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From inflation to acceleration, with phantom and canonical scalar fields in non-flat universe

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ABSTRACT

Motivated by the recent work of Elizalde et al. [E. Elizalde, S. Nojiri, S.D. Odintsov, D. Saez–Gomez, V. Faraoni, Phys. Rev. D 77 (2008) 106005], we generalize their work to the non-flat case. We consider a universe filled with matter and show that it is possible to obtain both inflation and accelerated expansion at late times by using a single scalar field. Realistic examples are worked out in order to illustrate this fact. Then we extend the problem to the interacting case.

1. Introduction

Nowadays it is plainly believed that the universe is experiencing an accelerated expansion. Recent observations from type Ia supernovae [1] in associated with Large Scale Structure [2] and Cosmic Microwave Background anisotropies [3] have provided main evidence for this cosmic acceleration. In order to explain why the cosmic acceleration happens, many theories have been proposed. Although theories of trying to modify Einstein equations constitute a big part of these attempts, the mainstream explanation for this problem, however, is known as theories of dark energy.

The combined analysis of cosmological observations suggests that the universe consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although the nature and origin of dark energy are unknown, we still can propose some candidates to describe it. The most obvious theoretical candidate of dark energy is the cosmological constant λ (or vacuum energy) [4,5] which has the equation of state $w = -1$. However, as is well known, two difficulties arise from the cosmological constant scenario, namely the two well-known cosmological constant problems, the fine-tuning problem and the cosmic coincidence problem [6]. An alternative proposal for dark energy is the dynamical dark energy scenario. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. So far, a large class of scalar-field dark energy models have been studied, including quintessence [7], K-essence [8], tachyon [9], phantom [10], ghost condensate [11] and quintom [12], and so forth. But we should note that the mainstream viewpoint regards the scalar field dark energy models as an effective description of an underlying theory of dark energy. In addition, other proposals on dark energy include interacting dark energy models [13], braneworld models [14], Chaplygin gas models [15], and holographic dark energy [16], etc.

Some experimental data have implied that our universe is not a perfectly flat universe and recent papers have favoured a universe with spatial curvature [17]. As a matter of fact, we want to remark that although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [18]. Cosmic Microwave Background (CMB) anisotropy data provide the most stringent constraints on cosmic curvature k. Assuming that dark energy is a cosmological constant, the three-year WMAP data give $\Omega_k = -0.15 \pm 0.11$, and this improves dramatically to $\Omega_k = -0.005 \pm 0.006$, with the addition of galaxy survey data from the SDSS [19]. The effect of allowing non-zero curvature on constraining some dark energy models has been studied by [20]. Recently Clarkson et al. [21] have shown that ignoring Ω_k induces errors in the reconstructed dark energy equation of state, $w(z)$, that grow very rapidly with redshift and dominate the $w(z)$ error budget at redshifts ($z \succcurlyeq 0.9$) even if Ω_k is very small.

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In the present Letter motivated by the recent work of Elizalde et al. [22], some explicit examples are presented in which a unified, continuous description of the inflationary era and of the late-time cosmic acceleration epoch is obtained. We generalize examples of Elizalde et al. [22] to the non-flat case in the non-interacting then to the interacting cases respectively in Sections 2 and 3.

2. Unified inflation and late time acceleration in non-flat universe

We consider a universe filled with matter with equation of state $p_m = w_m \rho_m$ and a scalar field which only depends on time. The action is as,

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_m\right), \tag{1}$$

where $V(\phi)$ is the scalar potential, $\omega(\phi)$ is the kinetic function, and $\mathcal{L}_m$ is the matter Lagrangian density. As we work in the spatially non-flat Friedmann–Robertson–Walker (FRW) universe, the metric is given by

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\,d\Omega^2\right), \tag{2}$$

where $a = a(t)$ is the scale factor of the FRW universe and k denotes the curvature of space with $k = 0, 1, -1$ for flat, closed and open universe respectively.
The third Friedmann equation describes the time evolution of the energy densities of the dark components. These equations are actually the continuity equations for the matter and scalar field,

$$\dot\rho_m + 3H(\rho_m + p_m) = 0, \tag{3}$$

$$\dot\rho_\phi + 3H(\rho_\phi + p_\phi) = 0, \tag{4}$$

where the quantity $H = \dot a/a$ is the Hubble parameter. The corresponding FRW equations are written as,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_m + p_\phi), \tag{5}$$

$$\dot H - \frac{k}{a^2} = -4\pi G(\rho_m + p_m + \rho_\phi + p_\phi), \tag{6}$$

with ρ_ϕ and p_ϕ given by,

$$\rho_\phi = \frac{1}{2}\omega(\phi)\dot\phi^2 + V(\phi), \tag{7}$$

$$p_\phi = \frac{1}{2}\omega(\phi)\dot\phi^2 - V(\phi). \tag{8}$$

Combining Eqs. (5)–(8), we obtain,

$$\omega(\phi)\dot\phi^2 = -\frac{1}{4\pi G}\left(\dot H - \frac{k}{a^2}\right) - (\rho_m + p_m), \tag{9}$$

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{k}{a^2}\right) + \frac{1}{8\pi G}\left(\dot H - \frac{k}{a^2}\right) + \frac{(\rho_m - p_m)}{2}. \tag{10}$$

From Eq. (3), we get,

$$\rho_m = \rho_{m_0} a^{-3(1+w_m)}. \tag{11}$$

We now consider the theory in which $V(\phi)$ and $\omega(\phi)$ are,

$$\omega(\phi) = -\frac{1}{4\pi G}\left(f'(\phi) - \frac{k}{a_0^2}e^{-2F(\phi)}\right) - (w_m+1)F_0 e^{-3(1+w_m)F(\phi)}, \tag{12}$$

$$V(\phi) = \frac{3}{8\pi G}\left(f^2(\phi) + \frac{k}{a_0^2}e^{-2F(\phi)}\right) + \frac{1}{8\pi G}\left(f'(\phi) - \frac{k}{a_0^2}e^{-2F(\phi)}\right) + \frac{w_m - 1}{2}F_0 e^{-3(1+w_m)F(\phi)}, \tag{13}$$

where $f(\phi) = F'(\phi)$. Then, the following solution is found [23]

$$\phi = t, \qquad H(t) = f(t), \tag{14}$$

which leads to,

$$a(t) = a_0 e^{F(t)}, \quad a_0 = \left(\frac{\rho_{m_0}}{F_0}\right)^{\frac{1}{3(1+w_m)}}. \tag{15}$$

Using the FRW equations, the effective EoS parameter is defined as,

$$w_{\mathrm{eff}} = \frac{p_m + p_\phi}{\rho_m + \rho_\phi} = -1 - \frac{2}{3}\frac{\dot H - \frac{k}{a^2}}{H^2 + \frac{k}{a^2}}. \tag{16}$$

Now we consider above equations by following examples.

