Performance analysis of a handoff scheme for two-tier cellular CDMA networks

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Abstract A two-tier model is used in cellular networks to improve the Quality of Service (QoS), namely to reduce the blocking probability of new calls and the forced termination probability of ongoing calls. One tier, the microcells, is used for slow or stationary users, and the other, the macrocell, is used for high speed users. In Code-Division Multiple-Access (CDMA) cellular systems, soft handoffs are supported, which provides ways for further QoS improvement. In this paper, we introduce such a way; namely, a channel borrowing scheme used in conjunction with a First-In-First-Out (FIFO) queue in the macrocell tier. A multidimensional Markov chain to model the resulting system is established, and an iterative technique to find the steady-state probability distribution is utilized. This distribution is then used to find the performance measures of interest: new call blocking probability, and forced termination probability.

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1. Introduction

Handoff is the mechanism of transferring an ongoing call from the current cell to the neighbor cell as the Mobile Station (MS) moves through the coverage area of the cellular network. There are two important performance indices used in designing cellular communication systems. The first index is the blocking probability of a new call, which is the probability that a new call is denied due to the unavailability of free channels. The second index is the forced termination probability of handoff call, which is the probability that an ongoing call is ended abruptly while a handoff attempt is being made, again due to the unavailability of free channels. Typically, customers prefer having their new calls denied to having their ongoing ones ended. Suppose that the speed of MSs in a given cell...
A two-tier cellular network consists of a tier of cells with smaller size called microcells overlaid by a tier of cells with larger size called macrocells, with each macrocell covering $N$ microcells, as shown in Fig. 1. In two-tier cellular networks, high speed calls are assigned to macrocells, whereas stationary and low speed calls are assigned to microcells.

Handoff between cells in a two-tier cellular network occurs in three cases. The first case is when a call moves from a microcell to a macrocell (overflow), whether the macrocell is covering the microcell or a neighbor macrocell. The second case occurs when a call moves from a microcell to a microcell, whether in the same macrocell or not. The third case occurs when a call moves from a macrocell to a macrocell.

Stationary and low speed calls can be overflowed from a microcell to a macrocell in two cases:

1. When a call cannot find a free channel in a microcell.
2. When a call becomes high speed.

A two-tier cellular network can reduce the forced termination probability by two methods. The first method, called guard channels [10], is to reserve channels in the cell, either macrocell or microcell, exclusively for handoff calls, and the remaining channels are shared among new calls and handoff calls. The second method, queueing handoff calls [11], places in a queue requests for handoff calls that find no free channels in the destination cell.

There are two types of handoff: hard, used in Frequency-Division Multiple-Access (FDMA) and Time-Division Multiple-Access (TDMA) systems, and soft, used in code-division multiple-access (CDMA) systems [2]. In hard handoff the connection to the current cell is broken, then a connection to the new cell is made. This is known as break-before-make [5], where a MS is connected to only one Base Station (BS) at any given time. In soft handoff, which is the subject of this paper, where all cells use the same frequency [6], it is possible to make a connection to the new BS before the connection to the current BS is broken. This is known as make-before-break. That is, a MS call can be connected to two or more BSs at a time. For simplicity, we assume that each MS making a handoff call can connect to at most two BSs. It should be noted that a CDMA cell, either microcell or macrocell, is divided into two regions, Handoff Region (HR), and Non-Handoff Region (NHR). In an HR of a two-tier cellular CDMA network, one BS, which has a higher receiving power than another BS, is selected to demodulate the received signal. The selected BS is called the controlling BS, and the other BS is called the noncontrolling BS.

Stationary calls in a HR are divided into two groups: E-group and F-group. Let $P_{r,A}$ be the received power at BS A of a signal transmitted by a stationary MS call in handoff with cell A and B. Also, let $P_{r,B}$ be the received power at BS B of a signal transmitted by the MS call (Fig. 2). Then the E-group and the F-group can be defined as follows:

1. If $P_{r,A} > P_{r,B}$, that is, if BS is the controlling BS of the call, the call is in the E-group of cell A and the F-group of cell B.
2. If $P_{r,A} < P_{r,B}$, that is, if BS B is the controlling BS of the call, the call is in the F-group of cell A and the E-group of cell B.

In this paper, a handoff scheme based on the mobility of calls in a two-tier cellular CDMA network is proposed. This scheme is called a channel-borrowing handoff scheme. In this scheme, if there is a handoff call incoming to a cell with no free channels, a channel is borrowed from a call in the F-group of that cell, which corresponds to channel-borrowing from a call in the E-group of the current cell, and the channel is borrowed from the noncontrolling BS. If there are no channels in the F-group of the destination cell, the incoming handoff call is placed into a queue.

The rest of this paper is organized as follows: Section 2 describes the related works previously published in the literature. In Section 3, the system model is described. Section 4 analyzes the proposed model using Markov chain and compute performance measures of interest, and numerical results and analysis.
are presented in Section 5. Finally, in Section 6 we conclude the work and propose a future work.

2. Related works

Much research work has focused on the handoff problem in CDMA cellular networks. In [10], guard channels for handoff calls to reduce the forced termination probability. In [11], queues are used to achieve the same purpose. However, these two methods, while they reduce the forced termination probability, increase the new call blocking probability. This trade-off is inevitable in single tier systems. In [12], a combined guard channels/queueing approach is proposed. But then again, a single-tier system is assumed, and hence users’ mobility difference is not taken into account. In [13], a model is proposed for a two-tier cellular network is proposed whereby high and low speed calls are taken into account. In this model a first-in-first-out (FIFO) queue is used in one of the tiers to reduce the new call blocking probability and the forced termination probability. However, the assumed system uses FDMA not CDMA. Two tier systems with a FIFO queue in one of the tiers are considered also in [3,14,15] to take care of high and low speed calls, but in using TDMA. In [2], a two-tier cellular CDMA network with soft handoff queuing is developed. However, the forced termination probability in this model can be further reduced by applying such a technique as channel-borrowing, and the present work does just that. It should be noted that the channel borrowing technique was proposed in [9].

