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Procedure to determine the geometry of road alignment using GPS data

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#### Abstract

Traffic safety and road geometry are strictly interlinked because road geometry deeply influences the drivers' performance. So it is very important to know road alignments geometry. Road centreline data for the geometry definition can be generally collected from existing maps or by static measurements (traditional surveys) or by dynamic measurements (GPS receiver mounted on a car). The procedure to define the road geometry, independently from the survey technique and the data type, must be implemented considering the precision level necessary to road applications. This study tries to define intrinsic limits of this integrated data measurement and processing procedure, with the aim to define the reliability of road alignment geometry according to the final employing of road geometry recognition.


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## 1. Introduction

The knowledge of road geometry is one of necessary requirements to guarantee higher traffic safety standards. Drivers, in fact, are influenced by road geometry, because they adapt their way of driving to their perceptions, driving ability and accumulated experience in segments they have covered yet [1].

The knowledge of existing alignments and the study of vehicle-road interaction (mainly based on vehicle's dynamic balance) allow the diagnosis and the localization of incidental critical points, giving the possibility to choose better maintenance works to mitigate criticism.

In order to reach these targets, with the aim to increase traffic safety, the geometry of the centreline of the horizontal road alignment must be defined recognizing all the geometric elements that compose the road.

Actually, many countries are paying great attention to better awareness of the traffic dangers and a safer and sustainable mobility. The European Roads Safety Plan [2] has the aim to drastically reduce the number of road

[^0]accidents by means of a policy to make conscious users and spreading experiences and knowledge among road designers and administrators. This can be reached also by a deeper knowledge of the road.

This paper proposes a method for automatic and repeatable recognition of the geometric elements composing the layout: horizontal (tangents, circumferences and spiral curves) and vertical (vertical tangents and curves).

From data collected by dynamic positioning measurements, the proposed method allows the definition of the curvature referred to the curvilinear abscissa, by means of the punctual data elaboration with the Least Squares [3] regression method. The analysis result is the continuous sequence of the elementary geometric elements composing the examined road horizontal alignments.

The survey method must be previously set up because of difficulties to validate the results: vehicle speed and receiver acquisition time frequency defines the distance of surveyed points. These quantities must generally vary according to road category and the curvature value of the geometric element under measurement.

In addition, independently from the survey technique, the procedure to define road geometry must consider the precision level necessary to specific road application.

## 2. Data

The georeferred data of the road alignment can be collected from existing maps or by direct method based on GPS (Global Positioning System) receiver. In this study, data have been collected by survey system exploiting the Mobile Mapping System (M.M.S.) because of its high versatility in applications and low operational costs, [4],[5]. Generally, a M.M.S. use a vehicle with different integrated equipment: a GPS receiver, INS (Inertial Navigation System) and an odometer are the path component of the vehicle, [6], [7], [8]; other sensors (digital cameras, laser scanner, gyroscopes, etc.) acquire additional information on the road path. The development and integration level of the equipment varies with the survey aim and the applicable standards: in other words, according to the type of data to obtain and the quality of georeferenziation. The path component acquires the spatial coordinates $x, y, z$, in time frequency, in WGS84 reference system. At the same time, an high precision synchronizer combines the spatial location to the data coming from various sensors: so a system of georeferred data is realized.

The data quality strictly depends from the spatial resolution of used GPS and the difficulty of maintaining the vehicle path parallel to road centreline. The intrinsic errors of the GPS are always present also using the differential system with phase measurement (DGPS-on-the-fly [7]) and also accidental and systematic errors due to the imprecision of satellites absolute time, refraction phenomena or orbit calculation. The values of these errors can be also about ten meters. The errors due to electromagnetic interferences or multipath can be less important instead. Also the other components of the MMS can be affected by intrinsic errors, such as integration errors for the INS components or synchronization errors. In the best case, planimetric precision can be in the order of ten centimeters if an inertial system precision has been used: the precision of positional data depends on both the equipment quality and the sophistication of measurements elaboration systems [7].

