Alexandria Engineering Journal (2016) 55, 2183–2191

HOSTED BY
ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

# Application of differential transformation method in micropolar fluid flow and heat transfer through permeable walls

**A. Mirzaaghaian, D.D. Ganji** *

*Department of Mechanical Engineering, Babol University of Technology, Babol, Iran*

Received 15 February 2016; revised 23 May 2016; accepted 9 June 2016
Available online 27 June 2016

**KEYWORDS**

Micropolar fluids;
Permeable channel;
Peclet number;
Heat transfer;
Mass transfer;
Differential Transformation Method (DTM)

**Abstract** In this paper, we applied Differential Transformation Method (DTM) to study micropolar fluid flow and heat transfer through a channel with permeable walls. In order to verify the accuracy and validity of the application of this method to this problem, comparison with numerical method (NUM) is taken into account. Results reveal that DTM is an appropriate method for approximating solutions of the problem while it is smooth and straightforward to implement. The effect of significant parameters such as the Reynolds number, micro rotation/angular velocity and the Peclet number on the stream function, temperature distribution and concentration characteristics of the fluid, is discussed.



## 1. Introduction

Micropolar fluids are fluids with microstructure. Micropolar fluids consist of rigid, randomly oriented particles with their own and spins and microrotations, suspended in a viscous medium. In micropolar fluids, rigid particles contained in a small element can rotate about the center of the volume element described by a micro rotation vector. Beside this type of fluid, an interesting phenomenon is nanofluid, that is applicable on heat transfer enhancement, thus attracts many researchers [1–6]. The theory of a micropolar fluid derives from the need to model the flow of fluids that contain rotating micro constituents. A micropolar fluid is a fluid with internal structures which is coupling between the spin of each particle and the macroscopic velocity field is taken into account. It is a hydro dynamical framework suitable for granular systems which consist of particles with macroscopic size. An interesting feature of this class of fluid is the sustenance of couple stress. Some anisotropic fluids, animal blood, and liquid crystals are the examples of micropolar fluids. The micropolar fluid theory is applicable to certain polymer solutions, lubricant fluids, colloidal expansions, and complex biological structures. Eringen [7] was the first pioneer of formulating the theory of micropolar fluids. His theory introduced new material parameters, an additional independent vector field – the microrotation – and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian fluid flow. The microrotation vector is a feature of this type of fluid that makes it applicable in the literature of modeling blood flow in an artery. Although the field of micropolar fluids is rich in literature, some gaps can be observed and need more study

* Corresponding author.
E-mail address: ddg_davood@yahoo.com (D.D. Ganji).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.06.011
1110-0168 

**Nomenclature**

| | | | |
|---|---|---|---|
| $C$ | species concentration (mol/m$^3$) | $\rho$ | Fluid density (kg/m$^3$) |
| $k_1$ | thermal conductivity (W/m k) | $\psi$ | stream function (m$^2$/s) |
| $g$ | dimensionless microrotation | $D^*$ | molecular diffusivity (m$^2$/s) |
| DTM | Differential Transformation Method | $f$ | dimensionless stream function |
| $(u, v)$ | Cartesian velocity components (m/s) | $h$ | half of channel width (m) |
| $N_{1,2,3}$ | dimensionless parameters | $j$ | micro-inertia density (m$^2$) |
| $Sh$ | Sherwood number | $\vec{N}$ | microrotation/angular velocity (m/s) |
| $p$ | pressure (Pa) | $Nu$ | Nusselt number |
| $Pe$ | Peclet number | $Sc$ | Schmidt number |
| $T$ | fluid temperature (k) | $Pr$ | Prandtl number |
| $(x, y)$ | Cartesian coordinate components parallel and normal to channel axis, respectively | $Re$ | Reynolds number |
| | | $s$ | microrotation boundary condition |
| | | $\theta$ | dimensionless temperature |
| *Greek symbols* | | $\kappa$ | coupling coefficient (Pa s) |
| $\eta$ | similarity variable | $\upsilon_s$ | microrotation/spin-gradient viscosity (N s) |
| $\mu$ | dynamic viscosity (Pa s) | | |

in this field. For instance, Gorla [8], Rees and Bassom [9] investigated the flow of a micropolar fluid over a flat plate. Also, Kelson and Desseaux [10] studied the flow of micropolar fluids on stretching surfaces. Heat and mass transfer have an important role in many industrial and technological processes such as manufacturing and metallurgical processes in which heat and mass transfer occur simultaneously. The influence of a chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving plate in a porous medium with heat generation was studied by Mohamed and Abo-Dahab [11]. Recently effect of using micropolar fluid, nanofluid, etc. on flow and heat transfer has been studied by several authors [12–23].

There are numerous amounts of publications about the application of semi-analytical methods on different phenomena. Such semi-analytical methods include Homotopy Analysis Method (HAM), Optimal Homotopy Asymptotic Method (OHAM), Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM) and Differential Transformation Method (DTM) that are popular because of their high accuracy and simplicity in obtaining the solution. DTM has some advantages in comparison with other semi-analytical methods. HAM needs to calculate auxiliary parameter $\hbar$, through h-curves and initial guesses [24,25], but DTM does not need any auxiliary parameter and initial guess. DTM does not need any auxiliary function like $H(p)$ that is used in OHAM [26,27]. This method does not need to obtain Adomian polynomial that is difficult to access for highly nonlinear terms in ADM [28,29]. Also, DTM does not need any small parameter like "$p$" in (HPM) for discretization, perturbation or linearization [30–32]. Differential Transformation Method (DTM) is a powerful analytical method which is well known as a high accurate technique for solving the differential equations while it is straightforward and easy to implement. This approach constructs an analytical solution in the form of a polynomial and formulizes the Taylor series in a totally different manner. Zhou [33] first introduced this method for solving linear and nonlinear initial value problems. He used this method to obtain semi-analytical solution for the electrical circuit analysis. A considerable research revealed that this approach is appropriate for various problems [34–40]. For instance, Sheikholeslami et al. [39] used DTM to the governing equations for the MHD fluid flow, heat and mass transfer between two horizontal parallel plates to count in the effects of Brownian motion and thermophoresis in the nanofluid model.

