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Endurance time method for multi-component analysis of steel elastic moment frames

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KEYWORDS
Endurance time method;
Intensifying acceleration functions;
Bi-directional excitation;
Three-dimensional dynamic analysis.

Abstract The Endurance Time (ET) method is a time history-based dynamic analysis procedure which uses special intensifying acceleration functions for evaluation of the seismic response of structures. One of the potential applications of the ET method is in the three-dimensional analysis of buildings under multi-directional excitations. In this paper, considering horizontal components of excitation, an algorithm for the multi-component analysis of building structures by the ET method is proposed, and results of the ET method for various steel moment frames with 1 to 7 stories are compared with results from time history analysis with real earthquakes. Results show that based on recommendations of structural codes for bi-directional time history analysis, which requires applying horizontal components of earthquakes simultaneously, the ET method can be used to predict the seismic response of structures with appropriate approximation.

1. Introduction

Earthquake-induced ground motions have three translational components that are directly recorded by accelerographs. There are some codified provisions that require consideration of the effects of ground motion components in the seismic analysis of sensitive structures. In this respect, three-dimensional analysis is obligatory for asymmetric, tall buildings or important structures such as dams, bridges and power plants [1]. In these circumstances, the most appropriate analysis procedure is time history analysis including components of consistent ground motions.

With the development of new computational tools, the capability of realistic dynamic modeling and complex analysis of structures has been increased and, in this situation, using improved and more complicated methods for seismic evaluation of structures has become a reasonable choice. Therefore, traditional two-dimensional static and response spectrum methods are gradually being replaced by nonlinear three-dimensional time history analysis. In response to this increasing demand for application of these complex methods, it is necessary to develop procedures for clear and logical use of these new approaches.

Three-dimensional analysis under actual recording has two major issues. Firstly, for a particular site specification, the number of available recorded earthquakes might not be sufficient and the selection of consistent accelerograms complicates the situation. Secondly, analysis of structures under these ground motions is time consuming, especially when considering critical orientation to be necessary. Moreover, interpretation of results for complex structures is quite difficult. Therefore, it is advantageous to use simpler methods that can estimate structural behavior under multi-directional excitation with satisfactory approximation and with less computational operations.

The Endurance Time method is a new method that is capable of being used in both the linear and nonlinear seismic analysis of structures [2]. One of the advantages of this method over other time history analysis procedures is its reduction of the required computational effort and its relative simplicity. In the ET method, the response of a structure is monitored against the intensity of excitation from beginning to collapse (somewhat similar to the Incremental Dynamic Analysis method [3]). The structure is then assessed based on its response at various equivalent excitation levels.

In this paper, application of the ET method in linear seismic analysis of structures is investigated. The ET method

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Nomenclature

$a_g(t)$	ground acceleration
ET	endurance time
ETacc	endurance time acceleration function
T	free vibration period (s)
S_a	acceleration response
$S_a(T,t)$	acceleration response for period T at time t
$S_{aC}(T)$	codified design acceleration spectrum for period T
$S_{aT}(T,t)$	target acceleration response for period T at time t
$S_u(T,t)$	displacement response for period T at time t
$S_{uT}(T,t)$	target displacement response value for period T at time t
t	time
t_{max}	time corresponding to the end of accelerogram
T_{max}	maximum free vibration period (s) to be considered in the optimization
t_{Target}	target time
α	weighing factor in optimization target function
MMF	moment resistant frame in both directions
CF	correction factor
$DI_{Avr\text{-}EQs}$	damage Index obtained from averaging the response of earthquakes
$DI_{ET@t=10}$	damage Index obtained under ET acceleration functions at $t = 10$ s
ρ	correlation coefficient
Mb	moment in beams
Pc	axial force in columns
MxC	moment at the end of column in X direction
MyC	moment at the end of column in Y direction
2DOF	two degree of freedom system

is evaluated by comparing results of ET analysis with results of time history analysis, using horizontal components of real ground motions, according to seismic analysis regulations, such as the Iranian National Building Code (INBC) [4] and ASCE 7-05 [5].

The first part of this paper is devoted to a brief review of code regulations and some investigations on the three-dimensional analysis of buildings. In the next section, various structures which are designed according to the INBC code are analyzed by both ET and time history analysis under real earthquakes. Finally, by comparing results of these two methods, an algorithm for code compliant ET analysis is proposed for simultaneous excitation in the perpendicular direction of structures. Even though time history analyses are seldom required in linear elastic analysis of structures, current research is aimed at laying the foundations for extension of the application of the ET method to seismic assessment, using three-dimensional dynamic models subjected to realistic multi-component ground motions. Obviously, the major benefits of the procedure can only be realized when dealing with complicated nonlinear models. Even though nonlinear two-dimensional analysis results, using currently available ET records, indicate that reasonable estimates can also be obtained in the non-linear range, non-linear multi-component ET analysis is beyond the scope of current research.

2. Review of code provisions and related research

Although there are some guidelines in seismic codes for multi-directional analysis under real ground motions, these methods are not routinely applied to the seismic analysis and design of common buildings [6]. Considerable research has been conducted in the past to clarify and simplify three-dimensional analysis. Naeim et al. [7] proposed the use of a genetic algorithm for selecting and scaling records. Many investigations have been performed to find characteristics of components of an earthquake [8–10] and the structural response due to two or three components of ground motion [11]. These efforts lead not only to suggestions for the time history analysis of structures, but also to recommendations for the application of components in static and response spectrum analysis [12,13].

Nearly all structural codes have essentially the same recommendations for the selection of earthquake records for the purpose of three-dimensional analysis. However, they are somewhat different in the scaling method and application of components of records. For example INBC, ASCE4-98 and EC8 recommend that analysis should be performed under components in the principal direction of buildings [14,15], but for columns or walls intersecting seismic force-resistant systems of a building located in category E and F, application of ground motion components in a critical direction in addition to analysis under horizontal components along principal directions is necessary. FEMA368 [16] recommends that each pair of time histories be applied at the same time to the model, considering the most disadvantageous location of mass eccentricity.

One way to consider critical directions is rotating the angle of induced excitation. By this procedure, an analysis requires a great deal of effort and time, which may not be justified for typical structures. To avoid such problems, simplified methods have been proposed to estimate the critical response of structures due to an earthquake, without rotating the angle of excitation [17]. However, rotating the angle of excitation is still more practical for considering the critical response of structures. The huge amount of computational effort required in three-dimensional response history analysis, using bi-directional excitation at multi levels can be prohibitive in many analysis and design situations. The ET method can considerably reduce the number of required analyses and, with appropriate approximation, provides a simple method for the three-dimensional analysis of structures. It should be noted again that in this paper, only linear behavior is investigated, where results at various excitation levels can be obtained by applying a scale factor. However, it should be obvious that this does not hold in general nonlinear cases.

3. Endurance time method

3.1. Basic concepts of the ET method

The Endurance Time method has been introduced as a new seismic analysis method [2], and application of this method to two-dimensional linear and nonlinear analyses of steel frames has been reported in literature [18,19]. In the ET method, structures are subjected to a set of specially designed intensifying accelerograms, known as "ET acceleration functions", and their seismic performance is judged based on their response at various equivalent dynamic excitation intensities.

