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The big constant out, the small constant in



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ABSTRACT

Some time ago, Tseytlin has made an original and unusual proposal for an action that eliminates an arbitrary cosmological constant. The form of the proposed action, however, is strongly modified by gravity loop effects, ruining its benefit. Here I discuss an embedding of Tseytlin's action into a broader context, that enables to control the loop effects. The broader context is another universe, with its own metric and dynamics, but only globally connected to ours. One possible Lagrangian for the other universe is that of unbroken AdS supergravity. A vacuum energy in our universe does not produce any curvature for us, but instead increases or decreases the AdS curvature in the other universe. I comment on how to introduce the accelerated expansion in this framework in a technically natural way, and consider the case where this is done by the self-accelerated solutions of massive gravity and its extensions.

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1. Introduction

Nearly a quarter-century ago, Tseytlin [1] has proposed an approach to the old cosmological constant problem, using an original idea by Linde [2], and certain string-theory developments of that time. The proposal is technically well-framed, while a highly unconventional nature of this approach is commensurate with the magnitude and longevity of the problem, hence suggesting the approach may have a chance of being viable.

While the proposed action of [1] enables one to eliminate an arbitrary cosmological constant, the action itself was argued to be unstable with respect to quantum corrections, therefore making the proposal not workable in its original form (see the note added in [1]). The goal of this work is to extend the proposal to avoid the quantum loop problem, and to incorporate the dark energy component into the theory in a technically natural way.

2. Tseytlin's proposal

To set the conventions, consider the action:

$$S = \int d^4x \sqrt{g} \left(\frac{1}{16\pi G_N} R + L(g, \psi_n) \right), \quad (1)$$

where ψ_n , $n = 0, 1, 2, 3, \dots$, denote all fields of the theory beyond the metric field $g_{\mu\nu}$. Ten Einstein equations can be decomposed as nine traceless and one trace equation:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T, \quad R + T = 0. \quad (2)$$

(Unless G_N or M_{Pl} are displayed explicitly, we use the $8\pi G_N = 1$ units.)

Instead of these equations, Tseytlin introduced a system where the trace equation is modified (see the corresponding action below in Eq. (6)):

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T, \\ R + T = \langle T \rangle - 2 \left\langle g^{\mu\nu} \frac{\partial L}{\partial g^{\mu\nu}} \right\rangle, \quad (3)$$

where $\langle \dots \rangle$ denotes a certain space-time average defined as follows:

$$\langle \dots \rangle \equiv \frac{\int d^4x \sqrt{g} (\dots)}{\int d^4x \sqrt{g}} \equiv \frac{[\dots]}{V_g}. \quad (4)$$

The modification of the trace equation in (3) is dramatic: local observables on the l.h.s.¹ are affected by space-time averaged quantities, where the averaging is done over the past and future. These averages, when nonzero, have pre-notion of future. In that sense, this is an acausal modification. Somewhat similar, but essentially different proposal was made in [3]; a subtle issues of defining the average, where there are more than one vacua, was also raised

¹ Throughout the paper we use commonly accepted acronyms: "l.h.s." and "r.h.s.", for left- and right-hand side respectively, "w.r.t." for with respect to, "UV" for ultra-violet, and "1PI" for 1-particle irreducible.

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there. To begin with, we envision a simple universe evolving in one vacuum state, and comment on possible generalizations later.

If $V_g \rightarrow \infty$, as in Tseytlin's approach, then for most of the stuff in the universe the r.h.s. of the trace equation in (3) is zero: For any observable, \mathcal{O} , that is localized either in space or in time, the average $\langle \mathcal{O} \rangle$ is zero due to the volume factor suppression. Hence, the acausality of the trace equation does not manifest itself in the dynamics of most of the stuff in the universe. On the other hand, for a constant Lagrangian, $L = c$, the r.h.s. of the trace equation (3) is proportional to the constant c itself, and the latter subtracts the equivalent part on the l.h.s., hence leaving the equation independent of c ! Therefore, the main consequence of the acausality might be that we don't observe the big cosmological constant in our universe [3].

