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Water Distribution Network Sectorisation Using Structural Graph Partitioning and Multi-Objective Optimization

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Abstract

Partitioning a water distribution network (WDN) into smaller sub-networks (called district metered areas, or DMAs) is a strategy to manage its complexity. A number of requirements for WDN partitioning make existing graph partitioning techniques inefficient at finding a good solution. There are also other structural and hydraulic constraints, such as partition size, minimum nodes’ elevation difference in partitions, and water velocity in pipes that make the identification of an efficient partitioning a challenging problem. In this paper, we propose a technique called WDN-Cluster to solve this partitioning problem for gravity-driven water distribution networks. WDN-Cluster applies a combination of structural graph partitioning and multi-objective optimization based on NSGA-II to find a good arrangement of nodes into DMAs.

Keywords: Water Distribution Network (WDN); District Metered Area (DMA); Graph Partitioning; Multiobjective Optimization; NSGA-II.

1. Introduction

A water distribution network (WDN) is the infrastructure that supplies drinking water to homes and businesses, linking water sources and consumers. Such networks are typically complex and dynamic, consisting of thousands of nodes with nonlinear hydraulic behavior (including reservoirs, tanks, and consumption nodes), linked by thousands of interconnecting links (including pipes, pumps and valves).

Partitioning a WDN (also known as a pressure zone, a discrete hydraulic sector, or a leakage district [1]) into smaller sub-networks is a strategy to manage the complexity of a WDN, as advised by the International Water
Association (IWA) [2]. Each sub-network, called a district metered area (DMA) [1], is defined as a discrete area of a distribution system, and is created by the closure of valves or by completely disconnecting pipes, in which the quantities of water entering and leaving the area are metered [2], [3]. The process of partitioning a WDN into a set of independent DMAs is referred to as water network sectorisation (WNS) [4].

There are many reasons for partitioning water networks into zones, but the core goal is to achieve better control over the distribution of water [5]. The main benefits of partitioning in this regard include: enhanced leakage and burst detection and management, a capacity to provide different pressure levels which help in the establishment of a permanent pressure control system (pressure zones), improved contamination spread control which is directly associated to water security, and enhanced rehabilitation and work planning [6]. Partitioning a water network is a major step towards a water smart grid, which is a transformation of the current water supply system using ICT capabilities [7], as it can provide a greatly enhanced monitoring and control capacity over the network.

Partitioning a WDN is a challenging problem. Considered as a graph-partitioning problem, it is NP-hard. Moreover, the combination of often-conflicting functional and non-functional characteristics and requirements of a water distribution add to the complexity. In particular, water security requirement imposes two important constraints on the partitioning of a WDN. First, water should not flow from one partition to another one. This requires that each distinguished partition must have direct access to a water source, and if there is no source in a partition, the paths from all nodes of the partition to a source must not contain nodes in other partitions. Second, there must be no flow exchange between different partitions. Besides security, also other factors should be taken into account when partitioning a WDN into a collection of DMAs. For example, the size of the partitions must be balanced and be in a predefined boundary (between 500-3000 customer connections is a typical configuration); the nodes in a partition must have similar hydraulic characteristics, especially in terms of elevation; preferably there should be a minimum cut size (both in terms of the number of the links that should be cut and equipped with meters and valves, and the diameter of them); and the partitioned network must satisfy the minimum service requirements for the customers. Quality water should be supplied at the required pressure at the required time to different customers continuously during different consumption scenarios (normal daily, peak daily, nightly, and fire flow). In addition, there are hydraulic constraints that further complicate the design of solutions: limited water velocity in pipes, limited pressure at nodes, and the levels of the tanks that should be in a boundary; while maximum network resilience and minimum dissipated power in the network are preferable [2], [8], [9]. We call these requirements DMA design criteria.