3. Model description

We consider a two-tier cellular network composed of microcells and macrocells covering the service area. The cell dwell time is defined as the time a MS spends in a cell before it is handed off to another cell. The mean cell dwell time $\frac{1}{\mu_2}$ can be calculated [2] as

$$\frac{1}{\mu_2} = \frac{\pi R^2}{24n},$$

where $R$ is the radius of the cell, and is the speed of the MS. The voice call holding time is defined as the total time of the call.

To simplify notation, all parameters related to microcell and macrocell use superscripts 1 and 2, respectively, and all parameters related to HR and NHR use subscripts 1 and 2, respectively.

We introduce the assumptions used in our system model as follows:

1. The arrival process of low speed new calls in a microcell is assumed to be Poisson with rate $\lambda_{m,1}^{(1)}$. The arrival rate $\lambda_{m,1}^{(1)}$ of low speed new calls in the HR (NHR) of a microcell is given by

$$\lambda_{m,1}^{(1)} = a \cdot \lambda_{m}^{(1)},$$

where $a(1-a)$ is the probability that a new call arrives at HR (NHR) of a microcell.

2. The arrival process of low speed handoff calls in a microcell is assumed to be Poisson with rate $\lambda_{m}^{(2)}$.

3. The arrival process of stationary new calls in a microcell is assumed to be Poisson with rate $\lambda_{m}^{(1)}$. The arrival rate $\lambda_{m,1}^{(1)}$ of stationary new calls in the HR (NHR) of a microcell is given by

$$\lambda_{m,1}^{(1)} = a \cdot \lambda_{m}^{(1)}, \quad \lambda_{m,2}^{(1)} = (1-a) \cdot \lambda_{m}^{(1)},$$

where $a(1-a)$ is the probability that a new call arrives at HR (NHR) of a microcell.

4. Related works

The arrival rate of stationary new calls at the $E$-group $\lambda_{q}^{(1)}$ of a microcell and participating in a soft handoff process can be calculated as follows:

$$\lambda_{q}^{(1)} = \lambda_{m,1}^{(1)} \left(1 - p_{bn}^{(1)}\right),$$

where $p_{bn}^{(1)}$ is the blocking probability of a new call in a microcell. The arrival rate of stationary new calls in a microcell and not participating in the soft handoff process $\lambda_{m}^{(1)}$ can be calculated as follows:

$$\lambda_{m}^{(1)} = \lambda_{m,2}^{(1)} \cdot \left(1 - p_{bn}^{(1)}\right),$$

4. The voice call holding time $T_h(T_a)$ for a low (high) speed call is assumed to be exponentially distributed with mean $\frac{1}{\mu_2}$.

5. The cell dwell time $T_{dl}^{(1)}$ for a low speed calls in a microcell is assumed to be exponentially distributed with mean $\frac{1}{\mu_{dl}^{(1)}}$. The mean cell dwell time $\frac{1}{\mu_{dl}^{(1)}} \left(\frac{1}{\mu_{dl}^{(2)}}\right)$ for low speed calls in the HR (NHR) of a microcell can be calculated [2] as

$$\frac{1}{\mu_{dl}^{(1)}} = \frac{16^{\log_{10}a}}{\mu_{dl}^{(1)}}, \quad \frac{1}{\mu_{dl}^{(2)}} = \frac{2^{\log_{10}(1-a)}}{\mu_{dl}^{(2)}}.$$

6. The transition rate of low speed calls from HR to NHR in a microcell $\mu_{dl}^{(1)}$ is given by

$$\mu_{dl}^{(1)} = \beta \cdot \mu_{dl}^{(1)},$$

where $\beta$ is the moving back (i.e., a call moves from HR to NHR) probability.

7. The departure rate of low speed calls from HR of the microcell $\mu_{dl}^{(1),t}$ is given by

$$\mu_{dl}^{(1),t} = (1 - \beta) \cdot \mu_{dl}^{(1)},$$

8. The arrival process of overflow low speed new calls in a macrocell is assumed to be Poisson with rate $\lambda_{h,m}^{(2)}$. The arrival rate $\lambda_{h,1}^{(2)} \left(\lambda_{h,2}^{(2)}\right)$ of overflow low speed new calls in the HR (NHR) of a macrocell is given by

$$\lambda_{h,1}^{(2)} = b \cdot \lambda_{h}, \quad \lambda_{h,2}^{(2)} = (1-b) \cdot \lambda_{h},$$

where $b(1-b)$ is the probability that a new call arrives at the HR (NHR) of a macrocell.

