Alexandria Engineering Journal (2016) 55, 2345–2350

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Fractional calculus approach to study temperature distribution within a spinning satellite

Jyotindra C. Prajapati [a,*], Krunal B. Kachhia [b], Shiv Prasad Kosta [c,d]

[a] *Department of Mathematics, Marwadi University, Rajkot-Morbi Highway, Rajkot 360003, Gujarat, India*
[b] *Department of Mathematical Sciences, Faculty of Applied Sciences, Charotar University of Science and Technology (CHARUSAT), Changa, Anand 388421, Gujarat, India*
[c] *Former affiliation: ISRO (Bangalore), Govt. of India, India*
[d] *CHARUSAT Center for Space Technology, Charotar University of Science and Technology, Changa, Anand 388421, Gujarat, India*

Received 10 March 2016; revised 29 April 2016; accepted 4 May 2016
Available online 24 May 2016

KEYWORDS

Temperature distribution;
Fractional calculus;
Fractional differential equation;
Wright generalized hypergeometric function;
Laplace transform

Abstract This paper deals with the temperature distribution within spinning satellites and problem is formulated in terms of fractional differential equation. Applying fractional calculus approach, solution of this equation is obtained in terms of Wright generalized hypergeometric function, a generalization of exponential function.

1. Introduction

A spinning satellite is a satellite which has the motion of one axis held (relatively) fixed by spinning the satellite around that axis, using the gyroscopic effect. The attitude of a satellite or any rigid body is its orientation in space.

- The entire satellite including its antenna(s) spins.
- This means that the satellite's attitude is very stable because the satellite as a whole is acting as a gyroscope.
- Spinning can be achieved using rods with coils of wire around them.
- Current passing through the rods, creating magnetic fields around the wires.
- When this rod's magnetic field interacts with Earth's magnetic field, the rod begins to spin.
- An advantage of spin stabilized is it requires very little power.
- A disadvantage is that the solar panels cannot all collect power all the time because they will spin, so they don't face the sun once every rotation.
- Also, the instruments can only take measurements in one direction once every rotation.

Liu et al. [11] studied temperature distribution using improved thermal network model. The conventional thermal network model for satellite surface temperature distribution is directly solved by a new solution method on the basis of

* Corresponding author.
E-mail addresses: jyotindra18@rediffmail.com (J.C. Prajapati), krunalmaths@hotmail.com (K.B. Kachhia), spkosta.adm@ecchanga.ac.in (S.P. Kosta).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.05.004
1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

the Monte Carlo Ray Tracing (MCRT) method. The solar direct incidence area, the solar radiation transfer coefficient, the infrared radiation transfer coefficient, and the network conduction and radiation coefficients are calculated by Monte Carlo statistical analysis rather than the Gebhart approach. The advantage of the MCRT method is that the surface material characteristics are taken into account in solving the process. The effect of absorptivity and emissivity for temperature distributions is analyzed in detail. Numerical simulation is carried out for the temperature distribution of the satellite surface with different solar incidences of round angle and zenith angle. In the unsteady stage, the cooling velocity of the main body surface is always smaller than that of the solar array surface and the maximum temperature of the main body is larger than the solar arrays. Under the same absorptivity–emissivity ratio, the solar incidence angle has a great effect on the uniform character of temperature distribution.

The temperature distribution in satellite is also studied by Venkataraman and Egalon [27], and Liu et al. [14].

The fractional Calculus is mathematics which deals with generalization of ordinary integrals and derivatives of integer orders to arbitrary ones. The role of this kind of calculus is to solve problems of complex systems that appear in various fields of sciences [25,9,17]. Recently, Prajapati and Kachhia [22] studied fractional model of temperature distribution and heat flux in the semi infinite solid. Kachhia and Prajapati [8] obtained solution of fractional partial differential equation aries in study of heat transfer through diathermanous materials. In the fractional modeling the fractional derivatives appear naturally in dealing with generalization of the existing classical models [1]. Machado and Galhano [15] used fractional calculus to study inductive phenomena based on the skin effect. Many applications of fractional calculus can be found in other diverse fields, etc. [2,7,5].

