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Probabilistic analysis of consolidation that considers spatial variability using the stochastic response surface method ☆

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Abstract

To obtain more accurate and reasonable results in the analyses of soil consolidation, the spatial variability of the soil properties should be considered. In this study, we analyzed the consolidation by vertical drains for soil improvement considering the spatial variability of the coefficients of consolidation. The coefficients for the variation in the vertical and horizontal coefficients of consolidation in Yeonjongdo, South Korea were evaluated, and the probability density function (PDF) was assumed by the Anderson–Darling goodness-of-fit test. Standard Gaussian random fields were generated based on a Karhunen–Loeve expansion, and then transformed using Hermite polynomials in the random field with the log-Gaussian PDF of the coefficient of consolidation. The average degree of consolidation was subsequently calculated using the finite difference method coupled with log-Gaussian random fields. In addition, the stochastic response surface method (SRSM) was applied for the efficient probabilistic uncertainty propagation. A sensitivity analysis was performed for the input parameters of the random field, and the spatial variability was considered using random variables from the Karhunen–Loeve expansion as the input data for the SRSM. The results indicated that when considering the spatial variability of soil properties, the probability of failure for the target degree of consolidation was smaller when the correlation distance was taken into account than when it was not. Additionally, the probability of failure decreased when the correlation distance decreased. Compared with the Monte Carlo simulation (MCS) results, the SRSM analysis can achieve results of similar accuracy to those obtained using the MCS analysis with a sample size of 100,000 (numerical runs), and a third-order SRSM expansion with only 333 numerical runs is sufficient for obtaining the probability with errors less than 0.01.

Keywords: Spatial variability; Consolidation; Stochastic response surface method; Monte Carlo simulation; Random field

1. Introduction

It is well-known that soil properties exhibit variation properties. Nearly all natural soils are highly variable in their properties and are rarely homogeneous. Soil heterogeneity can be classified into two main categories. The first category is lithological heterogeneity and the second is inherent spatial soil variability (Elkateb et al., 2002). The variability of soil properties shows a spatial correlation. Thus, the accuracy and reliability of the probabilistic analysis decrease when using only one random variable as the design parameter. Recently, a considerable number of studies have been conducted on geotechnical problems that consider the spatial variability of soil properties with the random field theory (Fenton and Griffiths, 2001; Sudret and Der Kiureghian, 2002; Popescu et al., 2005; Cho, 2007). Consolidation is one of the important geotechnical problems, and it is greatly influenced by the spatial variability. A few studies have been carried out to

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investigate the effects of spatial variability on soil consolidation, and 1-D consolidation analyses have been performed (Badaoui et al., 2007; Huang et al., 2008). Bari et al. (2012) investigated the effects of the spatial variability of soil permeability and volume compressibility on the consolidation of soft soil by prefabricated vertical drains. However, more research is needed through a variety of approaches which can consider their efficiency and applicability.

The probabilistic uncertainty propagation methods available for these analyses fall into three categories (a) the Monte Carlo simulation (MCS), (b) the analytical method, and (c) the response surface method (RSM) (Phoon and Huang, 2007). The MCS is a universal method that can be applied regardless of the complexities in the physical model and/or the input uncertainties. When the models are large or when there are numerous parameters, however, the MCS can be costly and time-consuming. Due to these shortcomings of the MCS, a variety of variance reduction techniques have been proposed to reduce the number of simulation runs. Alternately, the RSM was developed by Box and Draper (1987) to replace the expensive MCS numerical model with an approximation. Isukapalli et al. (1998) proposed the stochastic response surface method (SRSM). The SRSM can be viewed as an extension of the RSM. The main difference between the SRSM and the RSM is that the inputs for the SRSM are random variables, whereas the inputs for the RSM are deterministic variables.

The purpose of this paper is twofold. Firstly, the effects of the spatial variability of the coefficient of consolidation (c) on the consolidation by vertical drains was investigated. The statistical properties of the vertical and horizontal coefficients of the consolidation (c_v and c_h, respectively) in Yeonjongdo were evaluated by field data, and the probability density function (PDF) was assumed to be a three-parameter log-Gaussian distribution by the Anderson–Darling goodness-of-fit test. In particular, we generated the log-Gaussian random field considering the statistical properties of the field, and the positive correlation between c_v and c_h to obtain more reasonable results. A consolidation analysis was carried out by the finite difference method (FDM) coupled with log-Gaussian random fields, and a probabilistic analysis was performed using the MCS with a sample size of 100,000. Secondly, we assessed the performance of the SRSM for conducting the efficient uncertainty propagation of geotechnical problems with spatial variability. The standard random variables of the random field were subsequently used as input data for the SRSM analysis, and a sensitivity analysis was performed to assess the impact of changes in the input parameters on the output results. As a result, the accuracy of the SRSM analysis results was verified by comparing them with the MCS results.

2. Random field

2.1. Spatial variability of soil

The spatial correlation of soil properties in geotechnical problems is known to influence the soil behavior. As soil behaviors are utilized as locally averaged values, rather than extreme values, spatial variability has been recognized as an important issue. Additionally, traditional statistical parameters, such as mean and variance, are one-point statistical parameters that cannot accurately capture the features of the spatial structure of the soil (El-Ramly et al., 2002).

Vanmarcke (1983) used a scale of fluctuation to describe the spatial extent of the soil properties that show a strong spatial correlation. The spatial variations in the soil properties can be effectively described by their correlation structure within the framework of random fields. DeGroot and Baecher (1993) used the autocovariance distance, which is defined as the distance to which the autocovariance function decays to $1/e$ (where e is the base of the natural logarithms), to describe the spatial extent.

