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Emission and absorption of photons and the black-body spectrum in Lorentz-odd electrodynamics

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ABSTRACT

We study emission and absorption of radiation by non-relativistic electrons within the framework of a Lorentz-breaking electrodynamics in $(3+1)$ dimensions. We have realised that Planck-type law acquires extra terms proportional to the violating parameters: For the CPT-odd model, the leading extra terms appear to be linear or quadratic in these violating parameters according to the background vector is parallel or perpendicular to the photon wave-vector. In the CPT-even case a linear correction shows up. Besides these deviations in the black-body spectra, those violations may be also probed through a difference in the photon mean occupation number for the two modes. Our results also indicate that such violations are better probed at very low temperatures, where their effects on the thermal spectra are largely enhanced.

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Symmetries are keystones for building the modern theories describing particle physics. For instance, the symmetry structure of gauge groups is crucial for classifying the possible particle content in a given model whenever irreducible representations of the Lorentz group are concerned. On the other hand, continuous symmetries imply in dynamically conserved quantities, like energy–momentum and electric charge, whose conservation laws show up whenever the action remains invariant under space–time translations and local gauge transformations, respectively. Discrete symmetries are also very important in these frameworks, for instance, the combined CPT-invariance should be verified in all Lorentz-covariant local quantum field theories.

Although Lorentz and CPT invariances have been confirmed in several high precision tests, they are sometimes believed to be only approximate, not exact symmetries in Nature. Such a belief is partially supported by Lorentz violation in a number of frameworks trying to consistently describe quantum gravity, like string theories. Even though these violations take place only at the Planck scale, their reminiscent effects would survive at ordinary energies. Among such descendants, those which are power-counting renormalizable, preserve the usual $SU(3) \times SU(2) \times U(1)$ gauge structure and respect Lorentz symmetry under observer-type transformations (but not under particle-like ones) are collected in the so-called Standard Model Extension (SME), which is the Standard Model augmented by these non-standard terms [1]. Here, we shall consider two of them, responsible for breaking Lorentz symmetry at the Abelian electrodynamic level, say, in the radiation sector. One of these terms also violates CPT-operation, once it is parametrised by a constant vector-like background field which chooses a preferred direction in the space–time. Of course, such violations should be very small, once Lorentz and CPT symmetries have been confirmed to high precision in several experiments. Indeed, in the CPT-odd case, the background field magnitude is stringently constrained by astrophysical data, $|b_\mu| \lesssim 10^{-42}$ GeV [2,3]; in turn, CPT-even may take larger values: suitable combinations of the dimensionless rank-4 tensor parameter could be around 10^{-16} [4].

Several works have been devoted to study how those extra terms modify conventional results concerning radiation and matter physical properties. For example, in the presence of the Lorentz- and CPT-odd and/or CPT-even terms a number of usual results concerning classical and quantum aspects of electromagnetic radiation acquire (small) contributions which often are linear or quadratic in the violating parameters. Among them are the Cerenkov [5,6] and synchrotron radiations [7]. Quantum mechanical effects could be also probed by means of two-level system [8]. In turn, cos-

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mic microwave background (CMB) data have been also investigated to search for possible traces of these violations in the very early Universe [3,9]. An even more amazing possibility is the photon splitting into two or more other on-shell photons within such frameworks [10]. Furthermore, there is also a extensive literature dedicated to study how such violations can be someway probed in the matter sector, dealing with electrons [11], neutrinos [12], and so forth. Many other aspects have been also extensively investigated, like dimensional reduction [13], causality and unitarity [14], and so on. However, additional results are important, for instance, to wide the possible experimental ways for probing such subtle symmetry breakings.