With the above discussion in mind, in this study, we employed DTM to the governing coupled differential equations of micropolar fluid flow and heat transfer in a permeable channel. After that, we present a comparison with a numerical method to verify the accuracy and validity of this powerful method. As the Reynolds number ($Re$), Peclet number ($Pe$) and micro rotation/angular velocity play an important role in this problem, we focused on the effects of these parameters on the flow, heat transfer and concentration characteristics.

## 2. Problem description and governing equations

We considered the steady laminar flow of a micropolar fluid along a two-dimensional channel with porous walls through which fluid is uniformly injected or removed with speed $v_0$. The lower channel wall has a solute concentration $C_1$ and temperature $T_1$ while upper wall has solute concentration $C_2$ and

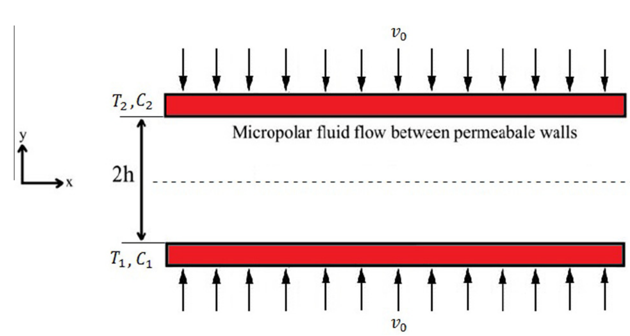


**Figure 1** The geometry of the problem.

temperature $T_2$ as shown in Fig. 1. Using Cartesian coordinates, the channel walls are parallel to the $x$-axis and located at $y = \pm h$ where $2h$ is the channel width. The relevant equations governing the flow in invariant form are [7,21] as follows:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \overrightarrow{V}) = 0, \quad (1)$$

$$\rho \frac{D\overrightarrow{V}}{Dt} = -\nabla p + (\mu + \kappa)\nabla^2 \overrightarrow{V} + (\kappa)\nabla \overline{N}, \quad (2)$$

$$\rho j \frac{D\overline{N}}{Dt} = -\kappa(2\overline{N} + \nabla \times \overrightarrow{V}) + \upsilon_s \nabla^2 \overline{N}, \quad (3)$$

$$\rho C_p(\overrightarrow{V}.\nabla T) = k_1 \nabla^2 T, \quad (4)$$

$$\overrightarrow{V}.\nabla C = D^* \nabla^2 C. \quad (5)$$

As the flow is 2-dimensional, the above equations in component form are as follows [41]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (6)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + (\mu + \kappa)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \kappa\frac{\partial N}{\partial y}, \quad (7)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + (\mu + \kappa)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \kappa\frac{\partial N}{\partial x}, \quad (8)$$

$$\rho\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = -\frac{\kappa}{j}\left(2N + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\upsilon_s}{j}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right), \quad (9)$$

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{k_1}{c_p}\frac{\partial^2 T}{\partial y^2}, \quad (10)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D^*\frac{\partial^2 C}{\partial y^2}. \quad (11)$$

where $u$ and $v$ are velocity components along the $x$- and $y$-axes, respectively, $\rho$ is the fluid density, $\mu$ is the dynamic viscosity, $N$ is the angular or micro rotation velocity, $\overline{N}$ is the component of micro rotation velocity, $p$ is the fluid pressure, $T$ and $c_p$ are the fluid temperature and specific heat at constant pressure, respectively, $C$ is the species concentration, $k_1$ and $D^*$ are the thermal conductivity and molecular diffusivity, respectively, $j$ is the micro-inertia density, $k$ is a material parameter and $\upsilon_s = \left(\mu + \frac{k}{2}\right)j$ is the micro rotation viscosity.

The appropriate boundary conditions are [42] as follows:

$$y = -h: \quad v = -v_0, \quad u = 0, \quad N = -s\frac{\partial u}{\partial y}_{|y=-h} \quad (12a)$$

$$y = +h: \quad v = v_0, \quad u = 0, \quad N = -s\frac{\partial u}{\partial y}_{|y=+h} \quad (12b)$$

where $v_0 > 0$ corresponds to suction, $v_0 < 0$ to injection and $s$ is a boundary parameter and indicates the degree to which the microelements are free to rotate near the channel walls. The case $s = 0$ represents concentrated particle flows in which microelements close to the wall are unable to rotate. Other interesting particular cases that have been considered in the literature include $s = 0.5$ which represents weak concentrations and the vanishing of the antisymmetric part of the stress tensor and $s = 1$ which represents turbulent flow. We introduce the following dimensionless variables [19]:

$$\eta = \frac{y}{h}, \quad \psi = -v_0 x\, f(\eta), \quad N = \frac{v_0 x}{h^2} g(\eta), \quad (13)$$

$$\theta(\eta) = \frac{T - T_2}{T_1 - T_2}, \quad \phi(\eta) = \frac{C - C_2}{C_1 - C_2}$$

where $T_2 = T_1 - Ax$, $C_2 = C_1 - Bx$ with $A$ and $B$ as constants. The stream function is defined in the usual way:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (14)$$

Eqs. (6)–(12b) reduce to the coupled system of nonlinear differential equations:

$$(1 + N_1)f^{IV} - N_1 g - Re(ff''' - f'f'') = 0, \quad (15)$$

$$N_2 g'' + N_1(f'' - 2g) - N_3 Re(fg' - f'g) = 0, \quad (16)$$

$$\theta'' + Pe_h f'\theta - Pe_h f\theta' = 0, \quad (17)$$

$$\phi'' + Pe_m f'\phi - Pe_m f\phi' = 0, \quad (18)$$

subject to the boundary conditions:

$$\begin{aligned} &\eta = -1: \; f = -1, f' = g = 0, \quad \theta = \phi = 1, \\ &\eta = +1: \; \theta = \phi = f' = g = 0, \quad f = 1. \end{aligned} \quad (19)$$