Some circumstances that make the survey less accurate (e.g the number of satellites available) can occur.
Another important aspect, that has been considered, is the frequency of data acquisition: it is chosen, as time frequency, in relation to the road importance and to the awaited minimum radius along road alignment.

In addition to equipment imprecision, the logged points from vehicle path don't coincide with road centreline because a certain transversal eccentricity is always present. Moreover, road centreline is not really visible, but it is conventionally coincident with the marking of median line.

## 3. Localization of road centreline

The definition of the road centreline from positioning data is practically impossible. This problem is unusual for rational scientific applications because, in addition to intrinsic errors of the measurement instruments, it is
related to a three-dimensional curve that cannot be materialized. This curve is conventionally referred to a line physically identified, e.g. the centreline of pavement road platform, the markings, or other; but these elements are variable during time because of wearing and maintenance works.

This research has developed some procedures to individuate the road centreline according to actual Italian Standard for Road Design[1]. This standard states that road centreline is differently defined according to the road cross section organization:

- single carriageway and two traffic ways; the centreline is the separation marking of the lines of opposite traffic way (it could be different from the geometric centreline of road platform);
- single carriageway and one traffic way; the centreline is the geometric centreline of the carriageway (it could be different from the geometric centreline of the road platform);
- two separate carriageways; if the design centreline is unique, the road centreline is the geometric centreline of the median, otherwise it is the centreline of each platform.

Two different algorithms, for single or double carriageway roads, have been implemented to define the point set that draw the centreline in the space.

For single carriageway and two traffic ways, two point sets has been logged, one for each traffic way, defined as: forward set and backward set and they have been logged by covering the road in each way. The algorithm, implemented in Visual Basic for Application, defines, for each $j$ point of the forward set, the nearest $k$ point of the backward set. Then the point with coordinates $\left(x_{i}, y_{i}, z_{i}\right)$ is estimated as the middle point located on the segment linking $j$ and $k$ points (1).

$$
\begin{equation*}
x_{i}=\frac{x_{j}+x_{k}}{2} \quad y_{i}=\frac{y_{j}+y_{k}}{2} \quad z_{i}=\frac{z_{j}+z_{k}}{2} \tag{1}
\end{equation*}
$$

where:

$$
x_{i} y_{i}, z_{i} \quad \text { middle point coordinates }
$$

$x_{j}, y_{j}, z_{j} \quad$ coordinates of forward set of points
$x_{k} y_{k} z_{k}$ coordinates of backward set of points
This point is considered to belong to the horizontal alignment centreline (Fig. 1).


Fig. 1. Estimated points of the centreline.

Successively, the same procedure is applied to the points belonged to the backward set and ruled out the previous phase for determining other points in the centreline.

In this way, the centreline points are defined in a unvocal way (by a method that is very similar to those implemented in other studies [9], [10], [11], [12], which is independent by the sequence of data analysis. This procedure assumes that the two vehicle paths have the same distance from the centreline. After this phase it is possible to plot the alignment.

In the case of two separate carriageways, two centrelines have been separately analysed, using the vehicle path separately.

## 4. Recognition of the road alignment geometry

A further specific algorithm of calculation has been compiled in V.B.A. in order to recognize horizontal and vertical road elements by elaborating and ordering the points succession representing the road centreline.

The planimetrical recognition aims to identify parts of the road that can be assimilated to any elementary horizontal elements used in the Italian Road Design [1]: Tangents, Circular Curves and Clothoids. The parameters that univocally characterize any of these elements in the plan have been individualized, determining: for the circular curves the Radius $R$ and the coordinates $\left(x_{c} y_{c}\right)$ of the centre, for the tangents the explicit equation of the line $y=m x+q$ and the coordinates of initial and final points of the segment. Furthermore, the recognition of transition curves has been effected beginning from the position of tangents and circumference arcs [13], .

The preliminary determination of the Tangents and the Arcs of Circumference has been effected through the iterative application of the Least Squares method to a mobile base $n$ constituted by a sufficient odd number of representative points of the road alignment.

The numerousness of the mobile base defines the number of points on which to determine, by means of regressions, the characteristics of elementary elements that better approximate the considered points. It assumes notable importance because it influences the effectiveness of the method: it is chosen in relationship to the results of the Sensibility Analysis described subsequently.