In the design of artificial satellites, it is important to determine the temperature distribution on the spacecraft's surface. An interesting special case is the temperature fluctuation in the skin due to the spinning of the vehicle. Lee et al. [10] studied analysis the unsteady-state temperature distribution of micro-satellite under stabilization effects. Liu et al. in [13] gave improved solution to thermal network problem in heat-transfer analysis of spacecraft, temperature distribution of satellite surfaces affected by solar absorptivity in [14] and numerical simulation on antenna temperature field of complex structure satellite in solar simulator in Liu et al. [12]. Yang et al. [28] studied thermal analysis for folded solar array of spacecraft in orbit.

If the craft is thin-walled so that there is no radial dependence, Hrycak [6] showed that he could approximate the nondimensional temperature field at the equator of the rotating satellite by

$$\frac{d^2T}{d\eta^2}+b\frac{dT}{d\eta}-c\left(T-\frac{3}{4}\right)=-\frac{\pi c}{4}\frac{F(\eta)+\frac{\beta}{4}}{1+\frac{\pi\beta}{4}}, \tag{1.1}$$

where

$$b=\frac{4\pi^2r^2f}{a},\quad c=\frac{16\pi S}{\gamma T_\infty}\left(1+\frac{\pi\beta}{4}\right),\quad T_\infty=\left(\frac{s}{\pi\sigma\epsilon}\right)^{\frac{1}{4}}\left(\frac{1+\frac{\pi\beta}{4}}{1+\beta}\right)^{\frac{1}{4}}, \tag{1.2}$$

$$F(\eta)=\begin{cases}\cos(2\pi\eta), & \text{for } 0\leqslant\eta\leqslant\frac{1}{4},\\ 0 & \text{for } \frac{1}{4}\leqslant\eta\leqslant\frac{3}{4},\\ \cos(2\pi\eta) & \text{for } \frac{3}{4}\leqslant\eta\leqslant 1,\end{cases} \tag{1.3}$$

a is the thermal diffusivity of the shell, f is the rate of spin, r is the radius of spacecraft, S is the net direct solar heating, β is the ratio of the emissivity of the interior shell to the emissivity of the exterior surface, ϵ is the overall emissivity of the exterior surface, γ is the satellite's skin conductance and σ is the Stefan–Boltzmann constant. The independent variable η is the longitude along the equator with the effect of rotation subtracted out $(2\pi\eta=\phi-2\pi ft)$. The reference temperature T_∞ equals the temperature that the spacecraft would have if it spun with infinite angular speed sp that the solar heating would be uniform around the craft. We nondimensionalized the temperature with respect to T_∞. We begin by introducing the new variables

$$y=T-\frac{3}{4}-\frac{\pi\beta}{16+4\pi\beta},\quad v_0=\frac{2\pi^2r^2f}{a\rho_0},\quad A_0=-\frac{\pi\rho^2}{4+\pi\beta}$$

and $\rho_0^2=c$ so that Eq. (1.1) becomes [4]

$$\frac{d^2y}{d\eta^2}+2\rho_0v_0\frac{dy}{d\eta}-\rho_0^2y=A_0F(\eta) \tag{1.4}$$

The following well-known facts are considered for studying the temperature distribution within a spinning satellite.

The Laplace Transform is defined [26] as

$$L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt\ (Re(s)>0) \tag{1.5}$$

The Caputo fractional derivative of order α is defined [17] as

$$D_t^\alpha f(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{f^{(n)}(\tau)}{(t-\tau)^{n-\alpha-1}}d\tau\ (n-1<\alpha<n) \tag{1.6}$$

The Laplace Transform of the Caputo fractional derivative is defined as [17]

$$\int_0^\infty e^{-st}D_t^\alpha f(t)dt=s^\alpha f(s)-\sum_{j=0}^{n-1}s^{\alpha-j-1}f^j(0)\ (n-1<\alpha<n) \tag{1.7}$$