An autocorrelation function for geotechnical problems was presented by Rackwitz (2000). In this study, the following exponential autocorrelation function is used:

$$\rho(x_1, x_2) = e^{-|x_1 - x_2|/r} \tag{1}$$

where r is the correlation distance.

2.2. Karhunen–Loeve expansion

In the late 1980 s, a number of techniques were developed to define discrete random fields, including the midpoint method, the spatial averaging method, and the shape function method. These early methods are relatively inefficient, and a large number of random variables are required to achieve a good approximation. Since that time, more efficient random field discretization methods, using the series expansion method, have been developed. Among these, the K–L expansion is preferred when an exponential autocorrelation function is used, because it provides the most accuracy (Sudret and Der Kiureghian, 2000).

The K–L expansion of a random field with a mean value (μ_ω) and variance (σ_ω^2) is written as (Spanos and Ghanem, 1989)

$$\omega(x,\theta) = \mu_\omega(x) + \sum_{i=1}^{\infty} \sigma_\omega \sqrt{\lambda_i} f_i(x) \xi_i(\theta), \; x \in \Omega, \tag{2}$$

where λ_i and $f_i(x)$ are, respectively, the eigenvalue and eigenfunction of the covariance function $C(x_1, x_2)$, and $\xi_i(\theta)$ presents the uncorrelated zero mean random variable. For practical implementation, the discretization of random field $\omega(x,\theta)$ is obtained by truncating the series expansion at the M-th term, namely,

$$\omega(x,\theta) = \mu_\omega(x) + \sum_{i=1}^{M} \sigma_\omega \sqrt{\lambda_i} f_i(x) \xi_i(\theta) \tag{3}$$

The number of truncated terms depends on the ratio of the correlation length to the domain size (Zhang and Lu, 2004). Generally, larger ratios of correlation length to domain size require smaller terms expansion.

2.3. Hermite polynomial chaos expansion

Hermite polynomials utilize a series of orthogonal polynomials to facilitate the stochastic analysis. The output Y is approximated by a Hermite polynomial chaos expansion, as given by

$$Y = a_0 H_0 + \sum_{i_1=1}^{n} a_{i_1} H_1(\xi_{i_1}(\theta)) + \sum_{i_1=1}^{n} \sum_{i_2=1}^{i_1} a_{i_1 i_2} H_2(\xi_{i_1}(\theta), \xi_{i_2}(\theta)) + \sum_{i_1=1}^{n} \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} a_{i_1 i_2 i_3} H_3(\xi_{i_1}(\theta), \xi_{i_2}(\theta), \xi_{i_3}(\theta)) + \cdots \quad (4)$$

where $\{\xi_{i_k}\}_{k=1}^{p}$ is a set of standard Gaussian random variables, a_i is the deterministic polynomial chaos expansion coefficient, and $H_p(\xi_{i_1}, \cdots, \xi_{i_p})$ are multi-dimensional Hermite polynomials of degree p. Eq. (4) can be written in a simpler form as

$$Y = \sum_{i=0}^{\infty} a_i H_i(\xi(\theta)) \quad (5)$$

2.4. Methodology of generating non-Gaussian random fields

Although Gaussian random fields are often used for modeling uncertainties with spatial variability for convenience or for a lack of available data, the Gaussian model is not applicable in many situations (Cho, 2010). In this study, Hermite polynomials are used to represent the non-Gaussian random fields. Phoon (2003) proved that any random variable can be expanded as a sum of the Hermite polynomials.

In Eq. (5), if μ_ω is zero and σ_ω is one, standard Gaussian random fields are generated by the K–L expansion. The probability distribution can be transformed using Hermite polynomials in the standard Gaussian space. Non-Gaussian random fields can be transformed to replace $\xi(\theta)$ in Eq. (5), with $\omega(x, \theta)$ in Eq. (2), as follows:

$$Y(x, \theta) = \sum_{i=0}^{\infty} a_i H_i(\omega(x, \theta)) \quad (6)$$

where $Y(x, \theta)$ is the non-Gaussian random field and i is the Hermite polynomial term. The probability distribution of the random field is determined by coefficient a_i.

3. Stochastic response surface method

3.1. Stochastic input data

The spatial variability can be assessed using uncorrelated random variables from the K–L expansion as the input data for the SRSM analysis. The number of random variables in the SRSM is equal to the number of terms used in the K–L expansion. The convergence and accuracy of the K–L expansion depend on the terms in the truncated K–L expansion. In physical systems, it is expected that the material properties vary smoothly at the scale of interest for most applications. Therefore, only a few terms for the K–L expansion can capture most of the uncertainty in the process (Huang et al., 2007).

3.2. Selection of collocation points

The collocation points are selected following the orthogonal collocation method proposed by Villadsen and Michelsen (1978). The collocation points can be replaced by the roots of the next higher order polynomial. As the polynomial chaos expansion uses the Hermite polynomial as a basis function, the collocation points are combinations of the roots of the Hermite polynomial. When 3rd order Hermite polynomals were used, the combinations of the roots of the fourth-order Hermite polynomial ($\pm\sqrt{3 \pm \sqrt{6}}$) and zero were used as the collocation points. This is because zero lies in regions of the highest probability, although zero is not one of the roots.

3.3. Implementation of SRSM

The implementation of the SRSM is to determine the coefficients of the polynomial chaos expansion. The coefficients of the polynomial chaos expansion are obtained using the model outputs at the selected collocation points.