From the application of the Least Squares method some Regression Quality Indicators have been determined in relation to the fitness of the regression curve. These indicators are generally dependent on the distance $d_{i}$ between the given point $x_{i}, y_{i}$ and the assumed one by the fit curve $C$.

Therefore, the best approximation is obtained minimizing the value of the indicator to vary some parameters proper of the fit curve.

This principle of Least Squares optimization has been applied for the recognition of circumferences and lines that better represent the part of the road considered with the respective mobile base. It needs to remember that the input data derive from survey, therefore any relationship doesn't exist between the coordinates x and y : this doesn't allow to consider one of them as independent variable, as show in Fig. 2.


Fig. 2. Least Squares method with x and y independent variables.

## 5. Circular Elements

The algorithm used to solve non-linear least squares problem is the method of Gauss-Newton, based on the expansion in Taylor series. By means of an iterative process this method allows to resolve the non-linear problem by its decomposition in a sequence of linear problems of least squares. This method has been implemented in previous similar studies [14].

In this way it is possible to determine the best fit circumference that better represents the part of road defined by the examined mobile base; this is the circumference that minimizes the sum of the squares of the distances from the points of the mobile base from the same circumference. This circumference coincides with the one that minimizes the sum of the squares of the distances from it and the mobile base points.

By the definition of an unknown quantity vector, $u=\left(u_{1}, \ldots, u_{n}\right)^{T}$ and by the formalization of the non-linear system of equations $f_{i}(u)=0$, the formulation of the non-linear problem is (4):

$$
\begin{equation*}
\sum_{i=1}^{m} f_{i}(u)^{2}=\min \quad \text { (minimization of non-linear system of equations) } \tag{4}
\end{equation*}
$$

where:

$$
f_{i}(u)=0 \quad \text { general non non-linear equation; }
$$

An algorithm implemented in the V.B.A. environment applies the not exact solution of the problem to the coordinates $\left(x_{i} y_{i}\right)$ of the $n$ belonging points to the mobile base selected for the circumference arc (Fig.3a). The coordinates of the best fit circumference centre $\left(x_{C}, y_{C}\right)$ and its radius $R$ are so calculated:

$$
\begin{align*}
& x_{C}=\frac{(d \cdot e)-(c \cdot f)}{(a \cdot d)-(b \cdot c)}, y_{C}=\frac{(a \cdot f)-(b \cdot e)}{(a \cdot d)-(b \cdot c)}  \tag{5}\\
& R= \pm \sqrt{\left(\frac{1}{n}\right) \cdot \sum_{i=1}^{n}\left[\left(x_{i}-x_{C}\right)^{2}+\left(y_{i}-y_{C}\right)^{2}\right]} \tag{6}
\end{align*}
$$

where the auxiliary quantities $a, b, c, d, e, f$, has been calculated by:

$$
\begin{array}{ll}
a=\sum_{i=1}^{n}\left[x_{i}\left(2 \cdot n \cdot x_{i}-2 \cdot \sum_{i=1}^{n} x_{i}\right)\right] & b=\sum_{i=1}^{n}\left[y_{i}\left(2 \cdot n \cdot x_{i}-2 \cdot \sum_{i=1}^{n} x_{i}\right)\right] \\
c=\sum_{i=1}^{n}\left[x_{i}\left(2 \cdot n \cdot y_{i}-2 \cdot \sum_{i=1}^{n} y_{i}\right)\right] & d=\sum_{i=1}^{n}\left[y_{i}\left(2 \cdot n \cdot y_{i}-2 \cdot \sum_{i=1}^{n} y_{i}\right)\right] \\
e=\sum_{i=1}^{n}\left[x_{i}\left(n \cdot x_{i}^{2}+n \cdot y_{i}^{2}-\sum_{i=1}^{n} x_{i}^{2}-\sum_{i=1}^{n} y_{i}^{2}\right)\right] & f=\sum_{i=1}^{n}\left[y_{i}\left(n \cdot x_{i}^{2}+n \cdot y_{i}^{2}-\sum_{i=1}^{n} x_{i}^{2}-\sum_{i=1}^{n} y_{i}^{2}\right)\right]
\end{array}
$$

where:

$$
x_{i}, y_{i} \quad \text { coordinates of generic given point }(i=1,2, \ldots, n)
$$

$$
n \quad \text { number of points. }
$$

In Eq. (6), the sign $\pm$ indicates if the circular curve is clockwise or anticlockwise respectively: it is necessary to know the planimetrical course of the road alignment and the curvature sign.

