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Supergravity and the knitting of the Kalb–Ramond two-form in eight-dimensional topological gravity

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Abstract

Topological Euclidean gravity is built in eight dimensions for manifolds with $Spin(7) \subset SO(8)$ holonomy. In a previous work, we considered the construction of an eight-dimensional topological theory describing the graviton and one graviphoton. Here we solve the question of determining a topological model for the combined system of a metric and a Kalb–Ramond two-form gauge field. We then recover the complete $N = 1$, $D = 8$ supergravity theory in a twisted form. We observe that the generalized self-duality conditions of our model correspond to the octonionic string equations.

1. Introduction

All types of superstring theories can be formally obtained by suitable anomaly free untwisting of a topological sigma-model [1]. This suggests the possibility that supergravities, which arise as low energy limits of superstrings, can be understood as topological gravities. In particular, $D = 11$ supergravity, which determines all known supergravities in lower dimensions, could be viewed as a topological theory.

In a previous work we have shown that, both in four [2] and in eight dimensions [3], the Einstein action plus the Rarita–Schwinger term (in a twisted form) can be obtained by constructing a topological quantum field theory (TQFT), which implements in a BRST invariant way the gravitational instanton equation.[1] These constructions only holds for manifolds with special holonomy, i.e., $SU(2) \subset SO(4)$ in four dimensions and $Spin(7) \subset SO(8)$ in eight dimensions.

In [3], we left open the delicate point of introducing a sector of the eight-dimensional TQFT involving a two-form gauge field. This case escapes the procedure displayed in [5]. In this Letter, the equations for the two-form gauge field appear mixed with the one for other fields. The basic idea is the construction of a TQFT for a general two-tensor field, which is the natural object stemming from the zero slope limit of topological sigma-models. The symmetric part of this tensor describes the metric, while its antisymmetric part gives the two-form field. We

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[1] In four dimensions, the twist of the complete action including the interaction terms has been discussed in [4].

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use the formalism where the metric is described as a vielbein modulo the Lorentz symmetry. This means adding and subtracting spurious degrees of freedom, a task which is by now familiar in the context of TQFT's. As in our previous model [3], we find that the topological theory is defined on manifolds with $Spin(7)$ holonomy. In fact, the $Spin(7)$-invariant four-form plays a central role in the determination of the topological gauge functions. Moreover, the presence on these manifolds of a covariantly constant spinor allows for the definition of a twist [3] which maps some of the fermionic ghosts and antighosts of the topological model on the spinors of $N=1$, $D=8$ supergravity. Finally, some ghosts of ghost and ghosts of antighosts can be untwisted into the commuting ghosts of local supersymmetry, explaining the emergence of local supersymmetry in our model.

2. Including the two-form in topological gravity

In this section we address the question of building a TQFT multiplet for a general tensor $A_{\mu\nu}$ of rank two. We consider an eight-dimensional manifold with holonomy group $Spin(7) \subset SO(8)$. The tensor $A_{\mu\nu}$ can be split into its symmetric and antisymmetric parts. If, as in [3], we only consider the symmetric part, which can be interpreted as a metric, we can construct a TQFT that contains the Einstein action, by using a gravitational octonionic self-duality equation. In the spirit of [2] we can also introduce a coupling to the TQFT for an Abelian graviphoton, which has ghost number two. This topological model determines, in a twisted form, a truncation of $N=1$, $D=8$ supergravity [3]. However, the dilaton, the Kalb–Ramond two-form, one graviphoton and their fermionic superpartners in the $N=1$, $D=8$ supergravity multiplet [6] escape this construction.

The difficulty of determining a TQFT for the antisymmetric part $B_{\mu\nu}$ of $A_{\mu\nu}$ can be appreciated as follows, following the ideas contained in [5]. The two-form $B_{\mu\nu}$ contains 28 components, which give 21 degrees of freedom modulo the gauge invariance $B_{\mu\nu} \sim B_{\mu\nu} + \partial_{[\mu}\Lambda_{\nu]}$. The field strength of $B_{\mu\nu}$ is a three-form $G_3 = dB_2$ containing $56 = \binom{8}{3}$ components. The analysis in [5] indicates that there is no natural way to chose an holonomy group for the eight-dimensional manifold which allows to write a self-duality equation for G_3, which, (i) would count for 21 topological independent equations and, (ii), would solve the relativistic wave equation of a two-form. Moreover, the 28 components of the topological ghost of a two-form cannot be rearranged in eight-dimensional spinor representations. Rather, to determine a TQFT involving the two-form in eight dimensions, we will see that it is necessary to combine the Lorentz invariance and the topological invariance of the two-form, and write self-duality equations that mix the dilaton, the eight-bein and the two-form. That this is not only possible, but a useful complement of the construction of [3] appears immediately from the fact that there were 56 components for the antighost for the vielbein, more than the 28 degrees of freedom it describes.

To start with, we enlarge the question of building a TQFT for $A_{\mu\nu}$ into that of building a TQFT for a vielbein e^a_μ, a Lorentz ghost Ω^{ab} and a two-form $B_{\mu\nu}$,[2]

$$A_{\mu\nu} = A_{\{\mu\nu\}} \oplus A_{[\mu\nu]} \to \left(e^a_\mu, \Omega^{ab}, B_{\mu\nu}\right).$$

The Lorentz symmetry can be used to set to zero the antisymmetric part of the matrix e^a_μ.[3] The number of degrees of freedom of the system $A_{\mu\nu} = A_{[\mu\nu]} \oplus A_{\{\mu\nu\}}$ is indeed equal to that of the $(e^a_\mu, B_{\mu\nu}, \Omega^{ab})$ system, when the components are algebraically counted, since the Lorentz ghost Ω^{ab} counts negatively. Eventually, we will interpret $A_{\{\mu\nu\}}$ as a metric $g_{\mu\nu}$, assuming that e^a_μ is an invertible matrix.

In contrast with our previous work, graviphotons are not separately introduced. Here, these fields naturally appear as ghosts for the topological ghosts of the two-form $B_{\mu\nu}$. An analogous situation holds for the dilaton,

[2] Throughout the Letter the Latin indices $a, b, \ldots$ denote flat $SO(8)$ tangent space indices, and $\mu, \nu, \ldots$ are eight-dimensional world indices.

[3] Strictly speaking, the two indices of the vielbein represent components in different spaces, so that one can only speak of the antisymmetric part of e^a_μ once a background vielbein has been chosen. This is not the case for other fields with the same indices as the vielbein, since the vielbein is then available to relate the two spaces.

which is a $Spin(7)$-invariant part of a ghost of ghost for the vielbein. Having obtained a truncation of supergravity in [3] is now understood as having consistently retained a part of the topological BRST multiplet.