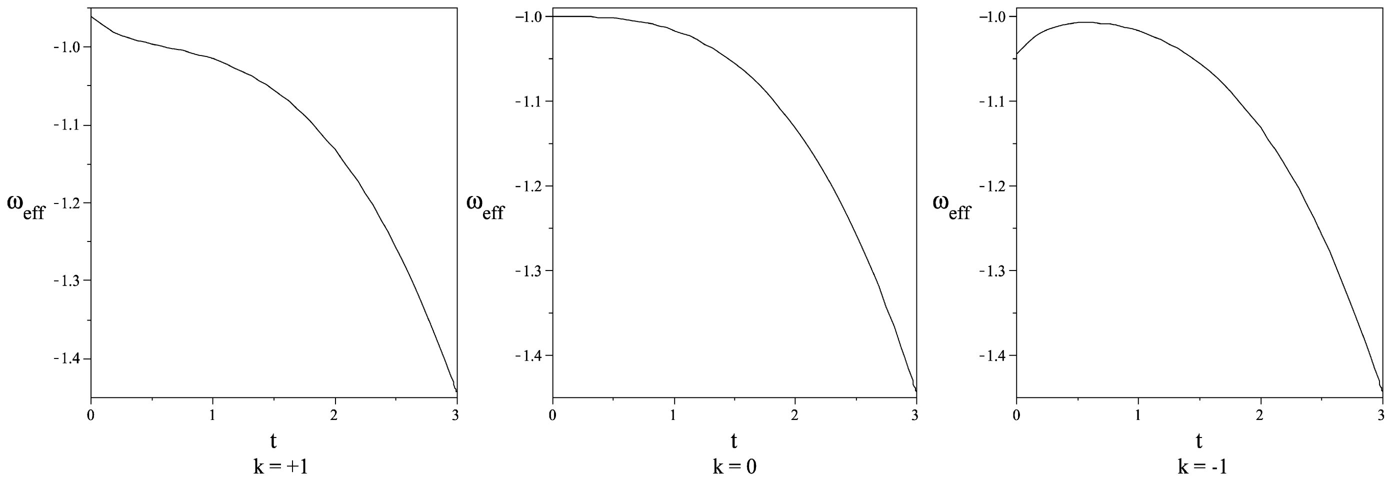

Fig. 1. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k=1$, $k=0$, and $k=-1$. Here in the numerical calculation we take $a_0=2, h_0=t_0=t_1=3$.

3. Example I

As a first example, we consider the following model [22],

$$f(\phi)=h_0^2\left(\frac{1}{t_0^2-\phi^2}+\frac{1}{\phi^2+t_1^2}\right). \tag{17}$$

Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=h_0^2\left(\frac{1}{t_0^2-t^2}+\frac{1}{t^2+t_1^2}\right)\Rightarrow \dot{H}=2th_0^2\left(\frac{1}{(t_0^2-t^2)^2}+\frac{1}{(t^2+t_1^2)^2}\right), \tag{18}$$

$$a(t)=a_0\left(\frac{t+t_0}{t_0-t}\right)^{\frac{h_0^2}{2t_0}}e^{\frac{h_0^2}{t_1}\arctan(t/t_1)}. \tag{19}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=-\frac{1}{\pi G}\frac{h_0^2(t_1^2+t_0^2)(\phi^2-\frac{t_1^2+t_0^2}{2})\phi}{(t_1^2+\phi^2)^2(t_0^2-\phi^2)^2}+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-(w_m+1)F_0e^{-3(w_m+1)F(\phi)}, \tag{20}$$

$$V(\phi)=\frac{h_0^2(t_1^2+t_0^2)}{8\pi G(t_1^2+\phi^2)^2(t_0^2-\phi^2)^2}\left[3h_0^2(t_1^2+t_0^2)+4\phi\left(\phi^2-\frac{t_1^2+t_0^2}{2}\right)\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}+\frac{w_m-1}{2}F_0e^{-3(1+w_m)F(\phi)}, \tag{21}$$

$F(\phi)$ is as,

$$F(\phi)=\frac{h_0^2}{2t_0}\ln\left(\frac{\phi+t_0}{t_0-\phi}\right)+\frac{h_0^2}{t_1}\arctan\frac{\phi}{t_1}. \tag{22}$$

By using Eqs. (18) and (19), the effective EoS parameter is obtained as following,

$$w_{\mathrm{eff}}=-1-\frac{2}{3}\frac{2a_0^2h_0^2t(\frac{1}{(t_0^2-t^2)^2}-\frac{1}{(t^2+t_1^2)^2})(\frac{t+t_0}{t_0-t})^{\frac{h_0^2}{t_0}}e^{\frac{2h_0^2}{t_1}\arctan(t/t_1)}-k}{a_0^2h_0^4(\frac{1}{t_0^2-t^2}-\frac{1}{t^2+t_1^2})^2(\frac{t+t_0}{t_0-t})^{\frac{h_0^2}{t_0}}e^{\frac{2h_0^2}{t_1}\arctan(t/t_1)}+k}. \tag{23}$$

Thus, when $t\to 0$ then $w_{\mathrm{eff}}\to -1$, $w_{\mathrm{eff}}<-1$, $w_{\mathrm{eff}}>-1$, respectively for $k=0$, $k=-1$, $k=1$. Therefore when $t\to 0$, we have an acceleration epoch for $k=0$, and $k=-1$. While for $t\to\infty$ we have $w_{\mathrm{eff}}\to -1$ for all cases, which can be interpreted as late time acceleration. See also Fig. 1.

