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Spectral properties of quadruple symmetric real functions

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ABSTRACT

This paper examined the spectral properties of quadruple symmetric real functions. Theorem is formulated, under which the complex spectra of such symmetric functions are conjugated. The properties of the trigonometric functions are used in the proof of the theorem. Numerical results are presented for illustrating of the proven theorem.

1. Introduction

It is well known, that the Fourier transformation is symmetric conjugation [2]. This property occurs in a Fourier transform of a real symmetric time function. In many areas of knowledge, there are processes, described by the quadruple symmetric real functions. These functions are complex conjugated by Fourier spectra. Spectral properties of the cite quadruple symmetric real functions are estimated in the paper.

2. Presentation

The following theorem for the quadruple symmetric real functions is formulated and proved in this section.

2.1. Theorem

Fourier spectra of $L_2\,[-\infty;+\infty]$ symmetric functions from first and second quadrants are complex conjugated. Fourier spectra of symmetric functions of the third and fourth quadrants are also complex conjugated. The four functions in four quadrants are frequency indistinguishable.

* The four function graphs are presented in Fig. 1. It is assumed that the function in second quadrant can be obtained by mirror flipping along the ordinate axis $A(t)$ of the function in first quadrant.

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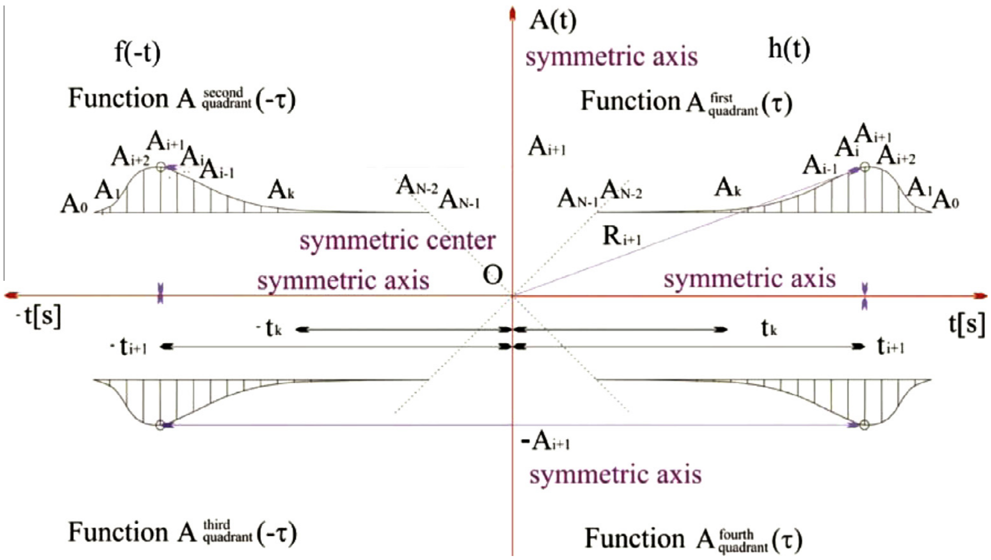


Fig. 1. Quadruple symmetric real functions.

- ∗ The functions representing in the third and fourth quadrants can be obtained by mirror flipping along the abscise axis t. All functions are normalized to a dimensionless unit 1.0000. The ordinates of the different graphs are identical, but the models ordinate orientations are variable. Between the various functions has mirror symmetry.
- ∗ Than the complex Fourier $G(j\omega)$ spectra of the symmetric functions in first and second quadrant are conjugated as well as.
- ∗ Moreover the complex Fourier $G(j\omega)$ spectra of the symmetric functions in third and fourth quadrants are conjugated respectively.
- ∗ The amplitudes of the functions in first and second quadrants are both positive, while these of the amplitudes for the functions for third and fourth quadrants are both negative. The functions under investigation could be of arbitrary amplitudes – negative and positive.

