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New Method for Image Segmentation

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Abstract

in this paper we describe a modified segmentation method applied to image. An EM algorithm is developed to estimate parameters of the Gaussian mixtures. Recently, researchers are focusing more on the study of expectation of maximization (EM) due to its useful applications in a number of areas, such as multimedia, image processing, pattern recognition and bioinformatics. The human visual system can often correctly interpret images that are of quality that they contain insufficient explicit information to do so. The difficulty is mainly due to variable brain structures, various MRI artifacts and restrictive body scanning methods. The IBSR image segmentation data set is used to compare and evaluate the proposed methods. In this paper, we propose a modified expectation of maximization (MEM) based on the properties of likelihood, while reducing number of iteration for a sick of fast converge to the center of cluster and your application to image segmentation. The experiments on real images show that: (1) our proposed approach can reduce the number of iterations, which leads to a significant reduction in the computational cost while attaining similar levels of accuracy. (2)The approach also works well when applied to image segmentation. A methodology for calculate is presented for making use the error between the ground truth, human-segmented image data sets to compare, develop and optimize image segmentation algorithms. This error measure is based on object-by-object comparisons of a segmented image and a ground-truth (reference) image. Experimental results for segmented images demonstrate the good segmentation performance of the proposed approach.

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Keywords: Image segmentation, Expectation of maximization, Image the resonance magnetic;

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1. Introduction

Image segmentation is one of the basic problems in image analysis. The importance and utility of image segmentation has resulted in extensive research and numerous proposed approaches such as intensity, color, texture, etc, both automatic and interactive. Although many segmentation techniques have been appeared in scientific literature, they can be divided into image-domain based, physics based and feature-space based techniques Martin & Fowlkes(2001). Many clustering algorithms have been developed for image segmentation (Kohonen, 1997; Wang, 2008; Zaixin, 2014) in particular, the FCM (Fuzzy C-Means) algorithm (Phan, 1999; Hange, 1996; Wilim, 1990) creates a fuzzy partition of the image, it identifies a cluster prototype as a point and determines the membership function of each pixel to a cluster. The unsupervised technique Fuzzy C-Means is the famous technique used to skin tumor border in medical image (Hartley, 1958;Wang, 2008). It is typically used to partition images into regions that are in some sense homogeneous, or have some semantic significance, thus providing subsequent processing stage high level information about scene structure. Different clustering algorithms, such as K-Means, EM (the Expectation Maximization algorithm), Dunn (1973) and SOM (self organizing map), Ruan (2002), and the mean shift algorithm are adopted to partition the image into K regions. In the paper, we focus on the unsupervised based image segmentation method, specifically, we explore a new kind of EM based approach called modified expectation maximization (MEM) to perform image segmentation which is the EM we first define a novel idea to limit the number of iteration for converge with a low time, which is obtained by compared the center obtained by EM algorithm and calculate the new center obtained by our method. A modified expectation of maximization (MEM) algorithm is a novel algorithm used in this paper, which is based on the choice center of cluster. The algorithm is based on: first choice the center of cluster second, compare to the new center which is obtunded bay additional a number of pixels which became near of the center of all pixels this astute minimize the number of iteration to converge to the center with a low time. In general evaluation methods for image segmentation can be classified into analytic and empirical goodness methods, in turn, can be classified into empirical goodness methods and empirical methods discrepancy methods. Research into methods to evaluating the quality of image segmentation has recently been recognized as an important topic. Several such segmentation evaluation benchmarks have been proposed Pohl (2002).In literature many segmentation algorithms for analyzing various have been studied. Hofmann (1998), have formulated the texture segmentation as a data clustering problem based on spar proximity data, they have derived the clustering algorithm using deterministic annealing and used this algorithm in the segmentation of synthetic data set and textured images. The expectation-maximization (EM) algorithm is a method for iteratively finding maximum likelihood estimates of parameters in probabilistic models, where the model depends on unobserved latent variables. All of these algorithms determine clusters in feature space. Due to the high intensity in homogeneities, clusters spread over large regions, thus increasing the sensitivity to initial conditions and local minima. Expectation maximization has been extensively used for image segmentation (Kapur, 1996;Leemput, 1999). To evaluate the results of image segmentation is calculated the error between the segmented image and a reference image (ground truth) image. The reference image is often obtained manually with the help of image segmentation is one of the most important research topics in computer vision and image under- standing. The task of image segmentation is to partition an image into a number of non-overlapping regions with homogeneous characteristics. Human expert for MR image and simple knowledge then human and the segmented image is obtained by algorithm. For comparison, these same sets of image were analyzed, the Structural Similarity (SSIM) metrics of Wang et al.(2006), and the Visual Information Fidelity (VIF) metric of Shekh, Inter-region contrast of Levine and Nazif, and in contrast, Martin et al. (2001) proposed an interesting empirical discrepancy measure for evaluating segmentation. In the present paper we well assume that the measures criteria have access to such segmentation algorithms. We selected four criterions for validation: a measure of structural similarity (SSIM) that compares local patterns of pixel intensities that have been normalized for luminance and contrast, the visual information fidelity (VIF) metric which is a specific and a quite successful implementation of the information fidelity, Martin measure, and Inter-region contrast of Levine and Nazif, to evaluate boundary accuracy. The remained of this paper is organized as follows. In section 2 we review each of the algorithms that were chosen for the image segmentation. In section 3 present the criteria for evaluation of image. In section 4 we discuss the user experiments, the data set and ground truth we used for the experiment, validate the selected evaluation measures, and discussion which criterion performed best. Finally, in the last section we present our conclusion and potential future work.