Here, we seek for possible effects of these violations in mechanisms of emission and absorption of quantum radiation by atoms. Namely, we realise that Planck law is sensitive to Lorentz breaking, accompanied by the CPT-odd and CPT-even terms. Although small, these deviations, linear or quadratic in the respective violating parameters, could be of prime importance once our results rely on mechanisms abundantly observed in nature. Indeed, the searching for those symmetry-breaking, based upon the present analysis (perhaps combined with others), include a very broad range of physical systems, from a relatively small number of atoms and photons to the CMB, which permeates the whole Universe. Namely, the observation of a tiny predominance of a given polarisation over the another in a thermal bath could be taken as a good indication of such violations, as some of our results claim. Additionally, our results clearly indicate that the probing for these small symmetry violations should be accomplished at very low temperatures, where thermal effects on the black-body-type spectra are enormously enhanced. This should be contrasted with similar thermal effects brought about by non-commutative geometry which demand very high energy scales [15] to be effectively probed.

Considering only the radiation sector of the (Abelian) SME, we realise that in this framework the Maxwell theory is modified by incorporating two additional Lorentz-odd terms, as below[1]:

$$\mathcal{L}_{\rm MED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}b_\alpha A_\beta \tilde{F}^{\alpha\beta} - \frac{1}{4}d_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$. Sources can be introduced in the usual way, $A_\mu j^\mu$. Those terms proportional to the parameters[2] b_μ and $d_{\mu\nu\alpha\beta}$ are responsible for the violation of the Lorentz symmetry, but keeping gauge invariance under usual local transformations, $A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x)$. Their roles are distinct under CPT-operation: once b_μ is a constant background vector field, $\partial_\alpha b_\mu = 0$, it implies in space–time anisotropy and ultimately in CPT-violation; contrary, $d_{\mu\nu\alpha\beta}$ respects this invariance. Additionally, it is dimensionless and bears the symmetries of the Riemann curvature tensor besides of having a vanishing double trace, so that only 19 components are independent. Another important difference between them lies on fact that while CPT-odd term yields a non-positive definite Hamiltonian, whenever $b_0 \neq 0$, all the CPT-even components leads to non-negative contributions to total energy (provided they are very small, as experiments strongly indicate; a good account on the latter is provided in Ref. [4]). The dynamical equations for A_μ read:

$$\partial^\nu F_{\mu\nu} + d_{\mu\nu\alpha\beta}\partial^\nu F^{\alpha\beta} + b^\nu \tilde{F}_{\nu\mu} = 0, \tag{2}$$

while the geometrical ones keep their usual form, $\partial_\mu \tilde{F}^{\mu\nu} \equiv 0$, stating the absence of magnetic sources. However, these exotic objects can be consistently introduced in this framework, provided that an extra electric current proportional to b_μ is induced [17]. From Eq. (2), the general dispersion relation may be obtained. However, treating each of the Lorentz-breaking term separately is more convenient and simpler. Thus, for the CPT-odd case (then, with $d_{\mu\nu\alpha\beta} = 0$), we have:

$$\left(k_\mu k^\mu\right)^2 + \left(k_\mu k^\mu\right)\left(b_\nu b^\nu\right) - \left(k_\mu b^\mu\right)^2 = 0, \tag{3}$$

which is valid for arbitrary wave-vector, $k^\mu = (k^0, \vec{k}) = (\omega, \vec{k})$, and b_μ. The coupling between both vectors yields the splitting of the frequency modes and eventually to distinct phase velocities, even in vacuum (light birefringence phenomenon; further details below). On the other hand, within the pure CPT-even framework (then, $b_\mu = 0$) we find, to leading order:

$$\omega_\pm^{\rm even} = (1 + \rho \pm \sigma)|\vec{k}|, \tag{4}$$

where $2\rho = -\tilde{d}_\mu{}^\mu$ and $2\sigma^2 = \tilde{d}_{\alpha\beta}\tilde{d}^{\alpha\beta} - 2\rho^2$, with $\tilde{d}_{\mu\alpha} = d_{\mu\nu\alpha\beta}\hat{k}^\nu\hat{k}^\beta$ and $\hat{k}_\mu = k_\mu/|\vec{k}|$. Once these modes move at different phase velocities, $v_{\rm ph} = \omega/|\vec{k}| = c(1 + \rho \pm \sigma)$, light experiences vacuum birefringence [1,4].