The determination of perpendicular distances $d_{i}$ allows to calculate the Regression Quality Index of the best fist curve; this index, $R Q I_{C}$, has been used for the best solution searching.

$$
\begin{align*}
& d_{i}=|R|-\sqrt{\left(x_{C}-x_{i}\right)^{2}+\left(y_{C}-y_{i}\right)^{2}}  \tag{10}\\
& R Q I_{C}=\sqrt{\frac{\sum_{i=1}^{n} d_{i}^{2}}{n}} \tag{11}
\end{align*}
$$



Fig. 3. (a) Circumference fit curve (b) line fit curve.

## 6. Linear Elements

The line that better fits the part of road defined by the examined mobile base has been conducted minimizing the sum of the orthogonal distances between the mobile base points and the line of the linear regression.

Therefore, the function to be minimized is the sum of the distances di of each point $\left(x_{j}, y_{j}\right)$ from the line of equation $y=a+b x$, (Fig.3b) using (12) and (13):

$$
\begin{align*}
& d_{j}=\frac{\left|y_{j}-\left(a+b \cdot x_{j}\right)\right|}{\sqrt{1+b^{2}}}  \tag{12}\\
& R_{\perp}=\sum_{j=1}^{m} \frac{\left|y_{j}-\left(a+b \cdot x_{j}\right)\right|}{\sqrt{1+b^{2}}} \tag{13}
\end{align*}
$$

Because of the absolute value function has not continuous derivative, it is not possible to minimize $R \perp$ with an analytical solution. This limit has been overcome using a closed form solution to the problem by means of the minimization of the Eq. (14):

$$
\begin{equation*}
R_{\perp}^{2}=\sum_{j=1}^{m}\left[\frac{y_{j}-\left(a+b \cdot x_{j}\right)}{\sqrt{1+b^{2}}}\right]^{2} \tag{14}
\end{equation*}
$$

This function has the minimum value when the following simultaneous equations (15) are verified:

$$
\left\{\begin{array}{l}
\frac{\partial R_{\perp}^{2}}{\partial a}=\frac{2}{1+b^{2}} \sum_{j=1}^{m}\left[y_{j}-\left(a+b \cdot x_{j}\right)\right](-1)=0  \tag{15}\\
\frac{\partial R_{\perp}^{2}}{\partial b}=\frac{2}{1+b^{2}} \sum_{j=1}^{m}\left[y_{j}-\left(a+b \cdot x_{j}\right)\right]\left(-x_{j}\right)+\sum_{j=1}^{m} \frac{\left[y_{j}-\left(a+b \cdot x_{j}\right)\right]^{2}(-1)(2 b)}{\left(1+b^{2}\right)^{2}}=0
\end{array}\right.
$$

These equations simultaneously verified become:

$$
\begin{equation*}
\left[y_{j}-\left(a+b \cdot x_{j}\right)\right]^{2}=y_{j}^{2}-2\left(a+b \cdot x_{j}\right) y_{j}+\left(a+b \cdot x_{j}\right)^{2}=y_{j}^{2}-2 a y_{j}-2 b x_{j} y_{j}+a^{2}+2 a b x_{j}+b^{2} x_{j}^{2} \tag{16}
\end{equation*}
$$