Fig. 1 shows a schematic depiction of the SRSM. The matrix of the SRSM can be expressed as

$$[H][a] = [Y] \quad (7)$$

where $[H]$ is a matrix of the Hermite polynomials, $[a]$ is a coefficient vector, and $[Y]$ is a response vector with the ith component given by $Y(\xi_{i_1}, \xi_{i_2}, \xi_{i_3}, \cdots, \xi_{i_d})$. Applying the least squares method, the analytical solution can be expressed as follows:

$$[a] = \left([H]^T[H]\right)^{-1}[H]^T[Y] \quad (8)$$

4. Probabilistic analysis

4.1. Study area

The study area is located in Yeonjongdo, South Korea. The groundwater level is almost close to the surface, so the ground is saturated. In accordance with the United Soil Classification

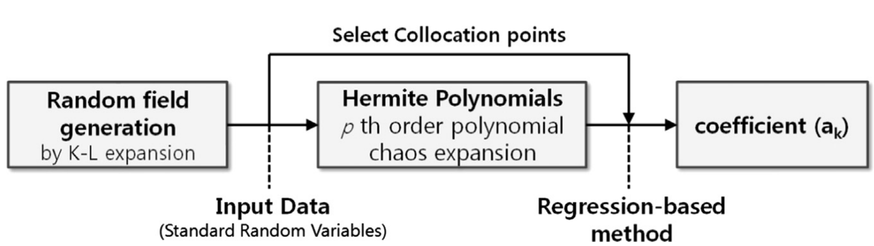

Fig. 1. Schematic depiction of SRSM.

System (USCS), most of the soils are classified as CL, ML, and SM. When the depth is more than 20 m, the soil types are weathered soil and weathered rock. Therefore, the consolidation analysis conditions assume that the analysis range is up to 20 m from the surface and that the drainage condition is in one direction (upward).

Fig. 2 shows c_v according to depth. When the depth increases, the soil layer changes, but there is no relationship between c_v and depth since the coefficient of correlation is -0.18. Therefore, 1459 data for c_v were used to analyze the statistical properties and applied to the probabilistic analysis.

c_h Data are small in quantity (only 32 data) and the measured depth is approximately 9–10 m. Therefore, the correlation does not consider the relationship between c_h and depth. Fig. 3 shows the probability density and cumulative distribution functions (CDF) for c_v and c_h.

To find the appropriate PDF, the Anderson–Darling goodness-of-fit test is performed for the c_v data. The results show that the three-parameter log-Gaussian distribution is the most appropriate when compared with other distributions (Gaussian, two-parameter Log-Gaussian, gamma, weibull, logistic, and general extreme value). The PDF of the three-parameter log-Gaussian distribution is given by

$$f(x;\mu,\sigma,\gamma)=\frac{1}{(x-\gamma)\sigma\sqrt{2\pi}}\exp\left\{-\frac{[\ln(x-\gamma)-\mu]^2}{2\sigma^2}\right\} \tag{9}$$

where μ is the scale parameter, σ is the shape parameter that affects the shape of the distribution, and γ is the threshold parameter. Then, $Y=\ln(x-\gamma)$has a Gaussian distribution with mean μ and variance σ^2. It is assumed that the PDF of c_h is the same as c_v, because the amount of data for c_h is insufficient for finding the PDF. The three parameters of the PDF are given in Table 1.

To assess how closely the two data sets (experimental CDF and fitted CDF) agree, the P–P (probability–probability) plot was plotted and is shown in Fig. 4. The P–P plots are approximately linear. Therefore, the fitted PDF is the correct model. The statistical properties of c_v and c_h are given in Table 2.

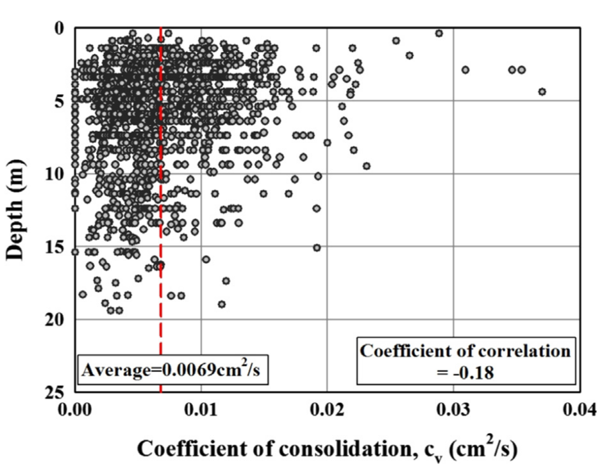

Fig. 2. c_v According to depth and coefficient of correlation.