In ET analysis, endurance time is considered to be when the maximum value of the specified design parameter exceeds its allowable limit. In order to decide whether or not the achieved performance can be considered adequate, structural response at the equivalent intensity of the imposed dynamic action

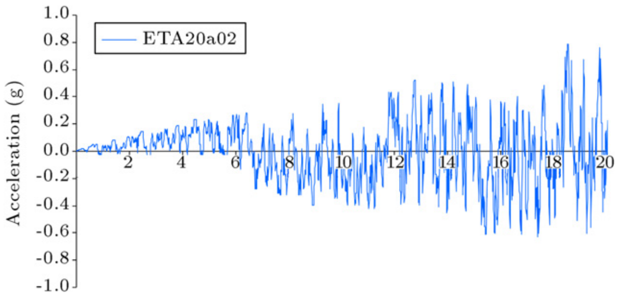


Figure 1: A typical ET acceleration function.

should be considered. Spectral acceleration is the most popular intensity measure used in practice and has been considered for calibrating the ET acceleration functions used in this study.

The acceleration functions are linearly intensifying by time. In this approach, if the target time is set to $t = 10$ s, ET acceleration functions are calibrated in such a way that their response spectra in a window from $t = 0$ to 10 s match the design spectrum with a scale of unity. When the window of an acceleration function is taken from $t = 0$ s to $t = 5$ s, its response spectrum corresponds to half the template spectra at all periods and, if an interval of $t = 0$ s to $t = 15$ s is taken, its response spectrum matches with 1.5 times the template spectrum and so on. Therefore, if for a certain structure, which is designed according to a design spectrum that matches the template spectrum with a scale factor of unity at $t = t_{\mathrm{Target}}$ (10 s in this research), the drift ratio exceeds its limit at $t = 15$ s. It can be concluded that the structure satisfies drift criteria, since its endurance time is more than that required by the code. A typical ET acceleration function is depicted in Figure 1.

For the generation of ET acceleration functions used in this study, the concept of the response spectrum has been directly applied. By scaling the ET acceleration functions, using a simple linear scale factor, Sa (and Sd) can be set to reach the required target level at any desired time. By applying this method, we define the target response of ET acceleration functions as follow:

$$S_{aT}(T,t) = \frac{t}{t_{\mathrm{Target}}} S_{aC}(T), \tag{1}$$

$$S_{uT}(T,t) = \frac{t}{t_{\mathrm{Target}}} S_{aC}(T) \times \frac{T^2}{4\pi^2}, \tag{2}$$

where $S_{aT}(T,t)$ is the target acceleration response at time t, T is the period of free vibration, $S_{aC}(T)$ is the codified design acceleration spectrum and $S_{uT}(T,t)$ is the target displacement response at time t. The problem of generating accelerograms with such characteristics was approached by formulating it as an unconstrained optimization problem in the time domain, as follows:

$$\text{Minimize } F(a_g) = \int_0^{T_{\max}} \int_0^{t_{\max}} \Big\{ [S_a(T,t) - S_{aT}(T,t)]^2 + \alpha [S_u(T,t) - S_{uT}(T,t)]^2 \Big\} dt\, dT, \tag{3}$$

where a_g is the ET accelerogram being sought and α is an optimization weighting parameter set to 1.0 in this study [2].

3.2. Characteristics of ET acceleration functions used in this study

Various sets of ET acceleration function have been developed based on the intended application. In general, these fall into two major categories: code compliant and ground motion compliant. Code compliant ET acceleration functions are based on a template spectrum that matches that of a particular design spectrum of a specified seismic code. These acceleration functions are mostly interesting from the design application perspective. On the other hand, ground motion compliant ET records are based on the average response spectrum of a set of ground motions pertaining to specific soil conditions, without any modifications, to provide a safety margin. These records are more suitable for comparative studies, when analyzing some inherent sources of inconsistency and scatter of the estimations obtained by the ET method. Major characteristics of ET acceleration functions, which have the greatest influence on structural response, match well with ground motions [20]. This is mostly due to the fact that ET acceleration functions are designed in such a way as to produce response spectrums matching those of ground motion. In the present article, ETA20f01-03 acceleration functions, whose template spectrum matches the average response spectrum of major components from 7 real accelerograms (listed in FEMA 440 for soil type C), at the target time of 10th second, are used. Similar to other sets of ET acceleration function, in this set, the response spectra of these acceleration functions increase with time. In Figure 2, the response spectra of the f series of ET acceleration functions are compared at different times. As shown in Figure 2, the linear intensification of response spectra at different times is apparent.

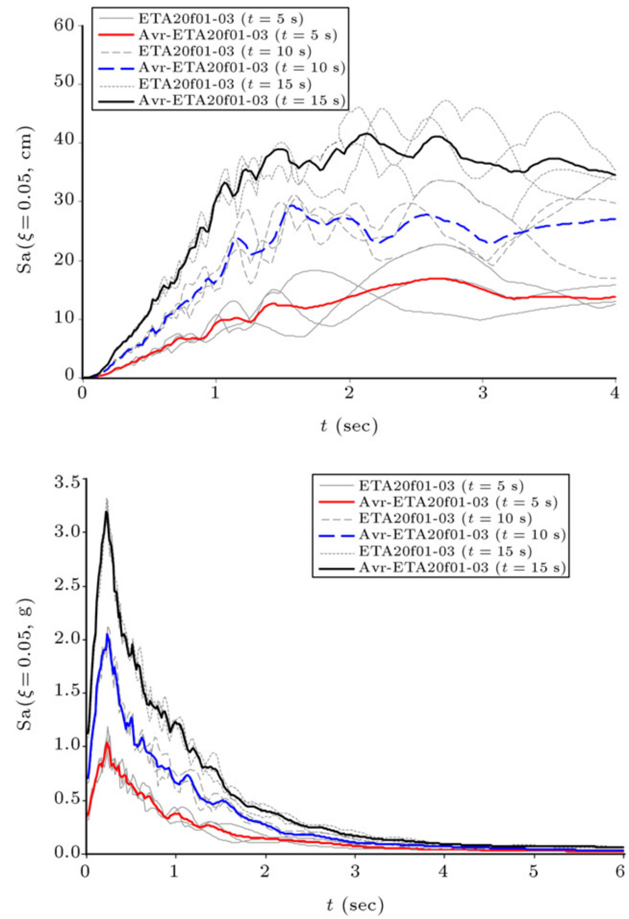


Figure 2: Response spectra of ETA20f01-03 at different times.