2.2. Proof

Let considered the Fourier complex spectra of two real symmetrical functions in the first and second quadrants:

$$h(t) = f(-t); \quad h,f \in L_2[-\infty, +\infty]; \tag{1}$$

The complex Fourier spectrum of the two functions can be written as well as:

$$h(t)e^{-i\omega t} = f(-t)e^{-i\omega t}; \tag{2}$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} h(t)e^{-i\omega t}dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(-t)e^{-i\omega t}dt; \tag{3}$$

$$H(\omega)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} h(t)e^{-i\omega t}dt; \tag{4}$$

$$F(-\omega)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(-t)e^{-i\omega t}dt; \tag{5}$$

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t); \tag{6}$$

$$H(\omega)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(-t)[\cos(\omega t) - i\sin(\omega t)]dt; \tag{7}$$

$$F(\omega)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)[\cos(\omega t) - i\sin(\omega t)]dt; \tag{8}$$

$$F(-\omega)\frac{1}{-1}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(-1)(-t)[\cos((-1)\omega t) - i\sin((-1)\omega t)]d(-1)t = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)[\cos(\omega t) + i\sin(\omega t)]dt$$
$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)\overline{\cos(\omega t) - i\sin(\omega t)}dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)\overline{e^{-i\omega t}}dt = \overline{F(\omega)}; \tag{9}$$

$$H(\omega) = F(-\omega) = \overline{F(\omega)}. \tag{10}$$

From statement (10) follows, that the spectrum $H(\omega)$ and the spectrum $F(-\omega)$ are conjugated. This means, that the first and the second quadrant functions have identical PSD. From everything said here follows, that the two functions are frequency indistinguishable.

Now let considered the other two functions: $A^{fourth}_{quadrant}(\tau)$ and $A^{third}_{quadrant}(-\tau)$. Those functions are shown in Fig. 1. The Complex Fourier spectrum $G(j\omega)$ could be written as well as:

$$G^{third}_{quadrant}(j\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(\tau)\cos(\omega\tau)d\tau - j\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(\tau)\sin(\omega\tau)d\tau = -G^{second}_{quadrant}(j\omega); \tag{11}$$

$$G^{fourth}_{quadrant}(j\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(\tau)\cos(\omega\tau)d\tau - j\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(\tau)\sin(\omega\tau)d\tau = -G^{first}_{quadrant}(j\omega). \tag{12}$$

From statements (11) and (12) follows, that the functions spectrum are equal to negative values of corresponding spectrums. This means that the functions have identical PSD. From everything said here follows: four functions are frequency indistinguishable. Those four functions, mentioned above, have identical PSD. The theorem is proved.

3. Applications of the theorem for spectral analysis of quadruple symmetric real functions

3.1. First application – development of the human emotion intensity

This application is connected with the process of development of the human emotion intensity in a short time period. The variability of the human affection reaction could be visualized by Fig. 1. The time investigated interval is about 5 min. The different persons are manifesting different state of affect. In the literature are presented different models for describing the emotional intensity in a state of affect for different types of persons [1]. They appreciate the variability of the emotional intensity as a function of time. It turns out that it is possible to obtain spectral dependencies between different models of the human affect. Two ''Consumer'' models are characterized by a sudden increase of the emotion intensity. After that the emotions subside gradually. The ''Consumer'' persons are going ''inside'' to the conflict situation. They are consumers of the conflict situation. In the opposite, for two ''Source'' models emotions gradually reach the maximum. Then they fell sharply. The ''Source'' persons are moving ''outside'' from the conflict situation. They are generators of the conflict situation. The emotional intensity function graphs are presented in Fig. 1. The four various types of models are as follows: (1) Source Stenetic Emotional Intensity Model; (2) Consumer Stenetic Emotional Intensity Model; (3) Consumer Astenetic Emotional Intensity Model; (4) Source Astenetic Emotional Intensity Model. The emotional intensity function graphs are presented in Fig. 1. It is assumed, that the function represented the emotional intensity development of the Source Stenetic Emotional Intensity Model can be obtained by mirror flipping along the ordinate axis $A(t)$ of the Consumer Stenetic Emotional Intensity Model. The functions representing the emotional intensity development of the Consumer Astenetic Model and the Source Astenetic Model can be obtained by mirror flipping along the abscise axis "t". All functions are normalized according to Section 2.1. The ordinates of the different graphs are identical, but the models ordinate orientations are different. Between the various functions has mirror symmetry. Proven theorem in Section 2 for quadruple symmetric real functions, allows obtaining some spectral dependencies between different models of the human affect. This theorem could be used for defining of the human emotional type. The definitions could be used for assessment of the interpersonal communications.

3.2. Second application – dynamical behavior of kinetic accumulators

This application is connected with the dynamical behavior of the kinetic accumulators according [5]. The theorem from Section 2 allows obtaining spectral dependencies between dynamical characteristics of one and multi disk battery of kinetic accumulators.