The parameters of primary interest are the buoyancy ratio $N$, the Peclet numbers for the diffusion of heat $Pe_h$ and mass $Pe_m$ respectively, the Reynolds number $Re$ where for suction $Re > 0$ and for injection $Re < 0$ and Grashof number $Gr$ given by

$$N_1 = \frac{\kappa}{\mu}, \quad N_2 = \frac{\upsilon_s}{\mu h^2}, \quad N_3 = \frac{j}{h^2}, \quad Re = \frac{v_0}{v}h, \quad Pr = \frac{v\rho c_p}{k_1},$$
$$Sc = \frac{v}{D^*}, \quad Gr = \frac{gB_T A h^4}{v^2}, \quad Pe_h = Pr Re, \quad Pe_m = Sc\ Re, \quad (20)$$

where $Pr$ is the Prandtl number, $Sc$ is the generalized Schmidt number, $N_1$ is the coupling parameter and $N_2$ is the spin-gradient viscosity parameter. In technological processes, the parameters of particular interest are the local Nusselt and Sherwood numbers. These are defined as follows:

$$Nu_x = \frac{q''_{y=-h} x}{(T_1 - T_2)k_1} = -\theta'(-1), \quad (21)$$

$$Sh_x = \frac{m''_{y=-h} x}{(C_1 - C_2)k_1} = -\phi'(-1), \quad (22)$$

where $q''$ and $m''$ are local heat flux and mass flux, respectively. And the shear stress on the wall and the local skin friction can be written as [21] follows:

$$\tau_w = (\mu + \kappa)\frac{\partial u}{\partial y}_{|y=-h} + \kappa N_{|y=-h}, \quad (23)$$

$$C_{f_x} = \frac{2\tau_w}{\rho u^2} \quad (24)$$

**Table 1** The fundamental operations of the differential transformation method [43].

| Original function | Transformed function |
|---|---|
| $x(t)$ | $X(k) = \alpha F(k) \pm \beta G(k)$ |
| $x(t) = \frac{d^m f(t)}{dt^m}$ | $X(k) = \frac{(k+m)! F(k+m)}{k!}$ |
| $x(t) = f(t)g(t)$ | $X(k) = \sum_{l=0}^{k} F(l)G(k-l)$ |
| $x(t) = t^m$ | $X(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k = m \\ 0, & \text{if } k \neq m \end{cases}$ |
| $x(t) = \exp(t)$ | $X(k) = \frac{1}{k!}$ |
| $x(t) = \sin(\omega t + \alpha)$ | $X(k) = \frac{\omega^k}{k!} \sin\left(k\frac{\pi}{2} + \alpha\right)$ |
| $x(t) = \cos(\omega t + \alpha)$ | $X(k) = \frac{\omega^k}{k!} \cos\left(k\frac{\pi}{2} + \alpha\right)$ |

**Table 3** Absolute error of the case $N_1 = N_2 = N_3 = 0.1$, $Pe_m = Pe_h = 0.1$, and $Re = 0.1$.

| $\eta$ | $f$ Abs. Error | $g$ Abs. Error | $\theta$ Abs. Error | $\phi$ Abs. Error |
|---|---|---|---|---|
| −1.0 | 0 | 0 | 0 | 0 |
| −0.8 | 0.000704 | 0.002633 | 0.010778 | 0.010778 |
| −0.6 | 0.001863 | 0.001747 | 0.020275 | 0.020275 |
| −0.4 | 0.002630 | 0.000191 | 0.027652 | 0.027652 |
| −0.2 | 0.002746 | 0.001042 | 0.032314 | 0.032314 |
| 0.0 | 0.002351 | 0.001843 | 0.033907 | 0.033907 |
| 0.2 | 0.002746 | 0.002526 | 0.031315 | 0.031315 |
| 0.4 | 0.002630 | 0.003354 | 0.027555 | 0.027555 |
| 0.6 | 0.001863 | 0.004219 | 0.020281 | 0.020281 |
| 0.8 | 0.000704 | 0.004072 | 0.010788 | 0.010788 |
| 1.0 | 0 | 0 | 0 | 0 |

## 3. Description of differential transformation method

In this section, we represent the basic idea of DTM. Let $f(t)$ be analytic in the time domain $T$, then it will be differentiated continuously with respect to time $t$,

$$\varphi(t,k) = \frac{d^k f(t)}{dt^k}, \quad \forall_t \in T. \tag{25}$$

For $t = t_i$, then $\varphi(t,k) = \varphi(t_i,k)$, where $k$ belongs to the set of non-negative integers, denoted as the $K$-domain. Therefore, Eq. (25) can be rewritten as

$$F(k) = \varphi(t_i,k) = \left[\frac{d^k f(t)}{dt^k}\right]_{|t=t_i}, \quad \forall_t \in K, \tag{26}$$

where $F(k)$ is called the spectrum of $f(t)$ at $t = t_i$ in the $K$-domain. If $f(t)$ can be represented by the Taylor series, then it can be represented as

$$f(t) = \sum_{k=0}^{\infty} [(t-t_i)^k / k!] F(k). \tag{27}$$

Eq. (27) is called the inverse transform of $F(k)$. Therefore, the Maclaurin series of $f(t)$ can be obtained by taking $t_i = 0$ in the above equation as follows:

$$f(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k f(t)}{dt^k}\right]_{|t=0}, \quad \forall_t \in T. \tag{28}$$

As explained in [34], the differential transformation of the function $f(t)$ is defined as follows:

$$F(k) = \sum_{k=0}^{\infty} \frac{H^k}{k!} \left[\frac{d^k f(t)}{dt^k}\right]_{|t=0} \tag{29}$$

The differential spectrum of $F(k)$ is confined within the interval $t \in [0, H]$ where $H$ is defined as a constant value. After that, the differential inverse transform of $F(k)$ is expressed as follows:

$$f(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H}\right)^k F(k) \tag{30}$$

The function $F(k)$ at values of argument $k$ is referred to a discrete issue which means $F(0)$ is known as the zero discrete, $F(1)$ as the first discrete and so on. The more the discrete is available and more precise and so it is possible to restore the unknown function. By choosing the right amount of $H$, it is observed the larger values of argument $k$, the less discrete of spectrum. The function $f(t)$ is expressed by a finite series and therefore Eq. (30) can be rewritten as

$$f(t) = \sum_{k=0}^{n} \left(\frac{t}{H}\right)^k F(k) \tag{31}$$

Some fundamental operations of the differential transformation method are listed in Table 1.