3.3. Comparison of ET method with other conventional seismic analysis approaches

In static analysis, by applying an equivalent load based only on a first mode shape, the effects of higher vibration modes of the structure are mostly excluded. By increasing the irregularities and complexities in buildings, the effects of dynamic specifications become remarkable, and static analysis will not be reliable [4]. The ET method is based on time history analysis and intrinsically takes all significant dynamic properties of the structure into account. Moreover, due to the fact that ET acceleration functions are intensifying with time, in each ET analysis, the strength of the structure can be predicted at different levels of intensity, while the analysis of a building with real accelerograms at different levels needs Incremental Dynamic Analysis (IDA) [3], which requires considerable computational effort. This advantage of the ET method cannot be realized in linear analysis which is the subject of this research. While some particular problems, such as optimal damper placement in linear systems, still require a response history-based analysis procedure, the major goal of this research should be considered as laying the necessary foundation for extension of the application of ET to multi-component seismic analysis.

4. Structural models

In the studied models, the endeavor is to focus on those parameters that have the most influence on three-dimensional analysis. Several steel moment resistant frames with 1, 3, 4, 5 and 7 stories, in three states of regular, irregular in one direction, and irregular in two directions for considering the effects of torsion are designed and investigated. It should be noted that for all frames, the storey height is 3.2 m and all spans are equally 6 m. Box sections are assigned to columns and HEA profiles are assigned to beams.

The names of the studied models are based on a lateral resistant system, the number of stories, spans in both directions and irregularities in each direction. All frame names begin with F3DMM, signifying that all are 3-D moment frames in both directions. This is followed by the letter, S, and a number that shows the number of stories. Then, the number of bays in X and Y directions is specified as $XnYm$, meaning n bays in X, and m bays in Y directions, respectively. The irregularity of the frame in X or Y or both directions is indicated next, e.g. IRX means irregular in the X direction, etc. For example, as shown in Figure 3(b), F3DS3X3Y3IRXY represents a 3-story moment resistant building with 3 spans in X and Y directions, and irregularities in both directions.

The equivalent static lateral force procedure, based on the provisions of INBC for soil condition type 2, has been used for the design of frames. Dead and live loads are assumed, 7500 and 2500 N/m^2, respectively, and an accidental eccentricity of 0.05 L (where L is the dimension of the building plan in each direction) is considered for the design as code requirements. The damping ratio for all frames is assumed to be 0.05, a typical value for this type of structure. Beam and column profiles are HEA and Box profiles, respectively. The importance factor is assigned to be 1, and the R factor is considered to be 7 in both directions, due to the moment resistant frame in both directions. Properties of frames and design assumptions are listed in Table 1. These buildings have predominant periods between 0.1 and 1.5 s. It seems that by covering a reasonable range of model variety, results of ET analysis can be extended for three-dimensional analysis of low rise steel moment frames.

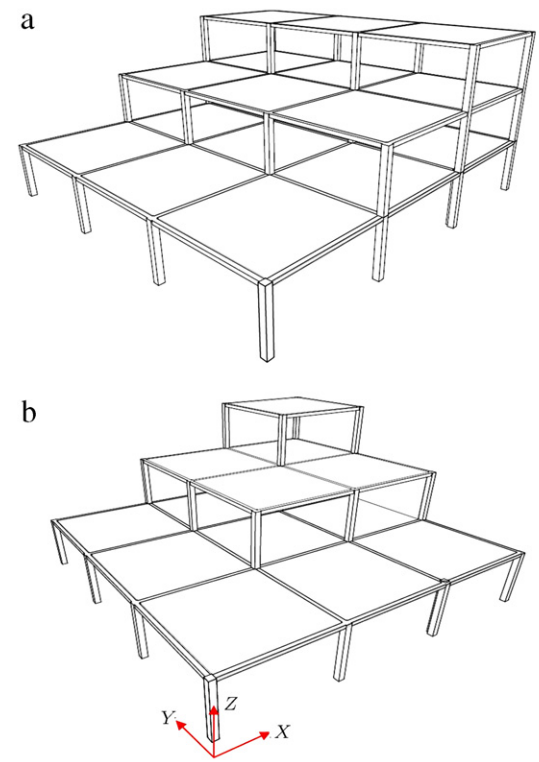


Figure 3: Investigated 3-storey models.

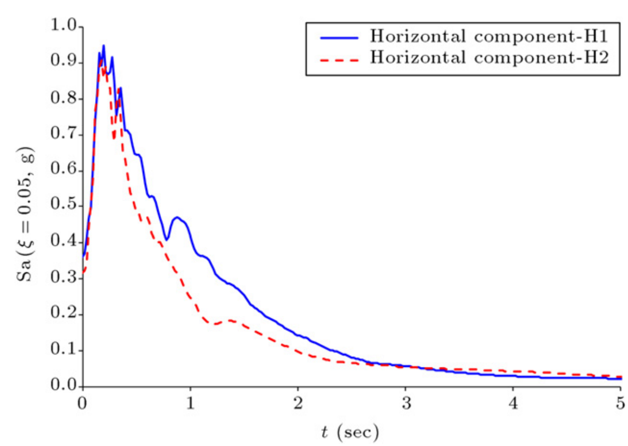


Figure 4: Average response spectra of horizontal components of selected accelerograms.

5. Selection of records

To verify results of the ET method with real earthquakes, seven real accelerograms are selected from 20 records listed in FEMA 440 for soil condition C. These records and their components are listed in Table 2. In this paper, the effect of a vertical component is not included. The average response spectra of these real accelerograms, which are scaled according to code requirements, are illustrated in Figure 4.

One important issue in Figure 4 is the difference between the response spectra of the horizontal components of each ground motion. Although the spectrum of each component is not the same at different periods, especially between 0.5 and 3 s, for the general purpose of seismic analysis, in this study, this difference is assumed to be insignificant due to the fact that each record

Table 1: Investigated frames, properties and design assumptions.

Name	No. of stories	No. of span X	No. of span Y	Seismic coefficient	T (s)	Base shear (kN)
F3DMMS3X3Y3IRX	3	3	3	0.125	0.722	735.004
F3DMMS3X3Y3IRXY	3	3	3	0.125	0.627	601.069
F3DMMS4X3Y3	4	3	3	0.119	0.93	1411.33
F3DMMS4X3Y3IRX	4	3	3	0.119	0.913	1269.93
F3DMMS4X3Y3IRXY	4	3	3	0.119	0.875	810.326
F3DMMS5X4Y4	5	4	4	0.106	1.08	2826.04
F3DMMS5X4Y4IRX	5	4	4	0.106	0.996	2209.49
F3DMMS5X4Y4IRXY	5	4	4	0.106	0.976	2142.8
F3DMMS7X3Y5	7	3	5	0.091	1.505	3159.01

Table 2: Properties of real accelerograms and their components.

Name	*Ms*	Station name	Abbreviation	Component (deg)	PGA (cm/s^2)
Landers	7.5	Yermo, fire station	LADSP000	0	167.80
			LADSP090	90	151.05
Loma Prieta	7.1	Saratoga, Aloha ave.	LPSTG000	0	494.50
			LPSTG090	90	317.90
Loma Prieta	7.1	Gilroy, Gavilon College Phys. Sch. Bldg.	LPGIL067	67	349.10
			LPGIL337	337	318.80
Loma Prieta	7.1	Santa Cruz, University of California	LPLOB000	0	433.10
			LPLOB090	90	387.00
Loma Prieta	7.1	Anderson Dam, downstream	LPAND270	270	239.40
			LPAND360	360	235.10
Morgan Hill	6.1	Gilroy #6, San Ysidro microwave site	MHG06090	90	280.40
			MHG06000	0	217.87
Northridge	6.8	Castaic, old ridge route	NRORR360	360	504.20
			NRORR090	90	557.30

Table 3: Scaling value of records components used in analysis of frames.

