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Solving a system of nonlinear fractional differential equations using Adomian decomposition

Hossein Jafari, Varsha Daftardar-Gejji*

Department of Mathematics, University of Pune, Ganeshkhind, Pune 411007, India

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Abstract

Adomian decomposition method has been employed to obtain solutions of a system of nonlinear fractional differential equations:

$$D^{\alpha_i} y_i(x) = N_i(x, y_1, \ldots, y_n), \quad y_i^{(k)}(0) = c_k^i, \quad 0 \leqslant k \leqslant [\alpha_i], \quad 1 \leqslant i \leqslant n \text{ and } D^{\alpha_i}$$

denotes Caputo fractional derivative.

Some examples are solved as illustrations, using symbolic computation.

Keywords: System of nonlinear fractional differential equations; Adomian decomposition; Adomian polynomials; Caputo fractional derivative; Fractional integral

1. Introduction

Various methods, for example, Laplace and Fourier transforms, have been utilized to solve linear Fractional Differential Equations [FDE] [11–13]. In contrast for solving the nonlinear FDE, one has to depend upon numerical solutions solely [8,9]. Recently developed technique of Adomian decomposition [2] has proven to be a powerful method, and has successfully been applied in a variety of problems. Biazar et al. [4] have employed the Adomian decomposition to solve a system of ordinary differential equations and system of Volterra integral equations [3] as well. Wazwaz [15] has explored this method to obtain solutions of wave equation. Shawagfeh [14] has employed Adomian decomposition in case of the nonlinear fractional differential equation: $D^\alpha y(x) = f(x, y)$, $y^{(k)}(0) = c_k$, $0 \leqslant k \leqslant [\alpha]$. Adomian decomposition offers certain advantages over routine numerical methods. Numerical methods use discretization which gives rise to rounding off errors causing loss of accuracy, and requires large computer power and time. Adomian decomposition method is better since it does not involve discretization of the variables hence is free from rounding off errors and does not require large computer memory or time.

Daftardar-Gejji and Babakhani [6] have presented analysis of system of FDE, wherein existence and uniqueness theorems for the initial value problem for the system of FDE have been proved. Following this Daftardar-Gejji and

* Corresponding author. Tel.: +91 25601272; fax: +91 205693899.
E-mail addresses: jafari_h@math.com (H. Jafari), vsgejji@math.unipune.ernet.in (V. Daftardar-Gejji).

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Jafari [7] have taken up the problem of finding explicit solutions for system of FDE and have developed decomposition method for the following system of linear FDE:

$$D^{\alpha_i} y_i(x) = \sum_{j=1}^{n} (\phi_{ij}(x) + \gamma_{ij} D^{\alpha_{ij}}) y_j + g_i(x), \quad y_i^{(k)}(0) = c_k^i,\ 0 \leqslant k \leqslant [\alpha_i],\ 1 \leqslant i \leqslant n,$$

where $\alpha_i, \alpha_{ij} \in \Re^+$.

The present paper is a sequel to this work [7] and here Adomian method has been applied to a more general case incorporating nonlinearities as well, *namely*

$$D^{\alpha_i} y_i(x) = N_i(x, y_1, \ldots, y_n), \quad y_i^{(k)}(0) = c_k^i,\ 0 \leqslant k \leqslant [\alpha_i],\ 1 \leqslant i \leqslant n,$$

where N_i's are linear/nonlinear functions of $x, y_1, \ldots, y_n$.

The paper has been organized as follows. Section 2 gives notations and basic definitions. Section 3 consists of main results of the paper, in which Adomian decomposition of the system of fractional differential equations has been developed. Some illustrative examples are given in Section 4 followed by the discussion and conclusions presented in Section 5. Mathematica commands used to compute Adomian polynomials and terms of the decomposition series, are given explicitly in Appendix.

2. Basic definitions

Definition 2.1. A real function $f(x),\ x > 0$ is said to be in the space C_α, $\alpha \in \Re$ if there exists a real number $p(> \alpha)$, such that $f(x) = x^p f_1(x)$ where $f_1(x) \in C[0, \infty)$. Clearly $C_\alpha \subset C_\beta$ if $\beta \leqslant \alpha$.

Definition 2.2. A function $f(x), x > 0$ is said to be in the space C_α^m, $m \in N \cup \{0\}$, if $f^{(m)} \in C_\alpha$.