If water network sectorisation is not performed in accordance with the DMA design criteria, it can cause supply problems, reduction of reliability and efficiency, and water quality issues. However, Murray et al. [9] showed that if the DMA design is performed appropriately, water network sectorisation can have advantages in terms of water security and leakage detection, without compromising the network reliability or water quality. However, current techniques for WDN partitioning prioritize only a subset of the full set of criteria, and as a result do not fully comply with the best practice as regards DMA design. In particular, water security (direct access to water source for each partition, and partition independence), minimum elevation difference in partitions, holistic pressure management considering different scenarios, water quality, and limited water velocity are generally neglected.

In this paper, we propose a novel WDN partitioning technique (WDN-Cluster) for solving this multifaceted partitioning problem for gravity-driven water distribution systems. WDN-Cluster applies a novel structural graph partitioning technique and a multi-objective optimization method based on NSGA-II to find a good partitioning arrangement of the nodes into different partitions.

This paper is organized as follows. Section 2 states the problem. Section 3 reviews the literature. The proposed method is explained in Section 4. Evaluation is discussed in Section 5, and finally, Section 6 concludes the paper and discusses future work.

2. Problem Statement

Partitioning a water distribution network into DMAs involves [10]:

1. Assigning all nodes in the network into different DMAs, ensuring each partition has direct access to a water source, and is also a “good” size,
2. Deciding how many and which links should be closed off to restrict the DMAs and how many and which links need to be left open and be equipped with flow meters. The goal here is that the DMAs are independent of each other, the reliability of the network is not decreased dramatically and the minimum levels of service for customers are guaranteed.

These two measures can be achieved by changing the status of the links; i.e., whether a link is open (=1) or closed (=0) can determine the DMAs’ boundaries. Therefore, WDN partitioning can be formulated as a least-cost optimization problem with a selection of links’ status as the decision variables. The links layout and their connectivity, nodal demand, and minimum head requirements are assumed to be known. In a general form, the optimization of WDN partitioning can be stated as: find the best combination of links’ status to partition the network into a collection of DMAs, while optimizing the following objectives and satisfying the following constraints.

2.1. Objectives

There are two sets of objectives in this partitioning problem: structural objectives, and hydraulic objectives. Structural objectives are related to the structure of the network and include minimum cut size (equation 1) and minimum boundary links diameters (equation 2).

Hydraulic objectives are related to hydraulic behaviour of the network after partitioning. There are a set of hydraulic optimization objectives, which are considered in the partitions’ boundary adjustments. In partitioning a WDN, these fitness measures must be checked to assess whether the partitioning is feasible. Hydraulic objectives include: minimum pressure requirements violations (equation 3), minimum dissipated power (equation 4), minimum elevation difference in partitions (equation 5), and maximum network resilience (equation 6).

2.2. Constraints

There are also structural and hydraulic constraints. Structural constraints are related to the structure of the network, i.e., connectedness, direct access to source, and partition size (DMA size). Connectedness (equation 7) means that each DMA should be a connected sub-graph and the whole network should be connected as well. Direct access to source (equation 8) means each partition must have direct access to a water source and water should not flow between partitions. DMA size constraints (equation 9) are either based on the number of customer connections (which should normally be between 500 and 3000) or total demand of the partitions. Hydraulic constraints govern the physics of the problem or the hydraulic limitations. They include conservation of mass (equation 10), conservation of energy (equations 11-12), pressure limitations (equations 13-14), flow limitations (equation 15), water velocity limitations (equation 16), tanks level limitations (equation 17), and link status bounds (equation 18) which states that the status of the links, which are the only decision variables in this problem are bounded to \{0, 1\}.

