ELSEVIER
Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia Environmental Sciences 10 (2011) 825 – 830
Procedia
Environmental Sciences

2011 3rd International Conference on Environmental Science and Information Application Technology (ESIAT 2011)

# Review of Statistical Induction Analysis Methods on Aircraft Vibration Test Data

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**Abstract**

The development of reliability and environment vibration test conditions depends on the processing of vibration test data. Because of randomness of vibration sources and different purposes of reliability testing and environmental testing, testing data should be dealt with different methods. This paper summarizes various statistical induction analysis methods for aircraft vibration test data, and indicates their advantages, disadvantages and applicability scope. Statistical analysis method on small sample, time-varying structures, non-stationary data processing and vibration environment prediction are the further research directions.


Selection and/or peer-review under responsibility of Conference ESIAT2011 Organization Committee.

*Keywords:*Reliability Test; Environmental Test; Vibration Test Data; Statistical Induction Analysis

## 1. Introduction

In working state, modern aircrafts are vulnerable to a variety of load combined effects. For example, during flight, the vibration, shock, noise and overload constitute an integrated mechanical environment. Because of violent vibration in this environment, the structure of the aircrafts will be easily destroyed and the instruments inside will perform badly or fall into malfunction.

Therefore, it requires dynamic environmental testing to ensure the high reliability of structure and equipment contained. Reliability test assesses the reliability of the aircraft, while environmental test assesses the environmental adaptability of the aircraft. Because of their different purposes, the data

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Selection and/or peer-review under responsibility of Conference ESIAT2011 Organization Committee.
doi:10.1016/j.proenv.2011.09.133

inductive methods of the two kinds of tests above should not be the same. Each statistical induction analysis method has its advantages, disadvantages and applicability scopes.

Vibration condition is the basis of aircraft design. In general, the establishment of appropriate dynamics environment condition depends on the previous practical data and data statistical induction analysis method. Vibration condition assists to avoid short-test and over-test, and meanwhile reduce design costs and shorten the development cycle, which makes the design more reasonable. This paper only considers statistical induction methods about preprocessed vibration data of all state with the same type in same region or within the same frequency band [1].

## 2. Non-parametric statistical inference

Non-parametric statistical inference has strong applicability and wide application because of its weaker assumption about samples distribution.

### *2.1. Extreme enveloping method*

Extreme enveloping method is the connection of maximum numbers of different groups of test data. It determines the vibration standard curve by connecting envelope curve in the vibration magnitude (amplitude or spectral density) - frequency image. Firstly, partition the types of aircrafts and installation regions of instruments, and then give the data of each region in the vibration value (amplitude or spectral density) frequency image, and finally, connect envelope curve and determine its vibration standard curve. Usually, the envelope curve contains the vast data points (about 95%).

The primary advantage of extreme envelope method is that it is easy to be applied. And this method reflects the absolute maximum vibration of the corresponding value the environment. Therefore, the enveloping method is very conservative and it is safe to make environmental test conditions with it. The disadvantages of extreme enveloping method are as follows:

- 1) It does not provide a specific probability that the envelope at a given frequency will exceed the spectrum for the response at another location of interest, i.e., the envelope, in itself, does not allow a quantitative evaluation of uncertainties.
- 2) It can produce somewhat different results depending on the frequency resolution of the predicted spectra.
- 3) For the usual case where the envelope is smoothed by a series of straight lines, the procedure becomes somewhat subjective in terms of the number of straight lines used to define the envelope, and whether all spectral peaks are enveloped or some are clipped.
- 4) This method does not provide an effective method to reasonably specify test time.

### *2.2. Risk rate method*

Vibration environment stress of reliability test cannot use the extreme enveloping method which is too conservative. Test risk rate can be obtained by using the sum of the frequency band in which actual vibration value exceed the test value to divide the test frequency range. And the test risk rate of a variety of airborne equipment should be determined according to their importance.

### *2.3. Equivalent energy method*

The area, which is surrounded by complex shape curve and frequency axis, expresses the excitation energy of rand vibration. And the area (or energy) can be computed by numerical integration method.

Equivalent energy method is to seek one "xdB" maximum envelop whose shape is similarity with the "0dB" maximum envelop or spectrum type given by common criteria and whose location.

In fact the equivalent energy envelop is the weighted average of power spectral density in frequency. Equivalent energy method is also better simulation method, but still has the problem that actual value are more than test value in some frequency bands. Specially, when the resonance frequency of equipment is more than the frequency range, the difference will be larger because of the amplification factor Q. But the problem will not be considered for the common criteria as the resonance frequency of equipment aren't non-regularity. It is possible to cutting for the difference reduction between the actual value and the test value for the special equipment test.