The convolution product of two functions $f(t)$ and $g(t)$ is defined [3] as

$$f(t)*g(t)=\int_0^t f(t-x)g(x)dx=\int_0^t f(x)g(t-x)dx \tag{1.8}$$

Theorem 1.1. *Let $f(t)$ and $g(t)$ be two functions defined for $t\geqslant 0$ and $\bar{f}(s)$ and $\bar{g}(s)$ be the Laplace transforms of $f(t)$ and $g(t)$ respectively. Then*

$$L^{-1}\{\bar{f}(s)\bar{g}(s)\}=f(t)*g(t) \tag{1.9}$$

The Wright generalized hypergeometric function is defined in Mathai et al. [16] as

$${}_p\Psi_q(z)={}_p\Psi_q\left[z\middle|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right]=\sum_{k=0}^\infty\frac{\Pi_{j=1}^p\Gamma(a_j+nA_j)}{\Pi_{j=1}^q\Gamma(b_j+nB_j)}\frac{z^n}{n!}, \tag{1.10}$$

where $a_i, b_j \in \mathbb{C}$ and $A_i, B_j \in \mathbb{R} = (-\infty, \infty); A_i, B_j \neq 0$, $i = 1, 2, \ldots, p, j = 1, 2, \ldots, q, \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j > -1$.

The various properties of generalized Wright function were studied by Prajapati et al. [24], Rao et al. [23], Prajapati and Gehlot [18,19], Prajapati et al. [20,21].

2. Formulation of Caputo fractional model

Let us consider a new model in the form of Caputo fractional differential equation

$$\frac{d^\alpha y}{d\eta^\alpha} + 2\rho_0 v_0 \frac{dy}{d\eta} - \rho_0^2 y = A_0 F(\eta) \tag{2.1}$$

where

$$1 < \alpha < 2 \tag{2.2}$$

On substituting $\alpha = 2$ then Eq. (2.1) reduces to ordinary second order differential Eq. (1.4).

3. Solution of problem

Applying Laplace transform to Eq. (2.1),

$$L\left\{\frac{d^\alpha y}{d\eta^\alpha}\right\} + 2\rho_0 v_0 L\left\{\frac{dy}{d\eta}\right\} - \rho_0^2 L\{y(\eta)\} = A_0 L\{F(\eta)\}$$

Using Laplace transform of $F(\eta)$ (see Appendix A.1) and (1.7), above equation reduces to

$$s^\alpha L\{y(\eta)\} - s^{\alpha-1} y(0) - s^{\alpha-2} \frac{\partial}{\partial\eta} y(0) + 2\rho_0 v_0 s L\{y(\eta)\} - 2\rho_0 v_0 y(0) - \rho_0^2 L\{y(\eta)\} = A_0\left[\frac{s + 2\pi e^{-\frac{s}{4}} + 2\pi e^{-\frac{3s}{4}} + se^{-s}}{s^2 + 4\pi^2}\right]$$

Taking $y(0) = C_0$ and $\frac{\partial}{\partial\eta} y(0) = C_1$, we get

$$(s^\alpha + 2\rho_0 v_0 s - \rho_0^2) L\{y(\eta)\} - C_0 s^{\alpha-1} - C_1 s^{\alpha-2} - 2\rho_0 v_0 C_0 = A_0\left[\frac{s + 2\pi e^{-\frac{s}{4}} + 2\pi e^{-\frac{3s}{4}} + se^{-s}}{s^2 + 4\pi^2}\right]$$

This gives,

$$L\{y(\eta)\} = \frac{C_0 s^{\alpha-1} + C_1 s^{\alpha-2} + 2\rho_0 v_0 C_0}{s^\alpha + 2\rho_0 v_0 s - \rho_0^2} + A_0\left[\frac{s + 2\pi e^{-\frac{s}{4}} + 2\pi e^{-\frac{3s}{4}} + se^{-s}}{s^2 + 4\pi^2} \frac{1}{s^\alpha + 2\rho_0 v_0 s - \rho_0^2}\right] \tag{3.1}$$

using Eq. (3.1) and calculation given in Appendix A.2, we obtain

$$\begin{aligned} L\{y(\eta)\} &= C_0 \sum_{k=0}^{\infty} \rho_0^{2k} \sum_{r=0}^{\infty} \binom{k+r}{r} (-2\rho_0 v_0)^r s^{r-\alpha r-\alpha k-1} \\ &+ C_1 \sum_{k=0}^{\infty} \rho_0^{2k} \sum_{r=0}^{\infty} \binom{k+r}{r} (-2\rho_0 v_0)^r s^{r-\alpha r-\alpha k-2} \\ &+ 2\rho_0 v_0 C_0 \sum_{k=0}^{\infty} \rho_0^{2k} \sum_{r=0}^{\infty} \binom{k+r}{r} (-2\rho_0 v_0)^r s^{r-\alpha r-\alpha k-\alpha} \\ &+ A_0\left[\frac{s + 2\pi e^{-\frac{s}{4}} + 2\pi e^{-\frac{3s}{4}} + se^{-s}}{s^2 + 4\pi^2} \frac{1}{s^\alpha + 2\rho_0 v_0 s - \rho_0^2}\right] \end{aligned} \tag{3.2}$$

