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An overview of coordinated control for multi-agent systems subject to input saturation☆

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Summary Coordinated control of multi-agent systems has widespread application in engineering such as unmanned aerial vehicles and unmanned ground vehicles. Due to the fact that input saturation can lead a control system to deterioration and instability, a lot of efforts have been devoted to investigating this subject of great importance. The present article offers a survey of recent developments on coordinated control of multi-agents systems subject to input saturation. Some preliminaries about graph theory, stability theory and input saturation are first provided, followed by some important results in the area, which are categorized into semi-global and global coordinated controls. Future research topics are finally discussed.

Introduction

Coordinated control of multi-agent systems is an active research field in both science and engineering, seemingly originated from distributed algorithms (Lynch, 1996) and decision-making (Degroot, 1974) the like. In 1987, Reynolds (1987) suggested a distributed behavioural model, known as the Biod model, to describe the aggregate motion of a flock of birds, a school of fish, and so on, in the natural world. In 1995, Vicsek et al. introduced a simple model with an innovative heading (directional) formula to investigate self-assemble behaviours of flocks, referred to as the Vicsek model today. Thereafter, Olfati and Murray (Saber and Murray, 2002, 2003a,b, 2004, 2007) studied the coordinated

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control of multi-agent systems by designing a specific control protocol for each agent, with formal formulations given in (Saber and Murray, 2003a,b) for flocking of multi-agent systems, which sublimed the Biod model mentioned above.

Briefly speaking, the investigation on coordinated control of multi-agent systems demands integrated knowledge from dynamical systems, control theory, graph theory, mechanics, distributed computation, and so on. Following the works mentioned above, coordinated control of multi-agent systems, including consensus, flocking and swarming, had become a research forefront in the last two decades on which a large volume of literature can be found (Hong et al., 2006; Jiang and Wang, 2010; Li et al., 2010, 2013a,b; Saber and Murray, 2002, 2003a,b, 2004, 2007; Su et al., 2009, 2011, 2014a,b; Tanner et al., 2004; Yu et al., 2010). Among the existing results, coordinated tracking, also called leader-following coordinated control, has received particularly significant attention, which aims at guiding all agents to track virtual/active leaders of the systems.

Roughly, research on coordinated control of multi-agent systems can be considered from three perspectives:

(i) Constructing distributed protocols that are much closer to practicality to drive all agents of the system to achieve consensus. Earlier model for consensus of multi-agent systems is of single-integrator type (Saber and Murray, 2004, 2007), which can be discrete-time or continuous-time single-integrator systems, perhaps with time delays. The consensus problem was then developed to systems with double-integrator dynamics (Hong et al., 2006; Su et al., 2009; Tanner et al., 2004). Lately, the investigation evolved to systems with linear dynamics (Li et al., 2010, 2013a,b) and as well nonlinear dynamics (Su et al., 2011, 2014a,b).
(ii) Developing new methodologies to achieve good performances. Good performances usually mean fast convergence speeds, minimum energy consumptions, great robustness, and so on. To realize these, many control algorithms were designed and tested, including such as pinning control which can guide all the followers to track a leader even only a small fraction of followers are informed directly by the leader (Su et al., 2009, 2014a,b), adaptive control (Li et al., 2013a,b; Su et al., 2011), and intermittent control (Wang and Wang, 2015).
(iii) Exploring the impact of the network topology on coordinated control. The local interactions among agents are influenced by the environment especially their communication network topology. Taking this fact into consideration, a great deal of effort has been devoted to the subject, ranging from undirected networks (Su et al., 2009, 2011) to directed ones (Yu et al., 2010), from fixed topologies (Hong et al., 2006; Saber and Murray, 2003a,b; Tanner et al., 2004) to switching ones (Jiang and Wang, 2010; Yu et al., 2010; Saber and Murray, 2004).

In the existing literature on coordinated control of multi-agent systems, for example those mentioned above and some references therein, it is assumed that no limitation is imposed to the movement of each agent in the consensus algorithms implementation. However, in reality, it is impractical for agents to move with absolutely freely during the process towards consensus. To tackle this kind of problems, Nedic et al. (2010) considered communication constraints, where every agent is restricted lying in a convex set in the motion space, and a distributed ''projected consensus algorithm'' is designed to successfully guide all agents to track the intersection of their individual constraint sets. As a direct extension, Lin and Ren (2014) analyzed the constrained consensus problem for multi-agent systems with an unbalanced topology and time delays.

