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Existence of periodic solutions for a periodic mutualism model on time scales ☆

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Abstract

By using Mawhin's continuation theorem of coincidence degree theory, sufficient criteria are obtained for the existence of periodic solutions of the mutualism model

$$\begin{cases} x^{\Delta}(t)=r_1(t)\left[\dfrac{K_1(t)+\alpha_1(t)\exp\{y(t-\tau_2(t,y(t)))\}}{1+\exp\{y(t-\tau_2(t,y(t)))\}}-\exp\{x\big(t-\sigma_1\big(t,x(t)\big)\big)\}\right],\\ y^{\Delta}(t)=r_2(t)\left[\dfrac{K_2(t)+\alpha_2(t)\exp\{x(t-\tau_1(t,x(t)))\}}{1+\exp\{x(t-\tau_1(t,x(t)))\}}-\exp\{y\big(t-\sigma_2\big(t,y(t)\big)\big)\}\right], \end{cases}$$

where $r_i, K_i, \alpha_i \in C(\mathbb{T}, \mathbb{R}^+)$, $\alpha_i > K_i$, $i=1,2$, $\tau_i, \sigma_i \in C(\mathbb{T}\times\mathbb{R}, \mathbb{T}^+)$, $i=1,2$, r_i, K_i, α_i, τ_i, σ_i $(i=1,2)$ are functions of period $\omega>0$.

Keywords: Time scales; Periodic solution; Coincidence degree; Mutualism model

1. Introduction

Consider the mutualism model

$$\begin{cases} \dfrac{dN_1(t)}{dt}=r_1N_1(t)\left[\dfrac{K_1+\alpha_1N_2(t)}{1+N_2(t)}-N_1(t)\right],\\ \dfrac{dN_2(t)}{dt}=r_2N_2(t)\left[\dfrac{K_2+\alpha_2N_1(t)}{1+N_1(t)}-N_2(t)\right], \end{cases} \tag{1.1}$$

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where $r_i, K_i, \alpha_i \in \mathbb{R}^+$ are constants and $\alpha_i > K_i$, $i = 1, 2$. Depending on the nature of K_i $(i = 1, 2)$, system (1.1) can be classified as facultative, obligate or a combination of both. For more details of mutualistic interactions we refer to [5,6,15]. A modification of system (1.1) leads to the time-lagged model

$$\begin{cases} \dfrac{\mathrm{d}N_1(t)}{\mathrm{d}t} = r_1 N_1(t)\left[\dfrac{K_1 + \alpha_1 N_2(t-\tau_2)}{1 + N_2(t-\tau_2)} - N_1(t)\right], \\ \dfrac{\mathrm{d}N_2(t)}{\mathrm{d}t} = r_2 N_2(t)\left[\dfrac{K_2 + \alpha_2 N_1(t-\tau_1)}{1 + N_1(t-\tau_1)} - N_2(t)\right], \end{cases} \tag{1.2}$$

where $\tau_1, \tau_2 \in [0, \infty)$ are constants. In system (1.2) the mutualistic or cooperative effects are not realized instantaneously but take place with time delays. For further ecological applications of system (1.2), we refer to [9] and the references cited therein.

The effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Thus the assumptions of periodicity of the parameters are a way of incorporating the periodicity of the environment (such as seasonal effects of weather, food supplies, mating habits and so forth). We refer to Pianka [14] for a discussion of the relevance of periodic environments to evolutionary theory.

Recently, Li [12], by using Mawhin's continuation theorem of coincidence degree theory [8], investigated the existence of positive periodic solutions for a periodic mutualism model

$$\begin{cases} \dfrac{\mathrm{d}N_1(t)}{\mathrm{d}t} = r_1(t) N_1(t)\left[\dfrac{K_1(t) + \alpha_1(t) N_2(t-\tau_2(t))}{1 + N_2(t-\tau_2(t))} - N_1\big(t-\sigma_1(t)\big)\right], \\ \dfrac{\mathrm{d}N_2(t)}{\mathrm{d}t} = r_2(t) N_2(t)\left[\dfrac{K_2(t) + \alpha_2(t) N_1(t-\tau_1(t))}{1 + N_1(t-\tau_1(t))} - N_2\big(t-\sigma_2(t)\big)\right], \end{cases} \tag{1.3}$$

where $r_i, K_i, \alpha_i \in C(\mathbb{R}, \mathbb{R}^+)$, $\alpha_i > K_i$, $i = 1, 2$, $\tau_i, \sigma_i \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R}^+)$, $i = 1, 2$, r_i, K_i, α_i, τ_i, σ_i $(i = 1, 2)$ are functions of period $\omega > 0$.