Further features may also be worked out for the pure CPT-odd model. For example, if $b_0 \neq 0$ a negative contribution to the Hamiltonian appears. Indeed, it was shown in Ref. [14] that a consistent quantisation of the radiation field is possible only for b_μ space-like, otherwise unitarity or causality is lost.[3] Therefore, we take hereafter $b^\mu = (b^0; \vec{b}) \equiv (0; m\hat{b})$, where m is a parameter with mass dimension while $\hat{b}$ is a constant unity vector pointing along a preferred direction in the three-dimensional space. [In this case, the term in the Lagrangian is T-odd, C- and P-even, so that a direction in time is chosen by the model.] In this situation, the dispersion relation (3) gets the form below:

$$\left(\omega_\pm^{\rm odd}\right)^2 = |\vec{k}|^2 + \frac{1}{2}m^2 \pm \frac{1}{2}m\sqrt{m^2 + 4(\vec{k}\cdot\hat{b})^2}. \tag{5}$$

In general, these modes carry distinct mass-like gap proportional to m. For $\vec{k}\cdot\hat{b} = 0$ one of the modes is massless while the another bears a mass-like gap equal to m. Note also that once their phase velocities are clearly different and depend on $m/|\vec{k}|$, they travel at distinct velocities even through vacuum. Actually, a number of results concerning emission and absorption of quantum radiation strongly depend on the frequency modes and how they couple to the wave-vector and other parameters, so that the dispersion relations (4) and (5) will be very important in our present work.

To canonically quantise the free radiation field, we expand A_μ in plane-waves, and use the Coulomb gauge, $\nabla\cdot\vec{A} = 0$ (no convenience seems to have in considering a covariant gauge, once Lorentz invariance is lost [14]), so that:

$$\vec{A}(\vec{x}, t) = \frac{1}{\sqrt{V}}\sum_k \left\{ \frac{1}{\sqrt{2\omega_+}}\left[a_+(\vec{k})\vec{\varepsilon}_+(\vec{k})e^{-i(\omega_+ t - \vec{k}\cdot\vec{x})}\right] + \frac{1}{\sqrt{2\omega_-}}\left[a_-(\vec{k})\vec{\varepsilon}_-(\vec{k})e^{-i(\omega_- t + \vec{k}\cdot\vec{x})}\right] + \text{ H.C.}\right\}, \tag{6}$$

where H.C. accounts for the Hermitian conjugate terms. Expansion above is valid even in the general case of having both CPT-even and CPT-odd terms provided that the two remaining modes be taken into account; $\vec{\varepsilon}_+$ and $\vec{\varepsilon}_-$ are the polarisation vectors for the distinct modes (for details, see Refs. [4,16]). The creation

[1] Our conventions read: μ, ν, etc. $= 0, 1, 2, 3$, $\text{diag}(\eta_{\mu\nu}) = (+, -, -, -)$, and $\epsilon^{0123} = -\epsilon_{0123} = 1$, etc. Natural units, with $\hbar = c = 1$, is used except where their presences are convenient.

[2] Instead of b_μ and $d_{\mu\nu\alpha\beta}$, it is more common to use $(k_{AF})_\mu$ and $(k_F)_{\mu\nu\alpha\beta}$, respectively. We justify our choosing for avoiding possible confusing of k with the wave-vector label, k_μ.

[3] Indeed, unitarity or causality is lost in the pure time-like case, while it is kept if b_μ is pure space-like. In the light-like case there still lacks a complete analysis about the consistent quantisation of the CPT-odd model, particularly, no definite answer has been given whether both unitarity and causality are preserved. Further details may be found, for instance, in Ref. [14].

and annihilation operators satisfy usual commutation relations, $[a_\pm(\vec{k}), a_\pm^\dagger(\vec{l})] = \delta_{+-}\delta_{\vec{k}\vec{l}}$, etc. The Fock space there follows immediately, once $a_\pm^\dagger(\vec{k})$ and $a_\pm(\vec{k})$ have their usual interpretation (for further details, see Ref. [16]).