	LADSP	LPSTG	LPGIL	LPLOB	LPAND	MHG06	NRORR
F3DMMS3X3Y3IRXY	1.805	0.988	1.042	1.017	1.158	1.078	0.533
F3DMMS3X3Y3IRX	1.734	0.986	1.097	1.082	1.203	1.062	0.527
F3DMMS4X3Y3	1.678	0.954	1.192	1.246	1.276	1.024	0.526
F3DMMS4X3Y3IRX	1.678	0.954	1.192	1.246	1.276	1.024	0.526
F3DMMS4X3Y3IRXY	1.678	0.954	1.192	1.246	1.276	1.024	0.526
F3DMMS5X4Y4	1.655	0.925	1.228	1.329	1.292	1.036	0.523
F3DMMS5X4Y4IRX	1.655	0.925	1.228	1.329	1.292	1.036	0.523
F3DMMS5X4Y4IRXY	1.655	0.925	1.228	1.329	1.292	1.036	0.523
F3DMMS7X3Y5	1.555	0.842	1.314	1.599	1.304	1.121	0.515

is applied to orthogonal directions, thus maximum response is the significant parameter. In all ET analyses in this study, ET acceleration functions with the same intensity and spectral shape are used in the bi-directional analysis of studied frames.

6. Multi-component analysis

6.1. Scaling procedure

There are different approaches for scaling earthquake records, such as the Square Root of the Sum of the Squares (SRSS), arithmetic and geometric mean, and the maximum spectral response. All types of averaging were primarily evaluated in this research, then among scaling methods, SRSS was selected because of a better fitness with the target spectrum. According to ASCE 7-05, horizontal components of ground motion should be scaled in such a way that the average Square Root of the Sum of the Squares (SRSS) spectrum from all horizontal component pairs, in the range of 0.2 T–1.5 T, where T is the predominant period of vibration for the studied structure, does not fall below 1.3 times the corresponding ordinate of the design spectrum by more than 10%. This approach is used for scaling the components of ground motion. These scaling values for used ground motions are illustrated in Table 3.

The scaling procedure for applying ET acceleration functions resembles the scaling of actual records, i.e. mentioned methods are used to obtain the scale factor for ET acceleration functions, considering their response spectrum at the target time. For example, for a pair of ET acceleration functions that consist of ETA20f01 and ETA20f02, the acceleration response spectrum for each ETAF is calculated at the target time. Using the SRSS method mentioned above, these response spectra are then combined, and compared to the amplified design spectrum (1.3 times the design spectrum), and the scaling factor could be calculated, which should be applied for both ET acceleration functions used. In this way, not only did the results from all scaling approaches lead to almost the same factor for ET acceleration functions, but also this scaling factor did not change significantly from one frame to another, while these scale factors were considerably different under different ground motions, due to their specific response spectrum. The major reason for such consistency of scaling methods in ET acceleration functions is that they inherently comply with the design response spectrum and, so the shape of the response spectrum is almost the same in different acceleration functions

Table 4: Scaling value for pairs of ET acceleration functions.

	ETA20f01 02	ETA20f02 03	ETA20f03 01	Average ET
F3DMMS3X3Y3IRXY	0.473	0.475	0.478	0.475
F3DMMS3X3Y3IRX	0.482	0.482	0.483	0.482
F3DMMS4X3Y3	0.480	0.483	0.483	0.482
F3DMMS4X3Y3IRX	0.480	0.483	0.483	0.482
F3DMMS4X3Y3IRXY	0.480	0.483	0.483	0.482
F3DMMS5X4Y4	0.485	0.488	0.484	0.486
F3DMMS5X4Y4IRX	0.485	0.488	0.484	0.486
F3DMMS5X4Y4IRXY	0.485	0.488	0.484	0.486
F3DMMS7X3Y5	0.495	0.499	0.497	0.497

belonging to the same set of records. The scale factors for pairs of ET acceleration functions are shown in Table 4. As shown in Table 4, the average scale factor of three pairs of ET acceleration function is used for all individual pairs in the analysis of each model.

6.2. Multi-component analysis by ET method

ET acceleration functions used in this study are designed in such a way that their response spectra increase by time. When used for response history analysis, most regulations set forth in the design codes, regarding general three-dimensional time history analysis, are also applicable to ET analysis. However, some special characteristics of ET acceleration functions require particular consideration. Although ET acceleration functions are statistically independent, all ET acceleration functions are produced in the same manner using the same assumptions, thus statistically the intensity and response spectrum at each time are theoretically the same for all ET acceleration functions in a set of ET acceleration functions. Therefore, definition of a major or minor component in the ET method is not relevant. Secondly, as the ET acceleration functions are produced synthetically, the critical angle or principal direction of excitation is also of little significance. Finally, because all ET acceleration functions in each set are statistically alike, in this study, pairs of ET acceleration function can be considered by swapping ET acceleration functions alternately with each other. For example, the first pair of ET excitations include ETA20f01 in the X direction and ETA20f02 in the Y direction; the second pair is a combination of ETA20f02 in the X direction and ETA20f03 in the Y direction, and the third pair is made up of ETA20f03 and ETA20f01 in X and Y directions, respectively. These pairs are applied to the structure alternately and results are averaged for final evaluation. A proposed algorithm for three-dimensional ET analysis is illustrated in Figure 5.

Following the flowchart in Figure 5, the designed frames were analyzed and compared with results from time history analysis under previously mentioned real accelerograms, in a situation where components of records are applied in principal directions of structures. For instance, displacements in X and Y directions of a three-story building, obtained from two methods, are compared.