### *2.4. The damage-based state inductive method*

The damage-based state inductive method [2] employs an extended response spectrum analysis and includes amplitude-cycle counts. Notable features of the analysis technique are a quantitative means for addressing fatigue damage potential in flight and a reduction in the overestimation of peak flight responses.

So, although the damage-based analysis process is more complicated than the maximal approach, the same resulting form of test specification is retained. In comparison to the traditional maximal approach, the significant advantages of the damage-based approach are believed to be the following: First, damage potential due to fatigue accumulation, as well as to peak response, is evaluated in a well-accepted, quantitative manner by using an extended response spectrum approach. Second, in comparison to the maximal approach, the damage-based method reduces the over-estimation of spectral peak values caused by brief periods of strong oscillation in the flight data. Third, processing errors resulting from the assumption of stationary, necessary for maximal processing, are eliminated. Finally, fatigue damage contributions from different phases of flight, such as liftoff and transonic events, are easily combined since the damage indicator values described herein are commutative.

### *2.5. Distribution-free tolerance limits*

Distribution-free tolerance limits circumvent the primary problem associated with normal tolerance limits, namely, the need to assume a lognormal distribution for the spatial variations of the structural response spectral values within the zone [3]. The assumption is eliminated by fixing the tolerance limit to the maximum spectral value in each of the m frequency resolution bandwidths The primary advantage of the distribution-free tolerance limit over the normal tolerance limit is that it is not sensitive to the spatial distribution of the spectral values for the responses within the zone, i.e., it is nonparametric. However, it does pose two problems not present in the normal tolerance limit, as follows:

- 1) The procedure does not permit an independent selection of the values for $\beta$ and $\gamma$ .
- 2) As for the envelope approach, the procedure can produce somewhat different results depending on the frequency resolution of the predicted spectra.

### *2.6. Empirical tolerance limits*

Empirical tolerance limit [5] determines an empirical distribution function for the predicted spectral values at $n$ locations first, and then selects some large percentile $x_\beta$ from the distribution (that value of $x$ which exceeds $\beta$ portion of the available values) to be a conservative limit. Ideally, this would be done independently for the spectral values in each frequency resolution bandwidth. In practice, however, it is rare to have predictions at enough locations to allow a determination of a large percentile for the values in each bandwidth; i.e., for $\beta$ =0.95, at least 20 values are needed to determine a limit. Hence, it is common to normalize the spectral values in each frequency resolution bandwidth to a common mean value, and then pool all the spectral values to arrive at a single distribution function for the determination of a limit that covers $\beta$ portion of the values. Of course, this pooling approach assumes the spatial distribution of the spectral values is the same in all the frequency resolution bandwidths.

The empirical tolerance limit approach does not assume a lognormal spatial distribution for the response spectral values within the zone and the empirical tolerance limit is not as sensitive as the envelope to the frequency resolution bandwidth of the predicted spectral values. However, the empirical tolerance limit does pose some potential problems, as follows:

- 1) If the spectral values in all frequency resolution bandwidths are pooled to arrive at the empirical distribution, it is sensitive to the assumption that the spatial distribution of the response spectral values is the same in all bandwidths.
- 2) It is most effective when response predictions are available at a relatively large number of locations (generally, $n > 10$ ).
- 3) It provides a limit with a confidence coefficient of $\gamma$ =0.50 only, unless further extensive computations are performed.

But non-parametric statistical inference does not make uniform inference based on the number of data samples and data distribution. So, non-parametric statistical inference is not as powerful as parameters statistic inference.

## 3. Parameters statistic inference

Parameters statistic inference, there are strict statistical interpretation, which is based on statistical theory and law of large numbers, not only to make statistical inferences on the parameters, but also in the time domain or frequency domain excitation loads, response.

If distribution assumption of sample meets the fact, it is allowed that using the sample observations to do statistical inference, otherwise, the conclusions are wrong or even absurd. So hypothesis testing Is need when using parameters statistic inference. Commonly used test methods are: the probability density function, Pearson $\chi^2$ test, Kolmogorov test, and the sleekness of normal distribution test kurtosis test.

### *3.1. Normal tolerance limits*

The normal tolerance limit approach is a more definitive inductive method and limits apply only to normally distributed random variables [1]. The reasons leading to the bias errors, and provided the upper tolerance factor formula with specifically confidence coefficient $\gamma$ and probability β.