Compared with the above-discussed communication constraints, input saturation (Lin, 1998, 1999; Saberi et al., 1996) is more important in practical situations, which can induce deterioration or instability of the underlying systems or networks, such as the windup phenomenon. Recently, a large number of works have been devoted to coordinated control of multi-agent systems subject to input saturation. In such a setting, the control input to agent u_i will always be limited within bound interval $[-\omega,\omega]$ and this can be described by a saturation function, $sat(u_i)$. One of the intuitive difficulties induced by input saturation is the nonlinearity. To handle this, the so-called parameterized low-gain feedback technique is introduced (Lin, 1999; Saberi et al., 1996). Roughly speaking, by low-gain feedback, the control input u_i for agent i can be turned to arbitrarily small to within the saturation bound interval so that the saturation nonlinearity can be avoided. There are two kinds of approaches to designing low-gain feedback laws, the eigenstructure assignment based design and the Algebraic Riccati equation based design. The current study on coordination control of multi-agent systems with input saturation mainly relies on the second approach (Chen et al., 2015; Fan, 2015; Su et al., 2013, 2014a,b, 2015a,b,c; Wang et al., 2015a,b; Wang and Wang, 2015; Yang et al., 2014a; Zhang et al., 2015; Zhao and Lin, 2014a,b). On the other hand, as pointed out by (Lin, 1999), parameterized high-gain feedback laws can lead the systems to achieve higher performances beyond consensus tracking, for example to achieve robust consensus tracking or robust swarm tracking (Su et al., 2015a,c; Wang et al., 2015a). With the help of the low-gain and high-gain feedback techniques, semi-global and global coordinated control of multi-agent systems with input saturation were studied in (Chen et al., 2015; Fan, 2015; Su et al., 2013, 2014a,b, 2015a,b,c,d; Wang et al., 2015a,b; Wang and Wang, 2015; Yang et al., 2014a; Zhang et al., 2015; Zhao and Lin, 2014a,b) and in (Meng et al., 2013; Yang et al., 2014b; Zhang and Chen, 2015; Zhang et al., 2014; Zhao and Lin, 2014a,b), respectively. Taking a panoramic view of the existing investigations on coordinated control of multi-agent systems with input saturation, the content of this survey focuses on analyzing the effects of the network topology on the coordinated control performance and finding distributed protocols to achieve consensus effectively, not only for general linear systems (Chen et al., 2015; Fan, 2015; Meng et al., 2013; Su et al., 2013, 2014a,b, 2015a,b,c,d; Wang et al., 2015a,b; Wang and Wang, 2015; Yang et al., 2014a,b; Zhang et al., 2014, 2015; Zhao and Lin, 2014a,b; Zhang and Chen, 2015); but also for nonlinear systems (Zhang et al., 2014).

The rest of the article is organized as follows. ''Preliminaries'' section provides some preliminaries. ''Semi-global coordinated control of multi-agent systems subject to input saturation'' and ''Global coordinated

control of multi-agent systems subject to input saturation'' sections introduce the semi-global and global coordinated control of multi-agent systems with input saturation, respectively. ''Conclusions'' section concludes the presentation.

Preliminaries

Notation

Throughout, N is the number of agents, R^+ and $R^{n\times m}$ are the sets of real numbers and $n\times m$ real matrices, respectively; for a square matrix A, A^T is its transpose; ARE means algebraic Riccati equation; **sign**$(\cdot)$ is the signum function; $\|\cdot\|_\infty$ denotes the infinite norm.

Graph theory

For a system consisting of N agents, labelled by 1, 2, ..., N, the switching directed interactions among agents can be described by a triplet $\mathcal{G}(t) = (\mathcal{V}, \mathscr{E}(t), \mathcal{A}(t))$ with node set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, edge set $\mathcal{E}(t) = \{(v_i, v_j) \in \mathcal{V}\times\mathcal{V}|$ if agent j can access the information of agent i at time $t\}$ and adjacent matrix $\mathcal{A}(t) = (a_{ij}(t) \in R^{N\times N})$ having

$$a_{ij}(t) = \begin{cases} 1, & \text{if } (v_i, v_j) \in \mathscr{E}(t); \\ 0, & \text{otherwise} \end{cases}$$