Since many authors [1,2,7,13] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations, also, since discrete time models can also provide efficient computational models of continuous models for numerical simulations, it is reasonable to study discrete time food-chain models governed by difference equations.

The theory of calculus on time scales (see [3,4] and references cited therein) was initiated by Stefan Hilger in his PhD thesis in 1988 [10] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work. It has been created in order to unify the study of differential and difference equations. Our purpose of this paper is to consider the model

$$\begin{cases} x^{\Delta}(t) = r_1(t)\left[\dfrac{K_1(t) + \alpha_1(t)\exp\{y(t-\tau_2(t, y(t)))\}}{1 + \exp\{y(t-\tau_2(t, y(t)))\}} - \exp\big\{x\big(t-\sigma_1\big(t, x(t)\big)\big)\big\}\right], \\ y^{\Delta}(t) = r_2(t)\left[\dfrac{K_2(t) + \alpha_2(t)\exp\{x(t-\tau_1(t, x(t)))\}}{1 + \exp\{x(t-\tau_1(t, x(t)))\}} - \exp\big\{y\big(t-\sigma_2\big(t, y(t)\big)\big)\big\}\right], \end{cases} \tag{1.4}$$

where $r_i, K_i, \alpha_i \in C(\mathbb{T}, \mathbb{R}^+)$, $\alpha_i > K_i$, $i = 1, 2$, $\tau_i, \sigma_i \in C(\mathbb{T} \times \mathbb{R}, \mathbb{R}^+)$, $i = 1, 2$, r_i, K_i, α_i, τ_i, σ_i $(i = 1, 2)$ are functions of period $\omega > 0$. $\mathbb{T}$ is a periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$.

Remark 1.1. Let $N_1(t) = \exp\{x(t)\}$ and $N_2(t) = \exp\{y(t)\}$. If $\mathbb{T} = \mathbb{R}$, then (1.4) reduces to the model

$$\begin{cases} \dfrac{\mathrm{d}N_1(t)}{\mathrm{d}t} = r_1(t) N_1(t)\left[\dfrac{K_1(t) + \alpha_1(t) N_2(t-\tau_2(t, N_2(t)))}{1 + N_2(t-\tau_2(t, N_2(t)))} - N_1\big(t-\sigma_1\big(t, N_1(t)\big)\big)\right], \\ \dfrac{\mathrm{d}N_2(t)}{\mathrm{d}t} = r_2(t) N_2(t)\left[\dfrac{K_2(t) + \alpha_2(t) N_1(t-\tau_1(t, N_1(t)))}{1 + N_1(t-\tau_1(t, N_1(t)))} - N_2\big(t-\sigma_2\big(t, N_2(t)\big)\big)\right]. \end{cases} \tag{1.5}$$

If $\mathbb{T}=\mathbb{Z}$, then (1.4) is reformulated as

$$\begin{cases} N_1(t+1)=N_1(t)\exp\left\{r_1(t)\left[\dfrac{K_1(t)+\alpha_1(t)N_2(t-\tau_2(t,N_2(t)))}{1+N_2(t-\tau_2(t,N_2(t)))}-N_1\big(t-\sigma_1\big(t,N_1(t)\big)\big)\right]\right\}, \\ N_2(t+1)=N_2(t)\exp\left\{r_2(t)\left[\dfrac{K_2(t)+\alpha_2(t)N_1(t-\tau_1(t,N_1(t)))}{1+N_1(t-\tau_1(t,N_1(t)))}-N_2\big(t-\sigma_2\big(t,N_2(t)\big)\big)\right]\right\}. \end{cases} \tag{1.6}$$