Inverse Laplace transform to Eq. (3.2) gives

$$\begin{aligned} y(\eta) &= C_0 \sum_{k=0}^{\infty} \rho_0^{2k} \sum_{r=0}^{\infty} \binom{k+r}{r} (-2\rho_0 v_0)^r L^{-1}\{s^{r-\alpha r-\alpha k-1}\} \\ &+ C_1 \sum_{k=0}^{\infty} \rho_0^{2k} \sum_{r=0}^{\infty} \binom{k+r}{r} (-2\rho_0 v_0)^r L^{-1}\{s^{r-\alpha r-\alpha k-2}\} \\ &+ 2\rho_0 v_0 C_0 \sum_{k=0}^{\infty} \rho_0^{2k} \sum_{r=0}^{\infty} \binom{k+r}{r} (-2\rho_0 v_0)^r L^{-1}\{s^{r-\alpha r-\alpha k-\alpha}\} \\ &+ A_0 L^{-1}\left\{\left[\frac{s + 2\pi e^{-\frac{s}{4}} + 2\pi e^{-\frac{3s}{4}} + se^{-s}}{s^2 + 4\pi^2} \frac{1}{s^\alpha + 2\rho_0 v_0 s - \rho_0^2}\right]\right\} \end{aligned} \tag{3.3}$$

Calculations of Appendices A.2, A.3, Theorem (1.1) and Eq. (1.8), lead to

$$\begin{aligned} y(\eta) &= C_0 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k}}{k!} \sum_{r=0}^{\infty} \frac{\Gamma(r+k+1)(-2\rho_0 v_0)^r}{\Gamma((\alpha-1)r+\alpha k+1)} \frac{\eta^{(\alpha-1)r+\alpha k+1}}{r!} \\ &+ C_1 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k}}{k!} \sum_{r=0}^{\infty} \frac{\Gamma(r+k+1)(-2\rho_0 v_0)^r}{\Gamma((\alpha-1)r+\alpha k+2)} \\ &\times \frac{\eta^{(\alpha-1)r+\alpha k+\alpha}}{r!} \\ &+ 2\rho_0 v_0 C_0 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k}}{k!} \sum_{r=0}^{\infty} \frac{\Gamma(r+k+1)(-2\rho_0 v_0)^r}{\Gamma((\alpha-1)r+\alpha k+\alpha)} \\ &\times \frac{\eta^{(\alpha-1)r+\alpha k+1}}{r!} + A_0 \int_0^\eta F(\eta-u) G(u) du \end{aligned} \tag{3.4}$$

We have,

$$G(\eta) = \sum_{k=0}^{\infty} \frac{\rho^{2k} \eta^{\alpha k+\alpha-1}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+\alpha, \alpha-1) \end{matrix}\right]$$

Thus,

$$\begin{aligned} y(\eta) &= C_0 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k} \eta^{\alpha k}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+1, \alpha-1) \end{matrix}\right] \\ &+ C_1 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k} \eta^{\alpha k+1}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+2, \alpha-1) \end{matrix}\right] \\ &+ 2\rho_0 v_0 C_0 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k} \eta^{\alpha k+\alpha-1}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+\alpha, \alpha-1) \end{matrix}\right] \\ &+ A_0 \int_0^\eta \left(\sum_{k=0}^{\infty} \frac{\rho^{2k} u^{\alpha k+\alpha-1}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 u^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+\alpha, \alpha-1) \end{matrix}\right]\right) \\ &\times F(\eta-u) du \end{aligned} \tag{3.5}$$