Considering the auxiliary quantity $\mathrm{B}(17)$ :

$$
\begin{equation*}
B=\frac{1}{2} \frac{\left[\sum_{j=1}^{m} y_{j}^{2}-\frac{1}{m}\left(\sum_{j=1}^{m} y_{j}\right)^{2}\right]-\left[\sum_{j=1}^{m} x_{j}^{2}-\frac{1}{m}\left(\sum_{j=1}^{m} x_{j}\right)^{2}\right]}{\frac{1}{m} \sum_{j=1}^{m} x_{j} \sum_{j=1}^{m} y_{j}-\sum_{j=1}^{m} x_{j} y_{j}} \tag{17}
\end{equation*}
$$

where:

$$
\begin{array}{ll}
x_{j}, y_{j} & \text { coordinates of generic given point }(j=1,2, \ldots, m) \\
m & \text { number of points. }
\end{array}
$$

The solution is $b$ (18):

$$
\begin{equation*}
b=-B \pm \sqrt{B^{2}+1} \tag{18}
\end{equation*}
$$

The exposed procedure has been applied to the coordinates $\left(x_{j} y_{j}\right)$ of the $m$ mobile base points of the linear regression. The determination of the perpendicular distances $d_{j}$ allows to calculate the Regression Quality Index of the best fist line (19); this index, $R Q I_{T}$, has been used for searching the best solution in the tangent case.

$$
\begin{equation*}
R Q I_{T}=\sqrt{\frac{\sum_{j=1}^{m} d_{j}^{2}}{m}} \tag{19}
\end{equation*}
$$

## 7. Vertical Elements

The vertical elements have been analysed by the same algorithms introduced for the horizontal elements: the vertical curves have been analysed in analogy with horizontal circular elements, while the vertical tangents have been analysed in analogy with the horizontal linear elements.

## 8. Planimetrical and Altimetrical Recognition

After the execution of the analyses previously described, the characterizing parameters of the approximating curve, linear or circular, have been associated to each point of the alignment series. It has been necessary to
define an evaluation criterion to determine which of the two elements realizes the best approximation: for each point of the centreline, the values of the RQI have been considered, $R Q I_{C}$ for arc and $R Q I_{T}$ for tangents.

Considering a number of points equal to the numerousness $n$ of the selected mobile base and the relative indicators it is possible to establish which is the element that realizes the best regression, both in the case of the line and of the circumference.

It has been defined the Local Minimum Point, PLM, as the point belonging to the mobile base that presents the relative minimum value for the indicator in examination. During the analysis, the parameters of the best fit curve corresponding to the $P L M$ are associated to remaining points of the mobile base.

If k is the half amplitude of the mobile base, the first step to search the planimetrical relative minimum PLM is comparing the index $R Q I_{C}$ of a point i with each point j belonging to the mobile base, with .

If the value $R Q I_{C}$ results as relative minimum, it is selected only if it is lower than $R Q I_{T}$, this happens only if the approximation obtained with a circumference is better than that obtained with a linear regression in the neighbourhood of the point $i$. In the same way, it is possible to operate on the $R Q I_{T}$ values to finally obtain the best approximations performed on the centreline series.

After this phase of analysis it's possible to associate to every point the parameters of the element that has given the best approximation: Radius of Curvature R and Centre $\left(x_{c}, y_{c}\right)$ in the case of circumference, angular coefficient $a$ and the intercept $b$ in case of line.

This parameters are assigned to all points belonging to the mobile base, assigning zero values of curvature if the approximating element is a line, non-null value if the better approximation is a circumference, according to the Eq. (20):

$$
\begin{equation*}
\rho=\frac{y^{\prime \prime}}{\sqrt[2]{\left(1+y^{\prime 2}\right)^{3}}}=\frac{1}{R} \tag{20}
\end{equation*}
$$

where:
$\rho: \quad$ curvature;
$y^{\prime}: \quad$ first derivate;
$y^{\prime \prime}: \quad$ second derivate;
$R$ : radius of curvature;
The procedure of calculation makes direct recognition of the tangents and for the arcs of circumference: for the transition curves has been realized an indirect recognition instead. It is obtained imposing linear variations of curvature with the progressive where the curvature has a variable value with the distance: those segments are included between two elements with constant curvature.

The difficulty found in the recognition of the horizontal transition curves is due to the analytical formulation of these curves and to the greatest randomness of the driver behaviour in correspondence of these elements. In effect, the greater or smaller sensibility of the driver to correctly rotate the steering gear approaching to circular curves can involve some deviations between vehicle path and centreline of the lane.