2. Preliminaries

In this section, we first recall some basic definitions, lemmas on time scales which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma,\rho:\mathbb{T}\to\mathbb{T}$ and the graininess $\mu:\mathbb{T}\to\mathbb{R}^+$ are defined, respectively, by

$$\sigma(t)=\inf\{s\in\mathbb{T}\colon s>t\},\qquad \rho(t)=\sup\{s\in\mathbb{T}\colon s<t\}\quad\text{and}\quad \mu(t)=\sigma(t)-t.$$

A point $t\in\mathbb{T}$ is called left-dense if $t>\inf\mathbb{T}$ and $\rho(t)=t$, left-scattered if $\rho(t)<t$, right-dense if $t<\sup\mathbb{T}$ and $\sigma(t)=t$, and right-scattered if $\sigma(t)>t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k=\mathbb{T}\setminus\{m\}$; otherwise $\mathbb{T}^k=\mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k=\mathbb{T}\setminus\{m\}$; otherwise $\mathbb{T}_k=\mathbb{T}$.

Let $\omega\in\mathbb{R}$, $\omega>0$, $\mathbb{T}$ is a ω-periodic time scale if $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$ such that $t+\omega\in\mathbb{T}$ and $\mu(t)=\mu(t+\omega)$ whenever $t\in\mathbb{T}$.

A function $f:\mathbb{T}\to\mathbb{R}$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be continuous function on $\mathbb{T}$. We define $C(J,\mathbb{R})=\{u(t)$ is continuous on $J\}$.

For $y:\mathbb{T}\to\mathbb{R}$ and $t\in\mathbb{T}^k$, we define the delta derivative of $y(t)$, $y^\Delta(t)$, to be the number (if it exists) with the property that for a given $\varepsilon>0$, there exists a neighborhood U of t such that

$$\big|\big[y\big(\sigma(t)\big)-y(s)\big]-y^\Delta(t)\big[\sigma(t)-s\big]\big|<\varepsilon\big|\sigma(t)-s\big|$$

for all $s\in U$.

If y is continuous, then y is right-dense continuous, and if y is delta differentiable at t, then y is continuous at t.

Let y be right-dense continuous. If $Y^\Delta(t)=y(t)$, then we define the delta integral by

$$\int_a^t y(s)\,\Delta s=Y(t)-Y(a).$$

Definition 2.1. (See [11].) We say that a time scale $\mathbb{T}$ is periodic if there exists $p>0$ such that if $t\in\mathbb{T}$, then $t\pm p\in\mathbb{T}$. For $\mathbb{T}\neq\mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 2.2. (See [11].) Let $\mathbb{T}\neq\mathbb{R}$ be a periodic time scale with period p. We say that the function $f:\mathbb{T}\to\mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega=np$, $f(t+\omega)=f(t)$ for all $t\in\mathbb{T}$ and ω is the smallest number such that $f(t+\omega)=f(t)$.

If $\mathbb{T}=\mathbb{R}$, we say that f is periodic with period $\omega>0$ if ω is the smallest positive number such that $f(t+\omega)=f(t)$ for all $t\in\mathbb{T}$.

Lemma 2.1. *If $a,b\in\mathbb{T}$, $\alpha,\beta\in\mathbb{R}$ and $f,g\in C(\mathbb{T},\mathbb{R})$, then*

(i) $\int_a^b[\alpha f(t)+\beta g(t)]\,\Delta t=\alpha\int_a^b f(t)\,\Delta t+\beta\int_a^b g(t)\,\Delta t$;
(ii) *if $f(t)\geqslant 0$ for all $a\leqslant t<b$, then $\int_a^b f(t)\,\Delta t\geqslant 0$;*
(iii) *if $|f(t)|\leqslant g(t)$ on $[a,b):=\{t\in\mathbb{T}\colon a\leqslant t<b\}$, then $|\int_a^b f(t)\,\Delta t|\leqslant\int_a^b g(t)\,\Delta t$.*

For convenience, we introduce the notation

$$\kappa=\min\{[0,\infty)\cap\mathbb{T}\},\qquad I_\omega=[\kappa,\kappa+\omega]\cap\mathbb{T},\qquad g^M=\max_{t\in\mathbb{T}}g(t),\qquad g^m=\min_{t\in\mathbb{T}}g(t),$$

$$\bar{g}=\frac{1}{\omega}\int_{I_\omega}g(s)\,\Delta s=\frac{1}{\omega}\int_{\kappa}^{\kappa+\omega}g(s)\,\Delta s,$$

where $g\in C(\mathbb{T},\mathbb{R})$ is an ω-periodic real function, i.e., $g(t+\omega)=g(t)$ for all $t\in\mathbb{T}$.