2. The Research Method

In this section the unsupervised deterministic classification and parametric probabilistic classification are used as based region image segmentation, then the modified algorithm MEM is presented. Finally, the criterion is applied between image segmentation with algorithm and ground truth given by expert.

2.1. Fuzzy C-Means (FCM)

The Fuzzy C-Means (FCM) clustering algorithm was first suggested by Dunn (1973) and later improved by Bezdek (1999). Unlike K-Means clustering, in which each observation has a clear-cut binary membership, the FCM method proposes a fuzzy membership that assigns a degree of membership for each class by iteratively updating the cluster centers and the membership degrees for each data point. Chen (2004) proposes a segmentation of MRI based on fuzzy clustering techniques. FCM methods have been proposed for the segmentation of MR Images Bezdek (1981) and for the segmentation of major tissues in (Dunn, 1973; Ruan, 2002) and possible tumor on T1-weighted volumes. The FCM is often used in medical image segmentation (Phan, 1999; Salzenstein, 1997). Chen (2004), have proposed an algorithm based on FCM for the correction of intensity in homogeneity and for segmentation of MRI images. The presence of noise requires using the Modified Fuzzy C-Means (MFCM) technique (Lucchese, 2001; Wells, 1996). An adaptive spatial fuzzy c-means algorithm (ASFCM) is proposed by Liew and Yan (2003). The concept of degree of membership in fuzzy clustering is similar to the posterior probability in a mixture modelling setting, by monitoring data points that have close membership values to existing classes, forming new clusters is possible; this is the major advantage of FCM clustering over K-Means clustering.

2.2. K-Means

K-Means algorithm is an unsupervised clustering algorithm that classifies the input data points into multiple classes based on their inherent distance from each other. The iterative k-means clustering algorithm was first proposed by MacQueen (1997). The algorithm aims at partitioning the data set, consisting of ℓ expression patterns {x1,…, xℓ} in an n-dimensional space, into k disjoint clusters {C_i}i=lk, such that the expression patterns in each cluster are more similar to each other than to the expression patterns in other clusters from Neal (1998). There are two popular partitioned clustering strategies: square-error and mixture modelling. The sum of the squared Euclidian distances between the samples in a cluster and the cluster center is called within-cluster variation. The sum of the within-cluster variations in a clustering scheme is used as a criterion in K-means clustering Coleman (1979). K-means are widely used in many applications such as data extraction and image segmentation from Jain (1988). The k-means method is an iterative algorithm that minimizes the sum of distances between each object and its cluster centroid. The distance used here is the usual Euclidean distance.