A directed path is a sequence of directed edges in the form of (v_1,v_2), (v_2,v_3), ..., with all $v_i \in \mathcal{V}$. A directed graph has a directed spanning tree if there exists at least one agent that has a directed path to every other agent. A directed graph is called strongly connected if any two distinct nodes of the graph can be connected via a path that follows the directions of the edges in the graph. The Laplacian matrix is defined by $\mathcal{L}(t) = \mathcal{D}(t) - \mathcal{A}(t)$, where $\mathcal{D}(t)$ is a diagonal matrix with the ith diagonal element equal to $\sum_{j=1,j\neq i}^{N} a_{ij}(t)$. Denote and order the eigenvalues of $\mathcal{L}(t)$ as $0 = \lambda_1(\mathcal{L}(t)) \le \lambda_2(\mathcal{L}(t)) \le \cdots \le \lambda N(\mathcal{L}(t))$. It is well known in reference Godsil et al. (2001) that $\lambda_2(\mathcal{L}(t)) > 0$ if and only if $\mathcal{G}(t)$ is connected.

Let $\bar{\mathcal{G}}(t)$ be an augmented graph generated by a leader (labelled by 0) and the above undirected graph $\mathcal{G}(t)$. The symmetric Laplacian matrix of $\bar{\mathcal{G}}(t)$ is denoted by $\mathcal{L}_{\pi(t)}$, where $\pi:[0,\infty) \to \Gamma$ is a switching signal whose value at time t is the index of the graph at time t, and Γ is finite. Moreover, define $\mathcal{H}_{\pi(t)} = \text{diag}(h_1(t), h_2(t), \ldots, h_N(t))$, where if agent i is a neighbour of the leader at time t, then $h_i(t) = 1$; otherwise $h_i(t) = 0$.

Note that the notation above will be independent of t if the topology of the graph is non-switching (i.e., fixed).

Semi-global coordinated control of multi-agent systems subject to input saturation

Coordinated control via low-gain feedback

The semi-global coordinated control of multi-agent systems with input saturation can be achieved by utilizing the parameterized ARE-based low-gain feedback design technique (Lin, 1999; Su et al., 2013, 2014a,b; Zhao and Lin, 2014a,b). ARE-based low-gain feedback means a family of feedback laws in which a parameterized gain matrix, $P(\varepsilon)$, approaches zero as the low-gain parameter $\varepsilon \in (0,1]$ approaches zero. By choosing a proper $\varepsilon \in (0,1]$, the control input would not go beyond the saturation domain and thus the nonlinearity induced by the saturation function can be avoided. The current investigations on this topic mainly include low-gain state feedback consensus tracking (Lin, 1999; Su et al., 2013; Zhao and Lin, 2014a,b) and low-gain output feedback consensus tracking (Lin, 1999; Su et al., 2013, 2014b) problems.

Specifically, consider a multi-agent system with N agents, labelled as 1, 2, ..., N, where all agents move in the n-dimensional Euclidean space according to the following dynamics:

$$\dot{x}_i(t) = A\dot{x}_i(t) + Bsat_\omega(u_i(t)) \tag{1}$$

where $x_i \in R^n$ is the state of agent i, $u_i(t)$ is the control input to i, $sat_\omega(\cdot)$ is the saturation function satisfying

(i) $sat_\omega(u_i(t)) = [sat_\omega(u_{i1}(t)), sat_\omega(u_{i2}(t)), \ldots, sat_\omega(u_{im}(t))]^T$;
(ii) $sat_\omega(u_{ij}(t)) = \textbf{sign}(u_{ij}(t))\min\{|u_{ij}(t)|,\omega\}$ for some constant $\omega > 0$.

Moreover, $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are constant matrices satisfying the following assumption.

Assumption 1. The pair (A,B) is asymptotically null controllable with bounded control (ANCBC), i.e.,

(1) All eigenvalues of A are in the closed left-half s-plane;
(2) The pair (A,B) is stabilizable.

The dynamics of the leader, labelled as 0, is described by

$$\dot{x}_0(t) = Ax_0(t), \tag{2}$$

where $x_0(t)$ is the state of the leader.

The network consisting of N agents and a leader, denoted as $\bar{\mathcal{G}}$, satisfies the following assumption.