Definition 2.3. The left sided Riemann–Liouville fractional integral of order $\mu \geqslant 0$, [10–13] of a function $f \in C_\alpha$, $\alpha \geqslant -1$ is defined as

$$I^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_0^x \frac{f(t)}{(x-t)^{1-\mu}}\, \mathrm{d}t,\ \ \mu > 0,\ \ x > 0,\ \ I^0 f(x) = f(x). \tag{1}$$

Definition 2.4. Let $f \in C_{-1}^m$, $m \in N \cup \{0\}$. Then the (left sided) Caputo fractional derivative of f is defined as [10,12]

$$D^\mu f(x) = \begin{cases} [I^{m-\mu} f^{(m)}(x)] & m-1 < \mu \leqslant m, \quad m \in \mathbb{N}, \\ \dfrac{\mathrm{d}^m f(x)}{\mathrm{d}x^m} & \mu = m. \end{cases} \tag{2}$$

Note that [10,12]

$$I^\mu I^\nu f = I^{\mu+\nu} f, \quad \mu, \nu \geqslant 0,\ \ f \in C_\alpha,\ \ \alpha \geqslant -1,$$

$$I^\mu x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\mu+1)} x^{\gamma+\mu}, \quad \mu > 0,\ \ \gamma > -1,\ \ x > 0,$$

$$I^\mu D^\mu f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{x^k}{k!}, \quad m-1 < \mu \leqslant m. \tag{3}$$

3. System of fractional differential equations and Adomian decomposition

In the present paper we consider the following system of fractional differential equations:

$$D^{\alpha_i} y_i(x) = N_i(x, y_1, \ldots, y_n), \quad y_i^{(k)}(0) = c_k^i, \quad 0 \leqslant k \leqslant [\alpha_i], \tag{4}$$

where $1 \leqslant i \leqslant n$, and $\alpha_i \in \Re^+$.

Applying I^{α_i} to both the sides of (4), we get

$$y_i = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{x^k}{k!} + I^{\alpha_i} N_i(x, y_1, \ldots, y_n), \quad 1 \leqslant i \leqslant n. \tag{5}$$

We employ Adomian decomposition method to solve the system of equations (4). Let

$$y_i = \sum_{m=0}^{\infty} y_{im} \tag{6}$$

and

$$N_i(x, y_1, \ldots, y_n) = \sum_{m=0}^{\infty} A_{im}, \tag{7}$$

where A_{im} are Adomian polynomials which depend upon $y_{10}, \ldots, y_{1m}, y_{20}, \ldots, y_{2m}, \ldots, y_{n0}, \ldots, y_{nm}$. In view of Eqs. (6) and (7), (5) takes the form

$$\sum_{m=0}^{\infty} y_{im} = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{x^k}{k!} + I^{\alpha_i} \sum_{m=0}^{\infty} A_{im}(y_{10}, \ldots, y_{1m}, \ldots, y_{n0}, \ldots, y_{nm}), \quad 1 \leqslant i \leqslant n. \tag{8}$$

We set

$$y_{i0}(x) = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{x^k}{k!};$$

$$y_{i,m+1}(x) = I^{\alpha_i} A_{im}(y_{10}, \ldots, y_{1m}, \ldots, y_{n0}, \ldots, y_{nm}), \quad 1 \leqslant i \leqslant n, \quad m = 0, 1, \ldots . \tag{9}$$

In order to determine the Adomian polynomials, we introduce a parameter λ and (7) becomes

$$N_i\left(x, \sum_{m=0}^{\infty} y_{1m}\lambda^m, \ldots, \sum_{m=0}^{\infty} y_{nm}\lambda^m\right) = \sum_{m=0}^{\infty} A_{im}\lambda^m. \tag{10}$$

Let $y_{i\lambda}(x) = \sum_{m=0}^{\infty} y_{im}(x)\lambda^m$, then

$$A_{im} = \frac{1}{m!}\left[\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N_{i\lambda}(y_1, \ldots, y_n)\right]_{\lambda=0}, \tag{11}$$

where

$$N_{i\lambda}(y_1, \ldots, y_n) = N_i(x, y_{1\lambda}, \ldots, y_{n\lambda}). \tag{12}$$