For a scalar field the Lagrangian can be decomposed into the part that depends on the metric (derivative terms, e.g., the kinetic term) and the one that is independent of $g_{\mu\nu}$:

$$L = L_g(g, \psi_n) - V(\psi_n). \quad (5)$$

Simplest examples of V are the vacuum energy term E_{vac}^4 , scalar mass term $m_\phi^2 \phi^2$, scalar potential $\lambda \phi^4$, or a linear combination of the above. The vacuum energy, or a constant part of a potential V , would give rise to a nonzero average, $\langle V \rangle = [\text{Const}]/V_g = \text{Const}$. Thus, this quantity would be subtracted from the trace T in (3). This is equivalent to the elimination of the cosmological constant!

The fact that a constant term in L is irrelevant, can also be seen by looking at the action

$$\bar{S} = \frac{S}{V_g}, \quad (6)$$

that Tseytlin introduced [1] as an object which has to be varied w.r.t. $g^{\mu\nu}$ to get the equations of (3). Any constant shift, $L \rightarrow L + c$, gives rise to a shift of the new action by the same constant, $\bar{S} \rightarrow \bar{S} + c$, that does not affect the equations of motion.

Furthermore, if the potential has two minima, one "false" and one "true", then what is being subtracted is the value in the "true" minimum, assuming that a transition from "false" to "true" is possible in finite time in the standard General Relativity context. Generically, what is being eliminated is what would have been the asymptotic future value of the vacuum energy density in GR, as discussed in detail in [3].

As to the second term on the r.h.s. of the trace equation in (3), it contains only the L_g part of the Lagrangian (5); for homogeneous scalar fields this part eventually decays on solutions for which the field settles in its minimum, therefore its average $\langle \dots \rangle$ is zero. Thus, inflation would generically proceed in a conventional way, except the phenomenon of self-replications is not straightforward to incorporate in this framework [2,4].

3. Problems with the loops

While the above approach appears to solve the big cosmological constant problem, at least in the limited context specified above, there are two important issues that it fails to address:

First, as mentioned already in [1], the loop corrections should be problematic, and they are indeed. They strongly renormalize the form of the action (6), and thus ruin the solution of the cosmological constant problem. Can the issue of the loops be resolved, by perhaps extending the proposal?

Second, Tseytlin's mechanism eliminates entirely the cosmological constant. Later on, it was discovered that the expansion of the universe is accelerating [5]. This acceleration can be accounted for by some form of dark energy, with an equation of state parametrized by $w = -1$. A cosmological constant has precisely

that equation of state. Then, the question arises: if one eliminates the cosmological constant how does one get to retain dark energy with $w = -1$? We'll discuss how the accelerated expansion can be accommodated in this scheme in a technically natural way; one option is to invoke massive gravity for this purpose.

We proceed in this work by discussing the path integral formulation of the theory more explicitly. This requires an introduction of a special algorithm for path integral quantization of (6). As a result, we'll end up with two different path integrals: one for all non-gravitational interactions quantized with the Planck's constant \hbar , and another path integral for gravity quantized with a different, dynamically determined Planck's constant.

In the absence of gravity – in the $M_{\text{pl}} \rightarrow \infty$ limit – \bar{S} differs from the standard action by an overall $\frac{1}{\infty}$ factor; the latter is field independent, and thus can be rescaled away.² Thus, in this limit one would quantize the theory (6) in a conventional way. When dynamical gravity is included, however, one needs to specify rules of quantization. One would not immediately worry about a UV completion, but a low-energy effective field theory quantization of gravity should certainly be a matter of concern: The Einstein gravity is a good low-energy effective quantum field theory below the Planck energy scale [6], and any of its extension should strive to retain this virtue, below a certain energy scale. It will be our goal to define such a theory in what follows.

We assume that gravity is quantized at some energy scale, M_{QG} (the Planck scale, or string scale), that is at least an order of magnitude higher than the UV scale, M_{SM} , of non-gravitational interactions, loosely referred below as Standard Model (SM) interactions. M_{SM} could be a scale at which the SM interactions themselves become UV complete, for instance, by grand-unifying into an asymptotically free theory. In such a setup it's not unnatural to have two orders of magnitude hierarchy between M_{QG} and M_{SM} ; if so, then gravity should be well approximated by a classical field theory below the energy scale M_{SM} . At these "low energies" the path integral can be defined with all the SM fields quantized using \hbar , while treating gravity as an external classical field, pending specification of the rules of quantization for gravity. The latter should give rise to further tiny corrections to already quantized SM processes (see below).