For analysing some features of the electromagnetic radiation emerging from these Lorentz-violating frameworks, we consider the case of emission and absorption of photons by non-relativistic atomic electrons. Explicitly, we consider the absorption process where an atom begins at a quantum state A, interacts with a photon, characterised by $(\omega_+, \vec{k})$, and ends at B (emission process and/or the transition involving a mode with ω_- there follow in similar ways). At first order, the absorption process is described by:

$$\langle B; n_{\vec{k},+} - 1|H_I|A; n_{\vec{k},+}\rangle = -\frac{e}{m}\sqrt{\frac{n_{\vec{k},+}}{2V\omega_+}}\sum_i \langle B|e^{i\vec{k}\cdot\vec{x}_i}\vec{p}_i\cdot\vec{\varepsilon}_+|A\rangle e^{-i\omega_+ t}, \tag{7}$$

from which it follows that it is impossible the absorption of a photon with polarisation distinct from that released from the electromagnetic field, as usual.

Now, suppose a number of such atoms interacting with a radiation field by means of reversible processes, $A \rightleftharpoons B + \gamma$, keeping thermal equilibrium. Thus, if the population number of initial state is $N(A)$ and of the final state is $N(B)$, we have that $N(B)/N(A) = e^{-E_B/k_B T}/e^{-E_A/k_B T} = e^{\hbar\omega/k_B T}$, where E_A, E_B represent the total photon energy of states A and B, respectively, while k_B is the Boltzmann constant. Usual expressions for the probability of emitting and absorbing photons may be obtained so that the mean occupation number at a given photon state is given by:

$$n_{\vec{k},\pm} = \frac{1}{e^{\hbar\omega_\pm/k_B T} - 1}. \tag{8}$$

Thus the average energy by photon state reads $\bar{E}_\pm = \hbar\omega_\pm n_{\vec{k},\pm}$. Along with the dispersion relations (4) and (5), these results clearly show that the radiation field is homogeneous, once the quantities above do not depend on vector position, for both CPT-even and CPT-odd frameworks. However, it is anisotropic in the CPT-odd case, since it depends on the relative orientation of $\vec{k}$ and the background field, $\vec{b}$. In addition, in both cases each polarisation state experiences Lorentz anisotropy differently, say, at a given temperature, T, and momentum, $\vec{k}$, we generally have $n_{\vec{k},+} \neq n_{\vec{k},-}$ (for details below).

Now, assuming usual periodic boundary conditions, the momenta of the radiation field enclosed in a cubic volume $V = L^3$ are given by $k_i = 2\pi n_i/L$ where $i = 1, 2, 3 = x, y, z$ and $n_x, n_y, n_z = \pm 1, \pm 2, \pm 3\ldots$. Then, the total number of quantum radiation oscillators (photons) with polarisation ω_+, energy between $[\hbar\omega_+, \hbar(\omega_+ + d\omega_+)]$, and propagating towards a direction enclosed by a solid angle $d\Omega$ equals the volume element in the three-dimensional n-space, $n^2\,dn\,d\Omega = \rho_{\omega,d\Omega}\,dE$ [18]. In the latter equality $\rho_{\omega,d\Omega} = \mathcal{N}\prod_i dk_i$ is the so-called density of allowed states per unity frequency, ω, while $\mathcal{N}$ is the number of polarisations. Therefore, energy density per unity frequency (and per polarisation) is given by the total energy enclosed in the volume times the density of states, $\int_V \rho_{\omega,d\Omega}\,dE$. Evaluating this expression we obtain (explicitly c and $\hbar$):

$$u(\omega_\pm) = \frac{4\pi}{(2\pi)^3}\frac{\hbar\omega_\pm}{e^{\hbar\omega_\pm/k_B T} - 1}\frac{k^2\,dk}{d\omega_\pm}. \tag{9}$$

The total energy density per frequency, $u(\omega)$, in a given radiation thermal bath is given by summing over the respective contributions from each polarisation. Although expression above has the usual form, it should be emphasised that differences actually appear once the term $\omega_\pm k^2\,dk/d\omega_\pm$ is dependent on the dispersion relations, as follows.