Assumption 2. The graph $\bar{\mathcal{G}}$ contains a spanning tree rooted at the leader.

To emphasize the role of the low-gain parameter ε in control input, $u_i(t)$ can be specifically expressed as $u_i(t,\varepsilon)$. By choosing a proper $\varepsilon^* \in (0,1]$, it can ensure $||u_i(t,\varepsilon^*)||_\infty \le \omega$.

Definition 1. For a system with agent dynamics (1) and leader dynamics (2), it is said to achieve semi-global consensus tracking if, for any *a priori* given bounded set $\chi \subset R^n$, there is an $\varepsilon^* > 0$ such that for any $\varepsilon \in (0,\varepsilon^*]$,

$$\lim_{t\to\infty} \left\|x_i(t) - x_0(t)\right\| = 0, i = 1, 2, \ldots, N,$$

provided that $x_i(0) \subset \chi$ for all $i = 0, 1, 2, \ldots, N$.

Su et al. (2013, 2014a,b) were the first to investigate the semi-global consensus tracking problem for multi-agent systems with input saturation, wherein both problems were

discussed on fixed and switching topologies, respectively. They showed that the problem of consensus tracking for system (1) can be addressed in two steps:

Step 1: For the chosen $\varepsilon \in (0,\varepsilon^*]$, solve the following parametric ARE for a positive definite matrix solution $P(\varepsilon)$:

$$A^TP(\varepsilon) + P(\varepsilon)A = \lambda P(\varepsilon)BB^TP(\varepsilon) + \varepsilon I_n = 0,$$

where constant $\gamma > 0$ corresponds to the system topology.
Step 2: Use the obtained $P(\varepsilon)$ to construct a distributed linear control law $u_i(t,\varepsilon)$ for agent $i(i = 1, 2, \ldots, N)$.

The above two steps show that the network topology and the control input both play critical roles in consensus problems for multi-agent systems. In Su et al. (2014a,b), Su et al. designed a distributed u_i as

$$u_i(t,\varepsilon) = -B^TP(\varepsilon)\sum_{j=1}^{N} a_{ij}(t)(x_i(t) - x_j(t)) - B^TP(\varepsilon)h_i(t)(x_i(t) - x_0(t)), i = 1, 2, \ldots, N. \quad (3)$$

The authors in reference Su et al. (2013) defined a Lyapunov function $V(t) = \sum_{i=1}^{N}[x_i(t) - x_0(t)]^TP(\varepsilon)[x_i(t) - x_0(t)]$, and then chose a proper $\varepsilon^* \in (0, 1]$ to guarantee that $\| - B^TP(\varepsilon^*)\sum_{j=1}^{N} a_{ij}(t)[x_i(t) - x_j(t)] - B^TP(\varepsilon^*)h_i(t)[x_i(t) - x_0(t)]\| \le \omega$. For *a priori* given bounded set $\chi \in R^n$, let $c > 0$ be a constant such that

$$c \ge \sup_{\varepsilon \in (0,1], x_i(0) \in \chi, i=0,\ldots,N} \times \sum_{i=1}^{N}[x_i(0) - x_0(0)]^TP(\varepsilon)[x_i(0) - x_0(0)],$$

$$L_V(c) = \{x_i(0) \in R^n(i = 0, \ldots, N) : V(0) \le c\}$$

Taking the derivative of $V(t)$ along with (1)–(3), it follows that, under Assumptions 1 and 2, control input (3) can guide all agents in system (1) to track the leader (2) semi-globally.

Furthermore, Wang and Wang (2015) provided a periodically intermittent type of controllers to deal with the semi-global consensus tracking problem for system (1), with the control input designed as follows:

$$u_i(t,\varepsilon) = \begin{cases} B^TP(\varepsilon)\left(\sum_{j=1, j\neq 1}^{N} a_{ij}[x_j(t) - x_i(t)] - h_i[x_i(t) - x_0(t)]\right), \\ t \in [mT, mT + k); \\ 0, t \in [mT + k, (m + 1)T), m = 0, 1, \ldots, \end{cases} \quad (4)$$

where $T > 0$ is a control time period and $0 < k < T$ is the control width. Then, Chen et al. extended the periodically intermittent consensus tracking (Wang and Wang, 2015) to a more general setting on event-based synchronization of discrete-time linear systems with input saturation. In event-triggered control, the controllers update information only on the occurrence period of an event. Specifically, the system uses the current information and the sampling information of the last step to determine when to make an update. In so doing, a concerned issue is how to remove the requirement for keeping track of the current information, moment by moment, so as to save control energy? Along this research line, the self-triggered control strategy was proposed (Wang et al., 2015b). Self-triggering control is a more feasible sample-data control method, where the system determines the next sampling instant according to the last sampling rather than doing sampling at pre-arranged instants.