As in ET analysis, time is a representative of intensity; it is obvious that results from the ET method are plotted over time, while responses of real accelerograms appear as points, with $+/-$ one standard deviation mark, and are extended by a line (representing linear analysis) for comparison. These values are compared with the ET method at target time, i.e. $t = 10$ s in this study (Figure 6). In this figure, Uxsti and Uysti determine the ith story displacement in X and Y directions, respectively. As shown, the results of ET analysis at $t = 10$ s are close to the results obtained from analysis under real accelerograms in

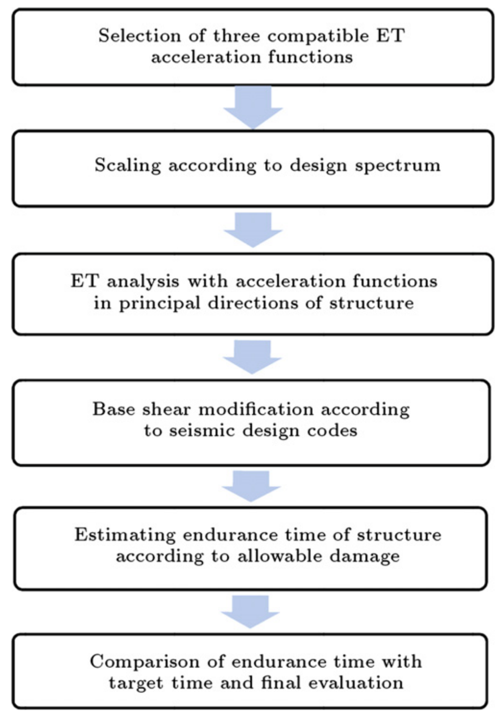
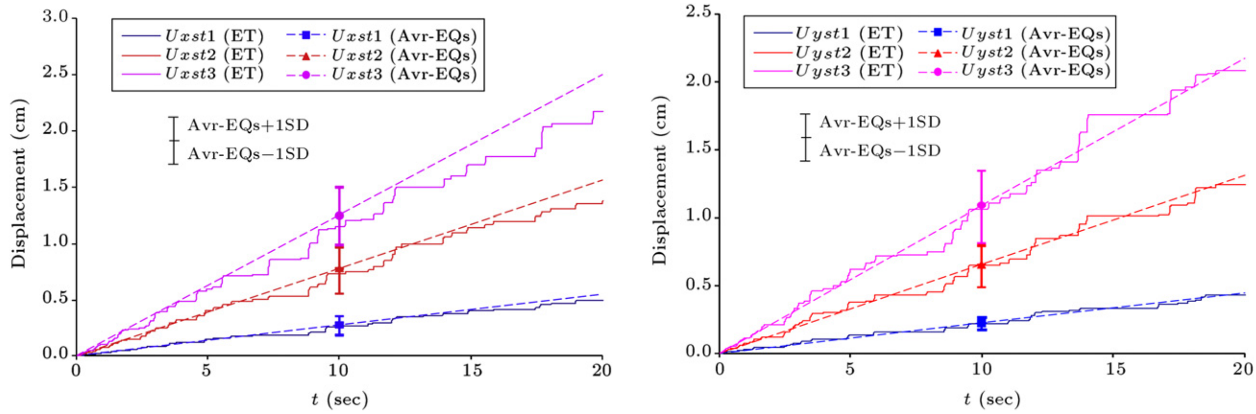


Figure 5: Proposed flowchart for bi-directional analysis of structures by ET method.

principal directions. It should be noticed that the curve is an average of results from ET analysis, and points are the average of results from real accelerograms. Further investigations show that other frames had similar results. For example, drifts of a seven-story building in the X direction, obtained from ET analysis at $t = 10$, are compared with results from real accelerograms (Figure 7).

In addition to displacement and drifts, internal forces of all members, e.g. moments in beams and columns, and axial force in columns are studied. In Figure 8, for a three-story building, moments and axial forces in some random beams and columns are sketched by time for ET analysis and compared with real accelerograms. In this figure, M_Bi and P_Cj refer to maximum moment in beam number i and maximum axial force in column number j, respectively. These elements are specified in Figure 3(b). It is obvious from Figure 8 that the response of all studied structural indices in the studied frames is approximately the same as in the ET method at target time ($t = 10$ s), and the horizontal components of real ground motions in principal directions. Obviously, there are some discrepancies that will be discussed later in this paper.

In addition to some random members, all members including all beams and columns were investigated to specify any



(a) Displacement in X direction.

(b) Displacement in Y direction.

Figure 6: Displacement responses at any time in ET analysis and comparison with real earthquakes, F3DMMS3X3Y3IRX.

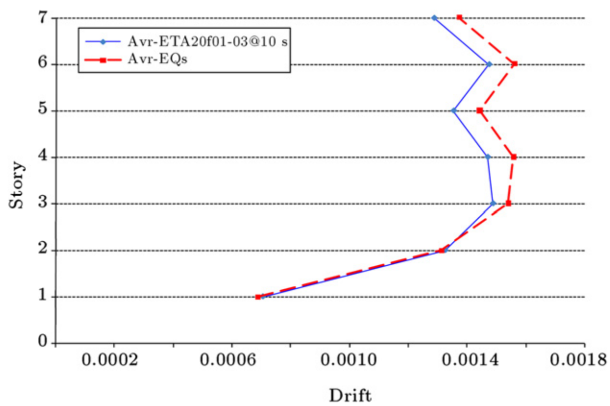


Figure 7: Comparison of drifts from ET analysis at $t = 10$ s and actual records, F3DMMS7X5Y3.

member which might behave differently from others. Furthermore, in this step, the correlation of results between the ET method and real earthquakes is derived. In Figure 9, drifts and displacements of stories in both directions for the average of real earthquakes are drawn versus the average of ET analysis at the target time for the regular 5-story frame. In addition, in Figure 10, the same figures are shown for the maximum moment in beams and the axial force in all columns of the 5-story building, which is irregular in both directions.

It is essential to note that the response of these structures is compared only under lateral loads and, in this state, the effect of vertical loads, such as gravity and the effect of vertical acceleration are ignored. Of course, considering gravitational load does not affect the conclusions obtained in this paper, which are based on the lateral load response.

As indicated from the figures, for studied damage criteria, the correlation of results from the ET method and real earthquakes is close to 1, and results from the average of earthquakes in principal directions can be estimated by a unique correction factor for each frame.

The correction factor is defined as the relation between results from real earthquakes and ET analysis at target time ($t = 10$ s), i.e.:

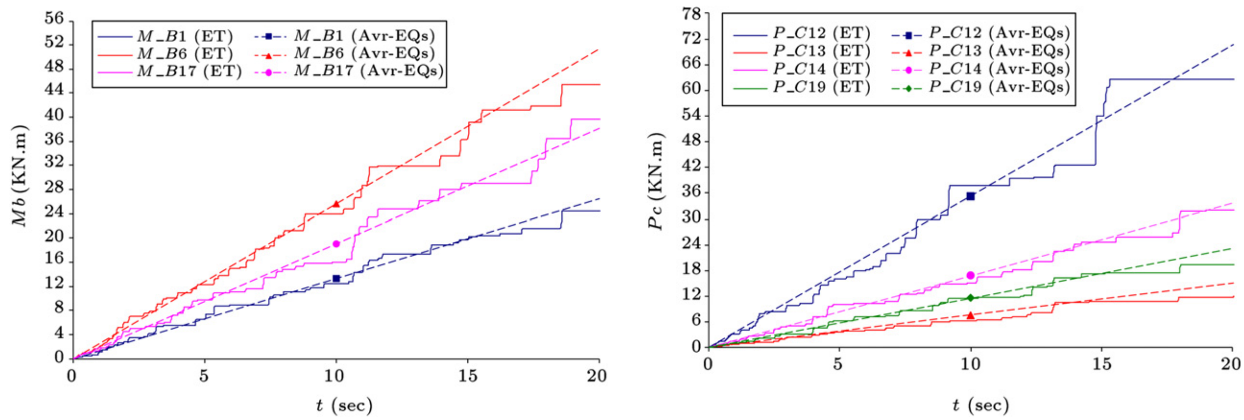
$$CF = \frac{DI_{\text{Avr-EQs}}}{DI_{\text{ET}@t=10}}. \quad (4)$$

Correction factors and correlation coefficients of all studied frames and most damage criteria for each frame are shown in Table 5.