3.3. Third application – efficiency discrete Fourier transform (DFT)

The basis trigonometric functions are periodic, symmetric and orthogonal. This properties and special properties of relationship with convolution, supplies the discrete Fourier transform (DFT) with enormous capacity for improvement of arithmetic efficiency [6]. There are main approaches in formulation of efficient discrete Fourier transformation. First approach breaks original one dimensional problem into multi-dimensional problem. The huge initial convolution is breaking down into multiple short convolutions. They could to be executed faster, than a direct implementation of the initial huge convolution. The efficiency of the approach is achieved by "Multi-dimensional Index Mapping", "Polynomial Reduction" and the "Chinese Remainder Theorem". Other approach for FFT connected with fast convolution or fast filtering. Rather than dealing with the transformation directly, the method converts the problem into a cyclic convolution. The third approach is factoring the Signal Processing Operators. Discrete Fourier transformation could be posed recursively as evaluating a DFT in terms of two half-length DFT, which are each in turn evaluated by quarter-length DFT and so one. Winograd's theorems are generators for creation of Winograd's Short DFT Algorithms. Cooly-Tukey FFT and Split-Radix FFT are other approaches for base of efficiency computation algorithms. Chirp algorithms, Goertzel's algorithm, QFT, NTT, SR-FFT, Approx FFT, Autogen and other programs and algorithms could to mention for the discussed area. The theorem proofed in the article could be used for develop new efficient algorithms for DFT in case of quadruple symmetric real functions. Those functions could be used for application in the recently developed concepts of quaternion Fourier transform (QFT), quaternion convolution (QCV), and quaternion correlation. The above mentioned concepts are based on quaternion algebra.

4. Numerical examples

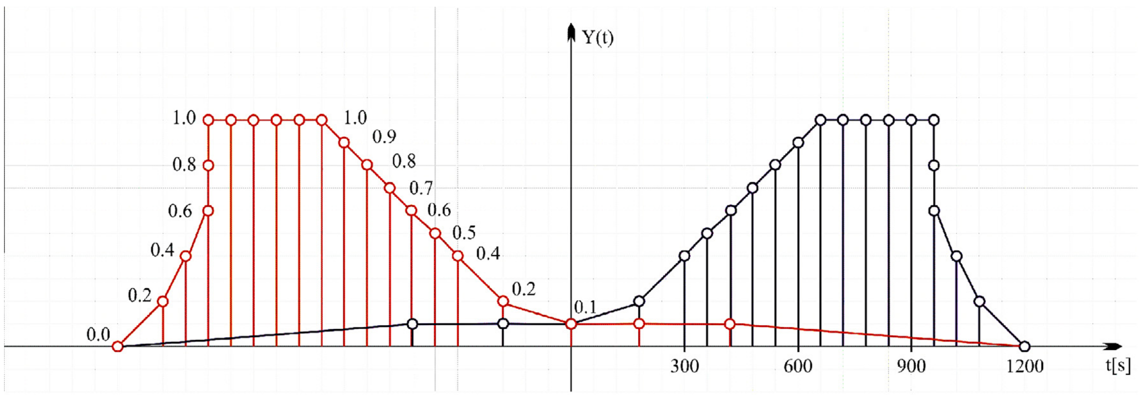
4.1. Development of the human emotion intensity in a short time period

It is well known that there is no body of people with the same characteristics of emotional development. Nevertheless, here it is assumed that for all the models the functions of emotional intensity varies between zero and one. It is assumed that the functions of emotional intensity development differ only in their orientation and in their positivity or negativity of the affect reaction. After these simplifications, it is possible to obtain the relationships between the different models. This example illustrates the connection between Source Stenetic Emotional Intensity Model and Consumer Stenetic Emotional Intensity Model. Corresponding functions are shown in Fig. 2.

The 128 samples of second quadrant function and 128 samples of first quadrant function are presented in Fig. 2. The time domain discrete step $\Delta\omega = 2.0$ (s^{-1}). The second quadrant function is presented in Fig. 2 by red color graphs and the first quadrant function is represented in the same figure by blue color graphs. The total durations of the signals of the presented numerical examples are 256 s. Fast Fourier Transformation (FFT) technique is presented for calculating of the direct transformation data Matlab [4]. This software system is used for obtaining of the Fourier coefficients. The real part of the second quadrant function is shown in Fig. 3. In the same figure the imaginary parts of the second quadrant and the first quadrant functions are presented. The power spectral density (PSD) of the functions is presented in Fig. 4. Frequency domain discrete step $\Delta\omega$ could be calculated by the formula:

$$\Delta\omega = \frac{2\pi}{N\Delta t}, \tag{13}$$

where the dimension of $\Delta\omega$ is (s^{-1}), N is dimensionless number of samples and the dimension of time domain discrete step Δt (s). In case of $N = 128$, $\Delta t = 2.0$ (s), $N\ \Delta t = 256$ (s), for the frequency domain discrete step is obtained the value: $\Delta\omega = 0.0245$ (s^{-1}). From this numerical results is seen, that the symmetric according coordinate axis $A(t)$ functions from Fig. 2 have conjugated complex spectra (Fig. 3) and identical power spectral density (Fig. 4).