2.3 Expectation Maximization

EM is an iterative optimization method to estimate some unknown parameters θ, given measurement data U. However, we are not given some "hidden" nuisance variables J, which need to be integrated out. In particular, we want to maximize the posterior probability of the parameters θ given the data U, marginalizing over J:

$$\theta^* = \arg\max_{\theta} \sum_{J \in \zeta^n} \left(\theta, \frac{J}{U} \right) \tag{1}$$

The intuition behind EM is an old one: alternate between estimating the unknowns θ and the hidden variables J. This idea has been around for a long time. However, instead of finding the best $J \in \zeta$ given an estimate θ at each iteration, EM computes a distribution over the space J. One of the earliest papers on EM from Hartley (1958), but the seminal reference that formalized EM and provided a proof of convergence is by Dempster, Laird, and Rubin

(1977). A recent book devoted entirely to EM and applications from Mclachlan (1977), where from Tanner (1996) is another popular and very useful reference. One of the most insightful explanations of EM, that provides a deeper understanding of its operation than the intuition of alternating between variables, is in terms of lower bound maximization from Tanner (1996). In this derivation, the E-step can be interpreted as constructing a local lower-bound to the posterior distribution, whereas the M-step optimizes the bound, thereby improving the estimate for the unknowns. This is demonstrated below for a simple example. The parameters found on the M step are then used to begin another E step, and the process is repeated until convergence. Mathematically for a given training dataset $\{x_1, x_2, x_3,,, x_n\}$ and model $p(x,z)$ where z is the latent variable, we have:

$$l(\theta) = \sum_{i=1}^{m} \log p(x_i, \theta) = \sum_{i=1}^{m} \log \sum_{z} p(x_i, z; \theta) \tag{2}$$

As can be seen from the above equation, the log likelihood is described in terms of x, z and θ. But since z, the latent variable is not known; we use approximations in its place. These approximations take the form of E & M steps mentioned above and formulated mathematically below.

E Step, for each i

$$Q_i(z^{(i)}) = p(\frac{z^{(i)}}{x^{(i)}}; \theta) \tag{3}$$

M Step, for all z:

$$\theta = \arg\max_{\theta} \sum_{i} \sum_{z^{(i)}} Q_{(i)}(z^{(i)}) \log \frac{p\left(x^{(i)}, z^{(i)}, \theta\right)}{Q_i(z^{(i)})} \tag{4}$$

where Q_i is the posterior distribution of $z^{(i)}$'s given theX $x^{(i)}$'s. Conceptually, The EM algorithm can be considered as a variant of the K Means algorithm where the membership of any given point to the clusters is not complete and can be fractional.

2. 4. Mean Shift (MS

Mean shift [8] is a powerful and robust tool for feature space analysis. It is an unsupervised nonparametric estimation algorithm of density gradient. MS was successfully applied by Mayer (1996), in clustering, segmentation and filtering of natural resources in 2D images Comaniciu (2002), using a paradigm adaptively to segment the brain MR images. The MS procedure-based image segmentation is a straight forward extension of the discontinuity preserving smoothing algorithm. Mean shift based FCM algorithm requires less computational time than established techniques listed in Zhou (2009).

2.5. Modified Expectation Maximization (MEM)

An Expectation-Maximization (EM) algorithm was proposed in Neil (1998) to model the homogeneities as a bias field of the image logarithm, has applied for the segmentation of brain MR image. The application of the EM algorithm to brain MR image segmentation was reported by Leemput (1999). A common disadvantage of EM algorithms is that the intensity distribution of brain images is modeled as a normal distribution, which is untrue,

especially for noisy images. The EM algorithm has demonstrated greater sensitivity to initialization than the KMeans or Fuzzy C-Means algorithms Dempster(1977);
The distribution which depends on x and z is given by:

$$p\left(\frac{z}{x},\theta\right)=\frac{p\left(z,\frac{x}{\theta}\right)p(z)}{\sum_{z}p\left(x,\frac{z}{\theta}\right)} \tag{5}$$