Eventually, we will recognize that we have a TQFT with an equivariance with respect to the Lorentz $\times$ diffeomorphism symmetry, whose gauge fields are the spin connection ω_μ^{ab} and the vielbein e_μ^a. Moreover, our topological model displays an equivariance with respect to the vector gauge symmetry of the two-form, $B_2 \sim B_2 + d\Lambda_1$. Finally, local supersymmetry will show up as a consequence of the symmetry of the topological ghost of the vielbein, defined modulo reparametrizations. Shortly speaking, the construction of a TQFT for a two-tensor yields all the fields of supergravity. As we will discuss in the next section, the topological gauge functions are given by self-duality equations which mix the symmetric and antisymmetric parts of the two-tensor.

Let us now proceed to the detailed construction of the BRST topological multiplets. Geometry determines the set of ghosts and our first guess for the complete set of fields for an $SO(8)$ invariant TQFT with a vielbein e_μ^a, a two-form $B_{\mu\nu}$ and a spin connection ω_μ^{ab} is:

$$
\begin{array}{ccccc}
 & & e_\mu^a & & \\
 & \Psi_\mu^{(1)a} & & \overline{\Psi}_\mu^{(-1)a} & \\
\Phi^{(2)a} & & \Phi^{(0)a}, b_\mu^{(0)a} & & \overline{\Phi}^{(-2)a} \\
 & \eta^{(1)a} & & \bar{\eta}^{(-1)a} &
\end{array}
$$

$$
\begin{array}{ccccc}
 & & \omega_\mu^{ab} & & \\
 & \widetilde{\Psi}_\mu^{(1)ab} & & \overline{\widetilde{\Psi}}_\mu^{(-1)ab} & \\
\widetilde{\Phi}^{(2)ab} & & \widetilde{\Phi}^{(0)ab}, \tilde{b}_\mu^{(0)ab} & & \overline{\widetilde{\Phi}}^{(-2)ab} \\
 & \tilde{\eta}^{(1)ab} & & \bar{\tilde{\eta}}^{(-1)ab} &
\end{array}
$$

$$
\begin{array}{ccccccc}
 & & & B_{\mu\nu} & & & \\
 & & \Psi_{\mu\nu}^{(1)} & & \overline{\Psi}_{\mu\nu}^{(-1)} & & \\
 & A_\mu^{(2)} & & A_\mu^{(0)}, b_{\mu\nu}^{(0)} & & A_\mu^{(-2)} & \\
R^{(3)} & & S^{(1)}, \Psi_\mu^{(1)} & & \overline{S}^{(-1)}, \overline{\Psi}_\mu^{(-1)} & & \overline{R}^{(-3)} \\
 & b^2_{S^{(1)}}, \Phi^{(2)} & & b^{(0)}_{\overline{S}^{(-1)}}, \overline{\Phi}^{(0)}, \Phi^{(0)} & & b^{(-2)}_{\overline{R}^{(-3)}}, \overline{\Phi}^{(-2)} & \\
 & & \eta^{(1)} & & \bar{\eta}^{(-1)} & &
\end{array}
$$

$$
\begin{array}{ccccccc}
\xi^{(1)\mu} & & \bar{\xi}^{(-1)\mu} & \Omega^{(1)ab} & & \overline{\Omega}^{(-1)ab} \\
 & b^{(0)\mu} & & & b^{(0)ab} &
\end{array}
\tag{1}
$$

For the sake of clarity, we have made explicit (as an upper index) the ghost number of the fields in the "pyramid" that describes the BRST topological multiplets. We could introduce a bigrading that separate the ghost number and antighost number, but this would make heavier the notations. In the above pyramids, the BRST symmetry acts on the south-west direction. The fields which are not on the left edge of each pyramid are topological pairs made of antighosts and their Lagrange multipliers. They satisfy trivial BRST equations.[4] Actually, each one of the fields that are labeled by a letter b or η, with various indices, is a bosonic or fermionic Lagrange multiplier field, and is essentially equal to the BRST variation of the antighost that is located at its upper right position, e.g., $s\Psi_{\mu\nu}^{(-1)} = b_{\mu\nu}^{(0)} + \cdots$, $s\overline{S}^{(-1)} = b^{(0)}_{\overline{S}^{(-1)}} + \cdots$. As an exception to this notational rule, we find useful to define the fields $\Psi_\mu^{(1)}$ and $\overline{\Psi}_\mu^{(-1)}$ as the fermionic Lagrange multipliers that stem from the BRST variation of the commuting ghosts of ghosts $A_\mu^{(0)}$ and $A_\mu^{(-2)}$ respectively, i.e., $sA_\mu^{(0)} = \Psi_\mu^{(1)} + \cdots$ and $sA_\mu^{(-2)} = \overline{\Psi}_\mu^{(-1)} + \cdots$.

[4] More precisely, all equations for the antighosts appearing in our field spectrum are of the type $\hat{s}\bar{g} = \lambda$, $\hat{s}\lambda = \mathcal{L}_\Phi \bar{g} + \delta_{\widetilde{\Phi}}\bar{g}$ with $sX = \hat{s}X + \mathcal{L}_\xi X + \delta_\Omega X$.

The fields which carry the essential geometrical information are on the left edge of the pyramids. Their topological symmetry is defined as:

$$
\begin{aligned}
&s e^a_\mu = \Psi^{(1)a}_\mu - \Omega^{ab} e^b_\mu + \mathcal{L}_\xi e^a_\mu, \qquad s\omega^{ab}_\mu = \widetilde{\Psi}^{(1)ab}_\mu + D_\mu \Omega^{ab} + \mathcal{L}_\xi \omega^{ab}_\mu,\\
&s\Psi^{(1)a}_\mu = -\Omega^{ab}\Psi^{(1)b}_\mu - \mathcal{L}_\Phi e^a_\mu + \widetilde{\Phi}^{(2)ab} e^b_\mu + \mathcal{L}_\xi \Psi^a_\mu,\\
&s\widetilde{\Psi}^{(2)ab}_\mu = -\Omega^{ac}\widetilde{\Psi}^{(2)cb} + D_\mu \widetilde{\Phi}^{(2)ab} - \mathcal{L}_\Phi \omega^{ab}_\mu + \mathcal{L}_\xi \widetilde{\Psi}^{(2)ab}_\mu,\\
&s\Phi^{(2)a} = \mathcal{L}_\xi \Phi^{(2)a} - \Omega^{ac}\Phi^{(2)a}, \qquad s\widetilde{\Phi}^{(2)ab} = -\Omega^{ac}\widetilde{\Phi}^{(2)cb} + \mathcal{L}_\xi \widetilde{\Phi}^{(2)ab},\\
&sB_{\mu\nu} = \Psi^{(1)}_{\mu\nu} + \mathcal{L}_\xi B_{\mu\nu}, \qquad s\Psi^{(1)}_{\mu\nu} = \mathcal{L}_\Phi B_{\mu\nu} + \partial_{[\mu} A^{(2)}_{\nu]} + \mathcal{L}_\xi \Psi^{(1)}_{\mu\nu},\\
&sA^{(2)}_\mu = \partial_\mu R^{(3)} + \mathcal{L}_\xi A^{(2)}_\mu, \qquad sR^{(3)} = \mathcal{L}_\xi R^{(3)},\\
&s\xi^\mu = f^\mu_a \Phi^{(2)a} + \xi^\nu \partial_\nu \xi^\mu, \qquad s\Omega^{ab} = \widetilde{\Phi}^{(2)ab} - \Omega^{ac}\Omega^{cb} + \mathcal{L}_\xi \Omega^{ab}.
\end{aligned}
\tag{2}
$$

In the variation of ξ^μ, the inverse of the vielbein e^a_μ appears and we denote it as f^μ_a. We have not introduced ghosts $V^{(1)}_\mu$ and ghost of ghost $m^{(2)}$ for the gauge invariance of the two-form gauge field, with the standard BRST symmetry $QB_{\mu\nu} = \partial_{[\mu} V^{(1)}_{\nu]}$, $QV^{(1)}_\mu = \partial_\mu m^{(2)}$, $Qm^{(2)} = 0$, which would yield $sV^{(1)}_\mu = A^{(2)}_\mu$ and $sm^{(2)} = R^{(3)}$. We choose instead to write equivariant BRST transformations with respect to this symmetry. This implies that the square of the BRST transformations on the field $B_{\mu\nu}$ is not zero, but corresponds to a reparametrization with parameter Φ^a and an Abelian transformation $B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\Lambda_{\nu]}$, with $\Lambda_\mu = A^{(2)}_\mu$. The BRST operator s is thus nilpotent only modulo gauge transformations for the two-form gauge field. Actually, the topological gauge functions we will use in Section 3 only involve the curvature $G_3 = dB_2$, so there is no need to give the details of the gauge symmetry of the two-form. We could further set $\xi^\mu = 0$, which would yield a BRST symmetry equivariant with respect to the reparametrization. In this case, the BRST symmetry would be nilpotent also modulo reparametrizations along the vector ghost of ghosts $e^\mu_a \Phi^{(2)a}$. As for the Lorentz invariance, we will instead carefully keep the Lorentz ghost dependence. Eventually, the corresponding gauge functions will be equivariant with respect to local Lorentz transformations for *Spin*(7) $\subset$ *SO*(8).

There is an $U(1)$ invariance for the fields $\Psi^{(1)}_\mu$ and $\overline{\Psi}^{(-1)}_\mu$. Also for this invariance we prefer to work with an equivariant BRST operator, and we do not write explicitly the related Faddeev–Popov ghosts $sc = \Phi^{(2)}$, $s\bar{c} = \overline{\Phi}^{(0)}$ in the BRST transformations

$$
\begin{aligned}
&sA^{(-2)}_\mu = \overline{\Psi}^{(-1)}_\mu + \mathcal{L}_\xi A^{(-2)}_\mu, \qquad sA^{(0)}_\mu = \Psi^{(1)}_\mu + \mathcal{L}_\xi A^{(0)}_\mu,\\
&s\overline{\Psi}^{(-1)}_\mu = \partial_\mu \Phi^{(0)} + \mathcal{L}_\xi \overline{\Psi}^{-1}_\mu, \qquad s\Psi^{(1)}_\mu = \partial_\mu \Phi^{(2)} + \mathcal{L}_\xi \Psi^{(1)}_\mu,\\
&s\Phi^{(0)} = \mathcal{L}_\xi \Phi^{(0)}, \qquad s\Phi^{(2)} = \mathcal{L}_\xi \Phi^{(2)},\\
&s\overline{\Phi}^{(0)} = \mathcal{L}_\xi \overline{\Phi}^{(0)} + \eta^{(1)}, \qquad s\overline{\Phi}^{(-2)} = \bar{\eta}^{(-1)} + \mathcal{L}_\xi \overline{\Phi}^{(-2)},\\
&s\eta^{(1)} = \mathcal{L}_\xi \eta^{(1)}, \qquad s\bar{\eta}^{(-1)} = \mathcal{L}_\xi \bar{\eta}^{(-1)}.
\end{aligned}
\tag{3}
$$

If we only retain a *Spin*(7) $\subset$ *SO*(8) invariance, we can redistribute the degrees of freedom of the antighosts and Lagrange multipliers of the BRST multiplets. In this way, we shall be able to define *Spin*(7) $\subset$ *SO*(8) invariant topological gauge functions, which is the key for building the eight-dimensional TQFT. Using the decomposition of a *SO*(8) valued two-form $M^{ab} = M^{ab^-} + M^{ab^+}$ in the *Spin*(7)-invariant representations $28 = 7 \oplus 21$, we get

$$
\begin{array}{ccccccc}
 & & & e^a_\mu & & & \\
 & \Psi^{(1)a}_\mu & & & \overline{\Psi}^{(-1)ab^-}_\mu,\ \bar{\chi}^{(-1)a} & & \\
\Phi^{(2)a} & & \sigma,\ \Phi^{(0)ab^-} & b^{(0)ab^-}_\mu,\ b^{(0)}_\mu & & \overline{\Phi}^{(-2)a} & \\
 & \chi^{(1)},\ \eta^{(1)ab^-} & & & \bar{\eta}^{(-1)a} & &
\end{array}
$$

$$
\begin{array}{ccccccc}
 & & & \omega_\mu^{ab} & & & \\
 & \widetilde{\Psi}_\mu^{(1)ab} & & & \overline{\widetilde{\Psi}}_\mu^{(-1)ab} & & \\
\widetilde{\Phi}^{(2)ab^\pm} & & \widetilde{\Phi}^{(0)ab^\pm}, \tilde{b}_\mu^{(0)ab} & & & \overline{\widetilde{\Phi}}^{(-2)ab^\pm} & \\
 & \tilde{\eta}^{(1)ab^\pm} & & & \bar{\tilde{\eta}}^{(-1)ab^\pm} & &
\end{array}
$$