Finally, we arrived at

$$\begin{aligned} y(\eta) &= C_0 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k} \eta^{\alpha k}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+1, \alpha-1) \end{matrix}\right] \\ &+ C_1 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k} \eta^{\alpha k+1}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+2, \alpha-1) \end{matrix}\right] \\ &+ 2\rho_0 v_0 C_0 \sum_{k=0}^{\infty} \frac{\rho_0{}^{2k} \eta^{\alpha k+\alpha-1}}{k!} {}_1\Psi_1\left[-2\rho_0 v_0 \eta^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+\alpha, \alpha-1) \end{matrix}\right] \\ &+ A_0 \sum_{k=0}^{\infty} \frac{\rho^{2k}}{k!} \int_0^\eta u^{\alpha k+\alpha-1} \left({}_1\Psi_1\left[-2\rho_0 v_0 u^{\alpha-1} \middle| \begin{matrix} (k+1, 1) \\ (\alpha k+\alpha, \alpha-1) \end{matrix}\right]\right) \\ &\times F(\eta-u) du \end{aligned} \tag{3.6}$$

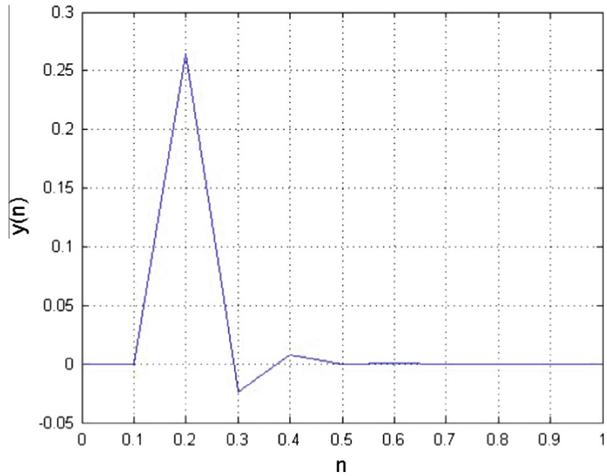

Figure 1 $\mu = 0.5, C_0 = 1, C_1 = 1.$

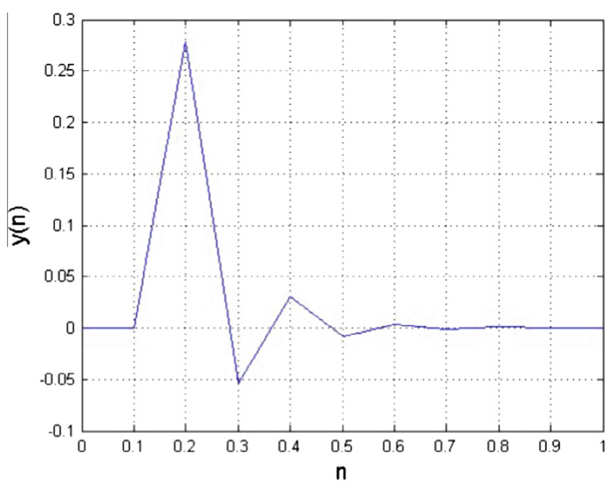

Figure 2 $\mu = 0.75, C_0 = 1, C_1 = 1.$

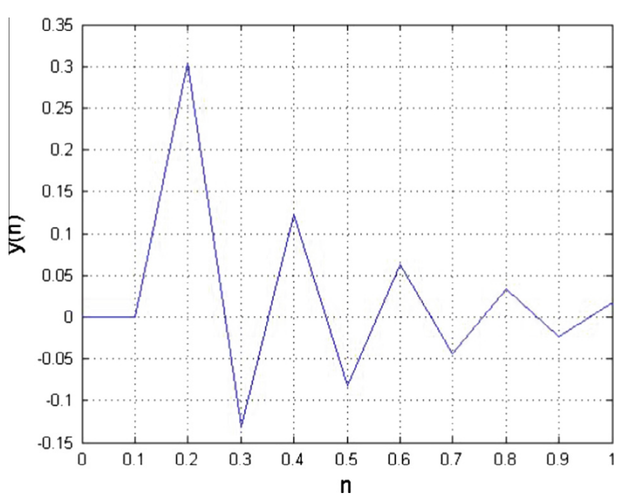

Figure 3 $\mu = 0.5, C_0 = 1, C_1 = 1.$

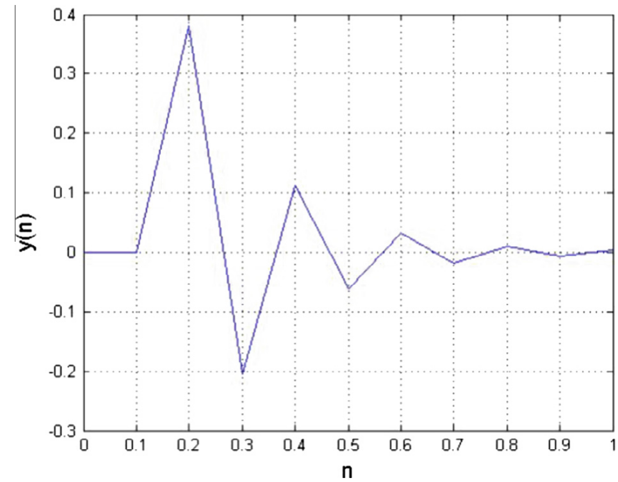

Figure 4 $\mu = 0.75, C_0 = 1, C_1 = 1.$

4. Graphical representation

Figs. 1 and 2: Taking $\alpha = 1.25$ and $\alpha = 1.5$ with $v_0 = 0$ (i.e. satellite is not spinning).

Figs. 3 and 4: Taking $\alpha = 1.25$ and $\alpha = 1.5$ with $v_0 = 0.375$ rpm (i.e. satellite spinning with some rate).

Above graphs show the variation of the non dimensional temperature as a function of η for the spinning rate v_0 and other parameters depend on characteristics of satellite with aluminum skin and fully covered with glass-protected solar cells. In above graphs, the fluctuations of temperature have a change in longitude along the equator when satellite is spinning or not spinning. Above graphs also represent that more fluctuations occur in temperature when satellite spins with some constant rate.