In view of (11), and (12) we get

$$\begin{aligned} A_{im} &= \frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m}[N_i(x, y_{1\lambda}, \ldots, y_{n\lambda})]_{\lambda=0} = \frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m}\left[N_i\left(x, \sum_{m=0}^{\infty} y_{1m}\lambda^m, \ldots, \sum_{m=0}^{\infty} y_{nm}\lambda^m\right)\right]_{\lambda=0} \\ &= \left[\frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N_i\left(x, \sum_{m=0}^{\infty} y_{1m}\lambda^m, \ldots, \sum_{m=0}^{\infty} y_{nm}\lambda^m\right)\right]_{\lambda=0}. \end{aligned} \tag{13}$$

Hence (9) and (13) lead to the following recurrence relations:

$$y_{i0}(x) = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{x^k}{k!}, \quad y_{i,m+1}(x) = I^{\alpha_i}\left[\frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N_i\left(x, \sum_{m=0}^{\infty} y_{1m}\lambda^m, \ldots, \sum_{m=0}^{\infty} y_{nm}\lambda^m\right)\right]_{\lambda=0}, \quad m = 0, 1, \ldots . \tag{14}$$

We can approximate the solution y_i by the truncated series

$$f_{ik} = \sum_{m=0}^{k-1} y_{im}, \quad \lim_{k\to\infty} f_{ik} = y_i(x).$$

For the convergence of the above method we refer the reader to the work of Abboui and Cherruault [1]. If system (4) admits unique solution, then this method will produce the unique solution. If system (4) does not possess unique solution, the decomposition method will give a solution among many (possible) other solutions.

4. Illustrative examples

To demonstrate the effectiveness of the method we consider here some systems of nonlinear fractional differential equations. Daftardar-Gejji and Babakhani [6] have presented analysis of such a system.

I. Consider the system of nonlinear fractional differential equations

$$\begin{cases} D^\alpha y_1 = y_1^2 + y_2, \\ D^\beta y_2 = y_2 \cos y_1, \end{cases} \quad y_1(0) = 0, \;\; y_2(0) = 1,$$

where $\alpha, \beta \in (0, 1)$. In view of the results obtained by Daftardar-Gejji and Babakhani [6], this system has unique solution. In order to solve the above system, we define the nonlinear terms by

$$N_1(\bar{y}) = y_1^2 + y_2 = \sum_{j=0}^{\infty} A_{1j}, \quad N_2(\bar{y}) = y_2 \cos y_1 = \sum_{j=0}^{\infty} A_{2j}.$$

In view of (13) and using Mathematica software, we evaluate the Adomian polynomials. They are as follows:

$$\begin{aligned}
A_{10} &= y_{10}^2 + y_{20}, \\
A_{11} &= 2y_{10}y_{11} + y_{21}, \\
A_{12} &= y_{11}^2 + 2y_{10}y_{12} + y_{22}, \\
A_{13} &= 2y_{11}y_{12} + 2y_{10}y_{13} + y_{23}, \\
A_{14} &= y_{12}^2 + 2y_{11}y_{13} + 2y_{10}y_{14} + y_{24}, \\
&\;\;\vdots
\end{aligned}$$

and

$$\begin{aligned}
A_{20} &= y_{20} \cos y_{10}, \\
A_{21} &= -y_{11}y_{20} \sin y_{10} + y_{21} \cos y_{10}, \\
A_{22} &= \tfrac{1}{2} y_{11}^2 y_{20} \cos y_{10} - y_{12}y_{20} \sin y_{10} - y_{11}y_{21} \sin y_{10} + y_{21} \cos y_{10}, \\
A_{23} &= \tfrac{1}{6}((y_{11}^3 y_{20} \sin y_{10} - 6y_{11}y_{12} \sin y_{10} - 6y_{13} \sin y_{10}) y_{20} \\
&\quad -3(y_{11}^2 \cos y_{10} + y_{12} \sin y_{10}) y_{21} - 6y_{11}y_{22} \sin y_{10} + 6y_{23} \cos y_{10}), \\
A_{24} &= \tfrac{1}{24}((y_{11}^4 \cos y_{10} + 12y_{11}^2 y_{12} \sin y_{10} - 24y_{11}y_{13} \cos y_{10} - 12(y_{12}^2 \cos y_{10} \\
&\quad +2y_{14} \sin y_{10})) y_{20} + 4(y_{11}^3 \sin y_{10} - 6y_{11}y_{12} \cos y_{10} - 6y_{13} \sin y_{10}) y_{21} \\
&\quad -12(y_{11}^2 \cos y_{10} + y_{12} \sin y_{10}) y_{22} - 24y_{11}y_{23} \sin y_{10} + 24y_{24} \cos y_{10}), \\
&\;\;\vdots
\end{aligned}$$