$$
\begin{array}{ccccccccc}
 & & & & B_{\mu\nu} & & & & \\
 & & \Psi_{\mu\nu}^{(1)} & & & & \overline{\Psi}_{\mu\nu^\pm}^{(-1)} & & \\
 & A_\mu^{(2)} & & & A^{(0)}, A_{\mu\nu^-}^{(0)}, b_{\mu\nu^\pm}^{(0)} & & & A_\mu^{(-2)} & \\
R^{(3)} & & S^{(1)}, \Psi_{\mu\nu^-}^{(1)}, \Psi^{(1)} & & & & \overline{S}^{(-1)}, \overline{\Psi}_\mu^{(-1)} & & \overline{R}^{(-3)} \\
 & b_{S^{(1)}}^{(2)}, \Phi^{(2)} & & & b_{\overline{S}^{(-1)}}^{(0)}, \Phi^{(0)}, \overline{\Phi}^{(0)} & & & b_{\overline{R}^{(-3)}}^{(-2)}, \overline{\Phi}^{(-2)} & \\
 & & \eta^{(1)} & & & & \eta^{(-1)} & &
\end{array}
\qquad (4)
$$

We do not reproduce the geometrical ξ and Ω^{ab} which are not modified. Let us now clarify the role of the various topological fields and their supergravity interpretation. To obtain the relevant set of propagating fields, a certain number of trivial gauge fixing must be done, which give algebraic terms of the type $s(AB) = (sA)B \pm A(sB) = \Psi_A A \pm A\Psi_B$. Such terms eliminate quartets of the form A, Ψ_A, B, Ψ_B in a BRST invariant way by their algebraic equations of motions. After some thinking, one understands that the following s-exact terms are needed

$$
\begin{aligned}
& s\big[\overline{\widetilde{\Phi}}^{(-2)ab^-}\big(\tilde{\eta}^{(1)ab^-} - \Omega^{ab^-}\big)\big] = \bar{\tilde{\eta}}^{(-1)ab^-}\big(\tilde{\eta}^{(1)ab^-} - \Omega^{ab^-}\big) + \overline{\widetilde{\Phi}}^{(-2)ab^-}\widetilde{\Phi}^{(2)ab^-}, \\
& s\big[\overline{\Psi}_{\mu\nu^+}^{(-1)}\widetilde{\Phi}^{(0)ab^+}e_a^\mu e_b^\nu\big] = \overline{\Psi}_{\mu\nu^+}^{(-1)}\tilde{\eta}^{(1)ab^+}e_a^\mu e_b^\nu + b_{\mu\nu^+}^{(0)}\widetilde{\Phi}^{(0)ab^+}e_a^\mu e_b^\nu, \\
& s\big[\overline{\Psi}_{\mu\nu^-}^{(-1)}A_{\mu\nu^-}^{(0)}\big] = \overline{\Psi}_{\mu\nu^-}^{(-1)}\Psi_{\mu\nu^-}^{(1)} + \bar{b}_{\mu\nu^-}^{(0)}A_{\mu\nu^-}^{(0)}, \\
& s\big[\overline{S}^{(-1)}A^{(0)}\big] = \overline{S}^{(-1)}\Psi^{(1)} + b_{\overline{S}^{(-1)}}^{(0)}A^{(0)}.
\end{aligned}
\qquad (5)
$$

The fields $\Phi^{(2)}, \overline{\Phi}^{(-2)}, \bar{\eta}^{(-1)}$ are also eliminated, together with the quartets for $A_{\mu\nu}^{(0)}$. The remaining fields are then:

$$
\begin{array}{ccccccc}
 & & & e_\mu^a & & & \\
 & \Psi_\mu^{(1)a} & & & \overline{\Psi}_\mu^{(-1)ab^-}, \bar{\chi}^{(-1)a} & & \\
\Phi^{(2)a} & & \sigma, \Phi^{(0)ab^-} & b_\mu^{(0)ab^-}, b_\mu^{(0)} & & \overline{\Phi}^{(-2)a} & \\
 & \chi^{(1)}, \eta^{(1)ab^-} & & & \bar{\eta}^{(-1)a} & &
\end{array}
$$

$$
\begin{array}{cccccc}
 & & \omega_\mu^{ab} & & & \\
 & \widetilde{\Psi}_\mu^{1ab} & & \overline{\widetilde{\Psi}}_\mu^{-1ab} & & \\
\widetilde{\Phi}^{2ab^+} & & \widetilde{\Phi}^{(0)ab^-}, \tilde{b}_\mu^{0ab} & & \overline{\widetilde{\Phi}}^{-2ab^+} & \\
 & & & \bar{\tilde{\eta}}^{-1ab^+} & &
\end{array}
$$

$$
\begin{array}{cccccccc}
 & & & B_{\mu\nu} & & & & \\
 & & \Psi_{\mu\nu}^{(1)} & & & & & \\
 & A_\mu^{(2)} & & & & A_\mu^{(-2)} & & \\
R^{(3)} & & S^{(1)} & & \overline{\Psi}_\mu^{(-1)} & & \overline{R}^{(-3)} & \\
 & b_{S^{(1)}}^{(2)} & & \Phi^{(0)}, \overline{\Phi}^{(0)} & & b_{\overline{R}^{(-3)}}^{(-2)} & & \\
 & & \eta^{(1)} & & & & &
\end{array}
\qquad (6)
$$

We will shortly see that the component $\Psi^{(1)}_{\mu\nu^+}$ of the topological ghost of the two-form $B_{\mu\nu}$ is gauge fixed in an algebraic way, leaving only $\Psi^{(1)}_{\mu\nu^-}$ as a propagating field.