4. Example II

As a second example, we consider the following model,

$$f(\phi)=\frac{H_0}{t_s-\phi}+\frac{H_1}{\phi^2}. \tag{24}$$

Using the solution (15), the Hubble parameter and the scale factor are given by,

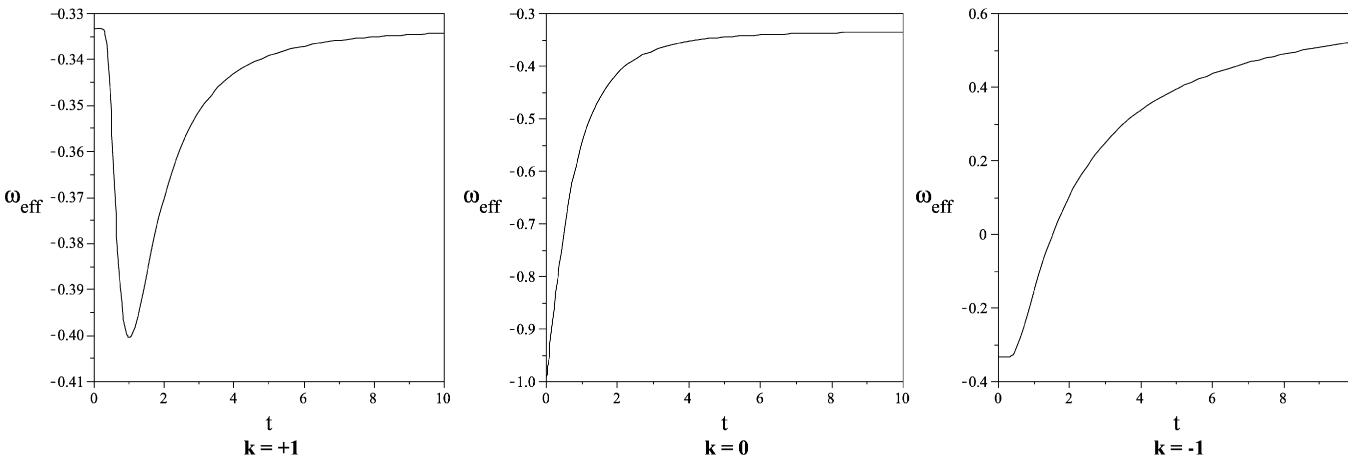

Fig. 2. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k=1$, $k=0$, and $k=-1$. Here in the numerical calculation we take $a_0=1$, $H_1=2$, $H_0=t_s=-1$.

$$H(t)=\frac{H_0}{t_s-t}+\frac{H_1}{t^2}\Rightarrow\dot{H}=\frac{H_0}{(t_s-t)^2}-\frac{2H_1}{t^3},\tag{25}$$

$$a(t)=a_0(t_s-t)^{-H_0}e^{\frac{-H_1}{t}}.\tag{26}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=-\frac{1}{4\pi G}\left[\frac{H_0}{(t_s-\phi)^2}-\frac{2H_1}{\phi^2}\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-(w_m+1)F_0e^{-3(w_m+1)F(\phi)},\tag{27}$$

$$V(\phi)=\frac{1}{8\pi G}\left[\frac{H_0(3H_0+1)}{(t_s-\phi)^2}+\frac{H_1}{\phi^3}\left(\frac{H_1}{\phi}-2\right)\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}+\frac{w_m-1}{2}F_0e^{-3(1+w_m)F(\phi)},\tag{28}$$

$F(\phi)$ is as,

$$F(\phi)=-\frac{H_1}{\phi}-H_0\ln(t_s-\phi).\tag{29}$$

By using Eqs. (25) and (26), the effective EoS parameter is obtained as following,

$$w_{\rm eff}=-1-\frac{2}{3}\frac{a_0^2(t_s-t)^{-2H_0}(\frac{H_0}{(t_s-t)^2}-\frac{2H_1}{t^3})e^{\frac{-2H_1}{t}}-k}{a_0^2(t_s-t)^{-2H_0}(\frac{H_0}{t_s-t}-\frac{H_1}{t^2})^2e^{\frac{-2H_1}{t}}+k}.\tag{30}$$

In Fig. 2 one can see that $w>-1$, namely this model is not phantom-like model.

5. Example III

As a third example, we consider the following model,

$$f(\phi)=H_0+\frac{H_1}{\phi^n},\tag{31}$$

where H_0 and $H_1>0$ are constant and n is a positive integer and constant. Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=H_0+\frac{H_1}{t^n}\Rightarrow\dot{H}=-\frac{nH_1}{t^{n+1}},\tag{32}$$

$$a(t)=a_0e^{[H_0t-\frac{H_1}{(n-1)t^{n-1}}]}.\tag{33}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=\frac{1}{4\pi G}\frac{nH_1}{\phi^{n+1}}+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-(w_m+1)F_0e^{-3(w_m+1)F(\phi)},\tag{34}$$

$$V(\phi)=\frac{1}{8\pi G}\frac{3}{\phi^{n+1}}\left[\frac{(H_0\phi^{n/2}+H_1)^2}{\phi^{n-1}}-\frac{nH_1}{3}\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}+\frac{w_m-1}{2}F_0e^{-3(w_m+1)F(\phi)},\tag{35}$$