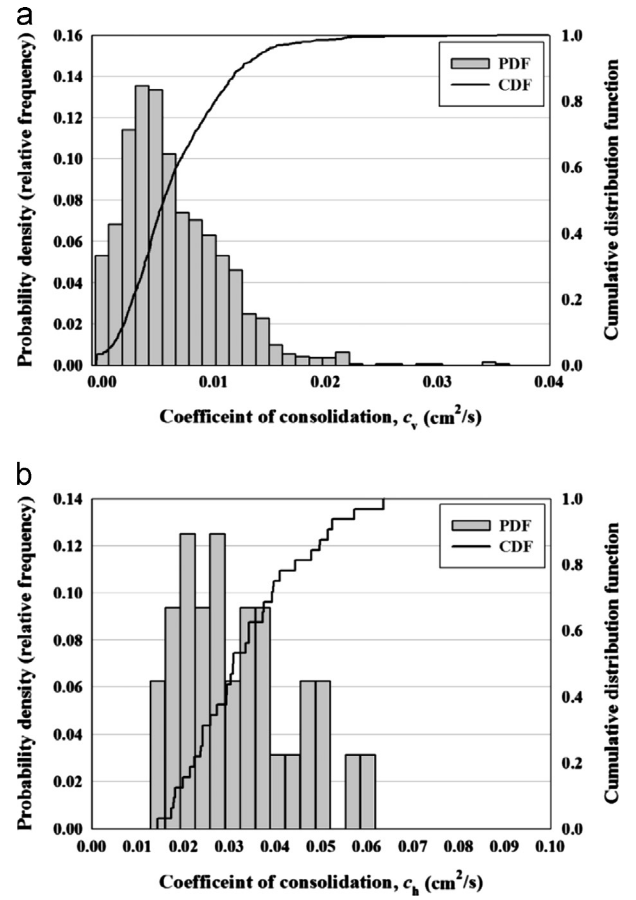

Fig. 3. Probability histogram and cumulative distribution function. (a) Vertical coefficient of consolidation, (b) horizontal coefficient of consolidation.

Table 1
The three parameters of the log-Gaussian distribution for the coefficient of consolidation.

Parameter	μ	σ	γ
c_v	-4.605	0.40618	-0.00394
c_h	-3.485	0.38076	0.00069

The correlation distance of the consolidation coefficient is not yet well known. For many soil properties, the vertical correlation distance is approximately 1–6 m, while the horizontal correlation distance is approximately 2–60 m (Huber et al., 2011; Phoon and Kulhawy, 1999). However, the horizontal domain is smaller than the vertical domain. In addition, a small correlation distance is more important for evaluating the applicability of the SRSM, because the number of terms in the K–L expansion increases for a specified accuracy as the correlation distance becomes smaller. Therefore, we assume that the correlation distances are 1 m and 2 m, and that the horizontal correlation distance is equal to the vertical correlation distance.

4.2. Generating log-Gaussian random fields

c_v and c_h have different variations. Therefore, each 1-D log-Gaussian random field must be generated for c_v and c_h. However, it is well known that there is a positive correlation between c_v and c_h. In the random field, the average and the distribution of properties are determined by random variables. If the same random variables are used for each term in the random fields, a positive correlation between c_v and c_h can be considered, and thus, more reasonable results can be obtained. Therefore, the standard Gaussian random fields were generated using the same random variables, and then transformed into log-Gaussian random fields using each Hermite polynomials with 5 terms. Fig. 5 shows the correlation between the average c_v and c_h in random fields.

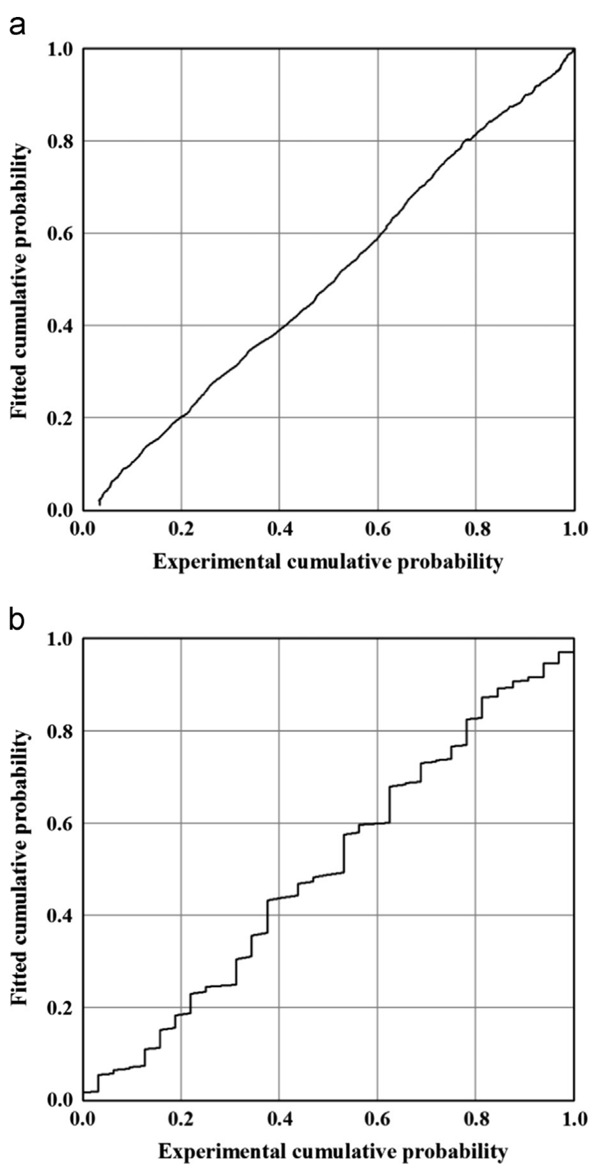

Fig. 4. P–P plot. (a) Vertical coefficient of consolidation, (b) horizontal coefficient of consolidation.

Fig. 6 shows that the CDFs of c_v and c_h in the generated log-Gaussian random field are in good agreement with the fitted log-Gaussian CDFs. Therefore, it can be seen that generated log-Gaussian random fields effectively reflect the statistical properties of c.

Table 2
Statistical properties of the coefficient of consolidation.