Thus, at low energies the path integral for quantized SM interactions reads as follows:

$$Z(g, J_n) = \text{const} \int d\mu(\tilde{\psi}_n) \times \exp\left(i \int d^4x \sqrt{g} (\mathcal{L}(g, \tilde{\psi}_n) + J_n \tilde{\psi}_n)\right), \quad (7)$$

where $d\mu(\tilde{\psi}_n)$ is a measure for all the SM fields $\tilde{\psi}_n$, that appropriately mods out gauge equivalent classes. The metric field g is an external field, and so are the sources, J_n 's, introduced for every single SM field. Then, the effective Lagrangian $L(g, \psi_n)$ used in (6) is defined as a Legendre transform of $W(g, J_n) = -i \ln Z(g, J_n)$

$$\int d^4x \sqrt{g} L(g, \psi_n) \equiv W(g, J_n) - \int d^4x \sqrt{g} J_n \psi_n, \quad (8)$$

where $\sqrt{g} \psi_n \equiv -i \delta \ln Z(g, J_n) / \delta J_n$, is \sqrt{g} times the vacuum expectation value of the SM field $\tilde{\psi}_n$, in the presence of a source J_n . The obtained quantum effective action (8) is a 1PI action. Thus, all the quantum corrections due to non-gravitational interactions are already taken into account in the effective Lagrangian L . This Lagrangian is then inserted into (6) to account for dynamical gravity.

² All four-volume infinities throughout the paper are assumed to be first regularized to yield finite quantities, and the regulator removed only after the equations of motion are derived.

Note that the effective quantum Lagrangian $L(g, \psi_n)$ depends on the classical fields, g and ψ_n 's, only. The difference between these two sets of fields is that the quantum corrections due to the SM interaction are already accounted for in the action for ψ_n 's, while the gravity quantum loops have not been taken into consideration yet. In what follows we will find it helpful to define an effective generating functional

$$\begin{aligned} Z_{\text{SM}}(g, \psi_n) &\equiv \exp\left(i \int d^4x \sqrt{g} L(g, \psi_n)\right) \\ &= Z(g, J_n) \exp\left(-i \int d^4x \sqrt{g} J_n \psi_n\right), \end{aligned} \quad (9)$$

that includes all the SM loops, but does not include quantized gravity.

In the end, $g_{\mu\nu}$ in (6) should also be quantized. The corresponding quantum effects are likely to become of order one at scales M_{QG} , and they should be taken care of by a putative UV completion of the theory, presumably via new degrees of freedom that could appear at energies $\sim M_{\text{QG}}$. These considerations can be postponed for a UV complete theory of gravity, such as string theory, perhaps along the lines proposed in [1]. However, there is an immediate issue, irrespective of the form of UV completion. It concerns the low-energy effective theory: the quantum gravity corrections should be small at scales well below M_{QG} for our approximations above to be meaningful. For instance, in Einstein's gravity, supplied with a diff-invariant UV cutoff for gravity loops (that requires additional counter-terms to retain diff invariance), one generates higher dimensional operators that make negligible contributions at energies below M_{QG} . In the present case, however, one first needs to define the rules of calculation of the gravity loops given that the classical action (6) has an unusual form. To define these rules, and check whether gravity loop corrections are small, is our goal in the remainder of this section.

We define an *extended action*:

$$\bar{S}_{q,\lambda} = \frac{1}{q} \int d^4x \sqrt{g} \left(\frac{1}{2} R + L \right) + \lambda (V_g - q), \quad (10)$$

and write down the path integral for gravity as follows

$$Z_g = \text{const} \int d\mu(g) dq d\lambda \exp(i\bar{S}_{q,\lambda}), \quad (11)$$

where $d\mu(g)$ is a measure over diff-inequivalent metric fields. Note, that the fields of the 1PI SM action, ψ_n 's, play a role of external fields in the path integral for gravity. Furthermore, one also integrates w.r.t. the *parameters* q and λ in this path integral.

The expression in (11) can be rewritten in terms of the path integral for the SM fields Z_{SM} given in (9):

$$Z_g = \text{const} \int d\mu(g) dq d\lambda \left(e^{iS_{\text{EH}}} Z_{\text{SM}}(g, \psi_n) \right)^{\frac{1}{q}} e^{i\lambda(V_g - q)}, \quad (12)$$

where S_{EH} is the Einstein–Hilbert action for gravity. The above path integral defines an algorithm for calculating quantum corrections – both due to the SM interactions and gravity: The SM loops are done in a conventional way, assuming the metric to be an external classical field; this gives rise to $Z_{\text{SM}}(g, \psi_n)$. Furthermore, for calculation of gravity loops one is invited to use an unconventional prescription specified either by (12), or equivalently, by (11).