**Table 2** Comparison between DTM and numerical method when $N_1 = N_2 = N_3 = 0.1$, $Pe_m = Pe_h = 0.1$, and $Re = 0.1$.

| $\eta$ | $f$ | | $g$ | | $\theta$ | | $\phi$ | |
|---|---|---|---|---|---|---|---|---|
| | Num | DTM | Num | DTM | Num | DTM | Num | DTM |
| −1.0 | −1 | −1 | 0 | 2.0E−11 | 1 | 1 | 1 | 1 |
| −0.8 | −0.943889 | −0.944593 | 0.125175 | 0.122542 | 0.909562 | 0.898784 | 0.909562 | 0.898784 |
| −0.6 | −0.791757 | −0.793620 | 0.161985 | 0.160238 | 0.818914 | 0.798639 | 0.818914 | 0.798639 |
| −0.4 | −0.567738 | −0.570368 | 0.138757 | 0.138566 | 0.726610 | 0.698958 | 0.726610 | 0.698958 |
| −0.2 | −0.295835 | −0.298581 | 0.078279 | 0.079321 | 0.631742 | 0.599428 | 0.631742 | 0.599428 |
| 0.0 | 0 | −0.002315 | 0 | 0.001843 | 0.533835 | 0.499928 | 0.533835 | 0.499928 |
| 0.2 | 0.295835 | 0.294229 | −0.078289 | −0.075763 | 0.432747 | 0.401432 | 0.432747 | 0.401432 |
| 0.4 | 0.567738 | 0.566842 | −0.138757 | −0.135403 | 0.328571 | 0.301016 | 0.328571 | 0.301016 |
| 0.6 | 0.791757 | 0.791387 | −0.161985 | −0.157766 | 0.221544 | 0.201263 | 0.221544 | 0.201263 |
| 0.8 | 0.943889 | 0.943804 | −0.125175 | −0.121103 | 0.111945 | 0.101157 | 0.111945 | 0.101157 |
| 1.0 | 1 | 1 | 0 | −2.0E−11 | 0 | 4.6E−11 | 0 | 4.6E−11 |

**Table 4** Comparison of DTM with HPM for the case of $N_1 = N_2 = N_3 = 0.1$, $Pe_m = 0.5$, $Pe_h = 0.2$, and $Re = 1$.

| $\eta$ | $\phi$ | |
|---|---|---|
| | DTM | HPM [19] |
| −1.0 | 1 | 1 |
| −0.8 | 0.910345 | 0.919513 |
| −0.6 | 0.819708 | 0.836043 |
| −0.4 | 0.727148 | 0.747686 |
| −0.2 | 0.635276 | 0.653564 |
| 0.0 | 0.536010 | 0.553641 |
| 0.2 | 0.435327 | 0.448552 |
| 0.4 | 0.333010 | 0.339347 |
| 0.6 | 0.227402 | 0.227374 |
| 0.8 | 0.117158 | 0.113923 |
| 1.0 | −3.2E−11 | 0 |

## 4. Implementation of differential transformation method

In this method, at first, we apply differential transformation to each differential equation based on the rule. By applying this transformation to Eqs. (15)–(18), we have

$$(1+N_1)(k+4)(k+3)(k+2)(k+1)F[k+4] - N_1 G[k] - Re\left[\sum_{l=0}^{k}(F[l](k-l+2)(k-l+1)F[k-l+2])\right] + Re\left[\sum_{l=0}^{k}((l+1)F[l+1](k-l+2)(k-l+1)F[k-l+2])\right] = 0, \tag{32}$$