$F(\phi)$ is as,

$$F(\phi)=H_0\phi-\frac{H_1}{(n-1)\phi^{n-1}}.\tag{36}$$

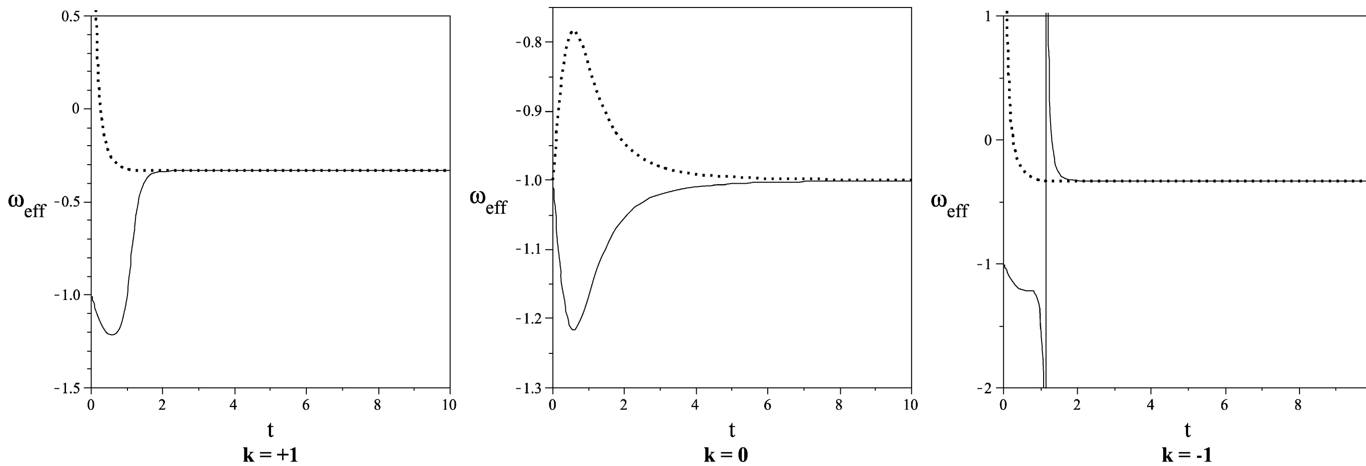

Fig. 3. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k=1$, $k=0$, and $k=-1$. Here in the numerical calculation we take $a_0=0.5$, $H_1=H_0=-2$ and $n=+2$ (line) and $n=-2$ (dot).

By using Eqs. (32) and (33), the effective EoS parameter is obtained as following,

$$w_{\rm eff}=-1+\frac{2}{3}\frac{a_0^2\frac{nH_1}{t^{n+1}}e^{2H_0t-\frac{2H_1}{(n-1)t^{n-1}}}+k}{a_0^2(H_0+\frac{H_1}{t^n})^2e^{2H_0t-\frac{2H_1}{(n-1)t^{n-1}}}+k}. \tag{37}$$

As one can see in Fig. 3, for $n=2$, when $t\to 0$ then $w_{\rm eff}\to -1$ and we have an acceleration epoch. While for $t\to\infty$, $w_{\rm eff}\to -1$ only in the case $k=0$, which can be interpreted as late time acceleration.

6. Example IV

As a fourth example, we consider the following model,

$$f(\phi)=\frac{H_i+H_lce^{2\alpha\phi}}{1+ce^{2\alpha\phi}}, \tag{38}$$

where H_i, H_l, c and α are positive constant. Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=\frac{H_i+H_lce^{2\alpha t}}{1+ce^{2\alpha t}} \Rightarrow \dot{H}=-\frac{2c\alpha(H_i-H_l)e^{2\alpha t}}{(1+ce^{2\alpha t})^2}, \tag{39}$$

$$a(t)=a_0\frac{(1+ce^{2\alpha t})^{\frac{H_l}{2\alpha}}}{(c+e^{-2\alpha t})^{\frac{H_i}{2\alpha}}}. \tag{40}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=\frac{1}{2\pi G}\frac{c\alpha(H_i-H_l)e^{2\alpha\phi}}{(1+ce^{2\alpha\phi})^2}+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-(w_m+1)F_0e^{-3(w_m+1)F(\phi)}, \tag{41}$$

$$V(\phi)=\frac{1}{8\pi G}\left[\frac{3(H_i+cH_le^{2\alpha\phi})^2}{(1+ce^{2\alpha\phi})^2}-\frac{2c\alpha(H_i-H_l)e^{2\alpha\phi}}{(1+ce^{2\alpha\phi})^2}\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}+\frac{w_m-1}{2}F_0e^{-3(w_m+1)F(\phi)}, \tag{42}$$

$F(\phi)$ is as,

$$F(\phi)=\frac{1}{2\alpha}\ln\frac{(1+ce^{2\alpha\phi})^{H_l}}{(c+e^{-2\alpha\phi})^{H_i}}. \tag{43}$$

By using Eqs. (39) and (40), the effective EoS parameter is obtained as following,

$$w_{\rm eff}=-1+\frac{2}{3}\frac{2a_0^2c\alpha(H_i-H_l)(1+ce^{2\alpha t})^{\frac{H_l}{\alpha}}e^{2\alpha t}+k(1+ce^{2\alpha t})^2(c+e^{-2\alpha t})^{\frac{H_i}{\alpha}}}{a_0^2(H_i+cH_le^{2\alpha t})^2(1+ce^{2\alpha t})^{\frac{H_l}{\alpha}}+k(1+ce^{2\alpha t})^2(c+e^{-2\alpha t})^{\frac{H_i}{\alpha}}}. \tag{44}$$

As one can see from Fig. 4, when $t\to 0$ then $w_{\rm eff}\to -1$, and we have an acceleration epoch, while for $t\to\infty$ we have $w_{\rm eff}\to -1$ for all cases, which can be interpreted as late time acceleration.