In this proposal, the parameter q may be regarded as a second Planck's constant that governs the gravity loops (recall that SM loops governed by the standard \hbar are already taken into account in (11)). Furthermore, one integrates w.r.t. the second Planck's constant, however, the value of the latter is also constrained by the value of the invariant four-volume due to integration w.r.t. λ . The

form of the extended action (10), unlike that in (6), is useful for thinking of the formulation of the path integral, or canonical momenta and Hamiltonian of the theory.³ Having the path integral set up in (11), we can integrate w.r.t. q and λ that would give rise to

$$Z_g = \text{const} \int d\mu(g) \exp(i\bar{S}), \quad (13)$$

with \bar{S} defined in (6).

The trouble with the gravity loops in the effective field theory approach, can be understood either in the language of (12) or of (13). The latter presentation is shorter, so we reiterate it here from [1] by observing that the $1/V_g$ factor in (6) is rescaling what would have been the Planck's constant for the gravity loops in a conventional effective field theory approach to Einstein's gravity; that is, we should take all the quantum gravity loop corrections calculated in the conventional approach and make a replacement, $\hbar \rightarrow \hbar V_g$ [1]. Adopting this procedure for the gravity loops, one would get:

$$\begin{aligned} \bar{S}_{\text{Ren}} &= \frac{1}{V_g} \int d^4x \sqrt{g} \left(\frac{1}{2} R + L(g, \psi_n) + V_g L_1(g, \psi_n) \right. \\ &\quad \left. + V_g^2 L_2(g, \psi_n) + \dots \right), \end{aligned} \quad (14)$$

where L_1, L_2, \dots contain all possible terms consistent with diffeomorphism and SM internal symmetries. The gravity loop corrections are huge, since V_g is huge. The new terms ruin the above-presented solution of the cosmological constant problem.

It should be noted, that there is yet another class of loop corrections if one quantizes graviton fluctuations in the theory (6) on a given background solution.⁴ To consider the effects of these fluctuations, let us decompose the metric as a background and fluctuation, schematically, $g = g_b + h$, where h is being treated as small. Then, the inverse volume factor, V_g^{-1} , multiplying the action S in (6), can also be expanded as follows: $V_g^{-1} = V_b^{-1} - V_b^{-2} H_h + \dots$, where $V_b = \int d^4x \sqrt{g_b}$ and $H_h \equiv \int d^4x \sqrt{g_b} \text{tr}(g_b^{-1} h)/2$, and so on. It is clear that the term, $-V_b^{-2} H_h$ (and all the other subsequent terms containing higher powers of h), will produce new unconventional interaction vertices when Wick-contracted with powers of h in the expansion of the action S . While an extra effort would be required to work out all these unconventional vertices, one should point out that all of them will be suppressed by powers of the background volume, V_b . Indeed, in the expression $-V_b^{-2} H_h$ one power of the inverse volume V_b^{-1} will be spent to offset the volume factor in the expression H_h , while the second power of V_b^{-1} will be suppressing the fluctuations of h . Thus, all the new vertices will come suppressed by as many powers of V_b^{-1} as the power of the fluctuation h arising from the expansion of V_g^{-1} in (6). This suggests that the loop effects discussed in the present paragraph could be assumed to be small and be neglected. Similar considerations apply to the proposal discussed in the next section.

4. Dealing with the problems

To avoid the difficulty with the quantum loops discussed in the previous section, let us introduce the following action instead of (6):

³ The idea of integration w.r.t. the parameters is adopted from [1], although the path integral here, and its interpretation, differ somewhat from that in [1].

⁴ I thank Arkady Tseytlin for bringing this point to my attention.

$$A = \frac{V_f}{V_g} S + \int d^D y \sqrt{f} \left(\frac{M_f^{D-2}}{2} R(y) + c_0 M^D + \dots \right), \quad (15)$$

where a second metric $f_{AB}(y)$, $A, B = 0, 1, 2, 3, \dots, D-1$, has been used, and $V_f = \int d^D y \sqrt{f(y)}$, in the $M_{\text{pl}} = 1$ units. Note that while the action S defined in (1) is four dimensional, the f -metric could live in $D \geq 4$ dimensions in general.