9. The arrival process of overflow low speed handoff calls in a macrocell is assumed to be Poisson with rate $\lambda_{h}^{(2)}$. The arrival rate $\lambda_{h,1}^{(2)} \left(\lambda_{h,2}^{(2)}\right)$ of overflow low speed handoff calls in the HR (NHR) of a macrocell is given by

$$\lambda_{h,1}^{(2)} = b \cdot \lambda_{h}^{(2)}, \quad \lambda_{h,2}^{(2)} = (1-b) \cdot \lambda_{h}^{(2)}.$$
10. The arrival process of overflow stationary new calls in a macrocell is assumed to be Poisson with rate $\lambda_m^o$. The arrival rate $\lambda_{m,1}^o$ (\(\lambda_{m,2}^o\)) of overflow stationary new calls in the HR (NHR) of a macrocell is given by

$$\lambda_{m,1}^o = b \cdot \lambda_m^o, \quad \lambda_{m,2}^o = (1 - b) \cdot \lambda_m^o. \quad (11)$$

The arrival rate of overflow stationary new calls at the E-group $\lambda_{eh}^o$ (F-group $\lambda_{eh}^o$) of a macrocell and participating in the soft handoff process is given by

$$\lambda_{eh}^o = \lambda_{m,1}^o \cdot \frac{1 - P_{th}^{(2)}}{2}, \quad (12)$$

where $P_{th}^{(2)}$ is the blocking probability of new call in a macrocell. The arrival rate of overflow stationary new calls in a macrocell and not participating in the soft handoff process is given by

$$\lambda_{eh}^o = \lambda_{m,2}^o \cdot \frac{1 - P_{th}^{(2)}}{2} + \lambda_{m,1}^o \cdot \frac{1 - P_{th}^{(2)}}{2} \cdot P_{th}^{(2)}. \quad (13)$$

11. The arrival process of high speed new calls in a macrocell is assumed to be Poisson with rate $\lambda_{hs}^{(2)}$. The arrival rate $\lambda_{hs}^{(2)}(\lambda_{hs}^{(2)})$ of high speed new calls in the HR (NHR) of a macrocell is given by

$$\lambda_{hs,1}^{(2)} = b \cdot \lambda_{hs}^{(2)}, \quad \lambda_{hs,2}^{(2)} = (1 - b) \cdot \lambda_{hs}^{(2)}. \quad (14)$$

12. The arrival process of low speed handoff calls in a macrocell is assumed to be Poisson with rate $\lambda_{lh}^{(2)}$. The arrival process of high speed handoff calls in a macrocell is assumed to be Poisson with rate $\lambda_{lh}^{(2)}$.

13. The cell dwell time $T_{dl}^{(2)}$ for low speed calls in a macrocell is assumed to be exponentially distributed with mean $\frac{1}{\mu_{dl}^{(2)}}$.

The mean cell dwell time for low speed calls in the HR (NHR) of a macrocell can be calculated [2] as

$$\frac{1}{\mu_{dl,1}^{(2)}} = \frac{1}{\mu_{dl}^{(2)}}, \quad \frac{1}{\mu_{dl,2}^{(2)}} = \frac{2}{\mu_{dl}^{(2)}}. \quad (15)$$

14. The cell dwell time $T_{dl}^{(2)}$ for high speed calls in a macrocell is assumed to be exponentially distributed with mean $\frac{1}{\mu_{dh}^{(2)}}$. The mean cell dwell time for high speed calls in the HR (NHR) of a macrocell can be calculated [2] as

$$\frac{1}{\mu_{dh,1}^{(2)}} = \frac{1}{\mu_{dh}^{(2)}}, \quad \frac{1}{\mu_{dh,2}^{(2)}} = \frac{2}{\mu_{dh}^{(2)}}. \quad (16)$$

15. The transition rate of low speed calls from HR to NHR in a macrocell $\mu_{ih}^{(2)}$ is given by

$$\mu_{ih}^{(2)} = \beta \cdot \mu_{ih}^{(2)}. \quad (17)$$

16. The transition rate of high speed calls from HR to NHR in a macrocell $\mu_{ih}^{(2)}$ is given by

$$\mu_{ih}^{(2)} = \beta \cdot \mu_{ih}^{(2)}. \quad (18)$$

17. The transition rate of high speed calls from HR to NHR in a macrocell $\mu_{ih}^{(2)}$ is given by

$$\mu_{ih}^{(2)} = \beta \cdot \mu_{ih}^{(2)}. \quad (19)$$

18. The departure rate of low speed calls from HR of the macrocell $\mu_{ilh}^{(2)}$ is given by

$$\mu_{ilh}^{(2)} = (1 - \beta) \cdot \mu_{ilh}^{(2)}. \quad (20)$$

19. The departure rate of high speed calls from HR of the macrocell $\mu_{ih}^{(2)}$ is given by

$$\mu_{ih}^{(2)} = (1 - \beta) \cdot \mu_{ih}^{(2)}. \quad (21)$$

20. The time line is divided into slots, each equal to the transmission time of one packet. Non-negative integers $k = 0, 1, \ldots$ are assigned to the individual slot boundaries. Slot $k + 1$ indicates the time interval $[k + 1]$.

21. The total number of channels in a microcell (macrocell) is $C_{micro}(C_{macro})$, and the queue size of the macrocell is $Q = q_i + q_{eh}$ where $q_i$ is the queue size for low speed calls and $q_{eh}$ is the queue size for high speed calls, and there is no queue in a microcell.