5. Conclusion

On substituting $\alpha = 2$ then Eq. (2.1) reduces to ordinary second order differential Eq. (1.4) having solution of [4]

$$\begin{aligned}
y(\eta) = &-\frac{A_0}{\pi\rho_0^2} - \frac{(4\pi^2+\rho_0^2)A_0\cos(2\pi\eta)}{2\left[(4\pi^2+\rho_0^2)^2+16\pi^2\rho_0^2v_0^2\right]} \\
&+\frac{2\pi\rho_0v_0A_0\sin(2\pi\eta)}{(4\pi^2+\rho_0^2)^2+16\pi^2\rho_0^2v_0^2} \\
&+\frac{2A_0}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n(16n^2\pi^2+\rho_0^2)\cos(2n\pi\eta)}{(4n^2-1)\left[64n^2\pi^2\rho_0^2v_0^2+(16n^2\pi^2+\rho_0^2)^2\right]} \\
&-16\rho_0v_0A_0\sum_{n=1}^{\infty}\frac{(-1)^n n\sin(2n\pi\eta)}{(4n^2-1)\left[64n^2\pi^2\rho_0^2v_0^2+(16n^2\pi^2+\rho_0^2)^2\right]}
\end{aligned} \tag{5.1}$$

An analytical solution of fractional differential Eq. (2.1) for temperature distribution within a spinning satellite was obtained in terms of Wright generalized hypergeometric function by using fractional calculus approach. This solution provides temperature field (the dashed line) of a non-rotating satellite where we neglect the effects of conduction and only radiation occurs. This is certainly more generalized solution for temperature distribution within a spinning satellite.

Acknowledgment

Authors are very much grateful to the reviewers for carefully reading the manuscript and suggesting a way of improvement for the betterment of this paper.

Appendix A. *A.1. Laplace transform of* $\boldsymbol{F}(\eta)$

We will evaluate Laplace transform of a function $F(\eta)$ defined as (1.3)

$$\begin{aligned}L\{F(\eta)\} &= \int_0^\infty e^{-\eta s}F(\eta)d\eta \\ &= \int_0^{\frac{1}{4}} e^{-\eta s}\cos(2\pi\eta)d\eta + \int_{\frac{3}{4}}^{1} e^{-\eta s}\cos(2\pi\eta)d\eta \\ &= \frac{s+2\pi e^{-\frac{s}{4}}+2\pi e^{-\frac{3s}{4}}+se^{-s}}{s^2+4\pi^2}\end{aligned}$$