In addition, Su et al. (2014b) constructed a state observer for each agent as follows:

$$\dot{\hat{x}}_i = A\hat{x}_i - F(y - C\hat{x}_i), \quad i = 0, 1, \ldots, N, \quad (5)$$

where $\hat{x}_i \in R^n$ and F is a matrix such that $A - FC$ is Hurwitz. The existence of such a matrix F is ensured by the following assumption.

Assumption 3. The pair (C,A) is detectable.

The observer-based feedback law for agent i was designed in Su et al. (2014b) as follows:

$$\dot{\hat{x}}_i = A\hat{x}_i - F(y - C\hat{x}_i) - BB^TP(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(t)(\hat{x}_i - \hat{x}_j) + h_i(t)(\hat{x}_i - \hat{x}_0)\right)$$

$$u_i(t,\varepsilon) = -B^TP(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(t)(\hat{x}_i - \hat{x}_j) + h_i(t)(\hat{x}_i - \hat{x}_0)\right), \quad (6)$$

$$i = 1, \ldots, N,$$

$$\dot{\hat{x}}_0 = A\hat{x}_0 - F(y_0 - C\hat{x}_0),$$

and semi-global consensus tracking on fixed and switching topologies, respectively, were analyzed in detail in Su et al. (2014a,b). Following Su et al. (2014b), Fan (2015) studied the semi-global consensus problem with relative output feedback and input saturation on systems with directed switching topologies, while Zhao and Lin (2014b) focused on semi-global consensus tracking of general linear multi-agent systems with position and rate saturation, with the agent dynamics described by

$$\begin{cases} \dot{x}_i = Ax_i + Bsat_p(v_i), \\ \dot{v}_i = sat_r(-v_i + u_i), \\ y_i = Cx_i, i = 1, 2, \ldots, N, \end{cases} \quad (7)$$

where $x_i \in R^n$ is the plant state, $y_i \in R^r$ is the output, $v_i \in R^m$ is the actuator state, $u_i \in R^m$ is the control input.

Notice that all the literature mentioned above focused on homogenous linear systems. Yang et al. (2014b) considered heterogeneous linear systems, where agents have different linear dynamics, with analysis on semi-global regulation of output synchronization for such networks subject to actuator saturation.

Differing from the objective of semi-global consensus tracking, which is to drive the followers to track the leader, containment control refers to controlling multi-agent systems with more than one leader and aims at guiding the

followers to move into the convex hull spanned by the multiple leaders in the motion space. Su et al. (2014a, 2015b,d) studied the containment problem for multi-agent systems with input saturation. Specifically, for system (1), suppose that there exist M leaders (labelled as $N+1, N+2, \ldots, N+M$) and each leader adjusts itself by following the linear dynamics

$$\dot{x}_i = Ax_i, \quad i = N+1, \ldots, N+M \tag{8}$$

The convex hull of the M virtual leaders is

$$\left\{ \sum_{k=N+1}^{N+M} \alpha_k x_k \,\middle|\, \forall k : \alpha_k \geq 0 \cap \left(\sum_{k=N+1}^{N+M} \alpha_k = 1 \right) \right\}$$

Containment of multi-agent systems with input saturation on a network having a fixed topology was studied in Su et al. (2014a). Then, as a direct extension, the focus in Su et al. (2015b) is on containment of systems with input saturation subject to switching topologies, while both state-containment and output-containment are discussed in Su et al. (2015d) by constructing intermittent low-gain feedback laws.

Beyond the ARE-based low-gain feedback, Zhang et al. (2015) proposed a model predictive flocking control scheme for a discrete-time second-order multi-agent system with input constraints based only on neighbouring measurements, which can direct a rigid-lattice flock with a non-zero final velocity to achieve the goal.