Mathematically the formulae can be stated as:

Minimize: \( OF_{CS} = |Cuts| \) \( (1) \)

Minimize: \( OF_{BLD} = \sum_{l \in \text{Cuts}} \text{diameter}(l) \) \( (2) \)

Minimize: \( OF_{PR} = \sum_{s \in \text{scen}} \sum_{j=1}^{N} C_{s,j}^{\text{pen}} \times \max\left(0, H_{s,j}^{\text{min}} - H_{s,j}\right) \) \( (3) \)

Minimize: \( OF_{DP} = \gamma \sum_{j=1}^{m_s} q_i \Delta H_i \) \( (4) \)

Minimize: \( OF_{el} = \sigma_{el} = \sum_{d=1}^{k} \sigma_d \) \( (5) \)

\( \sigma_d \) is the variance of elevation of nodes in \( dm_{a_d}. \)
Maximize: \[ OF_{NR} = I_m = \sum_{i=1}^{n} C_i P_{s,i} / P_{max} \]

Subject to:
- \( \forall u, v \in \text{nodes}, \text{there is a } u \rightarrow v \text{ walk in } dma_i \)
- \( \forall dma_i \in \text{DMA}, \forall u \in \text{nodes}, \exists s \in \text{Sources} \)
- where there is a \( u \leftrightarrow v \text{ walk in the clustered network, where for any } v \in \text{the } u \leftrightarrow v \text{ walk, } \delta_i \) \( j \) where \( v \in dma_j \)

\( \forall i : dma_{\text{MinSize}} < \text{size}(dma_i) < dma_{\text{MaxSize}} \)

\[ - \sum Q_{in} + \sum Q_{out} + DM = 0 \]

\[ \sum_{j=1}^{M} Q_{j,s} = Q_{c_{j,s}} \quad i = 1, 2, \ldots, N \]

\[ \Delta H_{j,s} = K_j Q_{j,s} \quad \text{for } M \text{ pipes} \]

\[ P_{s,\text{min}} \leq P_{i,s} \leq P_{s,\text{max}} \quad \forall i \in \text{Nodes}, t \in T \]

\[ H_{i,s} \geq H_{i,s}^{\text{min}} \quad i = 1, 2, \ldots, N \]

\[ Q_{s,\text{min}} \leq Q_{s,t} \leq Q_{s,\text{max}} \quad \forall i \in \text{Nodes}, t \in T \]

\[ V_{p,s} \leq V_{\text{max}} \quad \forall p \in \text{Pipes} \quad \text{and} \quad t \in T \]

\[ P_{s,\text{min}} \leq P_{i,s} \leq P_{s,\text{max}} \quad \forall i \in \text{ Tanks}, t \in T \]

\[ x_i = \text{LinkStatus}_i = \{0, 1\}, \quad (i \in \text{Links}) \]

In (6), the surplus power at node \( i \) is given by \( P_{s,i} = \gamma Q_i (H_i - H_i') \), where \( H_i' = z_i + \frac{1}{r_i} \) is the design pressure for the \( i \)-th node, and \( z_i \) is the elevation at node \( i \), and \( C_i = \frac{\sum_j n_{p,i} d_j}{n_{p,i} \text{ max} \{d_1 \ldots d_{p,j}\}} \)

where \( n_{p,i} \) is the number of pipes incident with the node \( i \) and \( d_j \) is the diameter of the incident pipe [12].

\[ DM \text{ is the demand at the node, } Q_{in} \text{ and } Q_{out} \text{ are the flow entering and leaving the node, respectively.} \]

\[ N = \text{number of nodes in the network;} \quad M = \text{number of links in the network;} \quad I = \text{network incidence matrix} \]

\[ (N \times M); \quad Q_{j,s} = \text{flow in pipe } j \text{ in scenario } s \text{ (m}^3\text{s)}; \quad Q_{c_{j,s}} = \text{consumption in node } i \text{ in scenario } s \text{ (m}^3\text{s)} \]

\[ \Delta H_{j,s} = \text{head loss in pipe } j \text{ in scenario } s \text{ (m)}; \quad K_j = \text{coefficient dependent on characteristics of pipe } j \text{ [13].} \]