Coefficient of consolidation	Mean (cm^2/s)	Standard deviation	Coefficient of variation (%)	PDF	Autocorrelation distance (m)
c_v	0.00692	0.00459	66.3	Log-Gaussian (3-parameter)	1.0 2.0 Not considered
c_h	0.03362	0.01254	37.3	Log-Gaussian (3-parameter)	1.0 2.0 Not considered

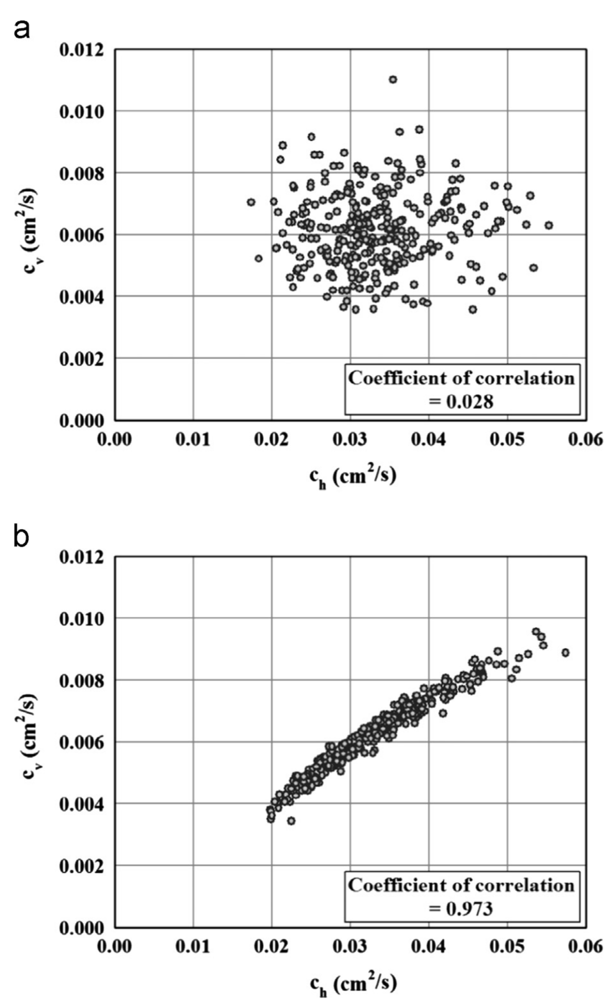

Fig. 5. Correlation between c_v and c_h in random fields. (a) Using different random variables, (b) using the same random variables.

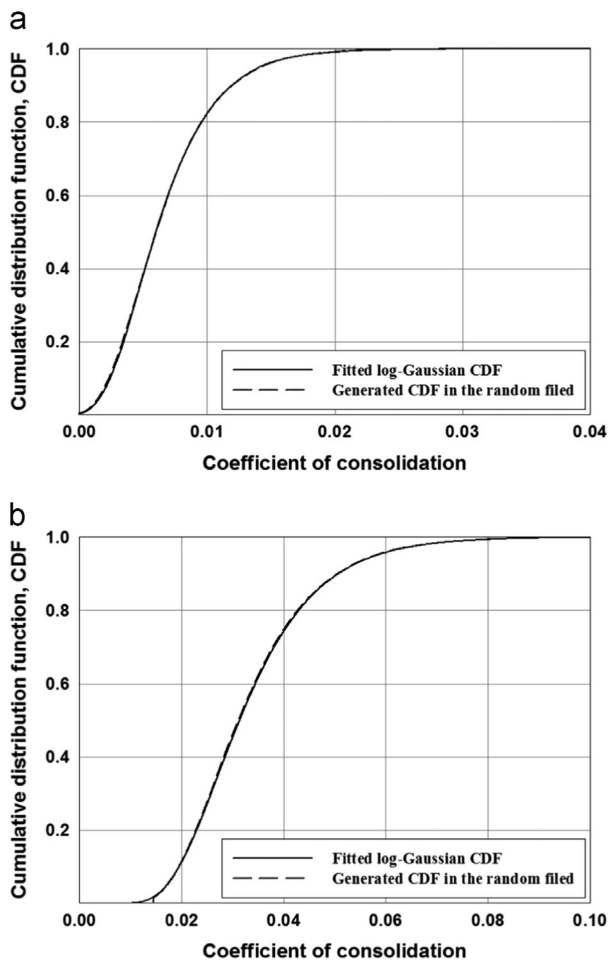

Fig. 6. Comparison of fitted log-Gaussian CDF and that generated in random field CDF. (a) Vertical coefficient of consolidation, (b) horizontal coefficient of consolidation.

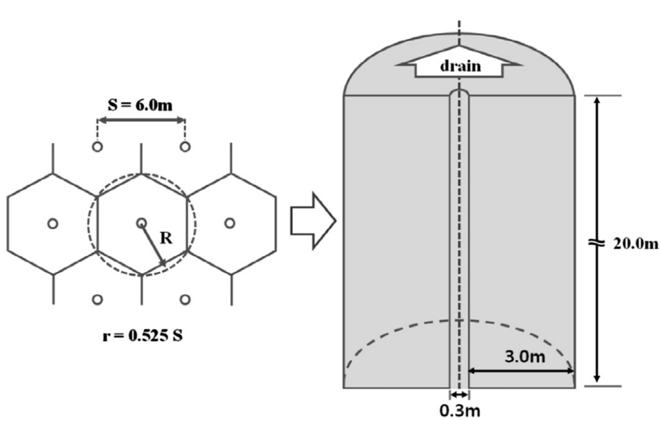

Fig. 7. Schematic view of soil cylinder with vertical drain.

4.3. Consolidation analysis

Preconsolidation is a technique used to minimize the effect of settlements on structures and to improve the strength of the soil. Vertical drains have been widely used to accelerate the consolidation process in soft soil under conditions of surcharge loading. Fig. 7 shows a soil cylinder with a vertical drain. Vertical drains are installed in a triangular pattern with a drain spacing (S) of 6.0 m and a drainage diameter of 0.15 m.

