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No fermion doubling in quantum geometry

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ABSTRACT

In loop quantum gravity the discrete nature of quantum geometry acts as a natural regulator for matter theories. Studies of quantum field theory in quantum space–times in spherical symmetry in the canonical approach have shown that the main effect of the quantum geometry is to discretize the equations of matter fields. This raises the possibility that in the case of fermion fields one could confront the usual fermion doubling problem that arises in lattice gauge theories. We suggest, again based on recent results on spherical symmetry, that since the background space–times will generically involve superpositions of states associated with different discretizations the phenomenon may not arise. This opens a possibility of incorporating chiral fermions in the framework of loop quantum gravity.

Loop quantum gravity has provided a non-trivial, anomaly free, finite theory of quantum general relativity coupled to matter [1]. In it, the quantum geometry plays the role of regulator of matter fields. Although this is suggested in the general theory, difficulties in making progress in general have hampered exhibiting detailed examples. On the other hand, detailed calculations are possible in the context of spherically symmetric space–times, where the theory can be solved completely in closed form [2]. In particular Hawking radiation has been recovered using quantum field theory in quantum space–time techniques [3]. Certain very high frequency trans-Planckian modes of the field are suppressed naturally as a consequence of the discreteness of the quantum space–time. This opens new possibilities for issues like the backreaction of Hawking radiation on black holes. What has been seen in the spherically symmetric case is that if one considers quantum field theories living on the quantum space–time, one ends up with a Fock-like quantization where the main ingredient is that the equations of the field theory become discretized. Although one can consider quantum geometries with sub-Planckian separations for the vertices of the spin networks (which play the role of points of the lattice), and therefore have an excellent approximation to the continuum theory, some important differences arise. To begin with, one obtains dispersion relations similar to those of lattices, which suppress the propagation of certain trans-Planckian modes of wavelength smaller than the lattice spacing. In turn, this helps eliminate the divergences that arise in ordinary field theories, for instance, when one computes the expectation value of the stress energy tensor.

A concern that may arise in this context is what happens when one considers fermions, particularly chiral ones. As is well known, fermions on the lattice have the problem of fermion doubling [4]. In fact, fermion doubling has already been noted in certain models of loop quantum gravity [5]. Under very general assumptions, the Nielsen–Ninomiya no-go theorem makes their appearance inevitable. Let us briefly recall how this problem arises. The dispersion relation for a fermion on a one dimensional lattice (or on a spherical quantum space–time) is given by,

$$\omega_n = \pm\frac{1}{\Delta}\sin(l_n)\,, \tag{1}$$

with $l_n = k_n\Delta$, $-\pi \le l_n \le \pi$ and

$$k_n = \frac{2\pi n}{N\Delta} \tag{2}$$

with N the number of vertices in the spin networks of the background quantum space–time and Δ the separation of the vertices of the spin network. The quantity $-N/2 \le n \le N/2$ is an integer that characterizes the wave number. For small values of k_n one recovers the linear dispersion relation of fermions in the continuum.

For small lattice spacings, the frequencies go as Δ^{-1}. So one will have finite frequencies will correspond to $l_n \to k_n\Delta$ or $l_n \pm \pi \to \pm k_n\Delta$. Waves near $l_n \to 0$ will correspond to long wavelength modes (compared to the Planck scale) that will correspond to the modes of the fermions in the continuum, with the usual linear dispersion relation. But what about the modes near $l_n \to \pm\pi$? They

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will also have a linear dispersion relation. In the case of a free field theory one can simply choose not to populate those modes and therefore one would recover at low energies the usual fermion behavior. But in an interactive theory that may not be possible. Following Susskind [4] we can consider an interaction term proportional to the charge density of fermions $\psi(m)^\dagger\psi(m)$ with m denoting the position on the lattice. In momentum space this will lead to an interaction term in the Hamiltonian of the form, in momentum space,

$$\psi(m)\psi^\dagger(m) = \int_{-\pi}^{\pi} \exp\left(im\left(l-l'\right)\right)\psi(l')^\dagger\psi(l)dldl'. \tag{3}$$

Such a term is likely to excite a pair with momentum $\pm k_n$ as with $(\pi/\Delta - k_n)$ or $(-\pi/\Delta + k_n)$. Generic couplings will all excite the low frequency modes near $k_n \to \pm\pi/\Delta$.