A.2. Series form

Since,

$$\begin{aligned}\frac{1}{s^\alpha+2\rho_0 v_0 s-\rho_0^2} &= \frac{s^{-1}}{s^{\alpha-1}+2\rho_0 v_0-\rho_0^2 s^{-1}} \\ &= \frac{s^{-1}}{(s^{\alpha-1}+2\rho_0 v_0)\left(1-\frac{\rho_0{}^2 s^{-1}}{s^{\alpha-1}+2\rho_0 v_0}\right)} \\ &= \frac{s^{-1}}{s^{\alpha-1}+2\rho_0 v_0}\sum_{k=0}^{\infty}\left(\frac{\rho_0{}^2 s^{-1}}{s^{\alpha-1}+2\rho_0 v_0}\right)^k \\ &= \sum_{k=0}^{\infty}\frac{\rho_0{}^{2k}s^{-k-1}}{(s^{\alpha-1}+2\rho_0 v_0)^{k+1}} \\ &= \sum_{k=0}^{\infty}\frac{\rho_0{}^{2k}s^{-\alpha k-\alpha}}{(1+2\rho_0 v_0 s^{1-\alpha})^{k+1}} \\ &= \sum_{k=0}^{\infty}\rho_0^{2k}s^{-\alpha k-\alpha}\sum_{r=0}^{\infty}\binom{k+r}{r}(-2\rho_0 v_0 s^{1-\alpha})^r\end{aligned}$$

i.e.

$$\frac{1}{s^\alpha+2\rho_0 v_0 s-\rho_0^2} = \sum_{k=0}^{\infty}\rho_0^{2k}\sum_{r=0}^{\infty}\binom{k+r}{r}(-2\rho_0 v_0)^r s^{r-\alpha r-\alpha k-\alpha}$$

A.3. Inverse Laplace transform

$$\begin{aligned}L^{-1}\left\{\frac{1}{s^\alpha+2\rho_0 v_0 s-\rho_0^2}\right\} &= L^{-1}\left\{\sum_{k=0}^{\infty}\rho_0^{2k}\sum_{r=0}^{\infty}\binom{k+r}{r}(-2\rho_0 v_0)^r s^{r-\alpha r-\alpha k-\alpha}\right\} \\ &= \sum_{k=0}^{\infty}\rho_0^{2k}\sum_{r=0}^{\infty}\binom{k+r}{r}(-2\rho_0 v_0)^r L^{-1}\{s^{r-\alpha r-\alpha k-\alpha}\} \\ &= \sum_{k=0}^{\infty}\frac{\rho_0{}^{2k}}{k!}\sum_{r=0}^{\infty}\frac{\Gamma(r+k+1)(-2\rho_0 v_0)^r}{\Gamma((\alpha-1)r+\alpha k+\alpha)}\frac{\eta^{(\alpha-1)r+\alpha k+1}}{r!} \\ &= \sum_{k=0}^{\infty}\frac{\rho_0{}^{2k}}{k!}\sum_{r=0}^{\infty}\frac{\Gamma(r+k+1)(-2\rho_0 v_0)^r}{\Gamma((\alpha-1)r+\alpha k+\alpha)}\frac{\eta^{(\alpha-1)r+\alpha k+1}}{r!} \\ &= \sum_{k=0}^{\infty}\frac{\rho_0{}^{2k}\eta^{\alpha k+\alpha-1}}{k!}{}_1\Psi_1\left[-2\rho_0 v_0\eta^{\alpha-1}\middle|\begin{matrix}(k+1,1)\\(\alpha k+\alpha,\alpha-1)\end{matrix}\right]\end{aligned}$$

Hence,

$$\begin{aligned}&L^{-1}\left\{\frac{1}{s^\alpha+2\rho_0 v_0 s-\rho_0^2}\right\} \\ &= \sum_{k=0}^{\infty}\frac{\rho^{2k}\eta^{\alpha k+\alpha-1}}{k!}{}_1\Psi_1\left[-2\rho_0 v_0 t^{\alpha-1}\middle|\begin{matrix}(k+1,1)\\(\alpha k+\alpha,\alpha-1)\end{matrix}\right]\end{aligned}$$

References

[1] D. Baleanu, K. Diethelm, E. Scalas, J.J. Trujillo, Fractional Calculus Models and Numerical Methods, Series on Complexity, Nonlinearity and Chaos, World Scientific, 2012.

[2] A. Carpinteri, F. Mainardi, Fractals and Fractional Calculus in Physics, Springer, Vienna, 1997.

[3] L. Debnath, Integral Transforms and their Applications, CRC Press, New-York-London-Tokyo, 1995.

[4] D.G. Duffy, Advanced Engineering Mathematics With Matlab, CRC Press, New-York-London-Tokyo, 2013.