For soft soil improvement by vertical drains, most of the settlement is due to radial drainage consolidation, and the vertical flow is often ignored. However, vertical drainage is included in the analysis when the 2-dimensional flow in the soil deposit is taken into account.

The general analysis of vertical drainage consolidation has used Terzaghi's theory (1923) to estimate the dissipation rate of the excess pore pressure and to evaluate the average degree of consolidation (U). Rendulic (1935) developed a solution for 1-D consolidation by radial flow based on Terzaghi's theory. The flow of these two drainage consolidation analyses can be expressed as

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} + c_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \tag{10}$$

where u, t, z, and r are the pore pressure, time, depth coordinate, and radial coordinate, respectively. In our study, we did not consider the smear effect, and U was obtained through the FDM.

4.4. Sensitivity analysis

The convergence and accuracy of the K–L expansion depend on the expansion terms, and the accurate random fields can be represented by many terms. However, this approach increases the computational effort. In particular, the computational efficiency of the SRSM is determined according to the number of random variables. Therefore, the sensitivity analysis was performed for random variables to determine the number of random variables in the SRSM.

Sensitivity is a measure of the impact of change in one input parameter on the output results. Complex models, for which analytical sensitivity equations are not easily derived, generally result in the need to use the perturbation method. The sensitivity coefficients are calculated using the following difference forms:

$$S_{i,j} = \frac{\partial y_i}{\partial x_j} \approx \frac{\Delta y_i}{\Delta x_j} \tag{11}$$

where i is the index for the i-th model dependent variable and j is the index for the j-th model input parameter. Δy_i is the change in y_i due to an infinitesimal change (Δx_j) in x_j.

The sensitivity gradient was calculated using the linear regression equation; a large sensitivity gradient means that the effect of the random variable on the consolidation analysis results is large. Fig. 8 shows the sensitivity gradient for each random variable. When the number of terms for the random field increases, the eigenfunction exhibits oscillations whose frequencies increase. However, the eigenvalue rapidly decreases, and the square root of the eigenvalue is used to represent the randomness of the random field. Therefore, the sensitivity rapidly decreases as the number of terms in the random field increases. The number of random variables for the SRSM analysis can be determined based on how many large errors are allowable. Assuming that a 30-term expansion

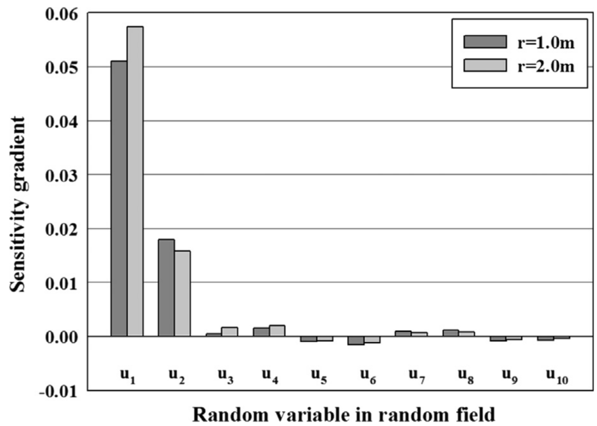

Fig. 8. Sensitivity gradient for random variables.

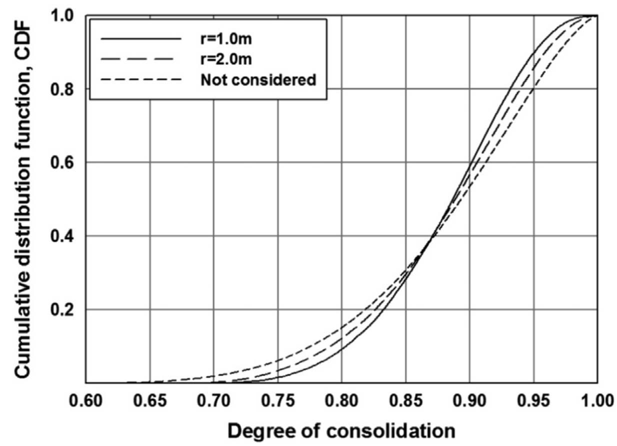

Fig. 10. Cumulative distribution functions of U for autocorrelation distance.

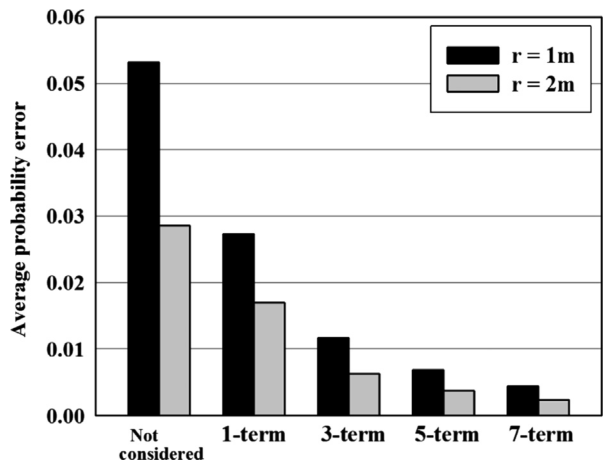

Fig. 9. Average errors of failure probability for number of terms and correlation distance.

is sufficient, the CDF becomes consistent with the CDF for which the K–L expansion with a sufficient number of terms increases. When compared with the results of the 30-term K–L expansion, the average errors of the failure probability in the range of 0.75–0.95 are shown in Fig. 9.