This fact depends, above all, on the presence identification of circular arc. Also it influences the reliability of the surveyed points series and the determination of the seeding of centreline series: in these particular parts, the approximation with geometric elements should not represent the real course of the centreline.

It is possible to represent the principal results of the exposed procedure in a graphic form. In Fig. 4 the comparison between $R Q I_{C}$ and $R Q I_{T}$ is represented and the result of the best fit curve choice is clearly visible as the envelope of the minimum values.


Fig. 4. Horizontal Regression Quality Index
In the Fig. 5 it is represented the horizontal curvature profile reconstructed through the described procedure: in fact it is possible to effect the recognition of the geometric elements of the road alignment through the analysis of such elaborate, in agreement with the approximations introduced by the used method.


Fig. 5. Horizontal Curvature Profile
The same procedure has been used for the analysis and the recognition of the vertical elements. The choice of the best approximation has been effected considering the values of the indicative $R Q I V_{T}$ for the vertical tangents and $R Q I V_{C}$ for the vertical curves that have been approximate with arcs of circumference. The procedure for the identification of the relative minimum has been the same one of the horizontal case, with the exclusion of the transition curves individualization, as represented in Fig. 6 and Fig.7.


Fig. 6. Vertical Regression Quality Index


Fig. 7. Vertical Curvature Profile

## 9. Analysis of Sensivity

The mobile base used for the recognition must be suitable to define the alignment elements. Choosing a limited set, the determination of the approximating geometric elements can have a notable ambiguity because high positioning errors of a single point would compromise the reliability of the entire method. On the contrary, very wide mobile base could compromise the recognition of circular curves with a small radius or short tangents.

For these reasons the defined procedure permits, as first, the calibration of the bases extension necessary to the regression and geometric elements definition, and then it allows the reliability verification of the method. Consequently, it is possible to assess a fit base extension a priori, using proper diagrams created through an
iterative process. These diagrams allow to know the minimum recognizable arc length by the algorithm starting from the maximum awaited value of highway Radius.

A large number of data has been analysed and statistically validated by means of many repetition implemented by a suitable algorithm for realizing these graphic tools.

These diagrams show the results of sensitivity analysis in relation to the category of road, the number $n$ of points of the mobile base and to the intrinsic error $E_{S}$ of the survey technique. For each value of Exact Radius $R_{I}$ it can be derived the minimum length of arc $L_{A R C}$ for a reliably recognition by the procedure; knowing this output data and the acquisition step of the surveyed points, it is possible to define the extension of the mobile base.

The data input are:

- $E_{S} \quad$ Accuracy of survey technique
- $R_{I} \quad$ Maximum value of typical radius of the road category
- $n \quad$ Number of points of the mobile base
- $L_{A R C}$ Length of the circular curves

The standard of choice is different for each parameter:
$E_{S}$ is a function only of the survey technique and the vehicle equipment: in order to ensure a compatible accuracy with the kinematic DGPS positioning mode, five values of error have been chosen, starting from 0.05 m to 1.00 m . In any case $n$ is an odd value, necessary to always have a central point of the base which refer the obtained result. The chosen values vary from 11 to 21 .
$R_{I}$ has different values depending on the road category: in fact considering the equation of dynamic equilibrium of the vehicle in curve [1], it is possible to fix the minimum design radius $R_{\text {min }}$ for each road category:

$$
\begin{equation*}
R_{\min }=\frac{V_{P \min }^{2}}{127 \cdot\left(q_{\max }+f_{t \max }\right)} \tag{21}
\end{equation*}
$$

where:
$V_{p m i n} \quad$ lower value of project speed range, $\mathrm{km} / \mathrm{h}$;
$q_{\text {max }} \quad$ maximum cross slope of the road section;
$f_{\text {tmax }} \quad$ maximum coefficient of transversal skid resistance.
The input data $L_{A R C}$ is interlinked to the driver's perception of the road. It must have a length corresponding to a travel time at least of 2.5 s (at design speed value of the curve) to correctly be perceived by the driver. However, it was chosen to consider a curve travel time of 5.0 s in order to include roads projected before entering into force the current Standards Project, [2].