The steps of the EM algorithm are as follows
1. Choose θ^{old}
2. E-Step: Evaluate $\psi(z|x, \theta^{old})$
Estimation of the unobserved z's, conditioned on the observation, using the values from the last maximization step:

$$p\left(\frac{z\ j=1}{x\ j},\theta_t\right)=\frac{p\left(z\ j=1,\frac{x\ j}{\theta_t}\right)}{p\left(\frac{x\ j}{\theta_t}\right)}$$
$$=\frac{p\left(\frac{x\ j}{z\ j=i},\theta_t\right)p\left(\frac{z\ j=i}{\theta_t}\right)}{\sum_{k=1}^{n}p\left(\frac{x\ j}{z\ j=k},\theta_t\right)p\left(\frac{z\ j=k}{\theta_t}\right)} \tag{6}$$

3. M-Step: Maximize the expected log-likelihood of the joint event

$$Q\left(\theta,\theta^{old}\right)=E-z[\log p(x,\frac{z}{\theta})]$$
$$=\sum_{z}p\left(\frac{z}{x},\theta^{old}\right)\log\left(x,\frac{z}{\theta}\right) \tag{7}$$

$$\theta^{new}=\arg\max_{\theta}\left(\frac{Q}{\theta},\theta^{old}\right) \tag{8}$$

4. Decision: If log p(x, z|θ) or θnew have converged, break. Otherwise, $\theta^{old} \leftarrow \theta^{new}$. It is important to remember that Qt(θ) is calculated in the E-step by evaluating $p(z|x, \theta^{old})$ using the current guess θ^{old}, whereas in the M-step we are optimizing Q(θ (q)) with respect to the free variable θ to obtain the new estimate θ^{new}. It can be proved that the EM algorithm converges to a local maximum of log p(x, θ), and thus equivalently maximizes the log-posterior log p(θ|x). But the question remains: why should this converge? We know

$$p\left(\frac{x}{\theta}\right)=\sum_{z}\left(x,\frac{z}{\theta}\right) \tag{9}$$

which implies

$$\log\left[p\left(\frac{x}{\theta}\right)\right] = L\left(\frac{q}{\theta}\right) + KL\left(q\|p\right) \tag{10}$$

For proof of this refer to the scribe the Kullback-Leibler Divergence.

Recall that:

$$L(q,\theta) = \sum_z q(z)\log\left[\frac{p\left(x,\frac{z}{\theta}\right)}{q(z)}\right] \tag{11}$$

$$KL\left(q\|p\right) = -\sum_z q(z)\log fracp\left[x,\frac{z}{\theta}q(z)\right] \tag{12}$$

We also know

$$\log p\left(x,\frac{z}{\theta}\right) = \log p\left(\frac{z}{x},\theta\right) + \log p\left(\frac{x}{\theta}\right) \tag{13}$$

In general, we know that KL > 0 which implies that log p(x|θ) will be smaller than the likelihood, L(q, θ), that we are interested in finding. So in the E-step, when KL = 0, this implies that $q(z) = p(z/x, \theta^{old})$. Once we fix q, we want to maximize the over the lower bound to get a new estimate. In the M-step, we obtain the new parameter value θ^{new}. After we get this new value, q(z) no longer equals $p(z/x, \theta^{old})$). Therefore KL 6= 0 which implies that KL >0. This means there is room for further improvement. Now we can take our lower bound and try to optimize further. We start by writing the likelihood as the Difference of logs

$$L(q,\theta) = \sum_z p\left(\frac{z}{x},\theta^{old}\right)\log p\left(x,\frac{z}{\theta}\right) - \sum_z p\left(\frac{z}{\theta^{old}},x\right)\log\left(\frac{z}{x},\theta^{old}\right) \tag{14}$$

Notice that the last term of this equation simply equals $Q\left(\theta,\theta^{old}\right)$, so

$$L(q,\theta) = Q\left(\theta,\theta^{old} \mp x\right) \tag{15}$$

Where x is a constant used for minimize the number of iteration, quickly converge to the center whether by adding the x or subtraction it.

3. Data base of images

In this paper, we used two different datasets of synthetic and real images. . We compared these criteria on segmentation results obtained by different algorithm. For all these criteria, the best segmentation is the lower score. Martin criterion to assess the consistency between two manual segmentations of a same image, this measure can be used to compare two segmentations one reference, the other obtained by an algorithm. The contrast between two adjacent regions Ri and Rj. Structure Similarity Index (SSIM) Measure loss of image structure, luminance, contrast and structure similarity. Visual Information Fidelity (VIF) measure loss of human perceivable information distortion in the process..