The action of the f -universe has a certain vacuum energy scale M , and a scale that determines the strength of its gravitational coupling is M_f . Depending on details of the theory – encoded in the dots in (15) – there may or may not be a stable hierarchy between the scales M_f and M (see below).

The main idea is that in (15) any shift of L by a constant, $L \rightarrow L + c$, converts c into a cosmological constant of the f -universe, thus removing it from the g -universe, where we presumably reside. Therefore, while the curvature in our universe is (nearly) zero, the other universe could be highly curved.

An analog of the extended action (10) now takes the form:

$$A_{q,\lambda} = \frac{1}{q} \int d^4 x \sqrt{g} \left(\frac{1}{2} R + L \right) + \lambda \left(\frac{V_g}{V_f} - q \right) + \int d^D y \sqrt{f} \left(\frac{M_f^{D-2}}{2} R(y) + c_0 M^D \right). \quad (16)$$

This can be used to define the path integral that includes integration w.r.t. q and λ , as discussed in detail in the previous section. Since all the essential steps of that construction carry through with a straightforward extension to include the dynamics of the second metric f , we will not repeat them here.⁵ Furthermore, in what follows we will use, for brevity, the form of the action (15), obtained from the extended action (16) by integrating out q and λ .

In order for the gravity loops not to ruin the crucial classical property of the action, one should make sure that $V_f \gg V_g$: then, the rescaling of what would have been the Planck's constant for gravity loops in a conventional approach is $\hbar \rightarrow \hbar(V_g/V_f)$, and the action including the gravity loop corrections would take the form

$$\frac{V_f}{V_g} \left[\frac{1}{2} R + L + \frac{V_g}{V_f} L_1 + \dots \right] + \int d^D y \sqrt{f} \left(\frac{M_f^{D-2}}{2} R + c_0 M^D + c_1 R^2 + c_2 \frac{\bar{S}^2}{M_f^D} + \dots \right). \quad (17)$$

As long as $V_f \gg V_g$, all the corrections proportional to V_g/V_f can be neglected. There are also terms similar to the ones discussed in the last paragraph on the previous section, but they are harmless for the same reasons as before.⁶ This is not all however, the gravity loop diagrams in the f -universe generate two groups of new terms – first, the terms containing higher powers and derivatives of curvatures $R(f)$'s, and second, terms containing powers of \bar{S} (and their products with powers of the R 's and derivatives); some of these terms are displayed in (17). All these terms, however, introduce small corrections, as it will be clear from the discussion on the hierarchy between the scales in the g - and f -universes given below.

In general, both V_f and V_g are divergent. It is sufficient for our purposes that the condition $V_f/V_g \gg 1$ is satisfied, even though V_f and V_g individually tend to infinity.⁷ For considerations of the

ratio, V_f/V_g , it is convenient to invoke the Euclidean space to get a sense of the ratio of the Euclidean four-volumes, V_f/V_g , as will be done below.

Then, how do we achieve the condition $V_f/V_g \gg 1$? To fulfill this we're going to explore technically natural hierarchies between parameters of the theory. First of all, we assume that the g -universe has supersymmetry broken at some high scale, and therefore, there is a natural value of its vacuum energy density proportional to E_{vac}^4 . The scale E_{vac} can be anywhere between a few TeV and the GUT scale, $\mu_{\text{GUT}} \sim 10^{16}$ GeV. As to the f -universe, it's presumably uncontroversial to set $M_f \sim M_{\text{pl}}$, but also we'd need the scale M to be somewhat higher than E_{vac} . The latter condition should be natural, since without special arrangements one would expect $M \sim M_f \sim M_{\text{pl}}$, and since $E_{\text{vac}} \ll M_{\text{pl}}$, one would also get $E_{\text{vac}} < M$. If so, then the vacuum energy of the g -universe, E_{vac}^4 , would make a small contribution to the pre-existing vacuum energy of the f -universe. In short, the vacuum energy density of the f -universe, $c_0 M^D$, would dominate over the vacuum energy density that gets delegated to the f -universe, from the g -universe.