As can be seen in Table 5, the correlation coefficient between the ET method and real accelerograms for various responses of studied frames is near 1. This means that all members conform to one correction factor and with application of this factor, the average response of real ground motions in principal directions can be estimated by the ET method. The next point in this table is that the correction factors are nearly the same for various response parameters in each frame, thus there is no need to apply different CF for different parameters. It is also found that discussed CFs for all frames are about unity (with maximum 15% tolerance), meaning that results from the ET method at target time are the same as results from the average of real accelerograms in principal directions of the structure.

As can be seen, there are some differences between results of ET acceleration functions and real earthquakes which occur due to the incompatibility of the response spectra of ET acceleration functions and actual ground motion. As can be seen in Figure 11, the average response spectrum obtained from the maximum response of two horizontal components for 7 selected earthquake ground motions is not exactly the same and, at most periods of vibration, is slightly greater than the average response spectrum of 3 ET acceleration functions. This inconsistency happens when, at some periods of vibration, the response spectrum of the second component of each ground motion is greater than the first component produced by the ET acceleration functions, with which it is compatible. Also the discrepancy is caused by the roughness of the target spectrum and optimization problems in generating ET acceleration functions.

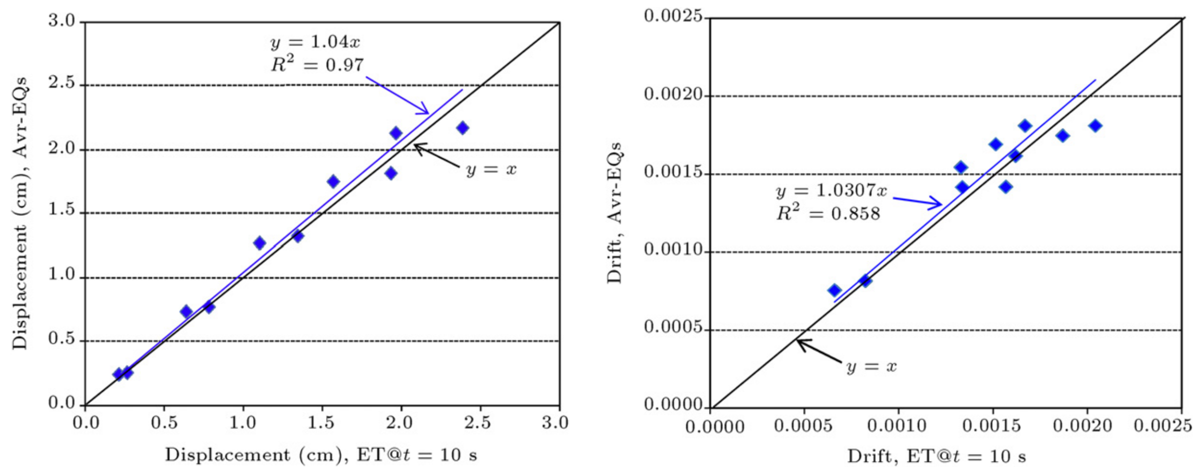
To reduce these discrepancies, the compatibility between two spectra should be improved. This goal can be achieved by producing more optimized ET acceleration functions or by using more than three acceleration functions in ET analysis. Also instead of considering the first component of earthquake ground motion, the maximum response of two horizontal components should be considered for generating or scaling ET acceleration functions. However, due to the fact that the ratio of intensities for two horizontal components is not determined, and there is no unique value for such a parameter, it could be assumed that the ET acceleration functions are produced to be compatible with the component that has greater intensity than another. By this assumption, depending on the structural period, the results of the ET method could be negligibly underestimated as compared to those obtained from actual ground motions. Due to the fact that this incompatibility can



(a) Moment in beams.

(b) Axial force in columns.

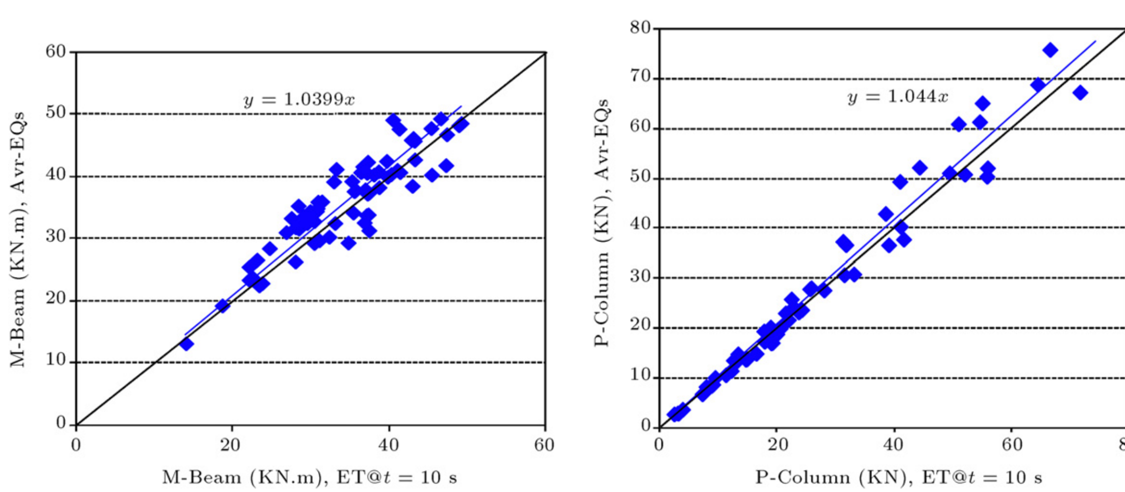
Figure 8: Internal force in members of F3DMMS3X3Y3IRXY in ET method and real earthquakes.



(a) Displacement.

(b) Drift.

Figure 9: Drifts and displacement values from earthquakes vs. ET results at $t = 10$ s for F3DMMS5X4Y4.



(a) Moment in beams.

(b) Axial force in columns.

Figure 10: Internal forces from earthquakes vs. ET results at $t = 10$ s for F3DMMS5X4Y4IRXY.

be ignored in the current study, considering the insignificance of the differences (maximum difference is 20%), these sets of ET acceleration function can be regarded as acceptable for reasonable response estimation.

Table 5: Correction factor and correlation coefficient of structural responses in the ET method and real earthquakes.