### 4. Model analysis

In this section, we analyze the performance of the microcell and the macrocell in a two-tier CDMA cellular network operating under the assumptions in Section 3. We will use for the analysis a Markov chain model. We will use extensively in the analysis the indicator function $\delta(x)$ defined as follows:

$$\delta(x) \begin{cases} 1 & x \text{ is true,} \\ 0 & x \text{ is false.} \end{cases} \quad (21)$$

#### 4.1. Analysis of the microcell

According to our assumptions, we define some random variables (RVs) which are all assumed non-negative integers. Let $P_{b}^{(2)}$ and $P_{eh}^{(2)}$ be two RVs denoting the number of low speed calls in HR and NHR of a microcell, respectively, in slot $k$. Let $P_{ih}^{(2)}$ and $P_{ih}^{(2)}$ be two RVs denoting the number of stationary calls participating in the soft handoff process in $E$-group and $F$-group of a microcell, respectively, in slot $k$. Let $P_{ilh}^{(2)}$ be a RV denoting the number of stationary calls not participating in the soft handoff process of a microcell in slot $k$. Clearly, $P_{b}^{(2)} + P_{eh}^{(2)} + P_{ih}^{(2)} + P_{ih}^{(2)} + P_{ilh}^{(2)} \leq C_{micro}$. Let the state of a microcell be defined by the vector $\vec{s} = (i, j, e, f, m)$. Then, the joint probability of the RV elements of the state vector $\vec{s}$ is

$$p_{s}^{K} = \Pr\{P_{b}^{(2)} = i, P_{eh}^{(2)} = j, P_{ih}^{(2)} = e, P_{ih}^{(2)} = f, P_{ilh}^{(2)} = m\}. \quad (22)$$

Let $M$ denote the number of channels in use in the cell while in state $\vec{s}$. Thus, $M = i + j + e + f + m$ and $M \leq C_{micro}$. Let $p_{s}^{K}$ be the probability that a microcell is in the borrowing state in slot $k$, and is calculated as follows:

$$p_{c}^{K} = \sum_{e} p_{s}^{K} \quad (23)$$

where

$$C1 = \{\vec{s} | M = C_{micro}, f > 0\}. \quad (24)$$

Referring to Fig. 3, which is the transition diagram of the microcell containing $C_{micro}$ channels, it can be seen that (5/6) $p_{s}^{K} \cdot p_{c}^{K}$ is the rate at which a channel is borrowed from an $E$-group of the current cell (which is at the same time the
F-group of the destination cell). This is because the destination cell is in the borrowing state and a there are 5 other cells, besides the current cell, exporting handoff calls to the destination cell. Now, \( j \cdot \mu^{(1)}_{ dl2} \cdot \frac{\lambda^{(1)}_{ ml2}}{\mu^{(1)}_{ ml2} + \mu} \) is the rate at which a channel is borrowed from an F-group of the current cell. This is because the destination cell is in the borrowing state, and the calls are moved from NHR of the current cell to HR of the same cell. Also, \( j \cdot \mu^{(1)}_{ dl2} \cdot \left( 1 - \lambda^{(1)}_{ ml2} \right) \) is the rate at which a channel is borrowed from one of the 5 other F-groups of the destination cell. This is because the destination cell is in the borrowing state, and the calls are moved from the NHR of the current cell to the HR of the destination cell. Let \( S \) represent the set of all feasible states of the current cell. Then, the arrival rate \( \lambda^{(1)}_{ ml2} \) of low speed handoff calls in a microcell in slot \( k \) can be calculated \([2]\) as
\[
\lambda^{(1)}_{ ml2} = \left( \sum_{ s} j \cdot \mu^{(1)}_{ dl2} \cdot \frac{k}{S} \right) \cdot \left( \frac{\mu^{(1)}_{ dl2} + \mu}{\mu^{(1)}_{ dl2}} \right) + \lambda^{(1)}_{ ml1} \cdot \left( 1 - P^{(1)}_{ ml} \right) \cdot P^{(1)}_{ ml}.
\] (23)

The corresponding state transition rates are
\[
L^{(1)}_{ ml} = \lambda^{(1)}_{ ml1} + \lambda^{(1)}_{ ml2}.
\]
\[
M^{(1)}(i) = i \cdot \left( \mu + \mu^{(1)}_{ dl1} \right).
\]

The balance equation for the joint probability of the RV elements of the state vector \( S \) of the current cell \( p_{ ml}^{(1)} \) in slot \( k + 1 \) can be written as follows:
\[
p_{ ml}^{(1)} + 1 = \frac{A}{B},
\] (24)

where
\[
A = \delta(M \neq C_{ micn}) \cdot \left[ M^{(1)}(i + 1) \cdot p^{(1)}_{ ml+1,f,m} + (j + 1) \cdot \mu \cdot p^{(1)}_{ ml+1,f,m} + (e + 1) \cdot \mu \cdot p^{(1)}_{ ml+1,f,m} + (m + 1) \cdot \mu \cdot p^{(1)}_{ ml+1,f,m} \right] + \delta(j = 0) \cdot \lambda^{(1)}_{ ml1} \cdot p^{(1)}_{ ml+1,f,m} + \delta(j = 0) \cdot \lambda^{(1)}_{ ml2} \cdot p^{(1)}_{ ml+1,f,m} + \delta(e = 0) \cdot \lambda^{(1)}_{ ml1} \cdot p^{(1)}_{ ml+1,f,m} + \delta(e = 0) \cdot \lambda^{(1)}_{ ml2} \cdot p^{(1)}_{ ml+1,f,m} + \delta(m = 0) \cdot \lambda^{(1)}_{ ml1} \cdot p^{(1)}_{ ml+1,f,m} + \delta(m = 0) \cdot \lambda^{(1)}_{ ml2} \cdot p^{(1)}_{ ml+1,f,m}.
\]
\[ + \delta(M = C_{\text{micro}}, m \neq 0) \cdot (5/6)^{\lambda_{mh}^{(1)}} \cdot p_{s_{1}h}^{(1)} \cdot p_{l_{1}j+1,f,m-1}^{(1)} \]
\[ + \delta(M = C_{\text{micro}}, i \neq 0, m \neq 0) \cdot (j+1) \cdot \mu_{d_{2}}^{(1)} \]
\[ \cdot \frac{p_{l}^{(1,k)}}{6} \cdot p_{l_{1}-i-j+1,e,f,m-1}^{(1)} \]
\[ + \delta(M = C_{\text{micro}}, j \neq 0) \cdot \mu_{j}^{(1)} \cdot \left(1 - \frac{p_{l}^{(1,k)}}{6}\right) \cdot p_{l_{1}-i-j+1,e,f,m}^{(1)} \]
\[ + \delta(M = C_{\text{micro}}, j \neq 0) \cdot (i+1) \cdot \mu_{j}^{(1)} \cdot p_{l_{1}-i-j+1,e,f,m}^{(1)} \]