While one could try to explore a case when the f -universe has a positive vacuum energy density, it seems more straightforward to make a mild assumption that the curvature due to the term $c_0 M^D$ in the f -universe is negative (AdS like) to begin with. In that case, the f -universe can be exactly supersymmetric, described by an unbroken supergravity.

For instance, if we were to consider $D = 4$, the f -universe could be described by supergravity with the "Planck scale" equal to M_f , and the quantity

$$3\bar{\lambda}^2 \equiv \bar{S} + c_0 M^4, \quad (18)$$

acting as its vacuum energy density. The action (15) completed to the one of the $N = 1$ AdS supergravity [7] would then be written as:

$$A_{\text{SUGRA}} = \int d^4 y \tilde{e} \left(\frac{M_f^2}{2} R(\tilde{e}, \tilde{\omega}) - \epsilon^{\mu\nu\alpha\beta} \tilde{\psi}_\mu \gamma_5 \gamma_\nu D_\alpha \psi_\beta + 3\bar{\lambda}^2 - \frac{2\bar{\lambda}}{M_f} \tilde{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \right), \quad (19)$$

where \tilde{e} is the determinant of the vierbein of the f -metric, $\tilde{\omega}$ is its spin connection, $D = \partial - \frac{1}{2} \tilde{\omega} \sigma$ is the covariant derivative, and ψ_μ is the Rarita-Schwinger field describing a f -gravitino. While a supergravity action is not the only one that can help reach our goal, the motivation to consider it can perhaps be attributed to the fact that supergravities naturally emerge in the low-energy limit of superstrings.

The quantity \bar{S} enters into $\bar{\lambda}$ in (19), while the latter defines the cosmological constant (with AdS sign) as well as a quadratic term for the gravitino.⁸ Thus, the entire g -universe enters this action via the parameter $\bar{\lambda}$ defined in (18). The gravitino bilinear term in (19) would also give a nonzero contribution into the equation of motion for the metric g , however, the respective new term will be proportional to the gravitino bilinear, which is zero on classical solutions. Thus this term will not change our conclusions on the cosmological constant.⁹

There is no reason for the parameter $\bar{\lambda}$ to be much smaller than M_f^2 ; quantum corrections would renormalize the former up to the scale of the latter, even if we started with a large hierarchy between them. On the other hand, we do need some small

⁵ As it's evident from the above, f is quantized in a conventional way with \hbar .

⁶ Thus, the gravity loop corrections to both gravity itself, and the standard model processes, either vanish or are very small in this prescription. However, the theory still needs UV completion to make sense of its unusual form for g and f gravities. I thank David Pirtskhalava for very useful discussions on these points.

⁷ As emphasized above, we assume that these infinities are first regularized, and the regularization is removed at the end of calculations.

⁸ This is certainly not a gravitino mass term [8].

⁹ For $\bar{\lambda}^2$ to be positive the scale M should be (somewhat) higher than the scale E_{vac} . This could be arranged without any fine tunings as discussed above.

hierarchy between M_f and $\bar{\lambda}^{1/2}$, essentially to be sure that AdS curvature of the f -universe can reliably be described in the supergravity approximation. For this, an order of magnitude hierarchy, $M_f \sim 10\bar{\lambda}^{1/2}$, would be more than sufficient. While this hierarchy could perhaps be attributed, without too much of anxiety, to the $4\pi^2$ loop factor's arising at various places, we note that it could be generated dynamically if we were to introduce more general supersymmetric theory with some matter fields in the Lagrangian: the N_m matter fields with a characteristic scale M_m would renormalize additively the Planck scale M_f via the Adler–Zee mechanism producing, $M_f^2 \rightarrow M_f^2 + \Delta M_f^2$, where $\Delta M_f^2 \sim N_m M_m^2$ [9,10], while renormalization of the cosmological constant $\bar{\lambda}$ due to the complete SUSY multiplets of matter would have been zero. Thus, we could adopt, $M_f \sim 10\bar{\lambda}^{1/2}$, as a technically natural choice. If so, then the hierarchies $M_{\text{Pl}} \sim M_f \sim 10M$, $M \gtrsim 10E_{\text{vac}}$, ensure that all the corrections in (17) are negligible in comparison with the terms in (15).