3.1 Synthetic images

In this paper, the synthetic images containing different levels of textures. The simulation summary brain tumors MRI images and examples of test data are presented in [www.ucnia.org] software and data. Figure.1 shows an example of different images: improved contrast T1w, T1w, T2 downward and axial views of MRI images of the synthetic data set from left to right: SimTumor001, SimTumor002, SimTumor003, SimTumor004, SimTumor005 IRM. Will be used to compare four criterions; of the five methods region-based segmentation used n this paper.

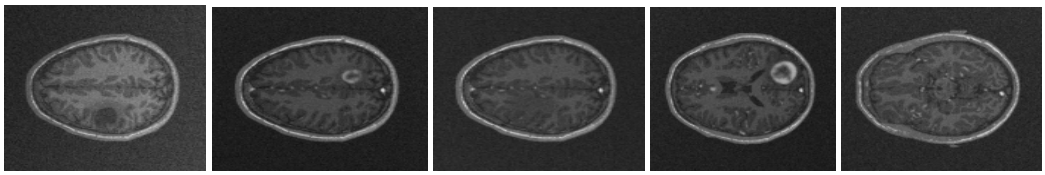


Fig. 1.Views axial images synthetic data set. : Contrast enhanced T1w , Sim Tumor001, SimTumor002, SimTumor003, SimTumor004 and sets SimTumor005 MRI data.

a b c d e f g

Fig. 2. From left to right : (a) image T1, (b) ground truth, (c) white matter, (d) gray matter, (e) CSF, (f) edema, (g) tumor.

The image is made up of: white matter (WM), gray matter (GM), cerebrospinal fluid (CSF), edema, tumor image of Mr. synthetic represented as a set of probability maps spatial fabric and pathology shown in Figure 4. in our laboratory from these issues, we created the truth.

3.2 Real images

The MR images used here is from the Internet Brain Segmentation Repository (IBSR). For the purpose of validating segmentation methods, we need a set of MR images the 3D pathological ground truth, these images serve as test data for the evaluation of segmentation methods.

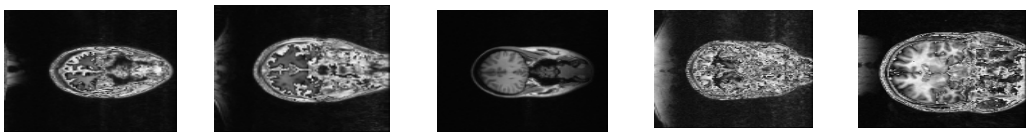
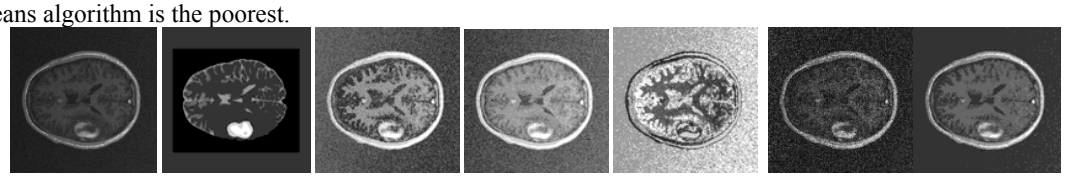


Fig. 3. Example of coronal slices from Internet Brain Segmentation Repository IBSR cases.

4. Results

In summary, our proposed approach is based on the theory of EM, the properties of expectation of maximization and a probabilistic formulation. As is well known, the objective of EM is to find center of each class is measured by the distances between center initial and the center of each class. In this paper, we have summarized the unsupervised deterministic classification and parametric probabilistic classification and evaluation criterion. We used for application two types of 50 images synthetic and 50 real images (MRI). We calculate the average for each measures of criterion for each segmentation algorithms. In Table 1 shows the results values for dataset images synthetic them appear clear that the best performing algorithms, in terms of measured accuracy, are the MS and EM algorithms, the K-Means algorithm is the poorest.



a b c d e f g

Fig. 4. Represented the Segmentation results of different methods: (a) original image, (b) ground truth image, (c) K-Means, (d) FCM, (e) MS, (f) EM, (g) MEM.