First, let us consider the CPT-odd framework. Using the dispersion relation (5) it is easy to show that (for $\omega \gg \omega_0$):

$$u^{\text{odd}}(\omega_\pm, T)|_{\omega\gg\omega_0} = \frac{4\pi\hbar}{(2\pi c)^3}\frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}\left(1 \pm \frac{\omega_0\cos\theta}{2\omega} + \frac{\omega_0^2}{4\omega^2}(2 - \cos^2\theta)\right) + \mathcal{O}(\omega_0^3), \tag{10}$$

where $\omega = \omega(k) = c|\vec{k}|$ and $\omega_0 \equiv mc^2/\hbar$ are the dynamical and rest-like (k-independent) frequencies. The non-vanishing ω_0 is related to the mass-like gap previously discussed which implies in a sort of rest energy for electromagnetic radiation in this case. The angle θ lies between the vectors $\vec{k}$ and $\vec{b}$. Now, the energy density per polarisation mode (thereof, the factor 1/2 below), $U_\pm = \int_0^\infty u(\omega_\pm, T)\,d\omega_\pm$, reads:

$$U_\pm(T) = \frac{1}{2}\sigma_0 T^4\left[1 \pm \sigma_1\omega_0\cos\theta T^{-1} + \sigma_2\omega_0^2(2 - \cos^2\theta)T^{-2}\right], \tag{11}$$

where $\sigma_0 = \pi^2 k_B^4/15(\hbar c)^3 \approx 7.56\times 10^{-16}$ J/m^3 K^4 is the Stefan–Boltzmann constant, while $\sigma_1 \equiv 15\hbar\zeta(3)/\pi^4 k_B \approx 1.41\times 10^{-12}$ K s ($\zeta(3) \approx 1.2$, $\zeta(x)$ is the ζ-Riemann function) and $\sigma_2 \equiv 5\hbar^2/\pi^2 k_B^2 \approx 3.69\times 10^{-41}$ K^2 s^2. Recalling that [3] $m \lesssim 10^{-42}$ GeV/c^2, what gives $\omega_0 \lesssim 10^{-17}$ Hz, then $\sigma_1\omega_0 \approx 10^{-30}$ K which is negligible compared to the unity, in equation above. However, considerable compensation may come from a very low temperature, say, $T \sim 10^{-9}$ K (achieved in laboratories), so that the leading correction associated to Lorentz and CPT violations is around 10^{-21} (similar results have been also obtained in the work of Ref. [19]). Black-body-type radiation has been also studied in non-commutative geometry frameworks, where additional terms appear proportional to higher powers of T, T^8 at leading order. Therefore, contrary to the our case, probing for a fundamental length in space–time structure should focus at very high temperature [15].

As a special case, consider $\cos\theta = \hat{k}\cdot\hat{b} = 1$. Then besides the linear correction in (10) we also see that if energy is equally distributed for the two modes, then ω_- mode over populates the thermal bath by around $n_-(\vec{k}) - n_+(\vec{k}) \sim \omega_0/2\omega$. Clearly, for $\cos\theta = 0$ deviations above go like ω_0^2/ω^2. On the other hand, for $\omega_0 \approx \omega$, we obtain the analogue of the Rayleigh–Jeans classical result:

$$u_\parallel^{\text{odd}}(\omega_\pm, T)_{\omega\approx\omega_0} \simeq \frac{4\pi}{(2\pi c)^3}\frac{2k_B T\omega^2}{(\sqrt{5}\pm 1)}, \tag{12}$$

which clearly indicate that at such regimes energy difference between the modes cannot be neglected.