Red Color Function - 128 Samples of the Signal – Function Second Quadrant
Blue Color Function - 128 Samples of the Signal – Function First Quadrant

Fig. 2. Numerical comparison between functions in first and second quadrants.

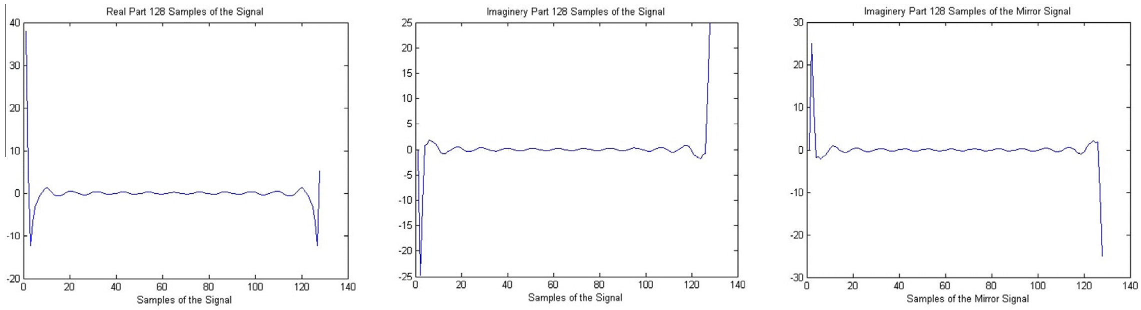


Fig. 3. 128 Samples real part and imaginary parts of the first and second quadrant functions.

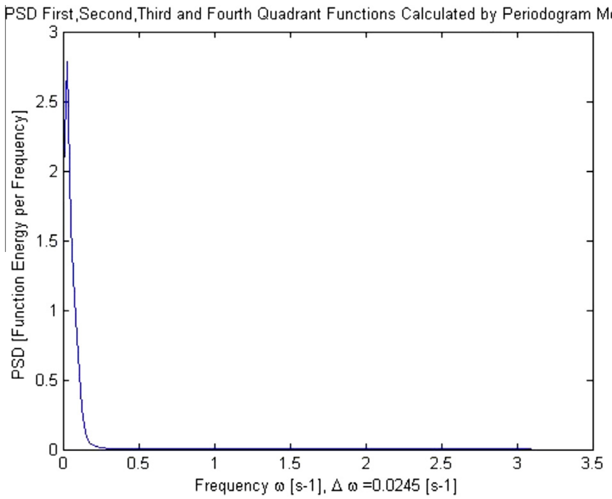


Fig. 4. PSD of the quadruple symmetric real functions.

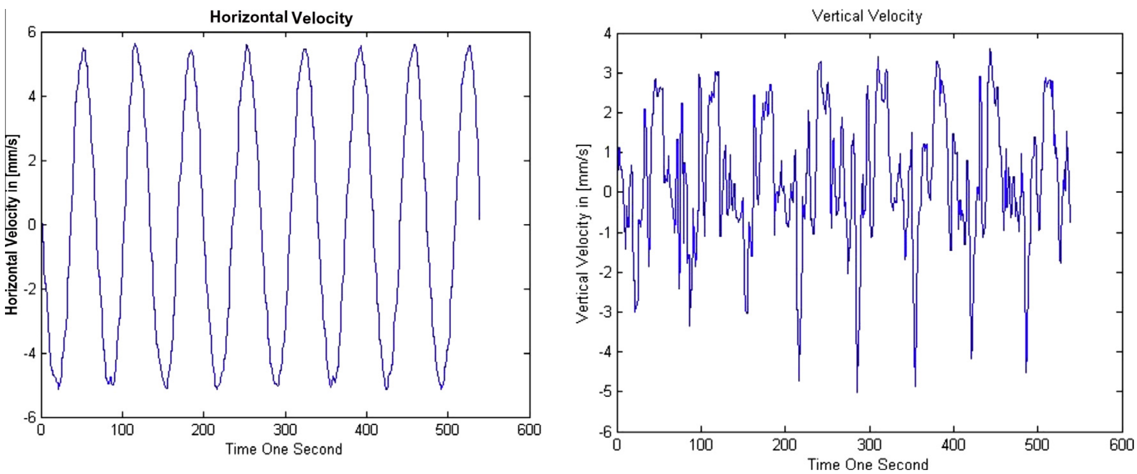


Fig. 5. Horizontal and vertical vibration velocities of the hull base of one disc kinetic accumulator.