	Displacement		Drift		*Mb*		*Pc*		*MxC*		*MyC*	
	CF	ρ	CF	ρ	CF	ρ	CF	ρ	CF	ρ	CF	ρ
F3DMMS3X3Y3IRX	1.039	1.000	1.026	0.989	1.021	0.948	1.017	0.985	1.033	0.902	0.974	0.990
F3DMMS3X3Y3IRXY	0.997	0.997	0.949	0.985	0.990	0.959	0.961	0.992	0.969	0.932	0.992	0.835
F3DMMS4X3Y3	1.041	0.994	1.029	0.977	1.080	0.994	1.092	0.997	1.077	0.977	1.090	0.973
F3DMMS4X3Y3IRX	1.055	0.990	1.059	0.963	1.122	0.995	1.113	0.994	1.156	0.998	1.153	0.997
F3DMMS4X3Y3IRXY	1.049	0.993	1.053	0.976	1.120	1.000	1.108	0.990	1.115	0.998	1.114	0.998
F3DMMS5X4Y4	1.040	0.986	1.031	0.942	1.125	0.994	0.987	0.977	1.158	0.999	1.148	0.991
F3DMMS5X4Y4IRX	0.988	0.998	0.978	0.987	1.000	0.987	0.969	0.996	0.985	0.992	1.078	0.995
F3DMMS5X4Y4IRXY	1.009	0.984	0.991	0.959	1.040	0.907	1.044	0.988	0.978	0.989	1.154	0.997
F3DMMS7X5Y3	1.089	0.978	1.068	0.923	1.153	0.991	1.134	0.985	1.155	0.999	1.106	0.999

Table 6: Correction factor and correlation coefficient of structural responses in the ET method and real earthquakes in their critical directions.

	Displacement		Drift		*Mb*		*Pc*		*MxC*		*MyC*	
	CF	ρ	CF	ρ	CF	ρ	CF	ρ	CF	ρ	CF	ρ
F3DMMS3X3Y3IRX	1.162	0.999	1.149	0.986	1.157	0.953	1.159	0.986	1.180	0.928	1.086	0.987
F3DMMS3X3Y3IRXY	1.120	0.996	1.069	0.990	1.139	0.961	1.107	0.993	1.122	0.919	1.137	0.764
F3DMMS4X3Y3	1.157	0.990	1.148	0.956	1.217	0.995	1.166	0.994	1.206	0.980	1.218	0.971
F3DMMS4X3Y3IRX	1.158	0.984	1.161	0.946	1.141	0.997	1.199	0.993	1.171	0.998	1.168	0.997
F3DMMS4X3Y3IRXY	1.185	0.983	1.193	0.941	1.272	1.000	1.200	0.989	1.166	0.999	1.166	0.999
F3DMMS5X4Y4	1.094	0.980	1.084	0.919	1.199	0.989	1.082	0.992	1.155	0.999	1.144	0.990
F3DMMS5X4Y4IRX	1.182	0.997	1.176	0.986	1.210	0.989	1.215	0.993	1.190	0.993	1.230	0.996
F3DMMS5X4Y4IRXY	1.162	0.983	1.118	0.959	1.182	0.904	1.228	0.989	1.172	0.991	1.222	0.997
F3DMMS7X5Y3	1.012	0.976	1.003	0.904	1.090	0.990	1.076	0.986	1.098	0.999	1.048	0.999

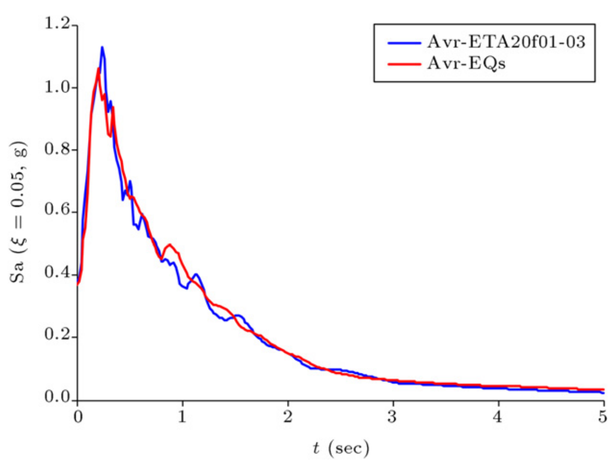
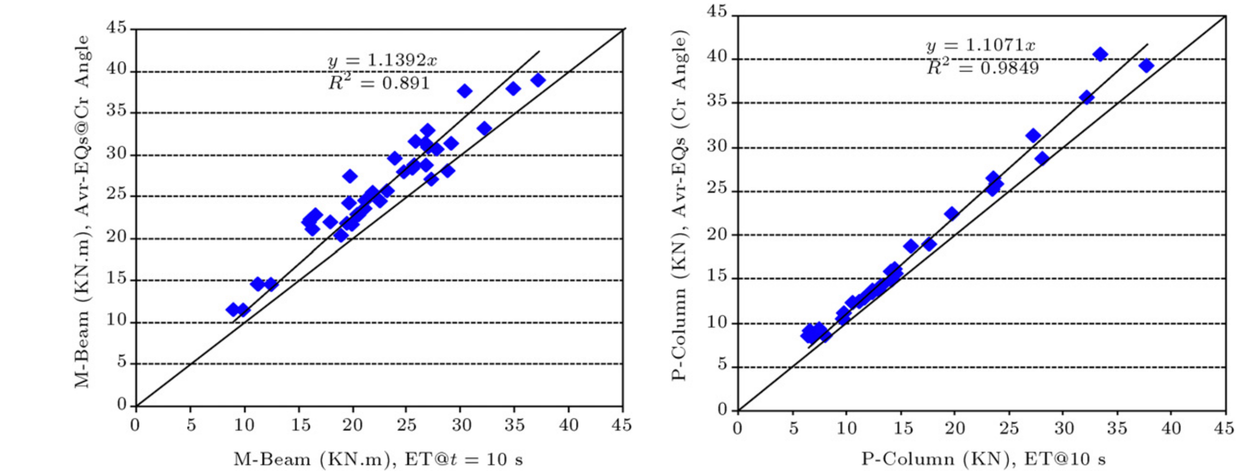


Figure 11: Response spectra of ET acceleration functions at $t = 10$ s and average of real earthquakes.

As stated at the beginning of this paper, most seismic codes accept application of seismic excitation only in principal directions. However, some structural codes, such as ASCE7-05, impose more stringent requirements, such as necessitating the analysis of members at intersections of two lateral resistant systems of building, which are located in E and F seismic categories, be performed in a critical direction. According to this requirement, engineers should analyze the structure under components of each earthquake in its critical direction. Then, the maximum values obtained from each record are averaged from 7 accelerograms. Although it is not likely that all members reach their maximum value simultaneously in the critical direction, and this approach seems to be conservative, it is necessary that this type of analysis be performed for vital structures. In the next step of the current paper, the average of the maximum structural response in the most adverse direction will be evaluated. However, more investigation is required in order to draw general conclusions in this regard. In Figure 12, the internal forces of all members for irregular three-storey frames are compared between the average of maximum results of each earthquake in their critical direction and the ET method at target time.

It is apparent that correlations of results from the two methods are high, and a correction factor can be applied to estimate the average of maximum results of earthquakes by the ET method. For studied frames, these correction factors are obtained and shown in Table 6. It should be noted that while strong correlation exists in each case, the correction factor varies based on the model, and no clear trend can be observed in order to propose a general correction factor.