and

\[ B = \delta(M \neq C_{\text{micro}}) \cdot [T_{h}^{(1)} + \lambda_{mh}^{(1)} + \lambda_{mh}^{(1)} + \lambda_{mh}^{(1)} + \lambda_{mh}^{(1)}] \]
\[ + \delta(i \neq 0) \cdot M_{(i)} + \delta(j \neq 0) \cdot j \cdot \mu + \delta(e \neq 0) \cdot e \cdot \mu \]
\[ + \delta(m \neq 0) \cdot m \cdot \mu \]
\[ + \delta(M = C_{\text{micro}}, e \neq 0) \cdot (5/6)^{\lambda_{mh}^{(1)}} \cdot p_{s_{2}}^{(1,k)} \]
\[ + \delta(M = C_{\text{micro}}, j \neq 0, e \neq 0) \cdot j \cdot \mu_{j}^{(1)} \cdot \frac{p_{l}^{(1,k)}}{6} \]
\[ + \delta(M = C_{\text{micro}}, j \neq 0) \cdot j \cdot \mu_{j}^{(1)} \cdot \left(1 - \frac{p_{l}^{(1,k)}}{6}\right) \]
\[ + \delta(M = C_{\text{micro}}, i \neq 0) \cdot i \cdot \mu_{j}^{(1)} \]

At steady state, the sequence \( p_{s_{k}}^{(1)} \) \( k \to \infty \) converges to a common distribution \( \tilde{p} \). The following algorithm shows how this distribution can be found iteratively.

Algorithm to find state distribution of a microcell without handoff queueing:

**Step 1:** \( k = 0 \) and set a suitable tolerance, \( tol \).

**Step 2:** Input \( a, R, v, \beta, \lambda_{hh}^{(1)}, \) and \( \lambda_{hh}^{(1)} \).

**Step 3:** Arbitrarily, initialize \( \tilde{p}_{2}^{(1)} \).

**Step 4:** Using Eqs. (1)–(8) and (21)–(23) in Eq. (24), plus the normalization condition to compute \( p_{s_{2}}^{(1)} \).

**Step 5:** If \( |p_{s_{2}}^{(1)} - p_{s_{1}}^{(1)}| > tol \), then set \( p_{s_{2}}^{(1)} = p_{s_{2}}^{(1)} \), and go to step 4.

**Step 6:** Output the value \( \tilde{p} = p_{s_{2}}^{(1)} \) (convergence reached).

Once \( \tilde{p} \) is obtained, then we can find our measures as follows:

1. **New call blocking probability for low speed call:** A low speed new call is blocked if \( M = C_{\text{micro}} \). Thus, the new call blocking probability in a microcell is

\[ \tilde{p}_{b}^{(1)} = \sum_{M = C_{\text{micro}}} \tilde{p}_{s} \]  \hspace{1cm} (25)

On the other hand, since a new call arriving at the HR of the current cell can attempt to obtain a free channel from both the current and destination cells, then the new call blocking probability in a microcell \( \tilde{p}_{b}^{(1)} \) can be calculated as follows:

\[ \tilde{p}_{b}^{(1)} = a \cdot \left( \tilde{p}_{b}^{(1)} \right)^{2} + (1-a) \cdot \tilde{p}_{b}^{(1)} \]  \hspace{1cm} (26)

2. **Forced termination probability for low speed call:** A low speed handoff call is forcefully terminated if \( M = C_{\text{micro}} \) and \( f = 0 \) to the unavailability of free channels and the fullness of the handoff queue. This probability is as follows:

\[ \tilde{p}_{h}^{(1)} = \sum_{M = C_{\text{micro}}, f = 0} \tilde{p}_{s} \]  \hspace{1cm} (27)

3. **Traffic overflow rate:** The rate of overflow of low speed new calls \( \tilde{p}_{o}^{(1)} \) can be found to be

\[ \tilde{p}_{o}^{(1)} = N \cdot \left[ \tilde{p}_{h}^{(1)} \cdot \left( \tilde{p}_{l}^{(1)} \right)^{2} + \tilde{p}_{h}^{(1)} \cdot \tilde{p}_{h}^{(1)} \right] \]  \hspace{1cm} (28)

The rate of overflow of stationary new calls \( \tilde{p}_{o}^{(1)} \) can be found to be

\[ \tilde{p}_{o}^{(1)} = N \cdot \left[ \tilde{p}_{h}^{(1)} \cdot \left( \tilde{p}_{l}^{(1)} \right)^{2} + \tilde{p}_{h}^{(1)} \cdot \tilde{p}_{h}^{(1)} \right] \]  \hspace{1cm} (29)

The rate of overflow of low speed handoff calls \( \tilde{p}_{o}^{(1)} \) can be found to be

\[ \tilde{p}_{o}^{(1)} = N \cdot \tilde{p}_{h}^{(1)} \cdot \tilde{p}_{h}^{(1)} \]  \hspace{1cm} (30)