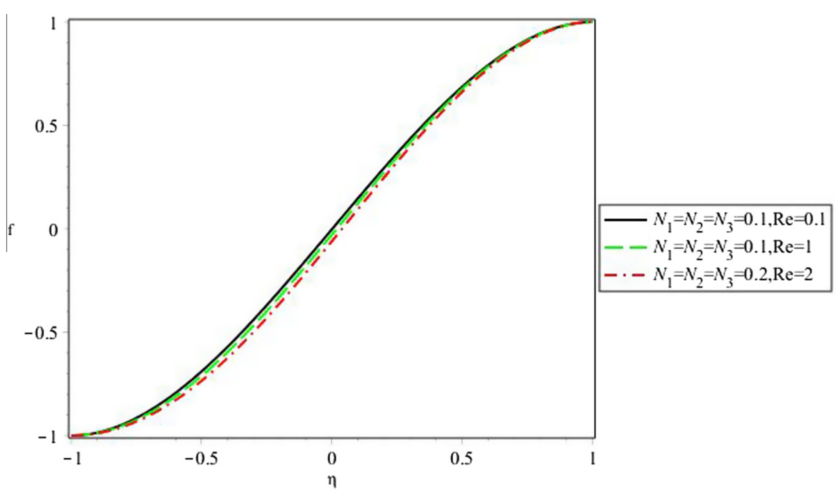


**Figure 2** Effect of the $N_1, N_2, N_3$ and $Re$ on the stream function.

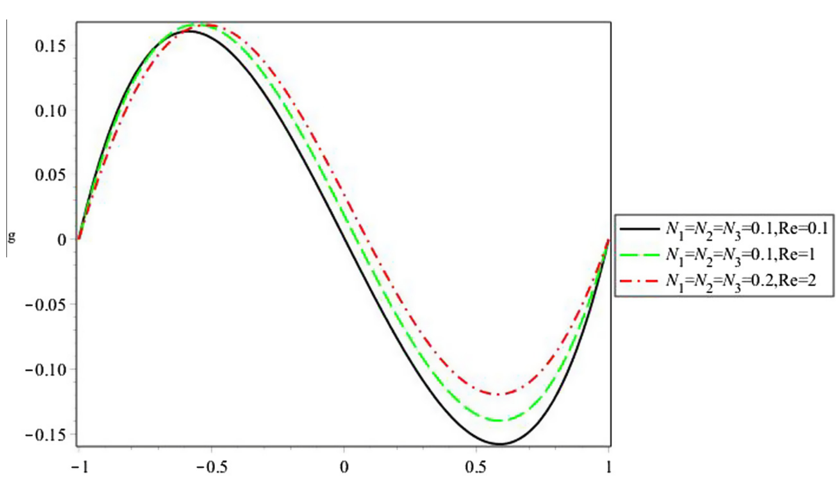


**Figure 3** Effect of $N_1, N_2, N_3$ and $Re$ on microrotation.

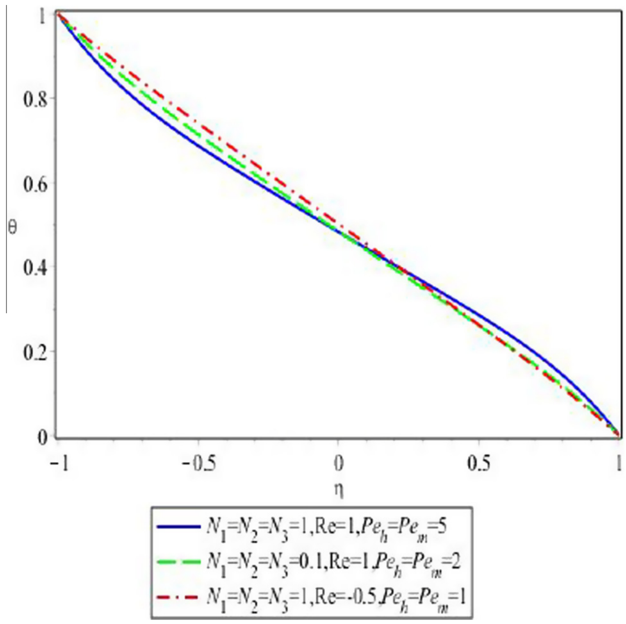


**Figure 4** Effect of various parameters on the temperature distribution.

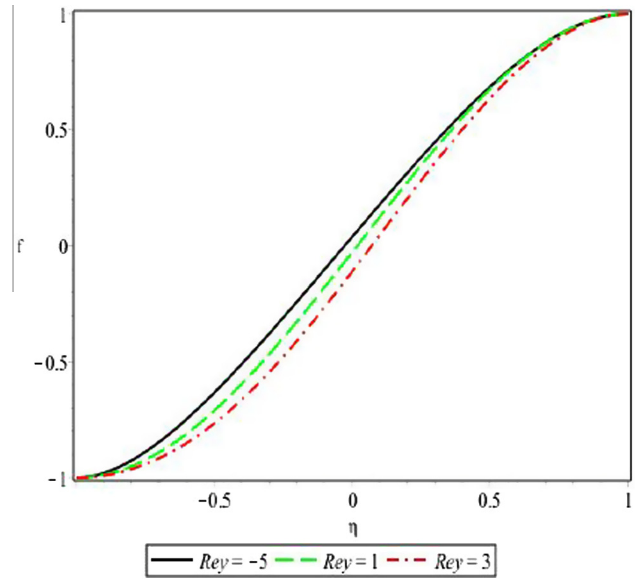


**Figure 6** Effect of Reynolds number on the stream function when $N_1 = N_2 = N_3 = Pe_h = Pe_m = 1$.

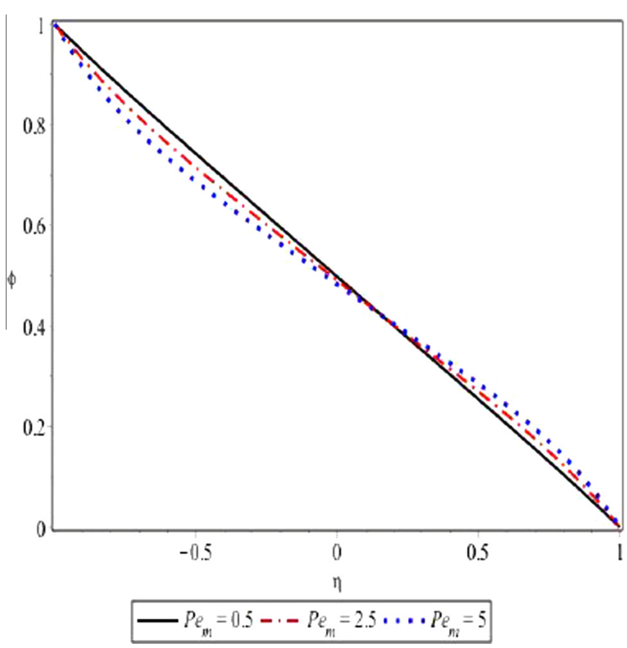


**Figure 5** Effect of Peclet number on the concentration when $N_1 = N_2 = N_3 = Pe_h = Re = 1$.

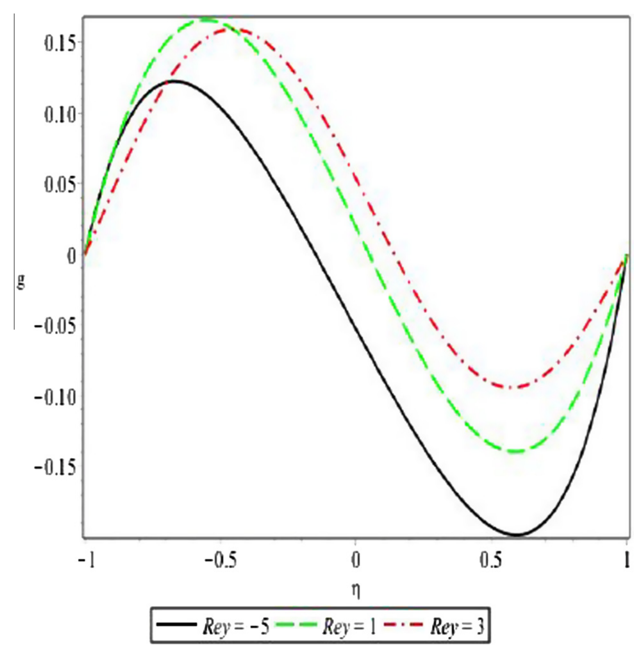


**Figure 7** Effect of Reynolds number on micro rotation when $N_1 = N_2 = N_3 = Pe_h = Pe_m = 1$.

$$N_2(k+2)(k+1)G[k+2] + N_1((k+2)(k+1)F[k+2] - 2G[k]) - N_3 Re\left[\sum_{l=0}^{k}(F[l](k-l+1)G[k-l+1])\right] + N_3 Re\left[\sum_{l=0}^{k}((l+1)F[l+1](k-l)G[k-l])\right] = 0, \tag{33}$$