Robust coordinated control via low-and-high-gain feedback

As a further extension of the low-gain feedback technique, the low-and-high feedback technique was developed by combining both low-gain and high-gain feedback laws together to deal with the coordinated tracking problem for multi-agent systems with input saturation and communication noise. Unlike the low-gain feedback, which is characterized by a parameter $\varepsilon \in (0,1]$, the high-gain feedback is equipped with a parameter $p \in (0+\infty)$. With the help of the high-gain feedback, a system can achieve better performance beyond consensus such as semi-global robust swarming and semi-global robust consensus. Su et al. (2015c) took the lead in investigating the robust coordinated tracking problem for multi-agent systems with input saturation and communication noise as well as a dead zone constraint. For system

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, 2, \ldots, N, \tag{9}$$

with a leader (2), the input control u_i is designed as

$$\begin{aligned} u_i(t,\varepsilon) = &-\sum_{i=1}^{N} a_{ij}\sigma[(1+p)B^TP(\varepsilon)(x_i - x_j) + g(x_i - x_j, t)] \\ &-h_i\sigma[(1+p)B^TP(\varepsilon)(x_i - x_0) + g(x_i - x_0, t), \end{aligned} \tag{10}$$

where $p > 0$ refers to the high-gain parameter and $\sigma : R^m \to R^m$ is the saturation function satisfies the following:

(i) $\sigma(s) = [\sigma_1(s_1), \sigma_2(s_2), \ldots, \sigma_m(s_m)]^T$, $s = [s_1, s_2, \ldots, s_m]^T \in R^m$;
(ii) for each $i = 1, 2, \ldots, m$,

$$\sigma_i(s_i) = \begin{cases} 0, & \text{if } |s_i| < b; \\ s_i - \mathbf{sign}(s_i)b, & \text{if } b \geq |s_i| < b+\delta; \\ \delta, & \text{if } |s_i| < b+\delta; \\ -\delta, & \text{if } |s_i| < -b-\delta, \end{cases}$$

where the constants $\delta > 0$ and $b > 0$.

Note that if $b = 0$, the saturation function $\sigma(\cdot)$ is the standard saturation function, but if $b > 0$, it is the standard saturation function with ideal dead-zone characteristics.

In (10), g: $R^n \times R_+ \to R^m$ represents disturbance uncertainties and satisfies the following assumption.

Assumption 4. The uncertain element $g(v,t) = [g_1(v,t), g_2(v,t), \ldots, g_m(v,t)] = -g(-v,t) \in R^m$ is piecewise continuous in t, locally Lipschitz in v, and its norm is bounded by a known function as

$$|g(v,t)| \leq g_0(|v|) + D_0, \forall (v,t) \in R^n \times R_+,$$

where D_0 is a known nonnegative constant and the known function g_0: $R_+ \to R_+$ is locally Lipschitz and satisfies $g_0(0) = 0$.

Su et al. (2015c) addressed the semi-global robust consensus and swarm tracking problems for multi-agent systems with input saturation and communication noise. For any *a priori* given bounded set $\chi \subset R^n$, with all the initial states of all the agents selected from χ, the robust semi-global consensus tracking and swarm tracking of system (9) and leader (2) can be achieved, respectively, if $\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0$, $i = 1, 2, \ldots, N$, holds and $\|x_i(t) - x_0(t)\|$, $i = 1, 2, \ldots, N$, remains in a prescribed set $\chi_0 \subset R^n$ with $0 \in \chi_0$ after some finite time.

Following Su et al. (2015c), Wang et al. (2015a) extended state coordinated tracking to output coordinated tracking, where to handle coordinated control of systems with input saturation and communication noise, a new low-and-high-gain feedback technique was developed by combining the du-low-gain feedback and the quasi-high-gain feedback together.

Global coordinated control of multi-agent systems subject to input saturation

One of the outstanding features of semi-global coordinated control is that the initial value of each agent is required to be in *a priori* given bounded set. To meet this presumption, one has to determine the values of the low-gain and high-gain parameters, ε and ρ, respectively, and the control processes are often difficult to carry out. To avoid such difficulties, a novel approach is referred to the scheduling low-and-high-gain feedback.