4.2. **Analysis of the macrocell**

We define some RVs. Let \( X_{hh}^{k} \) and \( X_{hh}^{k} \) be two RVs denoting the number of low speed calls in HR and NHR of a macrocell, respectively, in slot \( k \). Let \( \lambda_{hh} \) and \( \lambda_{hh} \) be two RVs denoting the number of handoff calls in HR and NHR of a macrocell, respectively, in slot \( k \). Let \( X_{hh}^{k} \) be a RV denoting the number of stationary calls participating in the handoff process in \( E \)-group and \( F \)-group of a macrocell, respectively, in slot \( k \). Let \( X_{hh}^{k} \) be a RV denoting the number of low speed handoff calls waiting in the macrocell queue in slot \( k \). Let \( X_{hh}^{k} \) be a RV denoting the number of high speed handoff calls waiting in the macrocell queue in slot \( k \). Clearly, \( X_{hh}^{k} + X_{hh}^{k} + X_{hh}^{k} + X_{hh}^{k} + X_{hh}^{k} + X_{hh}^{k} \leq C_{\text{macro}} \) and \( X_{hh}^{k} + X_{hh}^{k} \leq Q \). Let the state of a macrocell be defined by the vector \( v = (i, j, k, l, e, f, m, q_{1}, q_{3}) \). Then, the joint probability of the RV elements of the state vector \( v \) is

\[ p_{v}^{k} = \Pr[X_{hh}^{k} = i, X_{hh}^{k} = jX_{hh}^{k} = k, X_{hh}^{k} = l, X_{hh}^{k} = e, X_{hh}^{k} = f, \]
\[ X_{hh}^{k} = m, X_{hh}^{k} = q_{1}, X_{hh}^{k} = q_{2}] \].

Let \( M(v) \) denote the number of channels in use in state \( v \). Thus, \( M(v) = i + j + k + l + e + f + m \) and \( M(v) \leq C_{\text{macro}} \), and \( q_{1} + q_{2} \leq Q \). Let \( p_{v}^{k} \) be the probability that a macrocell is in the borrowing state in slot \( k \), and is calculated as follows:

\[ p_{v}^{k} = \sum_{c_{2}} \tilde{p}_{v}^{k} \], \hspace{1cm} (31)

where

\[ C_{2} = \{ v : M(v) = C_{\text{macro}}, f > 0 \} \].

By considering Fig. 4, which is the transition diagram of the macrocell containing \( C_{\text{macro}} \) channels and queue size \( Q = q_{1} + q_{2} \). Let represent the set of all feasible states. The arrival rate \( \lambda_{hh}^{(k)} \) of low speed handoff calls in a macrocell in slot \( k \) can be calculated as follows:

\[ \lambda_{hh}^{(k)} = \sum_{v} j \cdot \mu_{d_{2}}^{(1)} \cdot \left( \frac{p_{l}^{(1,k)}}{p_{l}^{(1,k)} + \mu} \right) \cdot \tilde{p}_{h}^{(1)} \cdot \left(1 - \frac{p_{l}^{(1,k)}}{6}\right) \]  \hspace{1cm} (32)

where \( \tilde{p}_{h}^{(1)} \) is the forced termination probability of the macrocell. The arrival rate \( \lambda_{hh}^{(k)} \) of high speed handoff calls in a macrocell in slot \( k \) can be calculated as follows:

\[ \lambda_{hh}^{(k)} = \sum_{v} j \cdot \mu_{d_{2}}^{(1)} \cdot \left( \frac{p_{l}^{(1,k)}}{p_{l}^{(1,k)} + \mu} \right) \cdot \tilde{p}_{h}^{(1)} \cdot \left(1 - \frac{p_{l}^{(1,k)}}{6}\right) \]  \hspace{1cm} (33)

where \( \tilde{p}_{h}^{(1)} \) is the forced termination probability of the macrocell.
\[ j^{2h}_{nk} = \left( \frac{2}{v} \right) \sum m \cdot \left( \frac{p^{(2)}_{zh,k} - 1}{h^{(2)}_{zh}} \right) \cdot \left( \frac{p^{(2)}_{zh,k} - 1}{h^{(2)}_{zh}} + \sum h^{(2)}_{zh} \right) \cdot (1 - p^{(2)}_{zh}) \cdot p^{(2)}_{zh} \]

The corresponding state transition rates are:

\[ L^{(2)}_{zh} = \lambda^{v}_{zh} + \lambda^{u}_{zh} + j^{2h}_{zh} \]
\[ H^{(2)}_{zh} = \lambda^{v}_{zh} + \lambda^{u}_{zh} \]

\[ M1(i) = i \cdot \left( \mu + p^{(2)}_{zh} \right) \]
\[ M2(k) = k \cdot \left( \mu + p^{(2)}_{zh} \right) \]
\[ M3(i, q_i) = i \cdot \left( \mu + p^{(2)}_{zh} \right) + q_i \cdot \left( \mu + p^{(2)}_{zh} \right) \]
\[ M4(k, q_k) = k \cdot \left( \mu + p^{(2)}_{zh} \right) + q_k \cdot \left( \mu + p^{(2)}_{zh} \right) \]