7. Unified inflation and late time acceleration in non-interacting case

In this section we consider the third Fridmann equation when there is interaction between dark energy density ρ_ϕ and the Cold Dark Matter (CDM) ρ_m. The corresponding continuity equations are now written as,

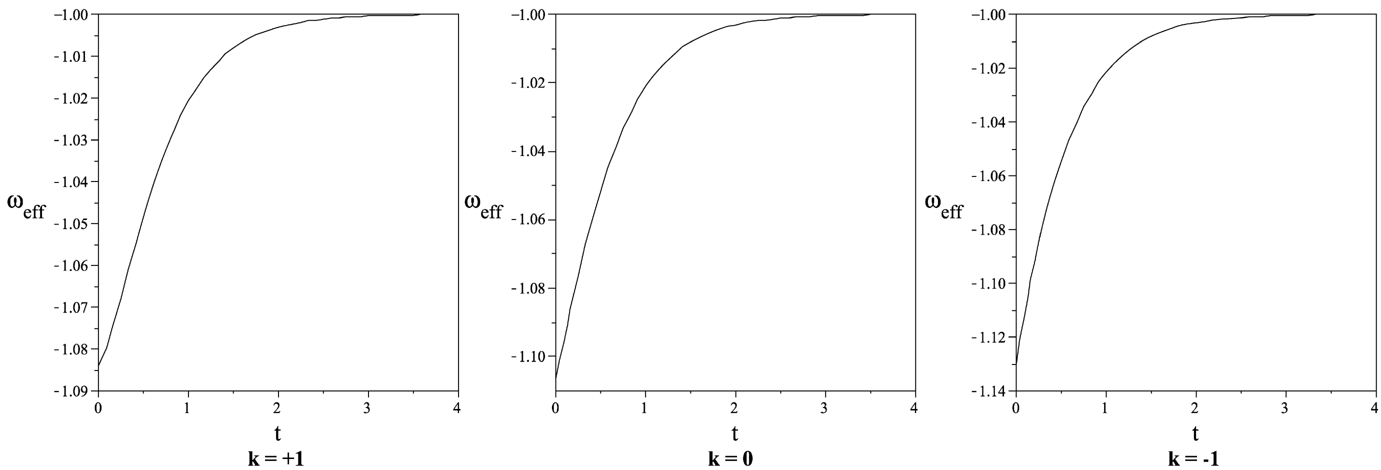

Fig. 4. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k = 1$, $k = 0$, and $k = -1$. Here in the numerical calculation we take $\alpha = 1, a_0 = c = H_l = 2, H_i = 1$.

$$\dot{\rho}_\phi + 3H(1 + \omega_\phi)\rho_\phi = -Q, \tag{45}$$

$$\dot{\rho}_m + 3H\rho = Q, \tag{46}$$

where the quantity Q expresses the interaction between the dark components. The corresponding FRW equations are written as,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_\phi), \tag{47}$$

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho_m + \rho_\phi + p_\phi), \tag{48}$$

with ρ_ϕ and p_ϕ given by,

$$\rho_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi), \tag{49}$$

$$p_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi). \tag{50}$$

Combining Eqs. (47)–(50), we obtain,

$$\omega(\phi)\dot{\phi}^2 = -\frac{1}{4\pi G}\left(\dot{H} - \frac{k}{a^2}\right) - \rho_m, \tag{51}$$

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{k}{a^2}\right) + \frac{1}{8\pi G}\left(\dot{H} - \frac{k}{a^2}\right) + \frac{\rho_m}{2}. \tag{52}$$

From Eq. (46), we get,

$$\rho_m = \rho_{m_0} a^{-3} + \frac{Q}{3H}. \tag{53}$$

We now consider the theory in which $V(\phi)$ and $\omega(\phi)$ are,

$$\omega(\phi) = -\frac{1}{4\pi G}\left(f'(\phi) - \frac{k}{a_0^2}e^{-2F(\phi)}\right) - F_0 e^{-3F(\phi)}, \tag{54}$$

$$V(\phi) = \frac{3}{8\pi G}\left(f^2(\phi) + \frac{k}{a_0^2}e^{-2F(\phi)}\right) + \frac{1}{8\pi G}\left(f'(\phi) - \frac{k}{a_0^2}e^{-2F(\phi)}\right) - \frac{1}{2}F_0 e^{-3F(\phi)}, \tag{55}$$

where $f(\phi) = F'(\phi)$. Then the following solution is found,

$$\phi = t, \qquad H(t) = f(t) \tag{56}$$

then we obtain,

$$a(t) = a_0 e^{F(t)}, \quad a_0 = \left(\frac{\rho_{m_0}}{F_0}\right)^{\frac{1}{3}} \tag{57}$$

EoS parameter is as,

$$w_{\text{eff}} = \frac{p_\phi}{\rho_m + \rho_\phi} = -1 - \frac{2}{3}\frac{\dot{H} - \frac{k}{a^2}}{H^2 + \frac{k}{a^2} + \frac{Q}{3H}}. \tag{58}$$

Now we consider above equations by following examples.

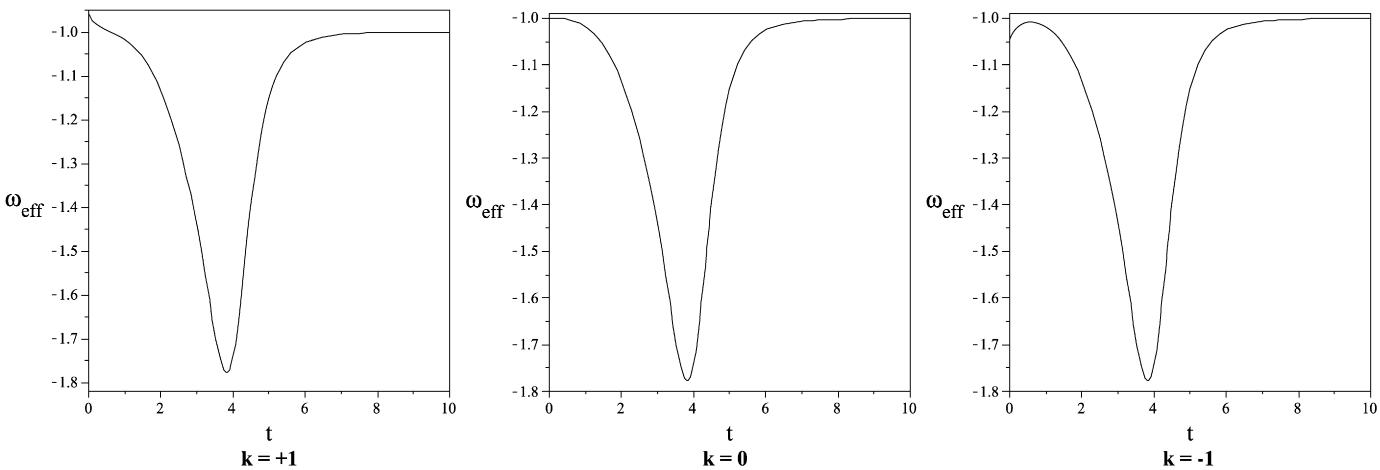

Fig. 5. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k=1$, $k=0$, and $k=-1$. Here in the numerical calculation we take $h_0=t_0=t_1=3$ and $Q=-1$.