In scheduling low-and-high-gain feedback, the low-gain parameter ε is a function of the states which can adjust so that the control input u_i is always located in the saturation

domain. Su et al. (2015a) studied the robust consensus tracking and robust swarm tracking of multi-agent systems with input saturation and communication noise via the ARE-based scheduling low-and-high-gain feedback approach. It requires information of all agents to determine the low-gain parameter ε. Zhao and Lin (2014a) suggested avoiding this problem by constructing a low-gain nonlinear feedback control law, in which each agent uses the information of other agents through multi-hop paths in the communication network. The number of hops each agent uses to obtain such information is not bigger than the largest algebraic multiplicity of the eigenvalues on the unit circle of the system matrix. Without using the low-gain feedback technique, the global consensus tracking problem for multi-agent systems subject to input saturation is studied in Meng et al. (2013) and Yang et al. (2014a) by carrying out state transformation to the model. Very recently, Chen et al. (2015) generalized this state transformation to event-based global synchronization of systems with input saturation.

Conclusions

This article offers a brief overview of the recent research and development in coordinated control of multi-agent systems subject to input saturation. To date, the investigations on this topic include semi-global and global coordinated controls, mainly from the perspectives of network topologies and distributed control algorithms. Low-gain feedback technique is widely used to solve the semi-global consensus tracking problem, while low-and-high-gain feedback is introduced to achieve better performance beyond consensus tracking, e.g., to robust consensus tracking and robust swarm tracking. The extension of converting semi-global results to global ones is not trivial, because it not only has to follow the distributed rule but also need to guarantee the control input to remain in the saturation domain. Future works are expected to address this important but challenging problem.

Conflict of interest

The authors declare that there is no conflict of interest.

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References

Degroot, M.H., 1974. Reaching a consensus. J. Am. Stat. Assoc. 69 (345), 118—121.

Fan, M.C., 2015. Distributed semi-global consensus with relative output feedback and input saturation under directed switching networks. IEEE Trans. Circuits Syst. II: Exp. Briefs 62 (8), 796—800.

Godsil, C.D., et al., 2001. Algebraic Graph Theory. Springer, New York.

Hong, Y., et al., 2006. Tracking control for multi-agent consensus with an active leader and variable topology. Automatica 42 (7), 1177—1182.

Chen, M.Z.Q., et al., 2015. Event-based synchronization of linear discrete-time dynamical networks. IET Control Theory Appl. 9 (5), 755—765.

Jiang, F., Wang, L., 2010. Consensus seeking of high-order dynamic multi-agent systems with fixed and switching topologies. Int. J. Control. 83 (2), 404—420.

Li, Z., et al., 2010. Consensus of multiagent systems and synchronization of complex networks: a unified viewpoint. IEEE Trans. Circuits Syst. I: Reg. Pap. 57 (1), 213—224.

Li, Z., et al., 2013a. Distributed consensus of linear multi-agent systems with adaptive dynamic protocols. Automatica 49 (7), 1986—1995.

Li, Z., et al., 2013b. Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols. IEEE Trans. Autom. Control 58 (7), 1786—1791.

Lin, P., Ren, W., 2014. Constrained consensus in unbalanced networks with communication delays. IEEE Trans. Autom. Control 59 (3), 775—781.

Lin, Z., 1998. Global control of linear systems with saturating actuators. Automatica 34 (7), 897—905.

Lin, Z., 1999. Low Gain Feedback. Springer.

Lynch, N.A., 1996. Distributed Algorithms. Morgan Kaufmann.

Meng, Z., et al., 2013. On global leader-following consensus of identical linear dynamic systems subject to actuator saturation. Syst. Control Lett. 62 (2), 132—142.

Nedic, A., et al., 2010. Constrained consensus and optimization in multi-agent networks. IEEE Trans. Autom. Control 55 (4), 922—938.

Reynolds, C.W., 1987. Flocks, herds and schools: a distributed behavioral model. ACM Siggraph Comput. Graph. 21 (4), 25—34.

Saber, R.O., Murray, R.M., 2002. Distributed cooperative control of multiple vehicle formations using structural potential functions. In: Proceeding on 15-th Triennial World Congress.

Saber, R.O., Murray, R.M., 2003a. Consensus Protocols for Undirected Networks of Dynamic Agents With Communication Time-delays.

Saber, R.O., Murray, R.M., 2003b. Flocking with obstacle avoidance: cooperation with limited communication in mobile networks. In: Proc. of the 42nd IEEE Conf. Decision Control, p. 2022-2028.