However, when one is contemplating a quantum field living on a quantum space–time, only for a measure zero set of quantum space–times will one encounter the situation above. That set would be quantum states with a single spin network leading to a single discretization of the quantum field. Generically, one would have superpositions with different separations between the vertices of the spin network Δ (and more in general superpositions with non-uniform spacings, here for simplicity we consider superpositions with uniform spacings). If one considers that case then it will not be true anymore that an interaction like the one considered above (and generic ones as well) will connect $k_n = 0$ with $k_n = \pm\pi$. Let us consider how the above interaction gets modified in this case. We consider a superposition of spin networks of spacing Δ, all of which contain a vertex at the point x. Then we have for the previously introduced interaction term,

$$\psi(x)\psi^\dagger(x) = \int_{\Delta_-}^{\Delta_+} d\Delta d\Delta' \int_{-\pi}^{\pi} \exp\left(ix\left(\frac{l}{\Delta} - \frac{l'}{\Delta'}\right)\right)\psi_\Delta(l')^\dagger\psi_{\Delta'}(l)dldl' \tag{4}$$

where we assume that the spacing Δ is in an interval $[\Delta_-, \Delta_+]$ such that $\Delta_+ - \Delta_- < \Delta \ll 1$ and that $\pi/\Delta_- - \pi/\Delta_+ \sim (\Delta_+ - \Delta_-)\pi/\Delta_-^2 \gg 2\pi$. These assumptions are quite reasonable if one assumes that the spacing of the lattices take continuous values and their properties resulted from very high energy physical processes occurring at early stages of the universe near the bounce. The generic situation would be in fact having superpositions of random lattices. And substituting $l \to k\Delta$ and $l' \to -\pi + k'\Delta'$, we get,

$$\psi(x)\psi^\dagger(x) = \int_{\Delta_-}^{\Delta_+} d\Delta d\Delta' \int_{-\pi/\Delta}^{\pi/\Delta} \int_{0}^{2\pi/\Delta'} \exp\left(-ix\frac{\pi}{\Delta'}\right) \times \exp\left(ix\left(k-k'\right)\right)\psi_\Delta(-\pi + k'\Delta')^\dagger\psi_{\Delta'}(k\Delta)\Delta\Delta' dkdk'. \tag{5}$$

And we see that unlike in the previous case, there appears a phase factor that when integrated over Δ' will vanish due to its oscillatory nature. Therefore the coupling between small k and $\pm(\pi/\Delta - k)$ vanishes.

In fact, there is not even the possibility of doubling in this context. If one computes the probability density $\psi(x)\psi^\dagger(x)$ for low energies, in the integrand the momenta of the order of inverse lattice scale will not contribute, whereas in the usual lattice treatment they do, leading to doubling.

The argument presented is in a two dimensional example, based on spherical symmetry, so we need to note that at this point this is only a suggestion. At the moment we do not have a similar framework in $3+1$ dimensions that would allow us to extend the argument there at any level of detail. It should be noted that others have argued that there is no fermion doubling in $3+1$ dimensions based on the use of random lattices, for instance, in the spin network context [6]. The mechanism we are suggesting here is a different one. A challenge in the use of random lattices is how to make sense of the continuum limit, which is usually achieved through a sum over lattices. Here we consider superpositions of background quantum space–times that provide an effective superposition of lattices, but one can choose the background quantum space–time to approximate a semiclassical situation and one does not need to sum over all possible lattices. Given the amount of interest in if loop quantum gravity can incorporate chiral fermions [7], it is encouraging to have more than one possible mechanism to address the problem. It may also lead to insights into the chiral anomaly in $1+1$ dimensions [8], were one could in principle carry out explicit calculations in the context of quantum geometries.

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