Nusselt number (Nu) decreases. Hence, rate of heat transfer at the wall decreases. Similarly, from Table 2 and Table 3, it is observed that as Nu increases, the rate of heat transfer at the wall increases.

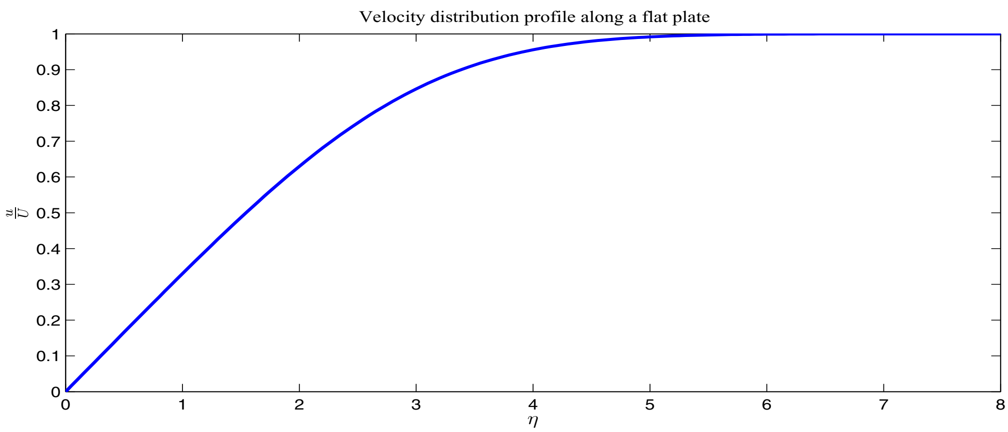

Fig. 2. Velocity distribution profile along a flat plate

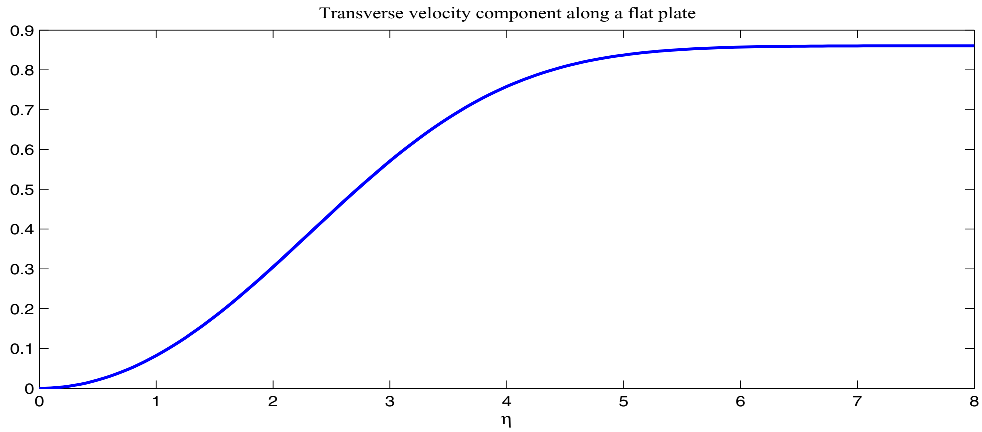

Fig. 3. The transverse velocity of boundary layer fluid flow over a flat plate

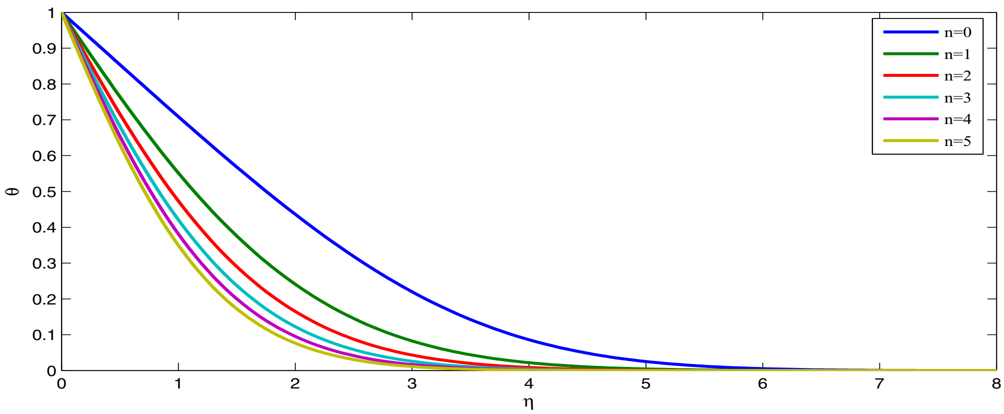

Fig. 4. Temperature distribution for Prandtl number = 0.7 and Eckert number = 0.0