The Adomian decomposition series (9) leads to the following scheme:

$$y_{10} = 0, \quad y_{1,m+1} = I^\alpha A_{1m},$$

$$y_{20} = 1, \quad y_{2,m+1} = I^\beta A_{2m}, \quad m = 0, 1, \ldots .$$

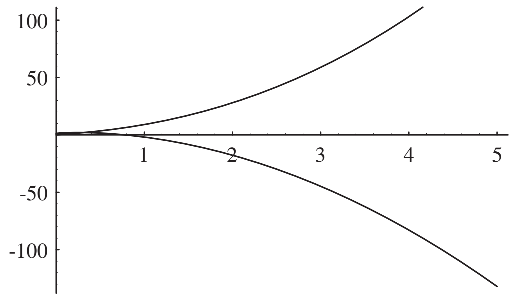

Fig. 1.

In the first iteration we have

$$y_{11} = I^{\alpha} A_{10} = \frac{x^{\alpha}}{\Gamma(\alpha+1)} \quad \text{and} \quad y_{21} = -I^{\beta} A_{20} = \frac{x^{\beta}}{\Gamma(\beta+1)}.$$

The subsequent terms are

$$y_{12} = I^{\alpha} A_{11} = I^{\alpha} \frac{x^{\beta}}{\Gamma(\beta+1)} = \frac{x^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)},$$

$$y_{22} = I^{\beta} A_{21} = I^{\beta} \frac{x^{\beta}}{\Gamma(\beta+1)} = \frac{x^{2\beta}}{\Gamma(2\beta+1)},$$

$$y_{13} = I^{\alpha} A_{12} = I^{\alpha} \left(\frac{x^{2\alpha}}{[\Gamma(\alpha+1)]^2} + \frac{x^{2\beta}}{\Gamma(2\beta+1)} \right) = \frac{\Gamma(2\alpha+1) x^{3\alpha}}{[\Gamma(\alpha+1)]^2 \Gamma(3\alpha+1)} + \frac{x^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)},$$

$$y_{23} = I^{\beta} A_{22} = I^{\beta} \left(-\frac{x^{2\alpha}}{2[\Gamma(\alpha+1)]^2} + \frac{x^{2\beta}}{\Gamma(2\beta+1)} \right) = -\frac{\Gamma(2\alpha+1) x^{2\alpha+\beta}}{[\Gamma(\alpha+1)]^2 \Gamma(2\alpha+\beta+1)} + \frac{x^{3\beta}}{\Gamma(3\beta+1)}.$$

Using the above terms

$$y_1 = \frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{x^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{\Gamma(2\alpha+1) x^{3\alpha}}{[\Gamma(\alpha+1)]^2 \Gamma(3\alpha+1)} + \frac{x^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + \cdots,$$

$$y_2 = 1 + \frac{x^{\beta}}{\Gamma(\beta+1)} + \frac{x^{2\beta}}{\Gamma(2\beta+1)} - \frac{\Gamma(2\alpha+1) x^{2\alpha+\beta}}{[\Gamma(\alpha+1)]^2 \Gamma(2\alpha+\beta+1)} + \frac{x^{3\beta}}{\Gamma(3\beta+1)} + \cdots.$$

In Fig. 1 we draw y_1 and y_2 for $\alpha = 0.5$, $\beta = 0.3$.

II. Consider the system of nonlinear fractional differential equations

$$\begin{cases} D^{\alpha} y_1 = 2y_2^2, \\ D^{\beta} y_2 = x y_1, \qquad y_1(0) = 0, \ \ y_2(0) = 1, \ \ y_3(0) = 1, \\ D^{\gamma} y_3 = y_2 y_3, \end{cases}$$

where $\alpha, \beta, \gamma \in (0, 1)$. In order to solve the above system, we define the nonlinear terms by

$$N_1(\bar{y}) = 2y_2^2 = \sum_{j=0}^{\infty} A_{1j}, \quad N_2(\bar{y}) = x y_1 = \sum_{j=0}^{\infty} A_{2j}, \quad N_3(\bar{y}) = y_2 y_3 = \sum_{j=0}^{\infty} A_{3j}.$$

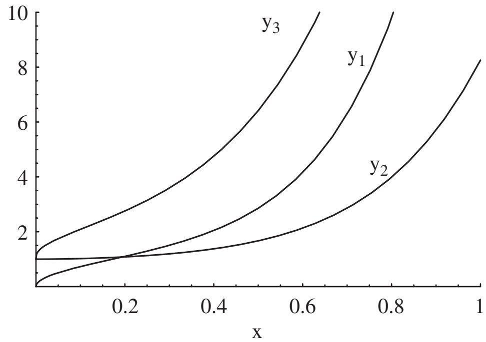

Fig. 2.