$$(k+2)(k+1)\Theta[k+2] + Pe_h\left[\sum_{l=0}^{k}((l+1)F[l+1](k-l)\Theta[k-l])\right] - Pe_h\left[\sum_{l=0}^{k}(F[l](k-l+1)\Theta[k-l+1])\right] = 0, \tag{34}$$

$$(k+2)(k+1)\Phi[k+2]+Pe_m\left[\sum_{l=0}^{k}((l+1)F[l+1](k-l)\Phi[k-l])\right]$$
$$-Pe_m\left[\sum_{l=0}^{k}(F[l](k-l+1)\Phi[k-l+1])\right]=0. \quad (35)$$

The boundary conditions can be written as

$$F(0)=a,\quad F(1)=b,\quad F(2)=c,\quad F(3)=d,$$
$$G(0)=e,\quad G(1)=r, \quad (36)$$
$$\Theta(0)=s,\quad \Theta(1)=t,\quad \Phi(0)=w,\quad \Phi(1)=z.$$

By solving the set of Eqs. (32)–(35) with the new form of the boundary conditions, the transformed functions at any other discrete number can be found. The unknown coefficients, $a$, $b$, $c$, $d$, $e$, $r$, $s$, $t$, $w$, and $z$ can be obtained after solving each transformed function and doing the inverse operation and then applying boundary conditions of the problem (Eq. (19)). For instance, the results obtained by $N_1=N_2=N_3=0.1$, $Pe_h=Pe_m=0.1$ and $\mathrm{Re}=0.1$ when we do the procedure until $F(7)$, $G(5)$, $\Theta(5)$ and $\Phi(5)$ are as follows:

$$f(\eta)=-0.00231570+1.50221914\eta+0.00348994\eta^2$$
$$-0.50490663\eta^3-0.00003279\eta^4+0.00315586\eta^5$$
$$-0.00114145\eta^6-0.00046836\eta^7 \quad (37)$$

$$g(\eta)=0.00184347-0.40259190\eta-0.00159986\eta^2$$
$$+0.37070946\eta^3-0.00024360\eta^4+0.03188243\eta^5 \quad (38)$$

$$\theta(\eta)=0.49992793-0.49748814\eta+0.00005760\eta^2$$
$$-4.45\times10^{-9}\eta^3-0.00001447\eta^4-0.00251185\eta^5 \quad (39)$$

$$\theta(\eta)=0.49992793-0.49748814\eta+0.00005760\eta^2$$
$$-4.45\times10^{-9}\eta^3-0.00001447\eta^4-0.00251185\eta^5 \quad (40)$$

The relevant constant coefficients are

$$a=-0.00231569,\quad b=1.50221914,\quad c=0.0034899457,$$
$$d=-0.50490663,\quad e=0.00184347,\quad r=-0.40259190,$$
$$s=0.49992793,\quad t=-0.49748814,\quad w=0.49992793,$$
$$z=-0.49748814. \quad (41)$$

As seen in terms of the above equations, the convergence of DTM is completely evident. In this study, we do the DTM procedure for four steps because the solution has a good agreement with the numerical method (it obviously can be inferred from Tables 2 and 3 in Section 5). Although it will converge to a more accurate one by the increasing number of steps, but it will take more time. Thus, we obtain the following results by four steps for the matter of time and accuracy.

## 5. Results and discussion

In this section, we represent the results that obtained by implementing DTM. In Tables 2 and 3, the validity and accuracy of DTM in comparison with the numerical method are depicted. The numerical method results were achieved by fourth order Runge-Kutta procedure with Maple software package. The results show that the DTM is a reliable approach for predicting the solutions of this problem. In Table 4, a comparison with HPM (that is done by Sheikholeslami et al.) is represented that shows the closeness of these results. The behavior of stream function, micro rotation, and temperature in the direction of the channel width ($\eta$), can be found from Figs. 2–4. Fig. 2 shows that the value of stream function decreases with increasing Reynolds number, but it increases with increase of the $N_1$, $N_2$ and $N_3$ values. It is worth mentioning that the Reynolds number indicates the relative significance of the inertia effect compared to the viscous effect. Thus the velocity boundary layer thickness decreases as Re increases. It can be inferred from Figs. 2 and 3 that Reynolds number has a significant effect on the stream function and micro rotation. Fig. 4 demonstrates the outcome of different parameters on the temperature and one can deduce that the effect of Peclet number is higher than others, because of the presence of Prandtl number. In Fig. 5, it can be concluded that species concentration value increases with the increase of Peclet number in top half of the channel and is the decreasing function of $\eta$. In Figs. 6 and 7 the effect of Reynolds number on the stream function and micro rotation is depicted and they show that with the increase of Reynolds number, stream function value decreases but micro rotation value increases.

## 6. Conclusion

In this investigation, micropolar fluid flow and heat transfer through permeable walls are considered. The DTM is used to obtain the solution of the governing equations. Comparisons between this method and numerical method reveal that this approach has a good agreement with the numerical method in this study. The effects of different parameters on the fluid flow, heat transfer and concentration characteristics are shown in detail. The results show that the Reynolds number has a little effect on the temperature and concentration fields. Also, as seen, the Reynolds number has a significant effect on the stream function value and stream function increases with the decrease of Reynolds number.