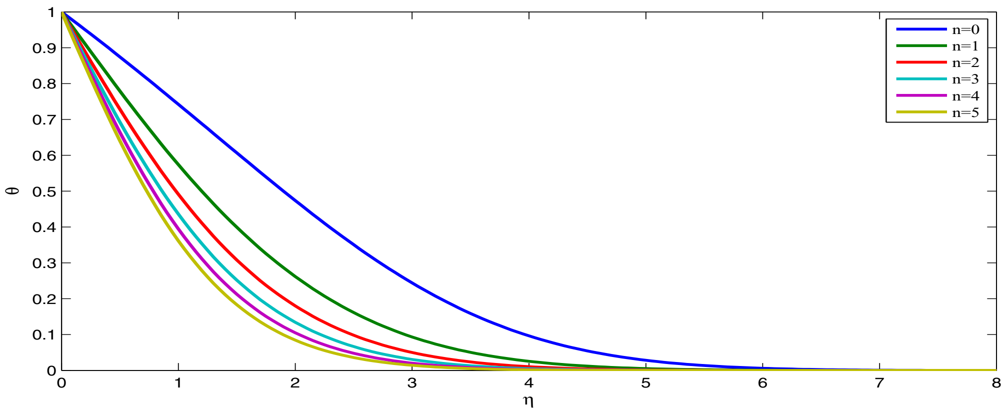

Fig. 5. Temperature distribution for Prandtl number = 0.7 and Eckert number = 0.4