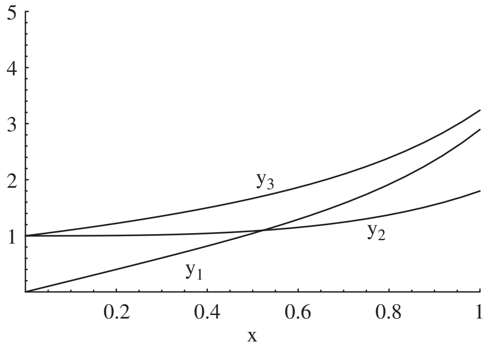

Fig. 3.

In view of (13) and using Mathematica software, we evaluate the corresponding Adomian polynomials $A_{ij}, i = 1, 2, 3$ and $j = 0, 1, \ldots$

$$
\begin{array}{ll}
A_{10} = 2y_{20}^2, & A_{20} = xy_{10}, \\
A_{11} = 4y_{20}y_{21}, & A_{21} = xy_{11}, \\
A_{12} = 2y_{21}^2 + 4y_{20}y_{22}, & A_{22} = xy_{12}, \\
A_{13} = 4y_{21}y_{22} + 4y_{20}y_{23}, & A_{23} = xy_{13}, \\
A_{14} = 2y_{22}^2 + 4y_{21}y_{23} + 4y_{20}y_{24}, & A_{24} = xy_{14}, \\
\vdots & \vdots
\end{array}
$$

$$
\begin{aligned}
A_{30} &= y_{20}y_{30}, \\
A_{31} &= y_{21}y_{30} + y_{20}y_{31}, \\
A_{32} &= y_{22}y_{30} + y_{21}y_{31} + y_{20}y_{32}, \\
A_{33} &= y_{23}y_{30} + y_{22}y_{31} + y_{21}y_{32} + y_{20}y_{33}, \\
A_{34} &= y_{24}y_{30} + y_{23}y_{31} + y_{22}y_{32} + y_{21}y_{33} + y_{20}y_{34}, \\
&\vdots
\end{aligned}
$$

The Adomian decomposition series (9) has the following terms:

$$
\begin{cases} y_{10} = 0, \\ y_{1,m+1} = I^{\alpha} A_{1m}, \end{cases} \quad \begin{cases} y_{20} = 1, \\ y_{2,m+1} = I^{\beta} A_{2m}, \end{cases} \quad \begin{cases} y_{30} = 1, \\ y_{3,m+1} = I^{\gamma} A_{3m}, \end{cases} \quad m = 0, 1, \ldots .
$$

In the first iteration we have

$$y_{11} = I^{\alpha} A_{10} = \frac{2x^{\alpha}}{\Gamma(\alpha+1)}, \quad y_{21} = I^{\beta} A_{20} = 0 \quad \text{and} \quad y_{31} = I^{\gamma} A_{30} = \frac{x^{\gamma}}{\Gamma(\gamma+1)}.$$

In Fig. 2, y_1, y_2 and y_3 are drawn for $\alpha=0.5$, $\beta=0.4$, $\gamma=0.3$, and in Fig. 3, y_1, y_2 and y_3 are drawn for $\alpha=\beta=\gamma=1$, respectively.

5. Discussion and conclusion

Adomian decomposition is a powerful tool which enables one to handle even nonlinear equations. Unlike in numerical methods, Adomian decomposition method is free from rounding off errors and does not require large computer memory or time. The real hard part of this method is computation of Adomian polynomials. It is demonstrated that using capabilities of Mathematica, Adomian polynomials and terms of Adomian decomposition series can be evaluated. By increasing the number of iterations one can reach desired accuracy. In the present work the method has successfully been applied to system of nonlinear fractional differential equations.