Next, let us recall the continuation theorem in coincidence degree theory. To do so, we need to introduce the following notation.

Let X, Y be real Banach spaces, $L:\operatorname{Dom}L\subset X\to Y$ a Fredholm mapping of index zero, and $P:X\to X$, $Q:Y\to Y$ continuous projectors such that $\operatorname{Im}P=\operatorname{Ker}L$, $\operatorname{Ker}Q=\operatorname{Im}L$, and $X=\operatorname{Ker}L\oplus\operatorname{Ker}P$, $Y=\operatorname{Im}L\oplus\operatorname{Im}Q$. Denote by L_P the restriction of L to $\operatorname{Dom}L\cap\operatorname{Ker}P$, $K_P:\operatorname{Im}L\to\operatorname{Ker}P\cap\operatorname{Dom}L$ the inverse (to L_P), and $J:\operatorname{Im}Q\to\operatorname{Ker}L$ an isomorphism of $\operatorname{Im}Q$ onto $\operatorname{Ker}L$.

Lemma 2.2. *Let $\Omega\subset X$ be an open bounded set and $N:X\to Y$ be a continuous operator which is L-compact on $\bar{\Omega}$. Assume*

(i) *for each $\lambda\in(0,1)$, $x\in\partial\Omega\cap\operatorname{Dom}L$, $Lx\neq\lambda Nx$;*
(ii) *for each $x\in\partial\cap\operatorname{Ker}L$, $QNx\neq0$, and* $\deg\{JQN,\Omega\cap\operatorname{Ker}L,0\}\neq0$.

Then $Lx=Nx$ has at least one solution in $\bar{\Omega}\cap\operatorname{Dom}L$.

Lemma 2.3. *(See [12].) Let*

$$f(x,y)=\left(a_1-\frac{a_1-b_1}{1+e^y}-c_1e^x,a_2-\frac{a_2-b_2}{1+e^x}-c_2e^y\right)$$

and $\Omega=\{(x,y)^T\in\mathbb{R}^2\colon\ |x|+|y|<M\}$, where $M,a_i,b_i,c_i\in\mathbb{R}^+$ are constants, $a_i>b_i$, $i=1,2$, and $M>\max\{|\ln(a_i/c_i)|,|\ln(b_i/c_i)|,\ i=1,2\}$. *Then*

$$\deg\{f,\Omega,(0,0)\}\neq0.$$

In order to achieve the a priori estimation in the case of dynamic equations on a periodic time scale $\mathbb{T}$, we first give the following inequalities, which will be very essential in this paper.

Lemma 2.4. *Let $t_1\in I_\omega$ and $t\in\mathbb{T}$. If $g:\mathbb{T}\to\mathbb{R}$ is ω-periodic, then*

$$\max_{t\in I_\omega}|g(t)|\leqslant|g(t_1)|+\int_{\kappa}^{\kappa+\omega}|g^\Delta(t)|\,\Delta t.$$

Proof. For $\forall t\in I_\omega$, by the definition of delta integral, we know that

$$\begin{aligned}|g(t)|&=\left|g(t_1)+\int_{t_1}^{t}g^\Delta(s)\,\Delta s\right|\leqslant|g(t_1)|+\left|\int_{t_1}^{t}g^\Delta(s)\,\Delta s\right|\leqslant|g(t_1)|+\int_{t_1}^{t}|g^\Delta(s)|\,\Delta s\\&\leqslant|g(t_1)|+\int_{\kappa}^{\kappa+\omega}|g^\Delta(s)|\,\Delta s,\end{aligned}$$

and hence

$$\max_{t\in I_\omega}|g(t)|\leqslant|g(t_1)|+\int_{\kappa}^{\kappa+\omega}|g^\Delta(s)|\,\Delta s.$$