Let us analyze the fields that remain after these eliminations. The fields that will a play a role as the classical fields of supergravity are:

$$\begin{aligned} &\text{bosons:} \quad e^a_\mu, \sigma, B_{\mu\nu}, A^{(2)}_\mu, A^{(-2)}_\mu, \\ &\text{fermions:} \quad \big(\Psi^{(1)a}_\mu, \overline{\Psi}^{(-1)ab^-}_\mu, \overline{\Psi}^{(-1)}_\mu\big), \big(\bar\chi^{(-1)a}, \Psi^{(1)ab^-}, \chi^{(1)}\big). \end{aligned} \tag{7}$$

Indeed, these are nothing but the fields of the $N=1$, $D=8$ supergravity multiplet, up to a twist. The most striking point of our construction is that the *topological* ghosts of ghosts of the two-form, $A^{(2)}_\mu$ and $A^{(-2)}_\mu$ can be interpreted as the propagating graviphotons of $N=1$, $D=8$ supergravity. The *Spin*(7)-scalar ghost of ghost σ, which has ghost number zero, can be interpreted as the dilaton. The corresponding topological ghosts can be recognized as the twisted version of the fermionic part of the spectrum. Modulo some field redefinitions that we will discuss in detail in the following section, the twisted gravitino can be identified with the ghosts $(\Psi^{(1)a}_\mu, \overline{\Psi}^{(-1)ab^-}_\mu, \overline{\Psi}^{(-1)}_\mu)$, and the twisted dilatino with $(\bar\chi^{(-1)a}, \Psi^{(1)ab^-}, \chi^{(1)})$. We remark that the BRST variation $sA^{(-2)}_\mu = \overline{\Psi}^{(-1)}_\mu$ appears in the twisted gravitino. This is in agreement with the analysis of [2,3], where we found that at least one graviphoton with nonzero ghost number is needed both in four and eight dimensions in order to introduce $\overline{\Psi}^{(-1)}_\mu$. As we will describe in the following section, the possibility of having *Spin*(7) decompositions allows one to do all relevant maps of tensors upon spinors.

The fields of the topological multiplets not appearing in Eq. (7) will be interpreted as the ordinary Faddeev–Popov ghosts and antighosts of the supergravity. The infinitesimal symmetry transformations of supergravity will be deduced from the topological BRST equations. After the eight-dimensional untwisting, one gets from Eq. (6) the propagating fields $(\Phi^{(2)a}, \Phi^{(0)ab-}, \Phi^{(0)})$, $(\overline{\Phi}^{(-2)a}, \widetilde{\Phi}^{(0)ab-}, \overline{\Phi}^{(0)})$ and $(\bar\eta^{(-1)a}, \eta^{(1)ab^-}, \eta^{(1)})$. These can be interpreted respectively as the twisted version of the Faddeev–Popov spinorial ghosts and antighosts for local supersymmetry and the corresponding Lagrange multipliers. The invariance of the untwisted theory therefore displays a variation of the vector-spinor by a derivative of these ghosts: this is presumably sufficient to ensure the full supersymmetry of the untwisted action. This is different from the case of topological Yang–Mills theory, where the complete supersymmetric invariance of the untwisted action is not automatic.

3. The topological action and its correspondence with supergravity

After having obtained the relevant field spectrum for the TQFT, the task is of finding the topological gauge functions.

A natural extension of the topological gauge function used in [3] in the presence of the two-form field $B_{\mu\nu}$ is given by imposing the octonionic self-duality condition

$$\tilde\omega^{ab} - \frac{1}{2}\Omega^{abcd}\tilde\omega^{cd} = 0, \tag{8}$$

on the extended connection

$$\tilde\omega^{ab} \equiv \omega^{ab} - G^{ab}_c e^c, \tag{9}$$

with torsion given by

$$\widetilde{T}^a = \widetilde{D}e^a = G^a_{bc}e^b e^c. \tag{10}$$

The condition (8) has to be considered together with the gauge function

$$d\sigma + 2 * (\Omega \wedge G_3) = 0 \tag{11}$$

involving the dilaton field. We thus write the topological action

$$\mathcal{L}_{e,B} = s\big[\overline{\Psi}^{(-1)ac^-}\big(b^{(0)cb^-} + \omega^{cb^-}(e) - G_d^{cb^-} e^d\big)\mathcal{V}_{ab} + \bar{\chi}^{(-1)a}\big(b_a^{(0)} + \partial_a\sigma + \Omega_{abcd}G_{bcd}\big)\big], \tag{12}$$

where we introduced the volume forms $\mathcal{V}_{a_1\ldots a_i} \equiv \frac{1}{(8-i)!}\epsilon_{a_1\ldots a_8} e^{a_{(i+1)}}\ldots e^{a_8}$. This topological action, after integration on the Lagrange multipliers $b^{(0)cb^-}$ and $b_a^{(0)}$, gives kinetic terms for the graviton, the two-form and the dilaton of $N=1$, $D=8$ supergravity.

First of all, the Einstein Lagrangian, written with the curvature $\widetilde{R}^{ab}$ of the extended connection $\tilde{\omega}$, is equal to the sum of the ordinary Einstein–Hilbert Lagrangian plus the squared norm of the field-strength G_3

$$\widetilde{\mathcal{L}} = \frac{1}{2}\widetilde{R}^{ab}\mathcal{V}_{ab} = \frac{1}{2}R^{ab}\mathcal{V}_{ab} - \frac{1}{2}G_c^{ab}G_c^{ab}\mathcal{V}. \tag{13}$$

The Bianchi identity on the field strength G_3 and (10) imply that

$$\widetilde{R}^{ab}e_b e_a + \widetilde{T}^a\widetilde{T}^a = 0. \tag{14}$$

By multiplying (14) by the self-dual four-form Ω we get

$$\frac{1}{2}\Omega_{abcd}\widetilde{R}^{cd}\mathcal{V}^{ab} = \Omega_{mnpq}G_{mn}^a G_{pq}^a\mathcal{V}. \tag{15}$$

The relation (15), together with the $Spin(7)$ decomposition

$$\widetilde{R}^{ab} = \widetilde{R}^{ab^+} + \widetilde{R}^{ab^-}, \qquad \frac{1}{2}\Omega_{abcd}\widetilde{R}^{ab} = \widetilde{R}^{cd^+} - 3\widetilde{R}^{cd^-}, \tag{16}$$

allows for the elimination of $\widetilde{R}^{ab^+}$ in the Lagrangian (13)

$$\begin{aligned}\widetilde{\mathcal{L}} &= \frac{1}{2}\widetilde{R}^{ab}\mathcal{V}_{ab} = 2\widetilde{R}^{ab^-}\mathcal{V}_{ab} + \frac{1}{2}\Omega_{abcd}G_e^{ab}G_e^{cd}\mathcal{V}\\ &= -4\tilde{\omega}^{ac^-}\tilde{\omega}^{cb^-}\mathcal{V}_{ab} + 2\tilde{\omega}^{ab^-}\widetilde{T}^c\mathcal{V}_{abc} + 2d\big(\tilde{\omega}^{ab^-}\mathcal{V}_{ab}\big) + \frac{1}{2}\Omega_{abcd}G_e^{ab}G_e^{cd}\mathcal{V},\end{aligned} \tag{17}$$