Roads with design speed equal to $140 \mathrm{~km} / \mathrm{h}$ are characterized by minimum length of arch calculable by (22):

$$
\begin{equation*}
L_{A R C}=v_{P \max } \cdot 5.0=\frac{140}{3.6} \cdot 5.0=194.44 m \cong 200.0 m \tag{22}
\end{equation*}
$$

While for roads with design speed equal to $80 \mathrm{~km} / \mathrm{h}$ the minimum length of arch is (23):

$$
\begin{equation*}
L_{A R C}=v_{P \max } \cdot 5.0=\frac{80}{3.6} \cdot 5.0=111.11 \mathrm{~m} \cong 110.0 \mathrm{~m} \tag{23}
\end{equation*}
$$

The calculation procedure to draw the diagrams considers a value of Radius $R_{I}$ and a length of curve $L_{A R C}$. Over the arc, n points are calculated and their exact initial position is varied by a random procedure, within a circular area which radius is equal to $E_{s}$. In this way the distribution of collected data has been simulated, with the added advantage to validate of the least squares regression method, knowing the theoretical initial radius $R_{i}$.


Fig. 8. Random generation of errors
Thus, applying the least squares regression procedure to the set of points, it is possible to determine the coordinates of circle centre and the value of the Radius. This value has been compared with the value of the initial Radius $R_{i}$ taking account of vehicle stability in curve, in relationship to the speed, the traversal skid resistance between tire and pavement and to the appropriate safety margins. So, it has been defined the radius value allowed tolerance by means of analytical form. By these diagrams it is possible to define the minimum length of the circular arc in order to get a suitable geometric reconstruction of centreline, in relation to characteristic radius and type of the road. The trend of the curves shows that to the growth of the Radius value requires a greater extension of the arcs to determine an acceptable solution that is inside the fixed range.

Fig. 9 shows the diagram realized using a mobile base of $n=17$ points. For example, for $R_{i}=1500 \mathrm{~m}$ and $E_{S}=0.10 \mathrm{~m}$ the minimum length for a suitable recognition of the curve will be equal to 123 m . The relationships between $R_{i}$ and $L_{A R C}$ is reliable only in the range of the experimental data, drawn with the continuous curve of regression. For values outside that range (dashed curves) there is not an experimental comparison, but only an inferential extension. In the example, the upper bound of reliability corresponds to a value of radius $R_{i}=3100 \mathrm{~m}$.


Fig. 9. Analysis of sensitivity $-\mathrm{n}=17$

The comparison among the minimum lengths corresponding to different values of $E_{s}$ shows that, for lower levels of accuracy, major extensions are required to make a reliable recognition. For example, if $E_{S}=0.20 \mathrm{~m}$ the minimum $L_{A R C}$ is 173 m .

Fig. 10 shows an analogous diagram, realized with a mobile base of $n=11$ points. In this case, considering a radius $R_{i}=1500 \mathrm{~m}$ and a precision $E_{S}=0.10 \mathrm{~m}$, at least the necessary length of arc is 132 m , while for $E_{S}=0.20 \mathrm{~m}$ $L_{A R C}=185 \mathrm{~m}$.

Increasing the number of points of the mobile base with a fixed values of the radius $R_{i}$ and the error $E_{S}$, the necessary length of the arc decreases, especially in the case of quite long Radius. Decreasing the accuracy of the survey, indeed, major arcs' extension is necessary in order to correctly determine the radius of the curve. The diagrams and analytical relations between the involved variables, have been used to critically examine the reliability of the results obtained from geometric recognition.


Fig. 10. Analysis of sensitivity $-\mathrm{n}=11$

## 10. Conclusion

The knowledge of the road geometry is a fundamental step to ensure higher safety standards in road circulation. In design phase, it allows to analyse the alignment, allowing to reduce or to eliminate critical points and to guarantee the necessary distance of unobstructed vision. In management phase, it allows to plan actions in order to improve the travelling safety, such as setting lower speed limits on critical roads. Furthermore, the user can get information about the road that he is covering, thanks to the recent development of the technology web2.0 and of the GPS receivers.