The balance equation for the joint probability of the RV elements of the state vector \( \bar{r} \) of the current cell \( p^{(2)}_{rk} \) in slot \( k + 1 \) can be written as follows:

\[ p^{(2)}_{rk} + 1 = \frac{C}{D} \tag{33} \]

where

\[ C = \delta \cdot M(\bar{r}) \cdot C_{macro} \cdot \left[ M1(i + 1) \cdot p^{(2)}_{i+1,j,k,l,m,q,r} \right. \]
\[ + (j + 1) \cdot \mu \cdot p^{i+1,j,k,l,m,q,r} + M2(k + 1) \cdot \mu \cdot p^{i+1,j,k,l,m,q,r} \]
\[ + (e + 1) \cdot \mu \cdot p^{i,j,k,l,m,q,r} + (f + 1) \cdot \mu \cdot p^{i,j,k,l,m,q,r} \]
\[ + (m + 1) \cdot \mu \cdot p^{i,j,k,l,m,q,r} + \delta(i \neq 0) \cdot L^{(2)}_{i+1,j,k,l,m,q,r} \]
\[ + \delta(k \neq 0) \cdot H^{(2)}_{i+1,j,k,l,m,q,r} \]
\[ + \delta(t \neq 0) \cdot \lambda^{v}_{zh} + \delta(t \neq 0) \cdot \lambda^{u}_{zh} \]
\[ + \delta(m \neq 0) \cdot \lambda^{v}_{zh} + \delta(m \neq 0) \cdot \lambda^{u}_{zh} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, m \neq 0) \cdot (5/6) \cdot (\lambda^{v}_{zh} + \lambda^{u}_{zh}) \cdot p^{(2)}_{i+1,j,k,l,m,q,r} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, q \neq 0) \cdot \lambda^{v}_{zh} \cdot p^{i,j,k,l,m,q,r} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, q 
eq 0) \cdot \lambda^{u}_{zh} \cdot p^{i,j,k,l,m,q,r} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, i \neq 0, m \neq 0) \cdot (j + 1) \cdot \frac{p^{(2)}_{i+1,j,k,l,m,q,r}}{6} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, i \neq 0) \cdot (j + 1) \cdot \frac{p^{(2)}_{i+1,j,k,l,m,q,r}}{6} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, k \neq 0, m \neq 0) \cdot (l + 1) \cdot \frac{p^{(2)}_{i+1,j,k,l,m,q,r}}{6} \]
\[ + \delta(M(\bar{r}) \neq C_{macro}, k \neq 0) \cdot (l + 1) \cdot \frac{p^{(2)}_{i+1,j,k,l,m,q,r}}{6} \]

Figure 4  Transition diagram of themacell containing C channels and queue size Q.
At steady state, the sequence \( \{ p_\infty^{(i)} \} \) converges to a common distribution \( p_\infty \). The following algorithm shows how this distribution can be found iteratively.

Algorithm to find state distribution of a macrowell with handoff queueing:

The results obtained from the iterative algorithm for microcell are provided to the iterative algorithm for macrocell without any handoff queueing.

**Step 1:** Set \( k = 0 \) and set a suitable tolerance, \( tol \).

**Step 2:** Input \( a, b, R, \nu, \beta, \mu, \nu_{\text{a}}, \nu_{\text{b}}, \nu_{\text{c}}, \nu_{\text{d}}, \) and \( \nu_{\text{e}} \).

**Step 3:** Arbitrarily initialize \( p_0^{(i)} \).

**Step 4:** Using Eqs. (1), (9)–(21) and (31)–(33) in Eq. (34), plus the normalization condition to compute \( p_\infty^{(i)} \).

**Step 5:** If \( |p_\infty^{(i)} - p_\infty^{(i-1)}| > tol \), then set \( p_\infty^{(i)} = p_\infty^{(i-1)} \), and go to step 4.

**Step 6:** Output the value \( p_\infty = p_\infty^{(i)} \) (convergence reached).

Once \( p_\infty \) is obtained, thus, we can find our measures as follows:

**1. New call blocking probability for high speed call:** A high speed new call is blocked if \( M(\overline{v}) = C_{\text{macro}} \). Thus, the new call blocking probability in a macrocell is given by

\[
p_\infty^{(2)} = \sum_{M(\overline{v}) = C_{\text{macro}}} p_{\overline{v}}.
\]

On the other hand, since the new call arriving at HR can attempt to obtain a free channel from both the current and destination cells, thus, the new call blocking probability in a macrocell \( p_\infty^{(2)} \) can be calculated as follows:

\[
p_\infty^{(2)} = b \cdot \left( p_\infty^{(2)} \right)^2 + (1 - b) \cdot p_{\overline{v}}.
\]  

**2. Forced termination probability for high speed call:** Two probabilities are defined. The first, is the probability \( p_f^{(2)} \) that a handoff call is forcefully terminated due to the unavailability of free channels and the fullness of the handoff queue. This probability is as follows:

\[
p_f^{(2)} = \sum_{M(\overline{v}) = C_{\text{macro}} + q_l = Q} p_{\overline{v}}.
\]

The second probability is the probability \( p_f^{(2)} \) that a low (high) speed handoff call is forcefully terminated due to the expiration of queueing time of the call before it can obtain a free channel from the destination cell. Thus, we can write the second probability as in [2]

\[
p_f^{(2)} = \sum_{M(\overline{v}) = C_{\text{macro}} - q_l = Q} p_{\overline{v}} \cdot (1 - \beta) \cdot p_{\overline{v}}
\]

Thus, the forced termination probability for low (high) speed handoff call \( p_{\overline{v}}^{(2)} \) in a macrocell can be written as follows:

\[
p_{\overline{v}}^{(2)} = p_f^{(2)} + p_{\overline{v}}^{(2)}
\]