[5] R. Hilfer, Applications of Fractional Calculus in Physics, World Scientific, Singapore, 2000.

[6] P. Hrycak, Temperature distribution in a spinning spherical space vehicle, AIAA J. 1 (1) (1963) 96–99.

[7] I.S. Jeses, J.A.T. Machado, Fractional control of heat diffusion systems, Nonlin. Dynam. 5 (3) (2008) 263–282.

[8] K.B. Kachhia, J.C. Prajapati, Solution of fractional partial differential equation aries in study of heat transfer through Diathermanous materials, J. Interdiscip. Math. 18 (1) (2015) 125–132.

[9] A.A. Kilbas, H.H. Srivastava, J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006.

[10] S.R. Lee, Z.C. Hong, K.T. Peng, L.S. Leu, Analysis the unsteady-state temperature distribution of micro-satellite under stabilization effects, COSPAR Colloq. Ser. 10 (1999) 324–332.

[11] D. Liu, X. Liu, G. Li, L. Jiang, Prediction on effects of absorptivity and emissivity for surface temperature distribution using an improved thermal network model, Heat Transf. – Asian Res. 39 (7) (2010) 539–553.

[12] Y. Liu, G.H. Li, L. Jiang, Numerical simulation on antenna temperature field of complex structure satellite in solar simulator, Acta Astronaut. 65 (7–8) (2009) 1098–1106.

[13] Y. Liu, G.H. Li, L.X. Jiang, A new improved solution to thermal network problem in heat-transfer analysis of spacecraft, Aerosp. Sci. Technol. 14 (4) (2010) 225–234.

[14] Y. Liu, X.X. Pan, G.H. Li, L. Jiang, Investigations on temperature distribution of satellite surfaces affected by solar absorptivity, Adv. Space Res. 46 (5) (2010) 593–603.

[15] J.A.T. Machado, M.S.F. Galhano Alexandra, Fractional order inductive phenomena based on the skin effect, Nonlin. Dynam. 68 (1–2) (2012) 107–115.

[16] A.M. Mathai, R.K. Saxsena, H.J. Haubold, The H-function Theory and Applications, Springer, New York Dordrecht Heidelberg London, 2010.

[17] I.J. Podulbuny, Fractional Differential Equations, Academic Press, New York, 1999.

[18] J.C. Prajapati, K.S. Gehlot, Fractional calculus of generalized K-Wright function, J. Fraction. Calcul. Appl. 4 (2) (2013) 283–289.

[19] J.C. Prajapati, K.S. Gehlot, On generalization of K-Wright functions and its properties, Pacif. J. Appl. Math. 5 (2) (2013) 1–8.

[20] J.C. Prajapati, K.S. Gehlot, B.P. Patel, Differential recurrence relation of generalized K-Wright function, Int. J. Pure Appl. Math. 87 (4) (2013) 603–610.

[21] J.C. Prajapati, K.S. Gehlot, B.P. Patel, Differential equation of generalized K-Wright function, Int. J. Pure Appl. Math. 87 (4) (2013) 611–619.

[22] J.C. Prajapati, K.B. Kachhia, Fractional modeling of temperature distribution and heat flux in the semi infinite solid, J. Fraction. Calcul. Appl. 5 (2) (2014) 38–43.

[23] J.C. Prajapati, S.B. Rao, Wright type hypergeometric function and its properties, Adv. Pure Math. 3 (3) (2013) 335–342.

[24] J.C. Prajapati, A.K. Shukla, S.B. Rao, A.D. Patel, Some properties of generalized hypergeometric function, Commun. Korean Math. Soc. 28 (2) (2013) 303–317.

[25] S.G. Samko, A.A. Kilbas, O.I. Marichev, Fractional Integrals and Derivatives-Theory and applications, Gordan and Breach, Linghorne, PA, 1993.

[26] I.N. Sneddon, Fourier Transform, McGraw-Hill, New York, 1951.

[27] N.S. Venkataraman, C.O. Egalon, Temperature distribution in satellite mounting plates due to conductive heat transfer, Acta Astronaut. 17 (10) (1988) 1127–1135.

[28] W.H. Yang, H.E. Cheng, A. Cai, Thermal analysis for folded solar array of spacecraft in orbit, Appl. Therm. Eng. 24 (4) (2004) 595–607.