Having the scales clarified, let us see how this plays out for the cosmological constant for a general D -dimensional f -universe. First we consider the case when f is not among the fields ψ_n , $n = 0, 1, 2, 3, \dots$. Then, the new terms in (15) or (19) do not affect the equations (3), except that they introduce a overall multiplier V_f . Thus, the cosmological constant is eliminated from the g -universe. There is, however, a new equation due to variation w.r.t. f :

$$M_f^{D-2} \left(R_{AB}(Y) - \frac{1}{2} f_{AB} R(Y) \right) = f_{AB} (\bar{S} + c_0 M^D) + \dots \quad (20)$$

The right-hand side contains the vacuum energy generated in our universe, $\bar{S} = \frac{|E_{\text{vac}}^4|}{V_g} = E_{\text{vac}}^4$, as well as that of the f -universe. According to our construction, the net energy density is negative, so that the f -universe has an AdS curvature. If so, then $V_f = \infty$ even in Euclidean space. Then, to reach our goal it is sufficient to have Euclidean V_g finite, so that $V_f \gg V_g$. A de Sitter universe with Euclidean $V_g = H_0^{-4}$ would fit the data and satisfy the above criterium.¹⁰ However, the entire cosmological constant has been eliminated from the g -universe, and thus it's not easy any more to get $V_g = H_0^{-4}$. We'll discuss below how this could nevertheless be achieved.

5. Getting the accelerated universe

One needs to get a dS metric in the g -universe without using a vacuum energy or a scalar potential. More precisely, one would need to get the small dS curvature due to the terms in the Lagrangian (5) that explicitly depend on g .

There might be a few ways of achieving that: e.g., by invoking Lorentz invariant condensates of some vector fields with a coherence length comparable with H_0^{-1} , or by using field theories with higher derivatives but no Ostrogradsky instabilities. Such proposals could produce dark energy due to terms that aren't potentials, but depend on the metric g , so that the last term on the r.h.s. of the trace equation (3), would define the cosmic speed-up.

We briefly comment here on a possibility to obtain this feature due to massive gravity. Nonlinear massive gravity [11,12], or some of its extensions [13–16], introduce graviton mass m as a small parameter, $m \sim H_0$, in a technically natural way [17]; these theories also produce self-accelerated solutions with a dS background [18]; moreover, the fluctuations on these backgrounds are healthy when

the pure massive graviton is amended with a dilaton-like field [15, 16] (for theory reviews of massive gravity, see [19,20]).

Let us briefly outline how massive gravity would produce $R \sim m^2$ in the trace equation (3). For this we put aside the matter Lagrangian and assume that L represents instead the diffeomorphism invariant potential of massive gravity [12]:

$$\begin{aligned} L &= M_{\text{Pl}}^2 m^2 U(K) \\ &= M_{\text{Pl}}^2 m^2 (\det_2(K) + \alpha_3 \det_3(K) + \alpha_4 \det_4(K)), \\ K &= 1 - \sqrt{g^{-1}\gamma}, \end{aligned} \quad (21)$$

where the matrix K is defined via an inverse of the metric g and a fiducial metric γ ; we chose γ to be a metric of Minkowski space, $\gamma_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$, written in an arbitrary coordinate system parametrized by ϕ^a , $a = 0, 1, 2, 3$. The $\phi^a(x)$ fields also represent the Stückelberg fields that guarantee diffeomorphism invariance of (21). The square root of a matrix and its traces are defined via its eigenvalues, and α_3, α_4 , are some free parameters. Note that all possible values of the three parameters of the theory, m, α_2, α_3 , are technically natural [17]. Furthermore, the quasidilaton is introduced by requiring that the rescaling of the ϕ^a coordinates w.r.t. the x^μ coordinates be promoted into a global symmetry; this amounts to adding into (21) the kinetic term for the quasidilaton σ (and possibly some other derivative terms [15]), and replacing $\gamma \rightarrow e^{2\sigma/M_{\text{Pl}}} \gamma$.