8. Example I

As a first example, we consider the following model,

$$f(\phi)=h_0^2\left(\frac{1}{t_0^2-\phi^2}+\frac{1}{\phi^2+t_1^2}\right). \tag{59}$$

Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=h_0^2\left(\frac{1}{t_0^2-t^2}+\frac{1}{t^2+t_1^2}\right)\Rightarrow \dot{H}=2th_0^2\left(\frac{1}{(t_0^2-t^2)^2}+\frac{1}{(t^2+t_1^2)^2}\right), \tag{60}$$

$$a(t)=a_0\left(\frac{t+t_0}{t_0-t}\right)^{\frac{h_0^2}{2t_0}}e^{\frac{h_0^2}{t_1}\arctan(t/t_1)}. \tag{61}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=-\frac{1}{\pi G}\frac{h_0^2(t_1^2+t_0^2)(\phi^2-\frac{t_1^2+t_0^2}{2})\phi}{(t_1^2+\phi^2)^2(t_0^2-\phi^2)^2}+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-F_0e^{-3F(\phi)}, \tag{62}$$

$$V(\phi)=\frac{h_0^2(t_1^2+t_0^2)}{8\pi G(t_1^2+\phi^2)^2(t_0^2-\phi^2)^2}\left[3h_0^2(t_1^2+t_0^2)+4\phi\left(\phi^2-\frac{t_1^2+t_0^2}{2}\right)\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-\frac{1}{2}F_0e^{-3F(\phi)}, \tag{63}$$

$F(\phi)$ is as,

$$F(\phi)=\frac{h_0^2}{2t_0}\ln\left(\frac{\phi+t_0}{t_0-\phi}\right)+\frac{h_0^2}{t_1}\arctan\frac{\phi}{t_1}. \tag{64}$$

By using Eqs. (18) and (19), the effective EoS parameter is obtained as following,

$$w_{\mathrm{eff}}=-1-\frac{2}{3}\frac{2a_0^2h_0^2t(\frac{1}{(t_0^2-t^2)^2}-\frac{1}{(t^2+t_1^2)^2})(\frac{t+t_0}{t_0-t})^{h_0^2/t_0}e^{\frac{2h_0^2}{t_1}\arctan(t/t_1)}-k}{a_0^2[h_0^4(\frac{1}{t_0^2-t^2}-\frac{1}{t^2+t_1^2})^2+Q/(3h_0^2(\frac{1}{t_0^2-t^2}-\frac{1}{t^2+t_1^2}))](\frac{t+t_0}{t_0-t})^{h_0^2/t_0}e^{\frac{2h_0^2}{t_1}\arctan(t/t_1)}+k}. \tag{65}$$

In Fig. 5, ω_{eff} is starting in a value less than -1, and ending asymptotically go to -1. Therefore expansion of the universe has an acceleration epoch for all cases.

9. Example II

Now we consider the following model,

$$f(\phi)=\frac{H_0}{t_s-\phi}+\frac{H_1}{\phi^2}. \tag{66}$$

Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=\frac{H_0}{t_s-t}+\frac{H_1}{t^2}\Rightarrow \dot{H}=\frac{H_0}{(t_s-t)^2}-\frac{2H_1}{t^3}, \tag{67}$$

$$a(t)=a_0(t_s-t)^{-H_0}e^{\frac{-H_1}{t}}. \tag{68}$$

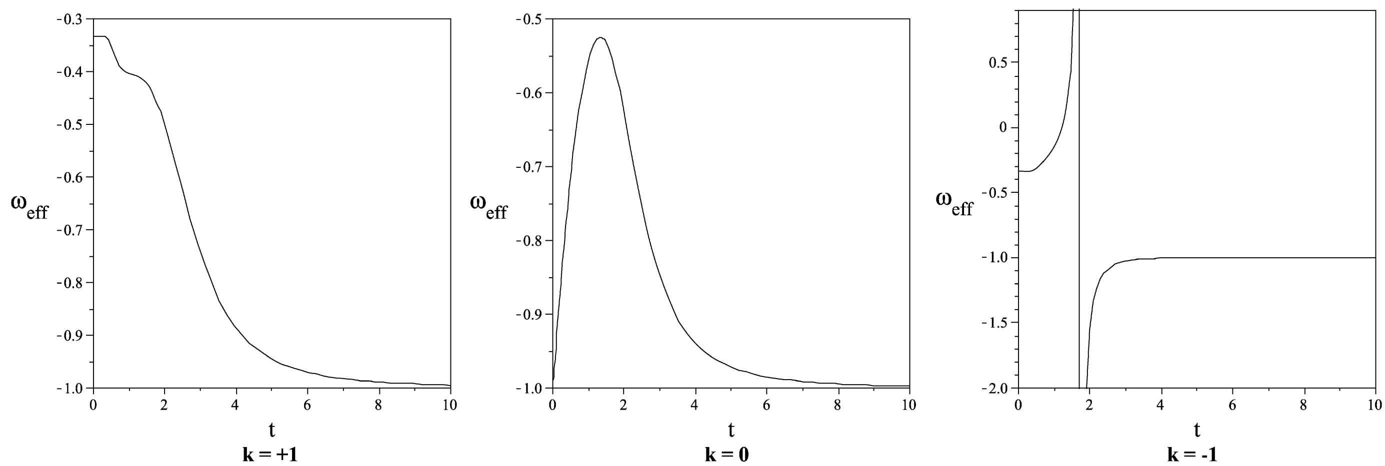

Fig. 6. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k=1$, $k=0$, and $k=-1$. Here in the numerical calculation we take, $a_0=1$, $H_1=2$,$H_0=t_s=-1$ and $Q=1$.