After calculating the new call blocking probability and the forced termination probability in a macrocell, we find the new call blocking probability, and the forced termination probability for the overall system for the two types of calls. Thus, the new call blocking probability for high speed call is \( p_{\overline{v}}^{(2)} \), which is given by (36). The forced termination probability for high speed call is \( p_{\overline{v}}^{(2)} \), which is given by (41). However, the new call blocking probability for low speed call \( p_{\overline{v}}^{(2)} \) is given by the following equation, using (36) for \( p_{\overline{v}}^{(2)} \) and (26) for \( p_{\overline{v}}^{(1)} \)

\[
p_{\overline{v}}^{(2)} = p_{\overline{v}}^{(2)} \cdot p_{\overline{v}}^{(2)}
\]

The forced termination probability for low speed call \( p_{\overline{v}}^{(2)} \) is given by the following equation, using (40) for \( p_{\overline{v}}^{(2)} \) and (27) for \( p_{\overline{v}}^{(1)} \):

\[
p_{\overline{v}}^{(2)} = p_{\overline{v}}^{(2)} \cdot p_{\overline{v}}^{(2)}
\]

5. Numerical results

In this section, we calculate, plot, and discuss the performance measures of interest: new call blocking probability and forced termination probability. We assume that every macrocell covers \( N = 7 \) microcells. We also assume that the low and high speeds of the mobile user are 3.6 and 36 km/h, respectively. Also, we assume \( C_{\text{macro}} = 4, C_{\text{micro}} = 5, Q = 3, \frac{1}{\mu} = 180 \text{ s}, R_{\text{macro}} = 300 \text{ m}, R_{\text{micro}} = 300^2 \text{ m}, a = b = 0.3, \beta = 0.37 \), and choose the tolerance \( tol \) to be \( 10^{-6} \). We will validate our results by comparing with the results of model C in Shun [2] which is similar to our model, except for channel
borrowing—the main feature of the present work. Thus in the graphs of this section we will plot always two curves: one for Shun Model C and one for our work, in order to show the impact of using channel borrowing on the system performance.

Figure 5  New calls blocking probability of low speed users for different arrival rates of low speed new calls.

Figure 6  Forced termination probability for low speed users for different arrival rates of low speed new calls.
The performance measures of low speed calls are shown in Figs. 5 and 6, respectively, with the high speed new call arrival rate fixed at 0.03. Fig. 5 shows that the new call blocking probability for low speed call of the proposed model is lower than that of the Shun model C by an average of 12%. This improvement is due to the channel-borrowing handoff scheme serving more moving handoff calls by borrowing channels from stationary calls to serve moving handoff calls.

Figure 7  New call blocking probability for high speed users for different arrival rates of high speed new calls.

Figure 8  Forced termination probability for high speed users for different arrival rates of high speed new calls.
Fig. 6 shows the forced termination probability \( p_{fh} \) of low speed calls vs. new high speed call arrival rate, in both the proposed model and model C in Shun [2]. It can be seen that the forced termination probability in the proposed model is lower than that of the Shun model C by an average of 0.1%. This improvement is admittedly insignificant. The performance measures of high speed calls are shown in Figs. 7 and 8, respectively, with the sum of stationary and low speed new call arrival rates fixed at 0.04. The improvement due to the channel borrowing scheme of the proposed models is made clear by plotting the same measure for both the proposed model and that of Shun model C.

Fig. 7 shows that the new call blocking probability \( p_{bn}^{(2)} \) of high speed calls in the proposed model lower than that in the Shun model C by an average of 9%. Similarly, Fig. 8 shows that the forced termination probability \( p_{fh}^{(2)} \) of high speed calls in the proposed model is lower than that in the Shun model C by an average of 26%. The improvement may not seem significant, but in view of the fact that call blocking and forced termination are highly undesired by mobile users, any amount of improvement is badly sought.

We can see that the improvement in performance due to the channel borrowing scheme is more in the case of high speed calls that low speed calls. This can be attributed to the fact that low speed calls make less handoffs than high speed calls and thus benefit less from the channel borrowing scheme introduced in the proposed model. Recall that low speed calls can operate both in macrocells and microcells. In the former case, they understandably need to handoff less often because they reach the border of the cell slowly. In the latter case, they reach the border of the cell quickly, but then they can be overflowed to a macrocell, avoiding many future handoffs.

6. Conclusions and future work

In this paper, we consider CDMA cellular networks of two tiers, where users with different mobility behaviors are assigned to the proper tier. Namely, high speed users are always assigned to the higher tier (macrocells), even if the speed decreases during the call to the low level. By contrast, low speed users are initially assigned to the lower tier (microcells), but if during their movement they reach the edge of the current (micro)cell and fail to get handed over to the neighbor (micro)cell, then they are assigned to the current higher tier (macrocell). For such networks a channel borrowing handoff scheme aiming at improving the quality of service without deteriorating the throughput of the system is proposed. To analyze the performance of the scheme, the system is modeled as a Markov chain, and utilize an iterative method to find the steady-state probability distribution. For validation and comparison purposes, numerical results have been obtained for both an example system using our scheme and an identical system not using the scheme. The results show that the scheme reduces the blocking probability of low speed calls by an average of 12%, and of high speed call by an average of 9%. On the other hand, the results shown that the scheme reduces the forced termination probability of low speed calls by an average of 0.1%, and of high speed calls by an average of 26%.

A two-tier CDMA cellular networks, utilizing the channel-borrowing handoff scheme without any queue in both microcell and macrocell is currently under investigation. Also, a two-tier CDMA cellular networks can be presented, utilizing the channel-borrowing handoff scheme with a FIFO queue in both macrocell and microcell.

References


Further reading