\[ P_{s,\text{min}} \text{ and } P_{s,\text{max}} \text{ are the maximum and minimum pressure requirements and } P_{i,s} \text{ is the service pressure in node } i \text{ at time } t \text{ in scenario } s; \quad \text{Nodes is the set of all nodes;} \quad T \text{ is the simulation period (usually 24 h, with step of 1 h).} \]

\[ H_{i,s} = \text{head for node } i \text{ for scenario } s; \quad H_{i,s}^{\text{min}} = \text{minimum admissible head for node } i \text{ for scenario } s \text{ (m) at time } t; \quad Q_{s,\text{min}} \text{ and } Q_{s,\text{max}} \text{ are the maximum and minimum flow requirements and } Q_{t,s} \text{ is flow in node } i \text{ at time } t \text{ in scenario } s \text{ at time } t. \]

\[ V_{p,s} = \text{the flow velocity in pipe } p \text{ at time } t \text{ (m/s) in scenario } s; \quad V_{\text{max}} = \text{the maximum flow velocity allowed (m/s).} \]

\[ L_{s,\text{min}} \text{ and } L_{s,\text{max}} \text{ are the maximum and minimum tank levels for each tank } i \text{ and } T_{i,s} \text{ is the level of tank } i \text{ at time } t \text{ in scenario } s \text{ at time } t. \]

\[ X = \{x_i\}, \forall i \in \text{Links}; \text{ the vector of decision variables.} \]

3. Related Work

A number of approaches proposed for partitioning a WDN into a collection of DMAs satisfy some subset of the specific requirements of this problem, e.g., [10], [14]–[17]. However, though these methods are designed specifically for WDN partitioning, they do not satisfy one of the most important requirements: direct access to a source, which is critical for water security. Only three approaches consider this requirement as a design factor.

Fernández et al. [6] use a multi-agent approach to divide a WDN into a set of DMAs. The work is limited to small size networks, as the number of DMAs identified equals the number of sources, and also, no hydraulic objective is considered. Di Nardo et al. [11] propose a methodology to design water network sectorisation based on graph theory principles. This methodology allows direct access to the water source for each DMA; however, it is also limited to small-sized networks, as the number of DMAs created is equal to the number of main sources. Ferrari et al. [18] propose a method which satisfies the direct access to source constraint, but has some limitations. If the size of an independent district is large and the number of connections between the district and the transmission mains are few, this method suggests adding some new pipes between the transmission mains and the district. This may not be
possible in many situations because of physical and infrastructure considerations, and even if it is possible, it is costly. Another limitation is that it does not consider nodes elevation as a design factor, despite the fact that this is an important decision criterion in real-world water distribution network partitioning projects.

As discussed, only three approaches consider direct access to water source as a design factor. No approach studies holistic pressure management considering all different scenarios (normal daily, peak daily, night-time, and fire flow). Elevation, despite its vital importance in real-world WDN partitioning projects, is considered as a design criteria in only two approaches [6], [17]. Limited water velocity is considered in just one [16]; network resilience in three [10], [11], [15]; and dissipated power and water quality in only two [11], [15].

4. WDN-Cluster

We propose WDN-Cluster as an approach to partitioning a graph with two types of nodes (sources and consumers) that addresses the combined challenges of (a) each partition having direct access to a source (i.e., if there is no source in a partition, the path from that partition to a source does not include any node in other partitions), (b) balancing the size of partitions in a predefined boundary, (c) partitioning without adding any edge to the graph. Fig. 1 shows the flowchart of WDN-Cluster, detailed as follows:

Step 1 (Initialization): The network data is read from an EPANET 2 [19] model, and some parameters are set, e.g., MainsSizeThreshold, which is the threshold size of the main pipes; dmaMinSize and dmaMaxSize, which are the minimum and maximum size of a partition; LinkCount, which is the number of links in the network; and template, which is a string of length LinkCount and will be used as a template for possible solutions.