The average errors of the failure probability decrease when the number of terms increases. Smaller errors are shown for a correlation distance of 2 m. This is because when the correlation distance is large, the eigenvalue rapidly decreases according to the terms expansion.

In this study, five random variables are considered for the SRSM analysis. The random fields of c_v and c_h are discretized into five random variables (ξ_1, ξ_2, ξ_3, ξ_4, ξ_5).

4.5. Effects of spatial variability of c

To analyze the difference in the results of U by the spatial variability, the MCS is performed for the correlation distance with a sample size of 100,000. Fig. 10 shows the CDF of U for an elapsed time of 25 days. If spatial variability is not considered, the correlation distance is considered to be infinite (∞ m). The probability of failure decreases when the correlation distance increases, and the highest probability of failure appears when the spatial variability is not considered. The reason is that c_v and c_h are the same at all points; and thus, they are treated as one random variable. Therefore, the probability that c_v and c_h are lower than the failure threshold at all points is higher than that when considering the spatial variability. When $\overline{U}_{target}$ is close to the average U, the difference between the probability of failure, according to the correlation distance, is small because the average U is the same regardless of the correlation distance, and the cumulative distribution curve is crossed at the average U.

4.6. Probabilistic analysis using SRSM

The SRSM is applied to consolidation analyses that consider the spatial variability. The five random variables (ξ_1, ξ_2, ξ_3, ξ_4, ξ_5) in the random fields are used as input data for the SRSM analysis. A polynomial chaos expansion is performed with the 2nd, 3rd, and 4th order expansions using Hermite polynomials, and the limit state function is Hermite polynomials in the SRSM. The number of unknown coefficients is 21, 56, and 126 for the 2nd, 3rd, and 4th order conditions, respectively.

The collocation points are selected as combinations of the root of the 3rd, 4th, and 5th order Hermite polynomials. As a result, there are 3^5 (=243), 5^5 (=3125), and 5^5 (=3125) possible collocation points for the random variables. As the number of possible collocation points is greater than the number of coefficients, the points closest to the origin are selected, because they lie in the regions of higher probability. In our study, the collocation points are divided into groups according to their distances from the origin. If the limit state function is linear, the distance can be regarded as a reliability index (β), which means that there is a range of possible probabilities. Therefore, the approximate range in probability can be inferred by the distance, although the true limit state function is non-linear.

Fig. 11 shows the range in U for the groups and the approximate probability range in the response surface.

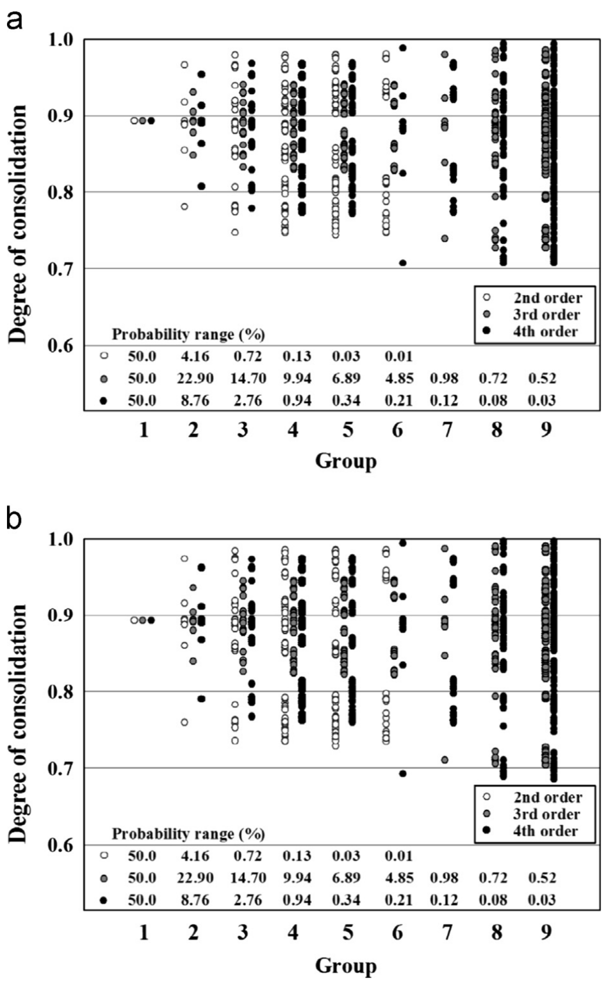

Fig. 11. Range in degree of consolidation for a group. (a) Correlation distance of 1 m, (b) correlation distance of 2 m.

Table 3
Probability that U is less than $\overline{U}_{target}$ ($\overline{U}_{target}=0.80$).

Correlation distance (m)	Method	The probability of failure (%) Groups					
		1–4	1–5	1–6	1–7	1–8	1–9
1.0	SRSM (2nd)	8.26	7.86	7.71	–	–	–
	SRSM (3rd)	17.16	9.26	1.65	9.11	8.86	–
	SRSM (4th)	–	–	–	–	8.61	8.89
	MCS	8.96					
2.0	SRSM (2nd)	10.86	10.61	10.56	–	–	–
	SRSM (3rd)	15.61	1.75	1.6	11.91	11.71	–
	SRSM (4th)	–	–	–	–	11.96	12.01
	MCS	11.96					

Table 4
Probability that U is less than $\overline{U}_{target}$ ($\overline{U}_{target}=0.85$).