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=-\frac{1}{4\pi G}\left[\frac{H_0}{(t_s-\phi)^2}-\frac{2H_1}{\phi^2}\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-F_0e^{-3F(\phi)}, \tag{69}$$

$$V(\phi)=\frac{1}{8\pi G}\left[\frac{H_0(3H_0+1)}{(t_s-\phi)^2}+\frac{H_1}{\phi^3}\left(\frac{H_1}{\phi}-2\right)\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-\frac{1}{2}F_0e^{-3F(\phi)}, \tag{70}$$

$F(\phi)$ is as,

$$F(\phi)=-\frac{H_1}{\phi}-H_0\ln(t_s-\phi). \tag{71}$$

By using Eqs. (25) and (26), the effective EoS parameter is obtained as following,

$$w_{\mathrm{eff}}=-1-\frac{2}{3}\frac{a_0^2(t_s-t)^{-2H_0}(\frac{H_0}{(t_s-t)^2}-\frac{2H_1}{t^3})e^{\frac{-2H_1}{t}}-k}{a_0^2(t_s-t)^{-2H_0}[(\frac{H_0}{t_s-t}-\frac{H_1}{t^2})^2+Q/(3(\frac{H_0}{t_s-t}-\frac{H_1}{t^2}))]e^{\frac{-2H_1}{t}}+k}. \tag{72}$$

As one can see in Fig. 6, we have an acceleration epoch in the $k=1$ case, but or $k=0,-1$ both phases deceleration and acceleration are possible.

10. Example III

As a third example, we consider the following model,

$$f(\phi)=H_0+\frac{H_1}{\phi^n}, \tag{73}$$

where H_0 and $H_1>0$ are constant and n is a positive integer and constant. Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=H_0+\frac{H_1}{t^n}\Rightarrow\dot{H}=-\frac{nH_1}{t^{n+1}}, \tag{74}$$

$$a(t)=a_0e^{[H_0t-\frac{H_1}{(n-1)t^{n-1}}]}. \tag{75}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=\frac{1}{4\pi G}\frac{nH_1}{\phi^{n+1}}+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-F_0e^{-3F(\phi)}, \tag{76}$$

$$V(\phi)=\frac{1}{8\pi G}\frac{3}{\phi^{n+1}}\left[\frac{(H_0\phi^{n/2}+H_1)^2}{\phi^{n-1}}-\frac{nH_1}{3}\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-\frac{1}{2}F_0e^{-3F(\phi)}, \tag{77}$$

$F(\phi)$ is as,

$$F(\phi)=H_0\phi-\frac{H_1}{(n-1)\phi^{n-1}}. \tag{78}$$

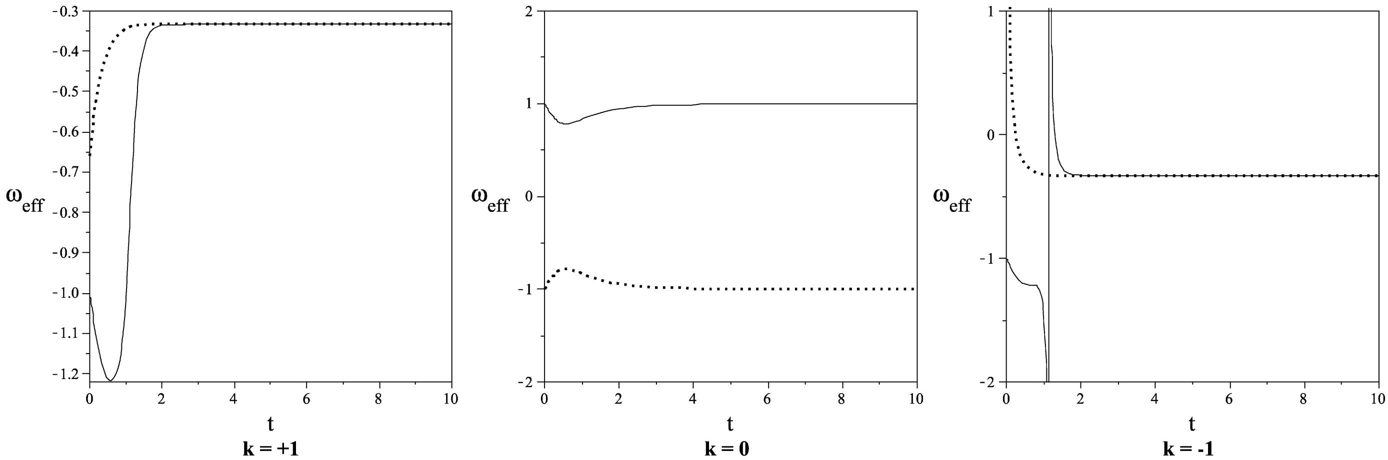

Fig. 7. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k=1$, $k=0$, and $k=-1$. Here in the numerical calculation we take $a_0=0.5$, $H_1=H_0=-2$, $Q=1$, $n=+2$ (line) and $n=-2$ (dot).

By using Eqs. (32) and (33), the effective EoS parameter is obtained as following,

$$w_{\rm eff}=-1+\frac{2}{3}\frac{a_0^2\frac{nH_1}{t^{n+1}}e^{[2H_0t-\frac{2H_1}{(n-1)t^{n-1}}]}+k}{a_0^2[(H_0+\frac{H_1}{t^n})^2+Q/(3(H_0+\frac{H_1}{t^n}))]e^{[2H_0t-\frac{2H_1}{(n-1)t^{n-1}}]}+k}. \tag{79}$$

As one can see, in case $n=2$, for all cases of k, we have two phase, one as deceleration epoch and other as acceleration epoch. But in $n=-2$ case, we have an acceleration phase for $k=1,-1$, in the other hand we have one acceleration epoch and the other a deceleration epoch for $k=0$. See also Fig. 7.