Step 2 (findAllPotSrcs): This process finds the major flow paths or transmission mains (transMains) from the main sources and considers nodes in these paths as new potential sources (ptnSrcs). The partitions can be identified starting from these potential sources, which can provide direct access to a source for each partition. The potential sources should not be part of any partition to ensure that partitions are independent. We start from the main sources (reservoirs) in the network and take advantage of a modified depth first search [20] algorithm in which the nodes are identified in the direction of flow when their connecting links are larger than a specified threshold (MainsSizeThreshold) and added to the list of ptnSrcs. All the links on the way are added to the transMains list and the connecting links’ status is set to 1 (open) in the template of solutions.

Step 3 (findPIG4aSrc): The potentially independent groups of nodes in the WDN connecting to each ptnSrcs will be identified and added to the list of indpGrps. After this step, some groups might be equal. This may happen for the groups of nodes which are connected to the transmission mains by more than one link. The equal sets will be removed and just one of them will remain.

Step 4 (Check Size): In this step DMA size constraint is examined. For each independent group in indpGrps list (let’s call it indpGrpI), if its size is less than the minimum allowable size of a DMA (dmaMinSize), it will be considered as an independent group of nodes (indpNodes) and there is no need to install meter for them. If its size is in the range of the allowable size of a DMA (between dmaMinSize and dmaMaxSize), it will be considered as a DMA, and the first link from the corresponding potential source will be added to Meters set. If its size is greater than the maximum allowable size of a DMA (dmaMaxSize), it must be partitioned into a number of DMAs all having direct access to the transmission mains, and consequently to a water source. Partitioning is done in a separate method called PartitionIG (Step 5).

Step 5 (PartitionIG): Fig. 2 shows the flowchart of PartitionIG method, which partitions large groups of nodes into smaller sub-networks, and has the following sub-steps:

Step 5.1 (Initializations): Read indpGrpI data, set some parameters related to optimization algorithm, e.g., number of variables which is LinkCount, number of objectives, maximum number of main loop iteration (MaxMainIt), maximum number of internal loop iteration (MaxSubIt), number of populations, probability of crossover, probability of mutation, and mutation ratio. For detail information about these parameters refer to [21].

Step 5.2 (Define ptnSrcs4IG): Let ptnSrcs4IG be the intersection of ptnSrcs and indpGrpI, and S be \[ \text{size(} \text{indpGroups} \text{)} / \text{dmaMaxSize} \], and T be the number of nodes in ptnSrcs4IG. If T is less than S, then we select the node with the largest links diameter and the maximum flow from ptnSrcs4IG, and using a BFS, we select the first S-T (S minus T) nodes from indpGrpI and add them to ptnSrcs4IG. If T equals S, we set MaxMainIt to 1.
Step 5.3 (Select S ptSources): In a loop of MaxMainIt iterations, we select S nodes from ptnSrcs4IG on random and put them into ptSources list. Note that T might be larger than S, so we use random selection.

Step 5.4 (DivideIntoSGroups): In this step, we divide indpGrpI into S Groups. Fig 3 shows the flowchart of this method. From each node i in ptSources we start a modified BFS in the direction of flow in parallel. If the neighboring node is still in indpGrpI, we add it to group; and remove it from indpGrpI and we set the connecting link status as 1 (open) in the solution template. If the neighboring node is not in indpGrpI, it means that it had joined another group (let’s call it group) before we reach it from node i. In this case we set the status of the neighboring link (the link between group, and group) and its two neighbors (one in group, and the other one in group) as X (unknown) in the solution template, so these links can choose their status in the next stages of the process. It should be considered that the status of one of these links should be set to 0 (closed) and the other ones to 1 (open). So a constraint is that the values of these variables should sum up to 2, i.e., one closed link (0) and two open links (1). At the end of this process, we will have a draft of the partitioning that should be realized by setting exact values for the status of the neighboring links.