Saber, R.O., Murray, R.M., 2004. Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Autom. Control 49 (9).

Saber, R.O., Murray, R.M., 2007. Consensus and cooperation in networked multi-agent systems. Proc. IEEE 95 (1), 215—233.

Saberi, A., et al., 1996. Control of linear systems with saturating actuators. IEEE Trans. Autom. Control 41 (3), 368—378.

Su, H., et al., 2011. Adaptive second-order consensus of networked mobile agents with nonlinear dynamics. Automatica 47 (2), 368—375.

Su, H., et al., 2013. Semi-global leader-following consensus of linear multi-agent systems with input saturation via low gain feedback. IEEE Trans. Circuits Syst. I: Reg. Pap. 60 (7), 1881—1889.

Su, H., et al., 2014a. Semi-global containment control of multi-agent systems with input saturation. IET Control Theory Appl. 8 (18), 2229—2237.

Su, H., et al., 2014b. Semi-global observer-based leader-following consensus with input saturation. IEEE Trans. Ind. Electron. 61 (6), 2842—2850.

Su, H., et al., 2015a. Global coordinated tracking of multi-agent systems with disturbance uncertainties via bounded control inputs. Nonlinear Dyn. 82 (4), 2059—2068.

Su, H., et al., 2015b. Multi-agent containment with input saturation on switching topologies. IET Control Theory Appl. 9 (3), 399—409.

Su, H., et al., 2015c. Robust semi-global coordinated tracking of linear multi-agent systems with input saturation. Int. J. Robust Nonlinear Control. 25 (14), 2375—2390.

Su, H., et al., 2015d. Semi-global containment control of multi-agent systems with intermittent input saturation. J. Franklin Inst. 352 (9), 3504—3525.

Su, H., et al., 2009. Flocking of multi-agents with a virtual leader. IEEE Trans. Autom. Control 54 (2), 443—455.

Tanner, H.G., et al., 2004. Leader to formation stability. IEEE Trans. Autom. Control 20 (3), 443—455.

Wang, X.L., Wang, X.F., 2015. Semi-global consensus of multi-agent systems with intermittent communications and low-gain feedback. IET Control Theory Appl. 9 (5), 766—774.

Wang, X., et al., 2015a. Observer-based robust coordinated tracking of multi-agent systems with input saturation. In: Proc. of the 54th IEEE Conf. Decision Control, Osaka, Japan, pp. 368—373.

Wang, X., et al., 2015b. Self-triggered based semi-global consensus tracking of multi-agent systems with input saturation. In: Proc. of the 34th Chinese Conf. Control, Hangzhou, China, pp. 7333—7338.

Yang, T., et al., 2014a. Global consensus for discrete-time multi-agent systems with input saturation constraints. Automatica 50 (2), 499—506.

Yang, T., et al., 2014b. Semi-global regulation of output synchronization for heterogeneous networks of non-introspective, invertible agents subject to actuator saturation. Int. J. Robust Nonlinear Control 24 (3), 548—566.

Yu, W., et al., 2010. Second-order consensus for multi-agent systems with directed topologies and nonlinear dynamics. IEEE Trans. Syst. Man Cybern. B Cybern. 40 (3), 881—891.

Zhang, H., et al., 2015. Model predictive flocking control for second-order multi-agent systems with input constraints. IEEE Trans. Circuits Syst. I: Reg. Pap. 62 (6), 1599—1606.

Zhang, L., Chen, M.Z.Q., 2015. Event-based global stabilization of linear systems via a saturated linear controller. Int. J. Robust Nonlinear Control, http://dx.doi.org/10.1002/rnc.3352 (in press).

Zhang, Z., et al., 2014. Asymptotic tracking control of uncertain nonlinear systems with unknown actuator nonlinearity. IEEE Trans. Autom. Control 59 (5), 1336—1341.

Zhao, Z., Lin, Z., 2014a. Global leader following consensus of a group of discrete-time linear systems using bounded controls. Proc. World Congress 19 (1), 263—268.

Zhao, Z., Lin, Z., 2014b. Semi-global leader-following consensus of multiple linear systems with position and rate limited actuators. Int. J. Robust Nonlinear Control 25 (13), 2083—2100.