The proof is complete. □

3. Main results

In order to explore the existence of periodic solutions of (1.4), first we should embed our problem in the frame of coincidence degree theory. Define

$$\Psi^\omega = \{(u,v) \in C(\mathbb{T}, \mathbb{R}^2)\colon\ u(t+\omega) = u(t),\ v(t+\omega) = v(t) \text{ for all } t \in \mathbb{T}\},$$
$$\|(u,v)\| = \max_{t\in I_\omega} |u(t)| + \max_{t\in I_\omega} |v(t)| \quad \text{for } (u,v) \in \Psi^\omega.$$

It is not difficult to show that Ψ^ω is a Banach space when it is endowed with the above norm $\|\cdot\|$. Let

$$\Psi_0^\omega = \{(u,v) \in \Psi^\omega\colon\ \bar{u} = 0,\ \bar{v} = 0\},$$
$$\Psi_c^\omega = \{(u,v) \in \Psi^\omega\colon\ (u(t), v(t)) \equiv (h_1, h_2) \in \mathbb{R}^2 \text{ for } t \in \mathbb{T}\}.$$

Then it is easy to show that Ψ_0^ω and Ψ_c^ω are both closed linear subspaces of Ψ^ω, $\Psi^\omega = \Psi_0^\omega \oplus \Psi_c^\omega$, and $\dim \Psi_c^\omega = 2$.

We now come to the fundamental theorem of this paper.

Theorem 3.1. *The system* (1.4) *has at least one ω-periodic solution.*

Proof. Take $X = Y = \Psi^\omega$ and define

$$N\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} r_1(t)\left[\dfrac{K_1(t) + \alpha_1(t)\exp\{y(t - \tau_2(t, y(t)))\}}{1 + \exp\{y(t - \tau_2(t, y(t)))\}} - \exp\{x(t - \sigma_1(t, x(t)))\}\right] \\ r_2(t)\left[\dfrac{K_2(t) + \alpha_2(t)\exp\{x(t - \tau_1(t, x(t)))\}}{1 + \exp\{x(t - \tau_1(t, x(t)))\}} - \exp\{y(t - \sigma_2(t, y(t)))\}\right] \end{bmatrix},$$

$$L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^\Delta \\ y^\Delta \end{bmatrix}, \quad P\begin{bmatrix} x \\ y \end{bmatrix} = Q\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}, \quad \begin{bmatrix} x \\ y \end{bmatrix} \in X.$$

Then $\operatorname{Ker} L = \Psi_c^\omega$, $\operatorname{Im} L = \Psi_0^\omega$, and $\dim \operatorname{Ker} L = 2 = \operatorname{codim} \operatorname{Im} L$. Since Ψ_0^ω is closed in Ψ^ω, it follows that L is a Fredholm mapping of index zero. It is not difficult to show that P and Q are continuous projections such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$. Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists and is given by

$$K_P\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} X - \bar{X} \\ Y - \bar{Y} \end{bmatrix} \quad \text{where } X(t) = \int_\kappa^t x(s)\,\Delta s \text{ and } Y(t) = \int_\kappa^t y(s)\,\Delta s.$$

Thus

$$QN\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\omega}\displaystyle\int_\kappa^{\kappa+\omega} r_1(t)\left[\dfrac{K_1(t) + \alpha_1(t)\exp\{y(t - \tau_2(t, y(t)))\}}{1 + \exp\{y(t - \tau_2(t, y(t)))\}} - \exp\{x(t - \sigma_1(t, x(t)))\}\right]\Delta t \\ \dfrac{1}{\omega}\displaystyle\int_\kappa^{\kappa+\omega} r_2(t)\left[\dfrac{K_2(t) + \alpha_2(t)\exp\{x(t - \tau_1(t, x(t)))\}}{1 + \exp\{x(t - \tau_1(t, x(t)))\}} - \exp\{y(t - \sigma_2(t, y(t)))\}\right]\Delta t \end{bmatrix}$$

and

$$K_P(I - Q)N\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \displaystyle\int_\kappa^t N_1(s)\,\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t N_1(s)\,\Delta s\,\Delta t - \left(t - \kappa - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}(t - \kappa)\,\Delta t\right)\bar{N}_1 \\ \displaystyle\int_\kappa^t N_2(s)\,\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t N_2(s)\,\Delta s\,\Delta t - \left(t - \kappa - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}(t - \kappa)\,\Delta t\right)\bar{N}_2 \end{bmatrix}.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Since X is a Banach space, it is easy to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$. Corresponding to the equation $Lx = \lambda Nx$, $Ly = \lambda Ny$, $\lambda \in (0,1)$, we have