Step 5.5 (NSGA-II): In this step, we find the status of the links in the partitions considering the objectives and constraints discussed in Section 2. To this aim, we use NSGA-II [21], which is a multi-objective genetic algorithm, with some modifications. The first modification is feasibility assessment of the solutions before their evaluations. If a generated solution is not structurally feasible, it will be removed from population. In feasibility assessment of a solution, we check structural constraints. A solution is feasible if it is connected, i.e., the whole graph is connected, so as the partitions; it partitions the main network into single fed DMAs; there is no flow exchange between different partitions; and partitions are directly connected to a source. The second modification is using template to generate solutions; i.e., any generated solution, either randomly generated ones in the initial population or next
generations which are generated based on last generations by applying genetic operators (crossover or mutation), must conform to the template. The third modifications is that we save the Pareto front at the end of each NSGA-II call, which is the internal iteration of PartitionIG method.

The initial solution population can be generated randomly considering the template and the constraint discussed in Step 5.4. For this, a link among each set of neighboring links should be considered as closed (the corresponding bit should be set to 0) on random and the rest of the links should be considered as open. This will make the genotypes (the particular set of genes contained in a genome) of different possible solutions. The phenotype (structural and hydraulic characteristics) of a solution can be obtained by setting the related genome as the status of the links in the algorithm, and asking the hydraulic solver (EPANET 2) to solve the network hydraulic equations for this new network. Then, we have the new state variables like pressure, flow, head, head loss, velocity, etc. of the solution (phenotypic characteristics) and we can calculate the fitness functions and check the constraint violations for it.

Then in a loop of MaxSubIt times (the loop of genetic algorithm), solutions being sorted and selected for crossover and mutation to generate next generations, and the new set of solutions are obtained (offspring). The new population, which is the last population plus new population generated by genetic operators (crossover and mutation), will be merged, sorted (using non-dominated sorting), and truncated. Finally, after MaxSubIt iterations, the method will provide a set of non-dominated solutions; i.e., no other solution is better than them in terms of all the objective functions, and they are better than any other solution in at least one objective function. This set is saved to be reported. It should be noted that we consider constraints as penalty functions and define them as objective functions that should be minimized. Totally, we have 10 objective functions in this problem.

Step 5.6: Decrease MaxMainIt by 1, and if it is greater than zero, go to Step 5.3; otherwise,

Step 5.7 (Finish and Report all Pareto Fronts): A set including all Pareto fronts will be returned to WDN-Cluster.

Step 4 and 5 will be repeated until all indpGrps are examined.

Step 6 (Finish and Report Results): The final results (including DMAs, indpNodes and all set of all Pareto fronts) will be given to domain experts to choose the best one based on other requirements of the problem (e.g., financial requirements, infrastructure constraints, etc.) and their experience.

5. Conclusion

WDN-Cluster is designed for the specific requirements of a water distribution network, to partition it into a set of DMAs, which are independent of each other and where water cannot flow between different partitions. The method provides direct access to water sources for each partition. Structural constraints (direct access to source, independence, and connectedness) and hydraulic constraints (water velocity, tanks’ levels limitations, and head, flow and pressure limitations in different consumption scenarios) are considered in this method. Although the method is in its preliminary steps of development, encouraging outcomes were achieved, but unfortunately, because of space limitations we could not report the results of the method.

Despite its advantages, WDN-Cluster has limitations. First, its focus is gravity-driven water distribution networks; i.e., the networks without pumps in the distribution mains. However, this method can be applied to networks with pumps in which pumps are used to fill the reservoirs or tanks. The second limitation is that water quality has not yet been considered. Finally, there is no guarantee that the partitioning solution is the optimal one; however, it is a good one, i.e., it is feasible and has good objective functions while having low constraint violations. In future work, we aim to extend WDN-Cluster for water distribution networks with pumps, and include water quality into consideration.

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