	KMeans	FCM	EM	MEM	MS
SSIM	0.5411	0.6103	0.6254	0.6616	0.8609
VIP	0.2313	0.2586	0.2430	0.2883	0.0089
Martin	0.1014	0.2775	0.2356	0.1661	0.3741
InetrIntra	0.5014	0.5013	0.5014	0.5015	0.5000

Table 1. Average criterion values for the MRI images of the five automatic segmentation algorithms

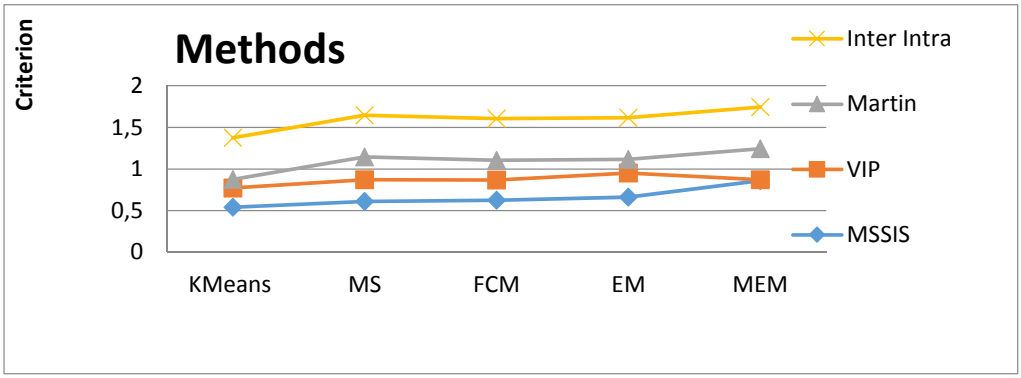
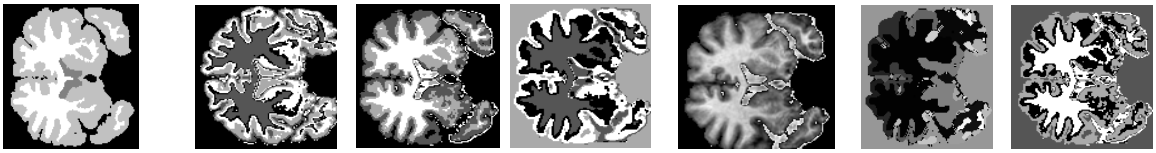


Fig. 5. Box plot average Mean of four measures for brain MRI data for each method.



a b c d e f g

Fig. 6. Representative examples of results obtained with the different segmentation methods: (a) ground truth image, (b) original image, (c) FCM, (d) KMeans, (e) MS, (f) EM, (g) MEM.

The image used here is from database IBSR characterized by T1 weighted MR image volumes from 50 subjects with age between 7 and 71 years were used (Figure 2). The size of the volume is 256×256×64 voxels with voxel size of 0.8×0.8×1.5.Table 2 shows the results for MR image.

	KMeans	FCM	EM	MEM	MS
SSIM	0.7079	0.5030	0.4811	0.7077	0.8155
VIP	0.0173	0.0346	0.0238	0.0113	0.0051
Martin	0.0698	0.07000	0.2308	0.2154	0.0618
InetrIntra	0.5000	0.5000	0.5000	0.5000	0.5000

Table 2. Criterion values for the MR images data set of the five automatic segmentation algorithms using four measures

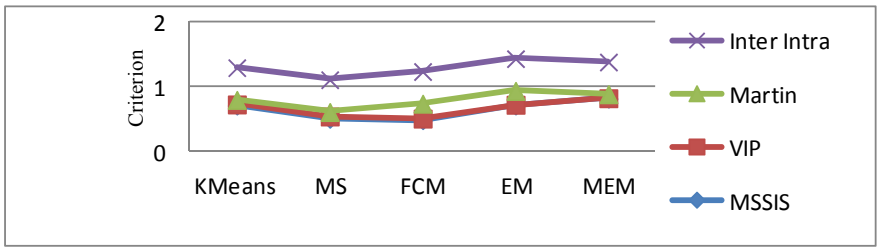


Fig. 7. Box plot average Mean of four measures for 50 images from the brain MRI data for each methods.