$$\begin{cases} x^\Delta(t) = \lambda r_1(t)\left[\dfrac{K_1(t)+\alpha_1(t)\exp\{y(t-\tau_2(t,y(t)))\}}{1+\exp\{y(t-\tau_2(t,y(t)))\}} - \exp\{x(t-\sigma_1(t,x(t)))\}\right], \\ y^\Delta(t) = \lambda r_2(t)\left[\dfrac{K_2(t)+\alpha_2(t)\exp\{x(t-\tau_1(t,x(t)))\}}{1+\exp\{x(t-\tau_1(t,x(t)))\}} - \exp\{y(t-\sigma_2(t,y(t)))\}\right]. \end{cases} \tag{3.1}$$

Assume that $(x(t), y(t))^T \in X$ is a solution of system (3.1) for a certain $\lambda \in (0,1)$. By integrating (3.1) over $[\kappa, \kappa+\omega]$, we obtain

$$\int_\kappa^{\kappa+\omega} r_1(t)\left[\frac{K_1(t)+\alpha_1(t)\exp\{y(t-\tau_2(t,y(t)))\}}{1+\exp\{y(t-\tau_2(t,y(t)))\}} - \exp\{x(t-\sigma_1(t,x(t)))\}\right]\Delta t = 0 \tag{3.2}$$

and

$$\int_\kappa^{\kappa+\omega} r_2(t)\left[\frac{K_2(t)+\alpha_2(t)\exp\{x(t-\tau_1(t,x(t)))\}}{1+\exp\{x(t-\tau_1(t,x(t)))\}} - \exp\{y(t-\sigma_2(t,y(t)))\}\right]\Delta t = 0. \tag{3.3}$$

It is easy to see that we can rewrite (3.2) and (3.3) respectively as

$$\int_\kappa^{\kappa+\omega} \frac{r_1(t)(\alpha_1(t)-K_1(t))}{1+\exp\{y(t-\tau_2(t,y(t)))\}}\,\Delta t + \int_\kappa^{\kappa+\omega} r_1(t)\exp\{x(t-\sigma_1(t,x(t)))\}\,\Delta t = \int_\kappa^{\kappa+\omega} r_1(t)\alpha_1(t)\,\Delta t \tag{3.4}$$

and

$$\int_\kappa^{\kappa+\omega} \frac{r_2(t)(\alpha_2(t)-K_2(t))}{1+\exp\{x(t-\tau_1(t,x(t)))\}}\,\Delta t + \int_\kappa^{\kappa+\omega} r_2(t)\exp\{y(t-\sigma_2(t,y(t)))\}\,\Delta t = \int_\kappa^{\kappa+\omega} r_2(t)\alpha_2(t)\,\Delta t. \tag{3.5}$$

Thus from (3.1) and (3.4), it follows that

$$\begin{aligned} \int_\kappa^{\kappa+\omega} \left|x^\Delta(t)\right|\Delta t &< \lambda \int_\kappa^{\kappa+\omega} r_1(t)\left[\frac{K_1(t)+\alpha_1(t)\exp\{y(t-\tau_2(t,y(t)))\}}{1+\exp\{y(t-\tau_2(t,y(t)))\}} + \exp\{x(t-\sigma_1(t,x(t)))\}\right]\Delta t \\ &< \int_\kappa^{\kappa+\omega} \frac{r_1(t)(\alpha_1(t)-K_1(t))}{1+\exp\{y(t-\tau_2(t,y(t)))\}}\,\Delta t \\ &\quad + \int_\kappa^{\kappa+\omega} r_1(t)\exp\{x(t-\sigma_1(t,x(t)))\}\,\Delta t + \int_\kappa^{\kappa+\omega} r_1(t)\alpha_1(t)\,\Delta t \\ &= 2\int_\kappa^{\kappa+\omega} r_1(t)\alpha_1(t)\,\Delta t \triangleq M_1, \end{aligned}$$

that is,

$$\int_\kappa^{\kappa+\omega} \left|x^\Delta(t)\right|\Delta t < M_1. \tag{3.6}$$