Many authors made researches concerning the geometric reconstruction of the road alignment from positional data collected by a vehicles MMS. Some of them [9], [10], [11], [12], [13], have obtained satisfactory results based on the use of polynomial curves, as spline curves, but with an high computational burdens and limited length of analyzed segment. Moreover these curves are admitted in design phase by the Italian Standard, [1], but their employment must be subjected to verification of the kinematic and dynamic parameters in order to ensure sufficient safety conditions. However this method does not offer a solution in line with current road design practice, based on simple geometrical elements, such as tangents, clothoids and circular curves in the horizontal alignment, and tangents and vertical curves in vertical alignment [11].

The presented method allows the recognition of the geometric road features, even for the whole centreline, using only the positioning information surveyed by a collected-data vehicle. This paper deals with some
observations about the modalities of collecting data, the setting of the acquisition frequency of surveyed points and measuring instruments in regard to road features (type, design speed, tortuosity, etc.). These matters have a potential of strictly close examination and it allows important development for the scientific search.

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## References

[1] Ministero delle Infrastrutture e Trasporti. Decreto ministeriale n. $6792 / 2001$ (2001). Norme funzionali e geometriche per la costruzione delle strade. S.O. n. 5 G.U. 4/1/2002, n. 3. (in Italian).
[2] European Commission. (2007). Road safety: European action plan continues to deliver results - target of saving 25000 lives on Europe's roads by 2010 is attainable.
[3] Ministero delle Infrastrutture e dei Trasporti .Decreto Ministeriale n.3484/2001 (2001). Modalità di istituzione ed aggiornamento del Catasto delle Strade. S.O. n. 6 G.U. 7/1/ 2002, n. 5.(in Italian).
[4] Palermo,C., Cera, L., Bidetta, F.(2007). L'evoluzione dei veicoli ad alto rendimento per il rilievo stradale. Strade\&Autostrade, 3, 1-4 (in Italian)
[5] Mussumeci, G.,\& Sigillato, G. Mobile Mapping System per il rilevamento delle strade. Strade\&Autostrade, online, http://www.stradeeautostrade.it/materiali/articolo.asp?arid=874.Accessed April 2009. (in Italian).
[6] Anthony, L. (1998). Modern Inertial Technology New York: Ed. Springer.
[7] Cina, A. (2003).GPS. Principi, modalità e tecniche di posizionamento. Torino:Celid, 2003 (in Italian).
[8] Cramer, M. (1997).GPS/INS Integration, Stoccarda :Photogrammetric Week '97.
[9] Castro, M., Iglesias, L., Ridrìguez-Solano, R., Sànchez J. A.(2006). Geometric Modelling of Higthways using Global Positioning System Data and Spline Approximantion. Transportation Research Part C 14, 223-243.
[10] Cantisani, G., Loprencipe, G., Dondi, D., Ranzo, A. (2004). Spline Curves for Geometric Modelling of Highway Design. Florence:II S.I.I.V. International Congress: New Technologies and Modeling Tools for Road applications to design and management. ISBN 8884532698.
[11] Jiménez, F., Aparicio F., Estrada, G. (2009). Measurement uncertainty determination and curve-fitting algorithms for development of accurate digital maps for advanced driver assistance systems. Transportation Research Part C 17,225-239.
[12] Guarino Lo Bianco C., Piazzi A. "Optimal trajectory planning with quintic G2-splines." Proceedings of the IEEE Intelligent Vehicle Symposium, Dearborn MI USA, 2000 pp. 620-625.]
[13] Crisman, B., Robba, A. (2004). Safety Evaluation: Practical Use Of Collected-Data Vehicle To Obtain Geometric Information Of Existing Roadways.Florence: II S.I.I.V. International Congress: New Technologies and Modeling Tools for Road applications to design and management. ISBN 8884532698.
[14] Santoponte, C. (2004). Ricostruzione della geometria dei tracciati stradali dai rilievo GPS. Master degree thesis. Sapienza, Università di Roma, Civil Engineering AA 2003-2004 (in Italian).


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