Obviously, from Table 6, correlations of all frames and all damage criteria are significantly high and for each frame, results from the ET method could be scaled up to results from the average of real accelerograms at their critical angles. One reason for this is that when maximum responses of earthquakes at the critical angle for each ground motion are averaged, the effective level of response spectrum, as an index of intensity, increases as a result of the statistical process of maximizing between more analysis cases. On the other hand, the probability of exceedance of seismic hazard is reduced [21]. For example, averages of the maximum response in the X direction of a 2DOF system under components of used earthquakes in their critical directions were computed and compared with that of ET acceleration functions at target time ($t = 10$ s) in Figure 13. It is obvious that ET acceleration functions are applied just at two orthogonal directions and will not be rotated; the critical angle for ET analysis is meaningless. It is seen that the response spectrum of ET acceleration functions is less than the response of the 2DOF system under horizontal components of real earthquakes at their critical angles at most periods of vibration. More studies are required in order to obtain effective response spectra pertaining to ground motions applied to all directions. In this way, ET acceleration functions can be developed based on these critical direction spectra, and



(a) Moment in beams.

(b) Axial force in columns.

Figure 12: Internal forces obtained for earthquakes at critical angle vs. ET method at target time F3DMMS3X3Y3IRXY.

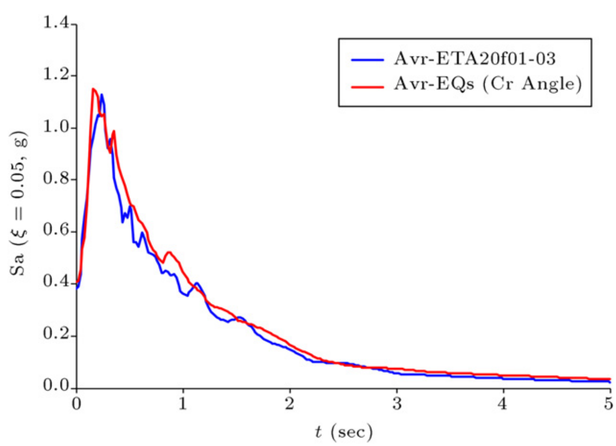


Figure 13: Response of 2DOF system under components of real earthquakes at their critical orientation and ET acceleration functions at $t = 10$ s.

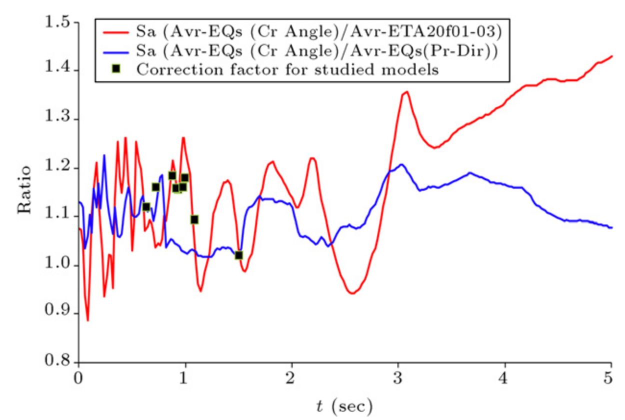


Figure 14: Ratio of response of 2DOF system under ET acceleration functions and horizontal components of earthquakes at critical orientation.

improved estimates can be made. However, it is also possible to improve the estimation by up-scaling current ET records, so that their response, in Figure 13, matches those of ground motions at their critical angle. The spectral ratio of horizontal components of earthquakes in critical and principal directions and ET acceleration functions are depicted in Figure 14.

It can be observed in Figure 14 that the ratios for actual ground motion vary between 1 and 1.2 and, for ETAF, between 0.9 and 1.4. Furthermore, it is seen that at periods $T > 3$ s, the ratio of the spectrum from earthquakes at a critical angle and ET acceleration functions increases. This matter can be expected from Figure 4. The response spectrum from the second component of earthquakes at higher periods, after $T = 3$ s, is greater than that of the first component, with which ET acceleration functions are consistent. Thus at higher periods, especial consideration should be given for determining the design spectrum based upon which the ET acceleration functions are selected or produced. Also it is seen that the curve obtained for actual ground motions is smoother than that of ETAF. This is due to the fact that the response spectra of ETAF and used records are not exactly the same, and there are always minor differences between ETAF and the target spectrum. Please note that this figure is consistent with results obtained from Table 6 where the scale factor varies between 1 and 1.23. Comparing the CF obtained from Table 6 with Figure 14, it is concluded that the differences between ET analysis and results of time history analysis at a critical angle can be interpreted by their response spectra. It seems that an appropriate scale factor, estimated from Figure 14, can be applied to studied frames, to estimate the average response of real earthquakes at their critical angle. It should also be noted that this required statistical correction factor is conceptually the same, considering either ground motions or ET analysis results. On the other hand, in ET analysis, this scale factor can be converted into its equivalent extra time; thus the average response of earthquakes at a critical angle can be estimated in the ET method by reading the response at a higher time, provided that this observation can be verified by more elaborate study. In any case, it should be noted that the response in a critical direction can be quite different from that obtained from analysis based on orthogonal direction excitation, and further research in this area is required if any general conclusion is to be achieved.

7. Conclusions

In this paper, application of the ET method in the analysis of buildings under bi-directional excitation was investigated, and a procedure for three-dimensional analysis by the ET method was proposed. Following seismic code recommendations, results of the time history analysis of buildings under horizontal components of earthquakes in principal and critical directions were compared with the results of ET analysis under pairs of ET acceleration functions applied to principal directions of the studied buildings. Based on the observations made in this study, the following conclusions can be drawn:

1. The response of structures, estimated by the proposed bi-directional ET analysis procedure, matches well with results from time history analysis, using real earthquake components, in principal directions of structures.
2. The average and minimum correlation coefficients for analysis results obtained from the ET method, and time history analysis, using real earthquakes, for investigated frames, are 0.97 and 0.80, respectively. Considering this strong correlation between results, it can be concluded that the average response to seismic excitation in a linear range can be predicted by the ET method with reasonable accuracy.
3. The response of structures studied in this research in critical directions of each earthquake can be correlated to their response using orthogonal direction analysis, by applying a correction factor of about 1.05 to 1.2 in the studied models. In these cases, results from ET analysis, at $t = 10$ s, can be multiplied by a correction factor or alternatively damage values should be read as a higher target time on the ET response curve for critical direction estimation. However, this observation cannot be extended to general cases, and more investigation is required before a reasonable conclusion can be made in this regard.
4. Based on the results from the studied models in this research, the response of steel moment frames subjected to multi-component seismic excitation can be predicted with reasonable accuracy using the proposed procedure. This procedure can reduce required computational effort when time history analysis is required, such as analysis of the effect of damping devices. However, in order to take advantage of the full potential of the ET method in multi-component seismic analysis, its application should be extended in the future to non-linear analysis.

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