Similarly, by (3.1) and (3.5), we have

$$\int_\kappa^{\kappa+\omega} \left|y^\Delta(t)\right|\Delta t < 2\int_\kappa^{\kappa+\omega} r_2(t)\alpha_2(t)\,\Delta t \triangleq M_2. \tag{3.7}$$

Moreover, from (3.4) it follows that

$$\int_{\kappa}^{\kappa+\omega} r_1(t)\alpha_1(t)\,\Delta t \geqslant \int_{\kappa}^{\kappa+\omega} r_1(t)\exp\bigl\{x\bigl(t-\sigma_1\bigl(t,x(t)\bigr)\bigr)\bigr\}\,\Delta t \geqslant \int_{\kappa}^{\kappa+\omega} r_1(t)K_1(t)\,\Delta t,$$

which implies that there exist a point $t_1' \in I_\omega$ and a constant $C_1 > 0$ such that

$$\bigl|x\bigl(t_1' - \sigma_1\bigl(t_1', x(t_1')\bigr)\bigr)\bigr| < C_1.$$

Suppose that $t_1' - \sigma_1(t_1', x(t_1')) = t_1 + n\omega$, $t_1 \in I_\omega$ and n is an integer, then

$$\bigl|x(t_1)\bigr| < C_1. \tag{3.8}$$

Similarly, by (3.5) we can obtain that there exist a point $t_2 \in I_\omega$ and a constant $C_2 > 0$ such that

$$\bigl|y(t_2)\bigr| < C_2. \tag{3.9}$$

Therefore it follows from (3.6)–(3.9) and Lemma 2.4 that

$$\max_{t\in I_\omega}\bigl|x(t)\bigr| \leqslant \bigl|x(t_1)\bigr| + \int_{\kappa}^{\kappa+\omega} \bigl|x^{\Delta}(t)\bigr|\,\Delta t < C_1 + M_1,$$

$$\max_{t\in I_\omega}\bigl|y(t)\bigr| \leqslant \bigl|y(t_2)\bigr| + \int_{\kappa}^{\kappa+\omega} \bigl|y^{\Delta}(t)\bigr|\,\Delta t < C_2 + M_2.$$

Clearly M_i and C_i $(i = 1, 2)$ are independent of λ. Denote $M = M_1 + M_2 + C_1 + C_2 + D$, where $D > 0$ is taken sufficiently large such that $M > \max\{|\ln(\overline{r_i\alpha_i}/\bar{r}_i)|, |\ln(\overline{r_iK_i}/\bar{r}_i)|,\ i = 1, 2\}$. Now we take $\Omega = \{(x(t), y(t))^T \in X\colon \|(x, y)^T\| < M\}$. This satisfies condition (i) in Lemma 2.2.

When $(x, y)^T \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap \mathbb{R}^2$, $(x, y)^T$ is a constant vector in $\mathbb{R}^2$ with $|x| + |y| = M$. Then

$$QN\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \overline{r_1\alpha_1} - \dfrac{\overline{r_1\alpha_1} - \overline{r_1K_1}}{1+\exp\{y\}} - \bar{r}_1\exp\{x\} \\[2ex] \overline{r_2\alpha_2} - \dfrac{\overline{r_2\alpha_2} - \overline{r_2K_2}}{1+\exp\{x\}} - \bar{r}_1\exp\{y\} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Furthermore, take $J = I : \operatorname{Im} Q \to \operatorname{Ker} L$, $(x, y)^T \mapsto (x, y)^T$. By Lemma 2.3, we have

$$\deg\bigl\{JQN(x, y)^T, \Omega, (0, 0)\bigr\} = \deg\bigl\{QN(x, y)^T, \Omega, (0, 0)\bigr\} \neq 0.$$

We now know that Ω verifies all the requirements in Lemma 2.2 and thus that (1.4) has at least one ω-periodic solution. The proof is complete. □

Remark 3.1. By Remark 1.1, we know that (1.5) and (1.6) have at least one positive periodic solution if (1.4) has at least one periodic solution.

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