Let us now look at the trace equation in (3): the trace of the stress-tensor, call it T^g , is obtained by the standard variation of $[\sqrt{g}L] = M_{\text{Pl}}^2 m^2 [\sqrt{g}U]$. On the self-accelerated solutions this trace equals to a constant, $T^g \sim M_{\text{Pl}}^2 m^2$. Therefore, T^g in the l.h.s. of (3) will cancel with $\langle T^g \rangle$ on the r.h.s.; the remaining trace equation will take the form

$$R = -2m^2 \left\langle g^{\mu\nu} \frac{\partial U(K)}{\partial g^{\mu\nu}} \right\rangle. \quad (22)$$

On the self-accelerated solutions, however, $g^{\mu\nu} \partial U(K) / \partial g^{\mu\nu}|_{\text{SA}} = -C(\alpha_2, \alpha_3)$, is also a constant, that depends on the parameters α_2 and α_3 . Therefore, its average yields the same constant, and we get $R = 2m^2 C(\alpha_2, \alpha_3)$. For a certain reasonable magnitudes, and certain signs of the parameters, one gets the dS curvature of the order, $m^2 \sim H_0^2$, in a technically natural way. Quasidilaton does not change this conclusion, it only affects (improves) dynamics of small perturbations above the solution [16]. Thus, to summarize, the above approach enables to remove the big cosmological constant, and to get a small space–time curvature determined by the graviton mass.

In the approach adopted above γ was taken to be independent of the f -metric, that was used to remove the big cosmological constant. We've discussed the case when f was an AdS metric, while γ was flat. However, neither of these choices are ordained – we only require that space–time described by f to have an infinite Euclidean volume. It is intriguing, therefore, to consider γ to be related (perhaps identified?) with f . In that case, γ cannot be fixed a priori, but will be determined by the f equation of motion (20); the latter will now be modified due to the terms in (21), but the modification is proportional to $m^2 \sim H_0^2 \ll M_f^2$, and should be negligible. If such a framework can be made to work in detail, this would provide an additional arguments for amending Tseytlin's approach by the f -metric, and conversely, would introduce an out-of-our-universe dynamics for the fiducial metric of massive gravity.

On a more sobering note, massive gravity and its extensions are strongly coupled theories at energies way below M_{Pl} ; while this may not be in conflict with observations in our universe due to the Vainshtein mechanism [21] in its intricate cosmological and

¹⁰ That $V_f^{\text{AdS}} / V_g^{\text{dS}} \rightarrow \infty$ can also be seen in Lorentzian signature, by calculating the ratio, e.g., in the global coordinate systems, for the universal covering of AdS, and the dS space.

astrophysical form [22–24], nevertheless, it still remains to be understood how to go above the strong scale, and show that superluminal phase and group velocities obtained on certain backgrounds probing this strong scale, are indeed artifacts to be removed in a complete treatment.

6. Conclusions and outlook

The proposed approach eliminates the cosmological constant, at least in a simple setup where there are a few (non-proliferating) vacua with well-separated hierarchy between their energy densities, and allowed transitions between them. What is eliminated is what would have been an asymptotic future value of the cosmological constant for such a potential in GR; for instance, for two vacua, “false” and “true”, with allowed transitions from “false” to “true”, the “true” vacuum energy is eliminated. This is similar to the proposal of [3], but here the action functional is available and it is stable w.r.t. quantum loop corrections, including loops of gravity in an effective field theory approach.

The dark energy component can be introduced via the Lorentz invariant condensates of vector fields, or via derivatively interacting scalar fields. We briefly discussed how the accelerated universe could be due to massive gravity in this approach.

The proposed scheme is rather unusual, as it involves nonlocal terms in otherwise local Einstein’s equations, making it difficult to be satisfied with this aspect. However, the cosmological constant problem is a long-standing enigma of a tremendous magnitude, and any insight into its possible dynamical solution within the well-defined rules of the low-energy field theory approach, is extremely important, and should be welcomed.

As an outlook, just three comments on the literature:

Ref. [3] has made arguments for a connection of the “high-pass filter” modification of gravity with a specific theory containing the averages $\langle \dots \rangle$. It might be interesting to see if the present proposal could also be connected to some “high-pass filter” modified gravities discussed in [3]. Conversely, one could then hope to find an action principle for the equations of [3], and address the issue of the gravity loops for them.

The original motivation of Tseytlin was to obtain the unconventional action (6) by including the winding modes of string theory. It would be interesting to see if any proposal along this idea can give the action (15), or a version of it.

Refs. [25] have recently discussed gravity equations involving the averages $\langle \dots \rangle$, with the goal to sequester the Standard Model vacuum energy. The equations and physical picture obtained in [25] are different from the ones discussed in the present work. It is argued that the particle physics loops are under control in [25], while the gravity loops were not considered. It could perhaps be interesting to apply the proposal of the present work to deal with the gravity loops in [25].

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