The normal tolerance limit approach offers three major advantages over the envelope procedure:

- 1) The normal tolerance limit has a rigorous statistical interpretation.
- 2) For any number of predictions, n, a normal tolerance limit can be computed for any independently selected values of $\beta, \gamma$ . For example, the normal tolerance limit is not restricted to the range of the predicted spectra used for its determination.
- 3) The normal tolerance limit is not as sensitive to the frequency resolution bandwidth as the envelope procedure.

The last advantage follows from the fact that the normal tolerance limit is computed from average values (the sample mean and standard deviation), which in turn are computed from the predicted spectral values in each frequency resolution bandwidth at all $n$ locations. Hence, the impact of frequency resolution bias errors on the sample mean and standard deviation is minimal. On the other hand, the envelope of the various predicted spectra is determined at most frequencies by a spectral peak at one location, meaning the frequency resolution bias error is often large.

Normal prediction limits are somewhat simpler to apply than any of the tolerance limits because they involve only one probability related parameter, namely, a confidence coefficient $\gamma$ . On the other hand, the normal prediction limit poses two problems, as follows:

- 1) It is sensitive to the assumption that the spatial distribution of the spectral values of the structural responses values within the zone is lognormal. This assumption might come into question for the higher values of $\gamma$ .
- 2) It provides a conservative limit for only one or some other specific number of future response predictions at randomly selected points within the zone, rather than to the response at all locations within the zone.

*3.2. Transforming non-normal distribution into normal distribution*

The spatial variation of structural responses to stationary, non-stationary, and transient dynamic loads is generally not normally distributed. But by simply making the logarithmic transformation or exponential transforms, a normal tolerance limit can be computed for the transformed predictions. Based on Johnson system distributions, an inductive method by transforming non-normal distribution data was presented and how to find the sample percentile point z was discussed.

*3.3. Application of other distribution functions*

Although a variety of distribution function can be turned into a normal distribution, but sometimes had to use non-normal distribution, for example the shock, vibration data may be subject to Weibull distribution of extreme value distribution theory. Telemetering vibration amplitude distribution is studied by using regression analysis and K test.

## 4. Conclusions and further research directions

*4.1. Small sample statistical inference*

Based on the sample, statistical tolerance method makes statistical inference under certain statistical model. Parameters statistical inference is based on the law of large Numbers. The actual vibration data obtained from aircraft is less, such as telemetry data. By comparing the maximum likelihood method, bilinear regression, probability weighted moments, the correlation coefficient optimization method and the gray parameter estimation, the following conclusion was drawn: 1) The larger the sample, the more accurate fitting results. 2) When the sample size is small, the gray estimation has still higher accuracy, while accuracy of the other several methods is down. In particular, when the number of samples is less than 10, in the above five parameter estimation method, gray estimation fitting results obtain best accurate.

Bayes statistical inference belongs to the parameters statistical models, which obtains the posterior distribution by utilizing the observation samples and the prior distribution.

Since the use of prior knowledge, Bayes statistical inference has better inference results for small sample also. How to turn traditional statistical theory based on the law of large numbers into statistical learning theory based on small sample and estimate probability density from small sample data is one of the current research focuses.

*4.2. Time-varying structure*

Aircraft structural parameters (stiffness, damping and quality, etc.) may change over time. For example, rocket, the fuel rapid consumption, and the quality characteristics of the rocket system is time varying. So far, there is not a mature method to design an equivalent test conditions from the measured time-varying vibration response data.

### *4.3. Non-stationary signal processing methods*

It needs to do stability test when preprocessing vibration data. The aircraft in flight vibration need to experience a variety of environments, such as non-stationary random vibration during lift-off and approximate stationary random vibration during free flight. Some methods were summarized about stability test. However, these induction methods and standards are applied to spectrum curve obtained by Fourier transform of experimental data, so inevitably bring great error, and cannot get the essential characteristics of non-stationary random vibration data. Although modern time-frequency analysis of non-stationary signals processing methods, such as short time Fourier transform, Gabor transform, wavelet transform, WVD, Cohen, HHT, fully describe the joint time-frequency characteristics, there is no mature method to make the equivalent design of test conditions according to individual sample records for non-stationary random vibration.

### *4.4. Vibration response environment prediction*

Dynamics environmental conditions can be obtained by applying inductive method to the experimental data and flight telemetry data. But for newly developed aircraft with a lot of uncertain parameters, without experimental data, it will bring great error that using the data of previous model to predict it. Therefore, during initial stage of the vehicle development program, it requires an effective prediction method for dynamic environment to analyze a variety of excitation source, and obtain relatively accurate vibration response, which provide a reliable basis for structural design and improvement [6]. Dynamics environment prediction has important engineering significance.

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