where in the last identity we used that $\widetilde{R}^{ab^-} = \widetilde{D}\tilde{\omega}^{ab^-} - 2\tilde{\omega}^{ac^-}\tilde{\omega}^{cb^-}$ and integrated by parts the term in $\widetilde{D}\tilde{\omega}^{ab^-}$. By comparing (17) with (13) and using (10) we can finally write the identity

$$4\tilde{\omega}^{ac^-}\tilde{\omega}^{cb^-}\mathcal{V}_{ab} = \mathcal{L}_{\mathrm{EH}} - \frac{1}{2}G_c^{ab}G_c^{ab}\mathcal{V} + \Omega_{abcd}G_f^{ab}G_f^{cd} + 4\omega^{ab^-}G_{ab}^c\mathcal{V}_c + 2d\big(\tilde{\omega}^{ab^-}\mathcal{V}_{ab}\big), \tag{18}$$

where we defined the Einstein–Hilbert action as $\mathcal{L}_{\mathrm{EH}} \equiv \frac{1}{2}R^{ab}\mathcal{V}_{ba}$. Let us consider now the square of the gauge function (11). By using the identity $\Omega_{abcd}\Omega^{afgh} = (6\delta_{bcd}^{[fgh]} - 9\Omega_{bc}^{[fg}\delta_d^{h]})$ is easy to find that

$$\left(\partial_a\sigma + \frac{1}{3}\Omega_{abcd}G^{bcd}\right)^2 = (\partial_a\sigma)^2 + \frac{2}{3}G_c^{ab}G_c^{ab} - \Omega_{abcd}G_f^{ab}G_f^{cd} + \frac{2}{3}\partial^a\big(\sigma\Omega_{abcd}G^{bcd}\big). \tag{19}$$

By summing (18) and (19) (multiplied by the suitable volume form $\mathcal{V}$), we finally get

$$\begin{aligned}&4\tilde{\omega}^{ac^-}\tilde{\omega}^{cb^-}\mathcal{V}_{ab} + \left(\partial_a\sigma + \frac{1}{3}\Omega_{abcd}G^{bcd}\right)^2\mathcal{V}\\ &\quad = \mathcal{L}_{\mathrm{EH}} + \left[(\partial_a\sigma)^2 + \frac{1}{6}G_c^{ab}G_c^{ab}\right]\mathcal{V} + 4\omega^{ab^-}G_{ab}^c\mathcal{V}_c + \text{ boundary terms.}\end{aligned} \tag{20}$$

This identity deserves some attention, since it shows that the sum of the 56 independent terms contained in $|G_{abc}(B)|^2$, plus those contained in the Einstein action can be obtained as a topological gauge-fixing term, stemming from 64 Lagrange multipliers.

We are now able to compare the topological terms (20) with the bosonic part of the action of $N=1$, $D=8$ supergravity. In the spirit of topological field theory, we can restrict our attention on the kinetic terms for the fields, which simplifies considerably the comparison. In fact, the basic requirement on the gauge-fixing conditions is that they must give a good definition for the propagators of the fields. Interaction terms can be then always added as BRST exact terms in order to get agreement with the complete twisted supergravity action. The first three terms of the topological action (20) correctly reproduce the kinetic terms for the graviton, the two-form and the dilaton of $N=1$, $D=8$ supergravity. Concerning now the mixed term $\omega_c^{ab^-}(e)G_{abc}$ in (20), it can be reduced in the quadratic approximation to

$$\partial_\mu e^a_\mu \Omega_{abcd} G_{bcd}. \tag{21}$$

This can be done by imposing the *Spin*(7) invariant Lorentz gauge condition $e_\mu^{[a} V^{b]^- \mu} = 0$, where $V^{b\mu}$ is an inverse 8-bein chosen as a reference system.[5] The expression (21) can be absorbed in the gauge-fixing term for the reparametrization invariance $s(\bar{\xi}^\mu \partial_\nu g_{\mu\nu})$. We thus conclude that the topological action (20) coincides with the bosonic part of the supergravity action in a given *Spin*(7)-invariant Lorentz gauge. This is not surprising since the condition (8) leaves only a residual *Spin*(7) symmetry group. It is remarkable that the kinetic energies of both the graviton and the two-form stem from the gauge-fixing term (8) that only comes from the topological freedom in the vielbein.

We now pass to the fermionic sector. The BRST variation of the first line in (12) gives part of the Rarita–Schwinger action, as in [3]. Notice that the condition $e_\mu^{[a} V^{b]^- \mu} = 0$, which must be enforced by the BRST exact term $s(\overline{\Omega}^{[ab]^-} e_\mu^{[a} V^{b]^- \mu})$, yields the condition $\Omega^{[ab]^-} = -\Psi_\mu^{[a} e^{b]^- \mu}$, and ensures the *Spin*(7) invariance of the fermionic part of the contribution of $\mathcal{L}_{e,B}$ to the Rarita–Schwinger action. There is actually a compensation between the BRST variations of $\omega_c^{ab^-}(e)$ and $G_c^{ac^-}(B)$, which is compulsory to enforce gauge invariance, since $\omega^{ab^-}(e)$ transforms as a connection for Lorentz transformations with self-dual parameter Ω^{ab^-}.

To determine the complete Rarita–Schwinger action, we must add to $\mathcal{L}_{e,B}$ the following term, as in [3]:

$$\mathcal{L}_{\overline{\Psi}_\mu} = s\big[\partial_{[\mu} A^{(-2)}_{\nu]^-} \Psi^{(1)a}_{[\mu} e^a_{\nu]}\big]. \tag{22}$$

Looking at the fermionic terms, using $sA_\mu^{(-2)} = \overline{\Psi}_\mu^{(-1)}$, $(\Psi_\mu^{(1)a}, \overline{\Psi}_\mu^{(-1)ab^-}, \overline{\Psi}_\mu^{(-1)})$ can be identified as the twisted gravitino, with 8 chiral and 8 antichiral components, as in [3].

To determine the propagation of $A_\mu^{(\pm 2)}$, we add:

$$s\big[\Psi^{(1)}_{\mu\nu} \partial_{[\mu} A^{(-2)}_{\nu]}\big] = \partial_{[\mu} A^{(2)}_{\nu]} \partial_{[\mu} A^{(-2)}_{\nu]} + \Psi^{(1)}_{\mu\nu} \partial_{[\mu} \Psi^{(-1)}_{\nu]}. \tag{23}$$

This identifies $A_\mu^{(\pm 2)}$ as the two graviphotons of $N=1, D=8$ supergravity. Notice that among the two graviphotons $A_\mu^{(-2)}$ and $A_\mu^{(2)}$, only the latter one has a "topological" transformation, since $sA_\mu^{(-2)} = \overline{\Psi}_\mu^{(-1)}$ while $sA_\mu^{(2)} = \partial_\mu R^{(3)}$. This is in agreement with the twisted supergravity transformations.