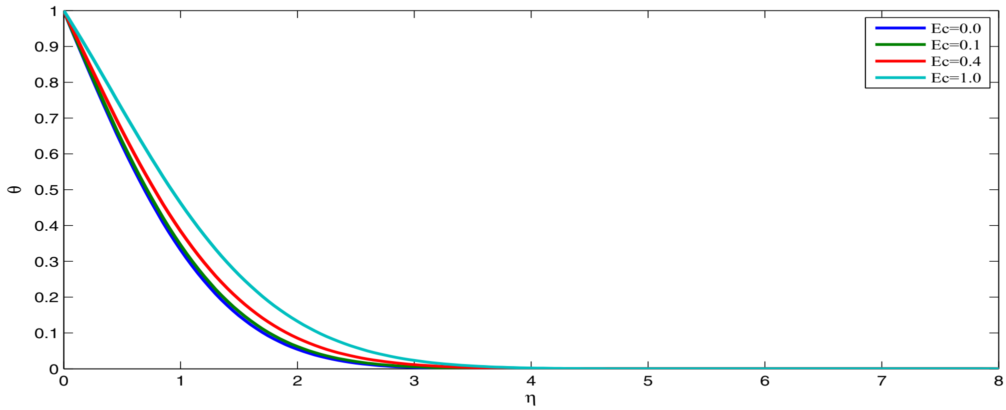

Fig. 6. Temperature distribution for Prandtl number = 3 and n=1

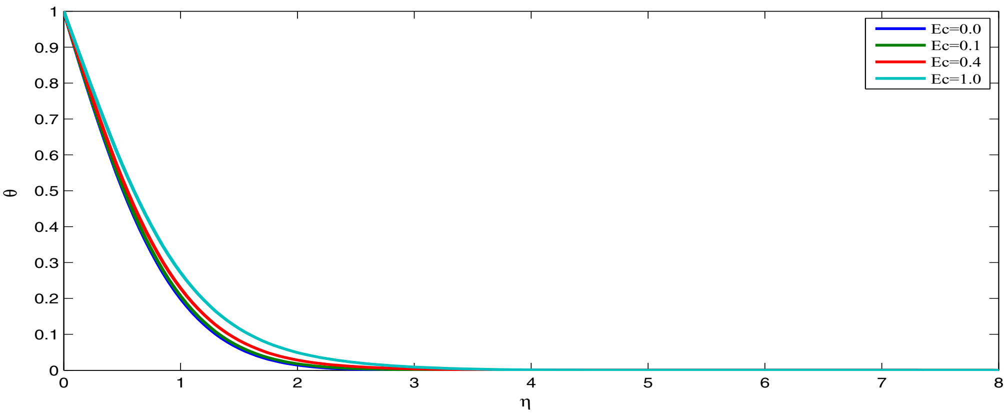

Fig. 7. Temperature distribution for Prandtl number = 3 and n=3

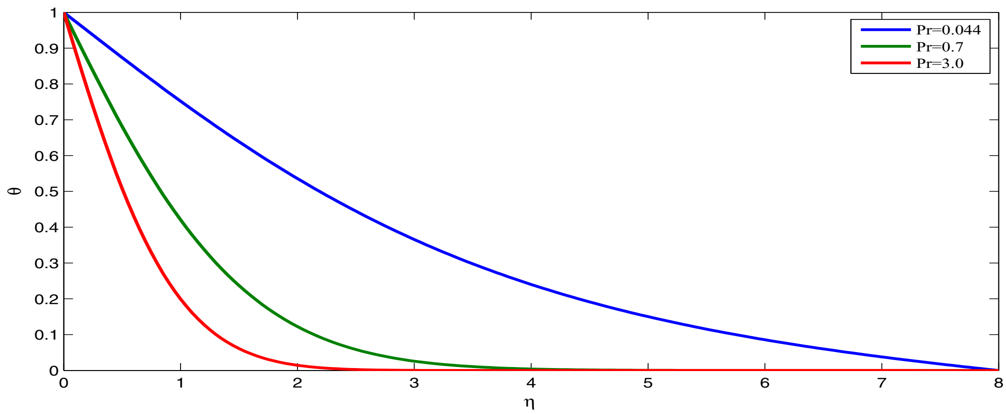

Fig. 8. Temperature distribution for Eckert number = 0.0 and n=3

Table 1. Variation of $-\theta'(0)$ for Prandtl number = 0.7 for variable temperature.

n	Ec=0.1	Ec=0.3	Ec=0.5	Ec=0.7
1	0.471081	0.45243	0.433778	0.415127
2	0.576896	0.56069	0.544484	0.528278
3	0.654579	0.639866	0.625153	0.61044
4	0.717506	0.703844	0.690181	0.676519

Table 2. Variation of $-\theta'(0)$ for Eckert number = 0.5 for variable temperature.

n	Pr=0.7	Pr=3	Pr=5
1	0.433778	0.645443	0.729576
2	0.544484	0.837255	0.964844
3	0.625153	0.974597	1.13158
4	0.690181	1.084292	1.264093

Table 3. Variation of $-\theta'(0)$ for n= 3 for variable temperature.

EC	Pr=0.7	Pr=3	Pr=5
0.1	0.654579	1.057992	1.250237
0.3	0.639866	1.016295	1.190909
0.5	0.625153	0.974597	1.13158
0.7	0.61044	0.9329	1.072252

5. Conclusion

In the present paper, the boundary layer equations over a flat plate are considered to observe the effect of viscous dissipation and convection of heat transfer. The governing boundary layer equations are transformed to nonlinear ordinary differential equations using similarity transformation. The Keller box method is applied to solve these equations and its numerical solutions are discussed. The numerical results are obtained by considering the various values of the Eckert number to observe the viscous dissipation effects. It is observed that as Ec increases the temperature distribution increases for variable temperature. To observe the rate of heat transfer at the wall, the Nusselt number (Nu) is calculated. It is found that the rate of heat transfer at the wall decreases in the case when Ec increases for fixed Prandlt number (Pr) and the rate of heat transfer at the wall increases in the case when Pr increases for specific variable temperature. Thus, the obtained numerical results by the Keller box method behave well with the physical phenomena of the problem.

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