The curve presents the different values of criterion for the five segmentation algorithms. In Table1 given the variety accuracies for each algorithm, we calculate the mean value criterion measures and search the best values. In addition, from the Table 3 in terms of measures, the best algorithms are the MS and MEM, which perform equally well on average.

Table 3. Presented the CPU time by different algorithms

	Image 180x180	Image 172x158	256×256 pixels
KMeans(CPUtime(s))	2.27	1.82	1.01
FCM (CPU time(s))	12.11	5.76	4.09
MS (CPU time(s))	0.35	0.46	0.39
EM (CPU time(s))	23.41	10.45	7.46
MEM (CPU time(s))	20.86	9.08	7.38

4.1. Computational time

The processing time for segmenting images is presented in Table 3, in which we list the CPU time in segmenting images. We can observe seen that the processing time for MEM is faster than the normally EM algorithm.

4.2. Discussion

In summary, the contributions of this work are as follows: The modified Expectation maximization (MEM) with the pruning strategy based on the maximum likelihood model is proposed to reduce the number of iteration. The advantage of this is observed in Figure 4 and Figure 6 and in the table 1, table 2 and table 3 in which the number of iteration is reduced when compared with traditional EM. The reduced number of iteration will in turn lead to the lower computational time of MEM. As shown in table 3 the computation time of MEM is significantly smaller than that of EM. The advantage of this is observed in Figure. 4 and 6, in which we observe that the segmentation results of the images obtained by MEM and EM are indistinguishable, while MEM requires a lower computational cost when compared with EM. As a result, MEM is the better choice when compared with traditional EM if we consider the efficiency and the segmentation quality together.

5. Conclusion

In this paper, we propose a new region-based modified MEM algorithm for region image segmentation. This method utilizes an astute to minimize the number of iteration to converge quickly a center a low time for segmented the image. The results of this paper prove that we can have good segmentation results by applying the MEM clustering method to real and synthetic images. Our major contribution is the new approach based on minimization the number of iteration and the execution time. The results from the segmentation algorithms are compared to the ground truth. Experimental results demonstrate desirable performance of our method for brain real MR and synthetic images with intensity in homogeneity. Additionally, it is robust to initialization, allowing fully automated applications. The results show that, MS and MEM methods are better than the others in segmentation image, which imply the best criteria are SSIM and Martin with lower degree of two others criterion. In future, we propose to test this method on color images in RGB and YUV spaces as well as on videos.

6. References

Bezdek J. C., Keller J. M., Krishnapuram R., Pal N. R., (1999). Fuzzy Models and Algorithms for Pattern Recognition and Image Processing. New York Springer.

Bezdek J.C. 1981. “Pattern Recognition with Fuzzy Objective Function Algorithms”, Plenum Press, New York.

Cai W., Chen S.C., Zhang D.Q., "Fast and robust fuzzy c-means clustering algorithms incorporating local information for image segmentation". Pattern Recognition 40 (2007) 825–838

Chen W., Giger M.L (2004)." A fuzzy c-means (FCM) based algorithm for intensity in homogeneity correction and segmentation of MR images".

Coleman G. B., Andrews H. C. (1979). " Image segmentation by clustering. Proc IEEE, 67, 773–785

Comaniciu D, Meer P, "Mean Shift: A robust approach toward feature space analysis". IEEE Trans. Pattern Analysis and Machine Intelligence, 24(5), 603-619 (2002).

Dempster A., Laird N, and Rubin D. "Maximum likelihood from incomplete data via the EM algorithm". Journal of the Royal Statistical Society, Series B, 39(1):1–38(1977).

Dunn, J. C.," A Fuzzy Relative of the ISODATA Process and Its Use in Detecting Compact Well-Separated Clusters". Journal of Cybernetics. 3, 32-57 (1973).

Jain, A.K, DUBES,"Algorithms for clustering data". Engle wood cliffs (1988).

Hance G., Umbaugh S., Moss R.H., Stocker W (1996)."Unsupervised color image segmentation with application to skin tumor borders". IEEE Engineering in Medicine and Biology. 15(1), 104-111.

Hartley H. (1958)." Maximum likelihood estimation from incomplete data". Biometrics, 14:174–194.