Part of the difficulty of this work was to understand the role of the ghost field $\Psi^{(1)}_{\mu\nu}$. On the one hand it is the field that generates by its ghost of ghost symmetry the second graviphoton of the supergravity. On the other hand, in supergravity, the Kalb–Ramond two-form only sees local supersymmetry through the gravitino and the dilatino, and it is challenging to uncover this from topological invariance. This leads to the conclusion that not all the components of $\Psi^{(1)}_{\mu\nu}$ are independent propagating fields. Only $\Psi^{(1)}_{\mu\nu^-}$ survives as an independent field stemming from the topological invariance of the two-form. To enforce the other components of $\Psi^{(1)}_{\mu\nu}$ in a BRST invariant

[5] In the physicist language, this means that one uses the Lorentz invariance to impose that 7 of the components of e^a_μ vanish in a *Spin*(7) invariant way. Eventually, the rest of the 21 Lorentz degrees of freedom can be used to enforce that e^a_μ be a symmetrical 8×8 matrix, that is $e^a_\mu = V^{b\mu}(\delta^{ab} + h^{ab})$, where h^{ab} is symmetrical in a and b. Then, there is a one-to-one mapping between the metric and this matrix.

way, we consider the following action, that exhausts the ghost of ghost symmetry in the Lorentz sector:

$$s\big[e^\mu_c e^\nu_d \overline{\widetilde{\Phi}}^{(-2)cd^+}\big(\Psi^{(1)}_{\mu\nu} - \Psi^{(1)a}_{[\mu} e^a_{\nu]}\big)\big] = e^\mu_c e^\nu_d \bar{\tilde{\eta}}^{(-1)cd^+}\big(\Psi^{(1)}_{\mu\nu} - \Psi^{(1)a}_{[\mu} e^a_{\nu]}\big) + \overline{\widetilde{\Phi}}^{ab^+}\big(\partial_{[a} A^{(2)}_{b]} - \widetilde{\Phi}^{(2)}_{ab} + \cdots\big). \quad (24)$$

This gauge-fixing allows one to identify $\Psi^{(1)}_{\mu\nu^+} = \Psi^{(1)a}_{[\mu} e^a_{\nu]^+}$. If we define

$$\Psi^{(1)}_{\mu\nu} = \chi^{(1)}_{\mu\nu^-} + \Psi^{(1)a}_{[\mu} e^a_{\nu]} \quad (25)$$

then $\chi^{(1)}_{\mu\nu^-}$ and $\chi^{(1)} = s\sigma$ are eight fermionic variables that can be identified with a twisted chiral component of the dilatino. $\bar{\chi}^{(-1)\mu}$ determines by twist the other chiral component. Moreover, Eq. (24) accomplishes the elimination of the ghost of ghost dependence in the Lorentz sector, by yielding an algebraic equation of motion for $\widetilde{\Phi}^{(2)ab^+}$ and $\overline{\widetilde{\Phi}}^{(-2)ab^+}$, in a *Spin*(7) invariant way.

Let us summarize the mapping between the fermionic degrees of freedom of the topological action defined by (12)–(24) and that of $N = 1$ supergravity. On a manifold with *Spin*(7) holonomy there exist a covariantly constant spinor (of norm one) ε,[6] which can be used to redefine as in [3] the gravitino $(\lambda, \bar{\lambda})$ and the dilatino $(\chi, \bar{\chi})$ of $N = 1$, $D = 8$ supergravity as[7]

$$\lambda = \Psi^a \gamma_a \varepsilon, \qquad \bar{\lambda} = \overline{\Psi}\varepsilon + \overline{\Psi}^{ab^-}\gamma_{ab}\varepsilon, \quad (26)$$

$$\chi = \bar{\chi}^a \gamma^a \varepsilon, \qquad \bar{\chi} = \chi\varepsilon + \chi^{ab^-}\gamma_{ab}\varepsilon. \quad (27)$$

On the l.h.s. of (26), (27) $(\lambda, \bar{\lambda})$ and $(\chi, \bar{\chi})$ are spinors of opposite chiralities. The eight-dimensional gamma matrices γ_a acts on spinors of definite chirality. Notice that the identification (25) implies the appearance in the topological action (12) of the mixed kinetic terms $\overline{\Psi}^{ac^-}\partial_d \chi^{cb^-} e^d \mathcal{V}_{ab}$, coming from the BRST-variation of the first line in (12), and $\bar{\chi}^a \Omega_{abcd}\partial_b \Psi_{cd}$, coming from the BRST-variation of the second line. In order to recover these terms from the twisted supergravity theory we have to impose the field redefinitions $\Psi^a_b \to \Psi^a_b + \chi^{ab^-}$ and $\overline{\Psi}^{ab^-}_c \to \overline{\Psi}^{ab^-}_c + \delta^{[a}_c \bar{\chi}^{b]^-}$. Moreover, since for the bosonic sector the equivalence of the topological and supergravity actions is valid only in a fixed *Spin*(7)-invariant Lorentz gauge, we expect that also some gauge-fixing terms of the same kind are involved in the comparison of the fermionic part. From Eqs. (26) and (27) we see that, modulo the above field redefinitions, the gravitino is mapped to the fields $(\Psi^a, \overline{\Psi}, \overline{\Psi}^{ab^-})$ of the topological model, while the dilatino is mapped to the fields $(\bar{\chi}^a, \chi, \chi^{ab^-})$.

What we have found is interesting. The topological gauge functions are such that the BRST transformation in the effective topological action for e and B is:

$$s e^a_\mu = \Psi^{(1)a}_\mu + \Omega^{ab} e^b_\mu + \cdots, \qquad s B_{\mu\nu} = \Psi^{(1)a}_{[\mu} e^a_{\nu]} + \chi^{(-1)}_{\mu\nu^-} + \cdots. \quad (28)$$

Only the symmetrical part of $\Psi^{(1)a}_\mu$ is involved in $s e^a_\mu$, while the Lorentz ghost Ω^{ab} allows one to put to zero the antisymmetrical part of e^a_μ. This explains how the supersymmetric transformation law of the two-form in the supergravity framework can be interpreted in a topological way, using a suitable gauge function for the topological invariance. What actually happens is that, when one twists the gravitino Ψ^α_μ into $\Psi^{(1)a}_\mu$, and defines $\Psi^{(1)}_{\mu\nu} = e_{a\nu}\Psi^{(1)a}_\mu$, then $\Psi^{(1)}_{\{\mu\nu\}}$ and $\Psi^{(1)}_{[\mu\nu]}$ are respectively the topological ghosts of the two-form $B_{\mu\nu}$ and of the metric $g_{\mu\nu}$. Here, the mapping between the spinors is not linear, since it involves various contractions by the vielbein. This a important distinction with the case of the Yang–Mills TQFT, where the mapping is a linear transformation.