## References


[1] M. Sheikholeslami, D.D. Ganji, Entropy generation of nanofluid in presence of magnetic field using lattice boltzmann method, Phys. A Stat. Mech. its Appl. 417 (2015) 273–286, http://dx.doi.org/10.1016/j.physa.2014.09.053.

[2] M. Sheikholeslami, K. Vajravelu, M.M. Rashidi, Forced convection heat transfer in a semi annulus under the influence of a variable magnetic field, Int. J. Heat Mass Transfer 92 (2016) 339–348, http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.08.066.

[3] M. Sheikholeslami, D. Domiri Ganji, M. Younus Javed, R. Ellahi, Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, J. Magn. Magn. Mater. 374 (2015) 36–43, http://dx.doi.org/10.1016/j.jmmm.2014.08.021.

[4] M. Sheikholeslami, D.D. Ganji, M.M. Rashidi, Ferrofluid flow and heat transfer in a semi annulus enclosure in the presence of magnetic source considering thermal radiation, J. Taiwan Inst. Chem. Eng. 47 (2015) 6–17, http://dx.doi.org/10.1016/j.jtice.2014.09.026.

[5] M. Sheikholeslami, H.R. Ashorynejad, P. Rana, Lattice Boltzmann simulation of nanofluid heat transfer enhancement and entropy generation, J. Mol. Liq. 214 (2016) 86–95, http://dx.doi.org/10.1016/j.molliq.2015.11.052.

[6] M. Sheikholeslami, S. Soleimani, D.D. Ganji, Effect of electric field on hydrothermal behavior of nanofluid in a complex geometry, J. Mol. Liq. 213 (2016) 153–161, http://dx.doi.org/10.1016/j.molliq.2015.11.015.
[7] A.C. Eringen, Theory of micropolar fluids, J. Math. Mech. 16 (1966) 1–18.
[8] R.S. Gorla, J. Eppichs, Heat transfer in micropolar boundary layer flow over a flat plate, Int. J. Eng Sci. 21 (1983) 791–798.
[9] D.A.S. Rees, A.P. Bassom, The blasius boundary-layer flow of a micropolar fluid, Int. J. Eng. Sci. 34 (1996) 113–124, http://dx.doi.org/10.1016/0020-7225(95)00058-5.
[10] N.A. Kelson, A. Desseaux, Effect of surface conditions on flow of a micropolar fluid driven by a porous stretching sheet, Int. J. Eng. Sci. 39 (2001) 1881–1897, http://dx.doi.org/10.1016/S0020-7225(01)00026-X.
[11] R.A. Mohamed, S.M. Abo-Dahab, Influence of chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium with heat generation, Int. J. Therm. Sci. 48 (2009) 1800–1813, http://dx.doi.org/10.1016/j.ijthermalsci.2009.01.019.
[12] M. Sheikholeslami, D.D. Ganji, M. Gorji-Bandpy, S. Soleimani, Magnetic field effect on nanofluid flow and heat transfer using KKL model, J. Taiwan Inst. Chem. Eng. 45 (2014) 795–807, http://dx.doi.org/10.1016/j.jtice.2013.09.018.
[13] M. Sheikholeslami, M. Gorji-Bandpy, S. Soleimani, Two phase simulation of nanofluid flow and heat transfer using heatline analysis, Int. Commun. Heat Mass Transfer 47 (2013) 73–81, http://dx.doi.org/10.1016/j.icheatmasstransfer.2013.07.006.
[14] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Numerical investigation of MHD effects on Al2O3-water nanofluid flow and heat transfer in a semi-annulus enclosure using LBM, Energy 60 (2013) 501–510, http://dx.doi.org/10.1016/j.energy.2013.07.070.
[15] M. Hatami, M. Sheikholeslami, D.D. Ganji, Nanofluid flow and heat transfer in an asymmetric porous channel with expanding or contracting wall, J. Mol. Liq. 195 (2014) 230–239, http://dx.doi.org/10.1016/j.molliq.2014.02.024.
[16] M. Sheikholeslami, M. Gorji-Bandpy, I. Pop, S. Soleimani, Numerical study of natural convection between a circular enclosure and a sinusoidal cylinder using control volume based finite element method, Int. J. Therm. Sci. 72 (2013) 147–158, http://dx.doi.org/10.1016/j.ijthermalsci.2013.05.004.
[17] M. Hatami, M. Sheikholeslami, M. Hosseini, D.D. Ganji, Analytical investigation of MHD nanofluid flow in non-parallel walls, J. Mol. Liq. 194 (2014) 251–259, http://dx.doi.org/10.1016/j.molliq.2014.03.002.
[18] A. Vahabzadeh, M. Fakour, D.D. Ganji, Study of MHD nanofluid flow over a horizontal stretching plate by analytical methods, Int. J. Part. Diff. Eq. Appl. 2 (2014) 96–104, http://dx.doi.org/10.12691/ijpdea-2-6-1.
[19] M. Sheikholeslami, M. Hatami, D.D. Ganji, Micropolar fluid flow and heat transfer in a permeable channel using analytical method, J. Mol. Liq. 194 (2014) 30–36, http://dx.doi.org/10.1016/j.molliq.2014.01.005.
[20] M. Sheikholeslami, D.D. Ganji, Three dimensional heat and mass transfer in a rotating system using nanofluid, Powder Technol. 253 (2014) 789–796, http://dx.doi.org/10.1016/j.powtec.2013.12.042.
[21] N.A. Abdul Latiff, M.J. Uddin, O.A. Beg, A.I. Ismail, Unsteady forced bioconvection slip flow of a micropolar nanofluid from a stretching/shrinking sheet, Proc. Inst. Mech. Eng. Part N J. Nanoeng. Nanosyst. (2015), http://dx.doi.org/10.1177/1740349915613817.
[22] M.J. Uddin, M.N. Kabir, Y.M. Alginahi, Lie group analysis and numerical solution of magnetohydrodynamic free convective slip flow of micropolar fluid over a moving plate with heat transfer, Comput. Math. Appl. 70 (2015) 846–856, http://dx.doi.org/10.1016/j.camwa.2015.06.002.
[23] M.M. Rahman, M.J. Uddin, A. Aziz, Effects of variable electric conductivity and non-uniform heat source (or sink)on convective micropolar fluid flow along an inclined flat plate with surfaceheat flux, Int. J. Therm. Sci. 48 (2009) 2331–2340, http://dx.doi.org/10.1016/j.ijthermalsci.2009.05.003.
[24] M. Sheikholeslami, R. Ellahi, H.R. Ashorynejad, G. Domairry, T. Hayat, Effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium, J. Comput. Theor. Nanosci. 11 (2014) 486–496, http://dx.doi.org/10.1166/jctn.2014.3384.
[25] M. Sheikholeslami, H.R. Ashorynejad, G. Domairry, I. Hashim, Flow and heat transfer of Cu-water nanofluid between a stretching sheet and a porous surface in a rotating system, J. Appl. Math. 2012 (2012), http://dx.doi.org/10.1155/2012/421320.
[26] M. Sheikholeslami, D.D. Ganji, Magnetohydrodynamic flow in a permeable channel filled with nanofluid, Sci. Iran. 21 (2014) 203–212.
[27] M. Shekholeslami, H.R. Ashorynejad, D. Domairry, I. Hashim, Investigation of the laminar viscous flow in a semi-porous channel in the presence of uniform magnetic field using optimal homotopy asymptotic method, Sains Malays. 41 (2012) 1281–1285.
[28] M. Sheikholeslami, D.D. Ganji, H.R. Ashorynejad, H.B. Rokni, Analytical investigation of Jeffery-Hamel flow with high magnetic field and nanoparticle by Adomian decomposition method, Appl. Math. Mech. 33 (2012) 25–36, http://dx.doi.org/10.1007/s10483-012-1531-7.
[29] M. Sheikholeslami, D.D. Ganji, H.R. Ashorynejad, Investigation of squeezing unsteady nanofluid flow using ADM, Powder Technol. 239 (2013) 259–265, http://dx.doi.org/10.1016/j.powtec.2013.02.006.
[30] M. Sheikholeslami, D.D. Ganji, Heat transfer of Cu-water nanofluid flow between parallel plates, Powder Technol. 235 (2013) 873–879, http://dx.doi.org/10.1016/j.powtec.2012.11.030.
[31] M. Sheikholeslami, H.R. Ashorynejad, D.D. Ganji, A. Kolahdooz, Investigation of rotating MHD viscous flow and heat transfer between stretching and porous surfaces using analytical method, Math. Probl. Eng. 2011 (2011), http://dx.doi.org/10.1155/2011/258734.
[32] M. Sheikholeslami, H.R. Ashorynejad, D.D. Ganji, A. Yldrm, Homotopy perturbation method for three-dimensional problem of condensation film on inclined rotating disk, Sci. Iran. 19 (2012) 437–442, http://dx.doi.org/10.1016/j.scient.2012.03.006.
[33] J.K. Zhou, Differential Transformation and Its Applications for Electrical Circuits, Huazhong University Press, Wuhan, China, 1986.
[34] A.R. Ahmadi, A. Zahmatkesh, M. Hatami, D.D. Ganji, A comprehensive analysis of the flow and heat transfer for a nanofluid over an unsteady stretching flat plate, Powder Technol. 258 (2014) 125–133, http://dx.doi.org/10.1016/j.powtec.2014.03.021.
[35] M. Hatami, D.D. Ganji, Natural convection of sodium alginate (SA) non-Newtonian nanofluid flow between two vertical flat plates by analytical and numerical methods, Case Stud. Therm. Eng. 2 (2014) 14–22, http://dx.doi.org/10.1016/j.csite.2013.11.001.
[36] A.A. Joneidi, D.D. Ganji, M. Babaelahi, Differential transformation method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity, Int. Commun. Heat Mass Transfer 36 (2009) 757–762, http://dx.doi.org/10.1016/j.icheatmasstransfer.2009.03.020.
[37] M. Sheikholeslami, D.D. Ganji, Nanofluid flow and heat transfer between parallel plates considering Brownian motion using DTM, Comput. Methods Appl. Mech. Eng. 283 (2015) 651–663, http://dx.doi.org/10.1016/j.cma.2014.09.038.