Hoffman T., Buhman J.M. (1998). Pairwise data clustering by deterministic annealing frame. IEEE Transactions on PAMI. 19(1), 1-13,

Kapur T., Grimson W., Wells W., and Kikinis R. 1996. "Segmentation of brain tissue from magnetic resonance images". Medical Image Analysis, vol. 1, no. 2, pp. 109.127,

Kohonen T., "Self-Organizing Maps", 2nd edition, Springer-Verlag, Heidelberg, 1997.

Pohl K, Wells W, Guimond A., Kasai K., Shenton M., Kikinis R., Grimson W., and Warfield S., "Incorporating non-rigid registration into expectation maximization algorithm to segment MR images", in MICCAI'2002, 2002, pp. 564.572.

Leemput K. V., Maes F., Vandermeulen D., Suetens P., "Automated model-based tissue classification of MR images of the brain". IEEE Trans. Med. Imag., 18(10), 897–908, (1999).

Liew A.W. C., Yan H.,"An adaptive spatial fuzzy clustering algorithm for 3D MR image segmentation". IEEE Transaction on Medical Imaging 22(9) (2003).

Lucchese L., Mitra S.K. (2001). "Color image segmentation", a state of the art survey, in: Proceedings of the Indian National Science Academy on Image Processing, Vis. Pattern Recognition 67 (2) 207–221.

MacQueen J. B. (1967)."Some Methods for classification and Analysis of Multivariate Observations". Proceedings of 5-th Berkeley Symposium on Mathematical Statistics and Probability, Berkeley, University of California Press, 1:281-297

Martin D., Fowlkes C., Tal D., Malik J. (2001)."A database of human segmented natural images and its application to evaluating algorithms and measuring ecological statistics". ICCV, 416–423

Mayer A., Green H. (2006)." Segmentation of brain MRI by adaptive mean shift. International Symposium on Biomedical Imaging", Macro to Nano, pp 319–322,

McLachlan G. and Krishnan T. (1997). The EM algorithm and extensions. Wiley series in probability and statistics. John Wiley & Sons

Neal R. and Hinton G. (1998). "A view of the EM algorithm that justifies incremental, sparse, and other variants". In Jordan, M., editor, Learning in Graphical Models. Kluwer Academic Press.

Pham D., Prince J. L. (1999)."Adaptative Fuzzy Segmentation of Magnetic Resonance Images". IEEE Transactions on Medical Imaging. 18(9), 737–752,

Ruan S., Moretti B., Fadili J., Bloyet D. (2002)."Fuzzy Markovian Segmentation in Application of Magnetic Resonance Images". Computer Vision and Image Understanding, 85, 54–69

Salzenstein. F and Pieczynski. W, 1997. "Parameter estimation in hidden fuzzy Markov random fields and image segmentation". Graph. Models Process. 59:205–220.

Shen S., Sandham W., Granat M, Sterr A, (2005). "MRI Fuzzy Segmentation of Brain Tissue Using Neighborhood Attraction With Neural-Network Optimization".IEEE Transactions on Information Technology In Biomedicine. 9(3), 459–467,

Wang Z., Bovik A., Sheikh H, (2006). "Simoncelli: Image quality assessment: from error visibility to structural similarity". IEEE Transactions on Image processing. 15(2), 430-444.

Wilim Y., Lee S, (1990)."On the color image segmentation algorithm based on the thresholding and fuzzy c-means techniques". Pattern Recognition. 23(9), 935-952.

Wang P., Wang H, (2008). "A Modified FCM Algorithm for MRI Brain Image Segmentation". Future Biomedical Information Engineering.

Wells W. M., Grimson W. E. L., Kikins R., Jolesz F, (1996). " Adaptive segmentation of MRI data". IEEE Trans. Med. Imag., 15, 429–442

Zhou H., Schaefer G, (2009). "Mean shift based FCM algorithm that requires less computational time than these established techniques". IEEE Journal of selected topics in signal processing, 3(1).

Tanner M, (1996). "Tools for Statistical Inference". Springer Verlag, New York. Third Edition.

Zaixin Z, Lizhi C, Guangquan C, 2014. Neighborhood weighted fuzzy c-means clustering algorithm for image segmentation, IET Image Processing , 8(3):150-161.