4.2. Spectral modeling of quadruple symmetric real functions in kinetic accumulator area

The modern kinetic accumulators are designed as multi disk batteries because of gyroscopic effect [5]. On the Fig. 5 are shown the experimental horizontal and vertical vibration velocities of the one disk kinetic accumulator structure. The signals

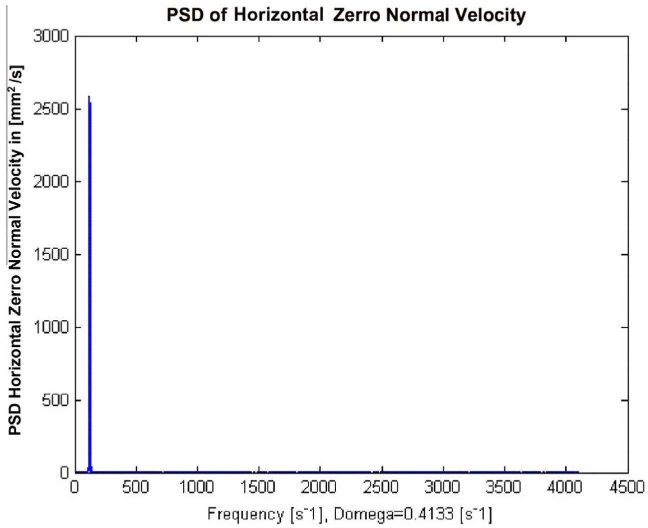


Fig. 6. PSD of the horizontal vibration velocity – one disk hull base kinetic accumulator case.

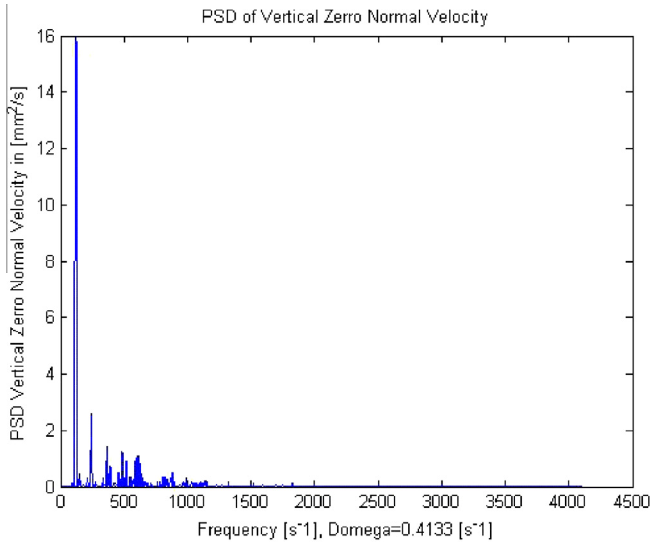


Fig. 7. PSD of the vertical vibration velocity – one disk hull base kinetic accumulator case.

are measured on the hull base of the one disk kinetic accumulator. On Fig. 6 is shown PSD of the horizontal vibration velocity and in Fig. 7 is shown PSD of the vertical vibration velocity of the described one disk structure. The theorem, proven in Section 2 of the article, allows obtaining the dynamic parameters of corresponding multi disk kinetic accumulator battery. The PSD of the all disks of multi disk structure will be identical (like Figs. 6 and 7). The time history and the complex Fourier spectra of the dynamical parameters (angle of rotation, velocities and accelerations of rotation) for the multi disk accumulator battery will follow the properties of quadruple symmetric functions from Fig. 1.

5. Discussion and conclusion

The evaluation of spectral properties of the quadruple symmetric functions could be performed in the frequency domain. PSD could be used in such evaluation. The theorem proven in Section 2 of the article states, that the patterns of the function under investigation are frequency indistinguishable. This requires, that the study of quadruple symmetric functions request using the complex Fourier spectra. They are complex conjugated in the case of axial symmetry of the corresponding functions. According proven in the paper theorem all symmetrical functions with respect to an axis possess the same PSD. Therefore, the assessment of the symmetric functions should be based on a complex Fourier spectrum. From [3] is seen, that the proved theorem is original and it is proved for the first time (see Figs. 1–4 of the present article).

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