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Appendix

We give Mathematica commands, using which the Adomian polynomials can be evaluated. For the sake of illustration, we consider the illustrative example II. We would like to comment here that, these commands are much easier in contrast to the lengthy algorithm given in [5].

(*Define the number of iterations by specifying explicit number in place of n.*)

$$\text{Itr} := n;$$

(* Define the functions Y_1, Y_2 and Y_3*)

$$Y_1[\lambda\_] = \sum_{i=0}^{\text{Itr}} y_{1,i}; \quad Y_2[\lambda\_] = \sum_{i=0}^{\text{Itr}} y_{2,i}; \quad Y_3[\lambda\_] = \sum_{i=0}^{\text{Itr}} y_{3,i};$$

(* Define the functions N_1, N_2 and N_3*)

$$N_1[\lambda\_] = 2y_2[\lambda]\hat{\ }2; \quad N_2[\lambda\_] = y_1[\lambda] * t; \quad N_3[\lambda\_] = y_2[\lambda] * y_3[\lambda];$$

$$B_{1,0} = N_1[0];\ A_{1,0} = B_{1,0}; \quad B_{2,0} = N_2[0];\ A_{2,0} = B_{2,0}; \quad B_{3,0} = N_3[0];\ A_{3,0} = B_{3,0};$$

(*Calculation of Adomian polynomials*)

$$\text{For } [i = 0,\ i < \text{Itr},\ B_{1,i} = \text{Simplify}\left(\left[\frac{1}{i!} * \text{ Derivative } [i][N_1][\lambda]\right)\right];$$

$$B_{2,i} = \text{Simplify}\left[\frac{1}{i!} * (\text{Derivative } [i][N_2][\lambda])\right]; B_{3,i} = \text{Simplify}\left[\frac{1}{i!} * (\text{Derivative } [i][N_3][\lambda])\right],$$

$$\lambda = 0;\ A_{1,i} = B_{1,i}/.\lambda \to 0;\ A_{2,i} = B_{2,i}/.\lambda \to 0;\ A_{3,i} = B_{3,i}/.\lambda \to 0;\ \lambda = .;\ i++]$$

(* Specifying the initial conditions *)

$$y_{1,0} = 0; \quad y_{2,0} = 1; \quad y_{3,0} = 1; \quad \alpha = 0.5; \quad \beta = 0.4; \quad \gamma = 0.3;$$

$$Y_{1,0} = y_{1,0}; \quad Y_{2,0} = y_{2,0}; \quad Y_{3,0} = y_{3,0};$$

(*Calculating the terms of the series*)

$$\text{Do}\ \Big[n = k+1;\ y_{1,n+1} = \frac{1}{\Gamma(\alpha)}\ \text{Integrate}\ [(x-t)^\wedge(\alpha-1) * A_{1,n}, (t,0,x),\ \text{Assumption}\ \to x>0];$$

$$y_{2,n+1} = \frac{1}{\Gamma(\beta)}\ \text{Integrate}\ [(x-t)^\wedge(\beta-1) * A_{2,n}, (t,0,x),\ \text{Assumption} \to x>0];$$

$$y_{3,n+1} = \frac{1}{\Gamma(\gamma)}\ \text{Integrate}\ [(x-t)^\wedge(\gamma-1) * A_{3,n}, (t,0,x),\ \text{Assumption}\ \to x>0];$$

$$y_{1,n+1} = y_{1,n+1}/.x \longrightarrow t;\ y_{2,n+1} = y_{2,n+1}/.x \longrightarrow t;\ y_{3,n+1} = y_{3,n+1}/.x \longrightarrow t;$$

$$Y_{1,n+1} = Y_{1,n} + y_{1,n+1};\ \ Y_{2,n+1} = Y_{1,n} + y_{1,n+1};\ \ Y_{3,n+1} = Y_{1,n} + y_{1,n+1};$$

$$\text{If}\ [n > \text{Itr},\ \text{Break}[\ ]], k, -1, \text{Itr}\Big]$$

$$\text{Print}\ [\text{“}\,Y1\text{”}, n+1, \text{“}\,=\,\text{”}, Y_{1,n+1}, \text{“}\,Y2\text{”}, n+1, \text{“}\,=\,\text{”}, Y_{2,n+1}, \text{“}\,Y3\text{”}, n+1, \text{“}\,=\,\text{”}, Y_{3,n+1}].$$

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