Some extra topological functions are needed in order to fix the symmetries of the gauge conditions used so far. These functions exhaust the remaining fields of the BRST topological multiplets. Let us briefly sketch

[6] This spinor can be used to define the self-dual four form as $\Omega_{abcd} = \varepsilon^T \gamma_{abcd}\varepsilon$ [7].

[7] Similar results concerning the twist of $N = 1$, $D = 8$ supergravity have been obtained by P. de Medeiros and B. Spence.

them. To take care of the gauge invariance of $\overline{\Psi}_\mu^{(-1)ab^-}$, which follows from the gauge functions, we redefine $b_\mu^{(0)ab^-} \to b_\mu^{(0)ab^-} + \partial_\mu \widetilde{\Phi}^{(0)ab^-}$. Then, to gauge fix the local supersymmetry, which pops up as the gauge invariance of the topological ghosts, we add, as in [2,3]

$$\mathcal{L}_{\text{ghosts}} = s\big[\sqrt{g}\,\big(\overline{\Phi}^{(-2)a} D_\mu \Psi_\mu^{(1)a} + \Phi^{(0)ab^-} D_\mu \overline{\Psi}_\mu^{(-1)ab^-} + \overline{\Phi}^{(0)} \partial_\mu \overline{\Psi}_\mu^{(-1)}\big)\big]. \tag{29}$$

The role of this redefinition of b^{ab^-} has been analyzed in [2,3]. It ensures the propagation of the field $\widetilde{\Phi}^{(0)ab^-}$. The expression of this action identifies $(\Phi^{(2)a}, \Phi^{(0)ab-}, \Phi^{(0)})$, $(\overline{\Phi}^{(-2)a}, \widetilde{\Phi}^{(0)ab-}, \overline{\Phi}^{(0)})$ and $(\bar{\eta}^{(-1)a}, \eta^{(1)ab^-}, \eta^{(1)})$ as the twisted version of the Faddeev–Popov spinorial ghost and antighosts for local supersymmetry, and their fermionic Lagrange multipliers, respectively.

We also use the topological gauge freedom of the spin connection ω^{ab} to eliminate this field in terms of e, by mean of the term

$$s\big[\overline{\widetilde{\Psi}}^{(-1)ab} e_b \wedge *T^a\big], \tag{30}$$

where $*T$ is the Hodge dual of the torsion $T = de + \omega \wedge e$. This gauge fixing, which can be improved by changing $T^a \to T^a + G^a_{bc} e^b e^c$, also trivially eliminates the dependence of the action upon the topological Lorentz ghosts $\widetilde{\Psi}_\mu^{(1)ab}$ and $\overline{\widetilde{\Psi}}_\mu^{(-1)ab}$, which disappear by their algebraic equations of motion. One must recognize that introducing the Lorentz symmetry is extremely useful, although most of its ingredients are eventually eliminated.

As for the fields $S^{(1)}$ and $R^{(-3)}$, they are used to fix the ordinary gauge symmetry of $A^{(-2)}$ and $A^{(2)}$ by the action

$$s\big[S^{(1)} \partial_\mu A_\mu^{(-2)} + \overline{R}^{(-3)} \partial_\mu A_\mu^{(2)}\big]. \tag{31}$$

To impose that the vielbein is a symmetrical matrix, and eliminate the Ω and $\overline{\Omega}$ dependence, we just add:

$$s\big[\overline{\Omega}^{ab^+} e_\mu^b V^{\mu a}\big] = \overline{\Omega}^{ab^+}\big(\Omega^{ab} + \cdots\big) + b^{ab^+} e_\mu^b V^{\mu a}, \tag{32}$$

keeping in mind that we had already used a term $s[\overline{\Omega}^{ab^-} e_\mu^b V^{\mu a}]$. After expansion, and a few field redefinitions, one gets that Ω^{ab} and $\overline{\Omega}^{ab}$ are eliminated by Gaussian integration. As said above, this necessitates the introduction of an inverse vielbein $V^{a\mu}$ as background.

Last of the last, we must add $s[\bar{\xi}^{-1\mu} \partial_\nu g_{\mu\nu}]$ to fix the reparametrization invariance.

The gauge-fixing of the gauge symmetries of topological gauge functions could have been done in a much more refined way, using the technology of equivariant cohomology.[8] This is a technicality that we have chosen not to present here. It would distract us from our main result, that is, we have finally end up our task of building a TQFT for the Kalb–Ramond field $B_{\mu\nu}$ within the context of topological gravity. The result is that the standard TQFT procedure has lead us to a twisted version of $N = 1$, $D = 8$ supergravity.

It is worth noticing that the topological gauge functions on the extended connection (8) and on the dilaton (11) are the same appearing in the octonionic superstring equations [9]. By coupling our model with a non-Abelian topological Yang–Mills theory, as it is defined in [7,8], one could thus obtain a topological theory which effectively describes the transverse properties of the octonionic superstring.

We believe that it will be interesting to study the possible dimensional reductions of the 8-dimensional topological gravity. This idea has already proven to be quite useful in the simpler case of the topological Yang–Mills theory. The reduction to seven dimensions of the octonionic self-duality conditions on the spin connection is of relevance for manifolds with (weak) G_2 holonomy [10]. The study of the dimensional reduction of the generalized self-duality conditions (8) and (11) could give a more general description of manifolds with G_2-structure.

[8] As remarked in the previous section, the equivariance is with respect to reparametrizations, $Spin(7) \subset SO(8)$ Lorentz invariance and two-form gauge symmetry.

Moreover, one may investigate the possibility of getting a generalization of the Seiberg–Witten theory involving gravity. Generalizing the flat space analysis of [7], this theory could be derived as an adequate dimensional reduction in four dimensions of the 8-dimensional topological gravity. Further dimensional reductions can give interesting models in 2 and 0 dimensions.

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