Correlation distance (m)	Method	The probability of failure (%) Groups					
		1–4	1–5	1–6	1–7	1–8	1–9
1.0	SRSM (2nd)	26.27	26.47	26.47	–	–	–
	SRSM (3rd)	26.22	27.32	27.32	27.27	27.27	–
	SRSM (4th)	–	–	–	–	27.27	27.45
	MCS	27.37					
2.0	SRSM (2nd)	28.12	28.22	28.42	–	–	–
	SRSM (3rd)	28.57	29.82	29.82	29.37	29.32	–
	SRSM (4th)	–	–	–	–	29.42	29.43
	MCS	29.28					

The groups are selected to include the target degrees of consolidation ($\overline{U}_{target}$) and added until the error (the difference between the results by the selected groups and the results by the next added group) is acceptably small. The results of the reliability analysis using the SRSM according to the collocation points (groups) and the expansion order are summarized in Tables 3 and 4.

The results show that approximately 211, 333, 573 collocation points are needed for the 2nd, 3rd, and 4th order SRSM, respectively.

Fig. 12 shows the CDFs of the degree of consolidation obtained from the MCS with a sample size of 100,000 and the SRSM analysis (2nd, 3rd, and 4th order). The SRSM approximates a limit state function by fitting the multi-dimensional polynomial. Therefore, the SRSM can give biased approximations of the failure probability for cases in which the true limit state function is highly nonlinear, and many input variables are considered. Although the biases are present, the results of the probabilistic analysis using the SRSM are in good agreement with those using the MCS, and very small numerical runs are required compared to the MCS. The absolute values for the difference in the probability of failure were about 0.05–1.40%, and when using high-order polynomials, smaller errors occurred. Thus, it can be concluded that the SRSM can achieve satisfactory results for a reliable analysis of U considering the fact that the spatial variability and the accuracy can be improved by adopting additional collocation points or using higher-order expansions.

5. Conclusions

This study has investigated the effects of the spatial variability of the consolidation coefficient on consolidation by vertical drains in the Yeonjongdo region, and has utilized the SRSM to conduct efficient uncertainty propagation of geotechnical problems with spatial variability. The statistical properties of c in the study area were analyzed by the field data. The results show that the appropriate PDF for c was the three-parameter log-Gaussian distribution, and that the coefficient of variation for c_v was greater than that of c_h. To consider the spatial variability and probabilistic distribution of c, standard Gaussian random fields were generated based on

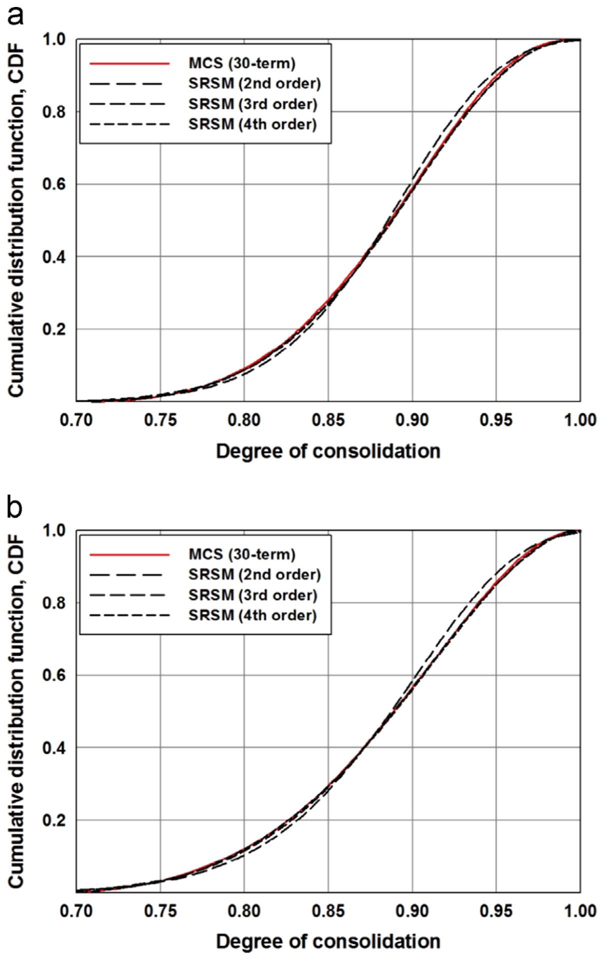

Fig. 12. Comparison of CDFs of degree of consolidation by reliability analysis methods. (a) Correlation distance of 1 m, (b) correlation distance of 2 m.

the K–L expansion, and transformed to log-Gaussian random fields using Hermite polynomials for vertical and horizontal consolidation. In particular, considering that the statistical properties of c_v and c_h are different, each 1-D random field was generated considering the positive correlation between c_v and c_h.

As a result, when considering the spatial variability of soil properties, the probability of failure for the target U is smaller than that determined with the conventional probabilistic analysis method, and as the correlation distance decreases, the probability of failure also decreases.

The SRSM analysis was used for an efficient probabilistic analysis of consolidation, and 2nd, 3rd, and 4th order Hermite polynomial expansions were used for the response surface function. We considered five random variables for the terms in the K–L expansion as the input data for the SRSM analysis. Compared with the MCS results, the SRSM analysis generated results with similar accuracy to those obtained using the MCS with a sample size of 100,000, the accuracy of the SRSM was able to be improved with a higher-order expansion, and the third-order expansion was found to be sufficient for obtaining a failure probability with errors less than 0.001. Although the SRSM also requires the generation of a large number of realizations, the realizations for the response surface function require very little computational effort compared to numerical model runs. In particular, the probabilistic analysis of geotechnical problems with a large ratio of correlation length to domain size can be quickly and accurately performed using the SRSM because the random field requires only a small number of terms in the K–L expansion.

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