Now, let us carry out the CPT-even case, where the modes are given, at leading order, by (4). After some algebra the Planck-like law is obtained to be:

$$u^{\text{even}}(\omega_\pm, T) = \frac{4\pi\hbar}{(2\pi c)^3}\frac{\omega^3}{e^{\hbar\omega_\pm/k_B T} - 1} \approx \frac{4\pi\hbar}{(2\pi c)^3}\frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}\left(1 - \frac{\hbar\omega}{k_B T}(\rho \pm \sigma) + \mathcal{O}\big((\rho\pm\sigma)^2\big)\right), \tag{13}$$

whose extra contributions appear linearly in the violating parameters, ρ and σ. If we sum over the modes with the assumption of equally partitioned energy between them, $\bar{E}(\omega_+) = \bar{E}(\omega_-)$, only the term proportional to ρ comes about at first order, say, $\hbar\omega\rho/k_B T$. For instance, for CMB radiation ($T \sim 3$ K, $\omega \sim 10^{11}$ Hz) we get $\frac{\hbar\omega}{k_B T}\rho \sim \rho$. Therefore, if ρ is constrained to be around 10^{-16}, the latter expression predicts a small deviation in the CMB power spectra around this latter value. Unfortunately, current data (provided by COBE) do restrict possible anisotropies in the CMB radiation due to thermal effects to be only about $\lesssim$0.5%. Of course, such a bound cannot be attributed to those anisotropies studied here, which are much smaller. However, Eq. (13) suggests that its deviation may be enhanced at very low temperatures. For example, a system at $T \sim 10^{-9}$ K would exhibit a deviation about 10^{-6}%, at $\omega \sim 10^2$ MHz (microwave radiation). The experimental challenge lies in the capability of measuring the spectrum of this system with such an accuracy, mainly at microwave frequencies.

Before concluding, we should point out that: (i) As $\vec{k} \to 0$ ($\omega \to 0$) then $u(\omega)$ identically vanishes for all cases (even in CPT-odd framework, with rest-like frequency), so that thermal equilibrium is achieved by means of radiation dynamics (at other special limits, as $T \to 0$ and $T \to \infty$, present results behave as their usual counterparts, as may be easily checked); (ii) Even though canonical quantisation cannot be carried out to obtain Planck-type law if b^μ is pure time-like, $b^\mu = (b_0 \neq 0; \vec{0})$, semi-classical analysis, along with detailed thermal equilibrium balance, may be used. The result is similar to the case where $\hat{k}\cdot\hat{b} = 0$, so that deviations appear only proportional to b_0^2.

In summary, we have studied a number of issues regarding the emission and absorption of photons by non-relativistic atomic electron, within a Lorentz-violating and CPT-odd or CPT-even electrodynamics. Our main results concern how Planck law is modified whenever those violations are incorporated in the usual Maxwell electromagnetism. We have realised that for the CPT-odd case the deviations appear to be linear or quadratic in the rest-like frequency, $\omega_0 = mc^2/\hbar$, according photon momentum and the background vector field are parallel or perpendicular each other. Actually, these corrections take place in the expressions for the energy density distribution for a given polarisation (mode). If a thermal bath with photons equally populated by each mode concerns, then even in the case where $\hat{b} \parallel \vec{k}$ the correction appears to be quadratic in ω_0, but a higher number of ω_--mode photons is in order by around $\omega_0/2\omega$. In the situation where CPT-symmetry is kept the leading order deviation appears linearly in the violating parameter. As a whole, our results clearly indicate that probing for these Lorentz violations by means of thermal effects demands very low temperature, where the associated deviations are enormously enhanced. For instance, CPT-even deviations (which are generally much larger than those from CPT-odd), in a system at $T \sim 10^{-9}$ K go around 10^{-6}%, at microwave frequencies, $\sim 10^2$ MHz.

As prospects for future investigation, we may quote the study of how such violations modify the analysis of the spin structure of photons. Indeed, in the CPT-odd situation, we have encountered several obstacles for carrying on this study, once even the *little group* of spatial rotations now depends upon the relative orientation of the photon momentum with the background field. In some cases, the little group seems to be reduced to the Abelian $SO(2)$-group [20], so that a quantisation (discretisation) of spin-like eigenvalues could be jeopardised.

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