[38] M. Sheikholeslami, M. Azimi, D.D. Ganji, Application of differential transformation method for nanofluid flow in a semi-permeable channel considering magnetic field effect, Int. J. Comput. Methods Eng. Sci. Mech. 16 (2015) 246–255, http://dx.doi.org/10.1080/15502287.2015.1048384.

[39] M. Sheikholeslami, M.M. Rashidi, D.M. Al Saad, F. Firouzi, H. B. Rokni, G. Domairry, Steady nanofluid flow between parallel plates considering thermophoresis and Brownian effects, J. King Saud Univ. – Sci. (2015), http://dx.doi.org/10.1016/j.jksus.2015.06.003.

[40] D. Domairry, M. Sheikholeslami, H.R. Ashorynejad, R.S.R. Gorla, M. Khani, Natural convection flow of a non-Newtonian nanofluid between two vertical flat plates, J. Nanomater. Nanoeng. Nanosyst. 225 (2012) 115–122, http://dx.doi.org/10.1177/1740349911433468.

[41] P. Sibanda, F. Awad, Flow of a micropolar fluid in channel with heat and mass transfer, Latest Trends Theor. Appl. Mech. Fluid Mech. Heat Mass Transfer (2010) 112–120. http://www.wseas.us/e-library/conferences/2010/Corfu/HEAPFL/HEAPFL-19.pdf.

[42] Z. Ziabakhsh, G. Domairry, Homotopy analysis solution of micro-polar flow in a porous channel with high mass transfer, Adv. Theor. Appl. Mech. 1 (2008) 79–94. <http://m-hikari.com/atam/atam2008/atam1-4-2008/domairryATAM1-4-2008.pdf>.

[43] Davood Domairry Ganji, Sayyid Habibollah Hashemi Kachapi, Analysis of nonlinear equations in fluids, Prog. Nonlinear Sci. 2 (2011).