11. Example IV

As a fourth example, we consider the following model,

$$f(\phi)=\frac{H_i+H_lce^{2\alpha\phi}}{1+ce^{2\alpha\phi}}, \tag{80}$$

where H_i, H_l, c and α are positive constant. Using the solution (15), the Hubble parameter and the scale factor are given by,

$$H(t)=\frac{H_i+H_lce^{2\alpha t}}{1+ce^{2\alpha t}}\Rightarrow\dot{H}=-\frac{2c\alpha(H_i-H_l)e^{2\alpha t}}{(1+ce^{2\alpha t})^2}, \tag{81}$$

$$a(t)=a_0\frac{(1+ce^{2\alpha t})^{\frac{H_l}{2\alpha}}}{(c+e^{-2\alpha t})^{\frac{H_i}{2\alpha}}}. \tag{82}$$

$\omega(\phi)$ and $V(\phi)$ are as,

$$\omega(\phi)=\frac{1}{2\pi G}\frac{c\alpha(H_i-H_l)e^{2\alpha\phi}}{(1+ce^{2\alpha\phi})^2}+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-F_0e^{-3F(\phi)}, \tag{83}$$

$$V(\phi)=\frac{1}{8\pi G}\left[\frac{3(H_i+cH_le^{2\alpha\phi})^2}{(1+ce^{2\alpha\phi})^2}-\frac{2c\alpha(H_i-H_l)e^{2\alpha\phi}}{(1+ce^{2\alpha\phi})^2}\right]+\frac{1}{4\pi G}\frac{k}{a_0^2}e^{-2F(\phi)}-\frac{1}{2}F_0e^{-3F(\phi)}, \tag{84}$$

$F(\phi)$ is as,

$$F(\phi)=\frac{1}{2\alpha}\ln\frac{(1+ce^{2\alpha\phi})^{H_l}}{(c+e^{-2\alpha\phi})^{H_i}}. \tag{85}$$

By using Eqs. (39) and (40), the effective EoS parameter is obtained as following,

$$w_{\rm eff}=-1+\frac{2}{3}\frac{2a_0^2c\alpha(H_i-H_l)(1+ce^{2\alpha t})^{\frac{H_l}{\alpha}}e^{2\alpha t}+k(1+ce^{2\alpha t})^2(c+e^{-2\alpha t})^{\frac{H_i}{\alpha}}}{a_0^2[(H_i+cH_le^{2\alpha t})^2+\frac{Q(1+ce^{2\alpha t})^3}{(H_i+cH_le^{2\alpha t})}](1+ce^{2\alpha t})^{\frac{H_l}{\alpha}}+k(1+ce^{2\alpha t})^2(c+e^{-2\alpha t})^{\frac{H_i}{\alpha}}}. \tag{86}$$

As one can see from Fig. 8, when $t\to 0$ then $w_{\rm eff}\to -1$, and we have an acceleration epoch, while for $t\to\infty$ we have $w_{\rm eff}\to -1$ for all cases, which can be interpreted as late time acceleration.

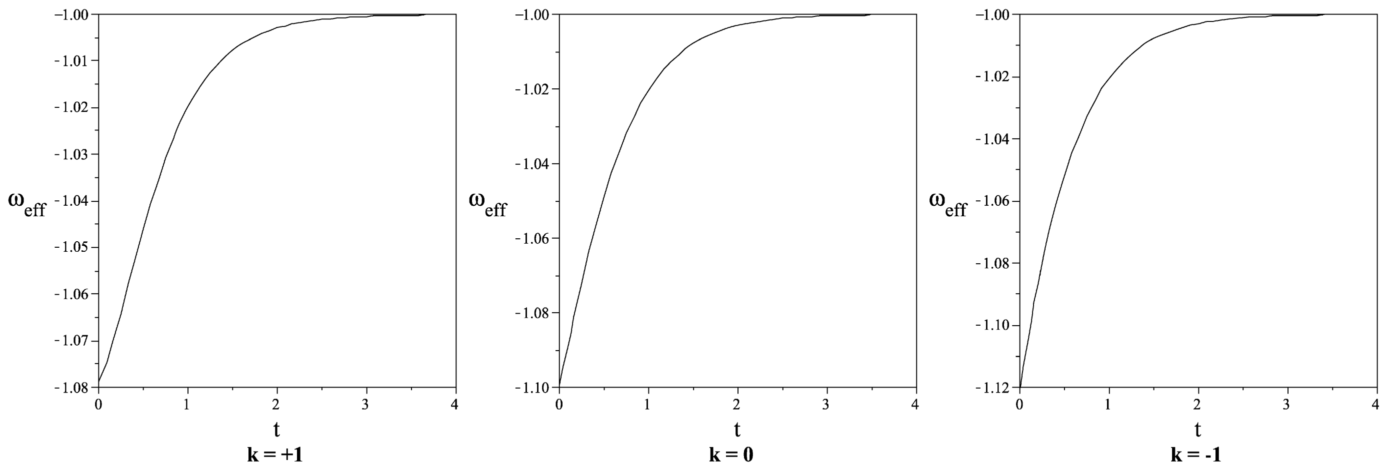

Fig. 8. Graphs for the effective EoS parameter: from the left to the right one has the graphs for $k = 1$, $k = 0$, and $k = -1$. Here in the numerical calculation we take $a_0 = 2$, $H_l = 2$, $H_i = 1$, $\alpha = 1$, $c = 2$ and $Q = 1$.

12. Conclusions

In order to solve cosmological problems and because of the lack of knowledge, for instance for determining what could be the best candidate for DE for explaining the accelerated expansion of the universe, cosmologists try to approach the best results as precisely as they can by considering all the possibilities they have. The flat universe is usually assumed when constraining the time dependence of the equation of state. The assumption is often justified by invoking a prediction of the inflation or by resorting to confirmation by cosmological observations. However, we should test the inflationary paradigm by measuring the curvature of the universe, and observational evidence of a flat universe is often obtained assuming a cosmological constant for dark energy. Therefore, not knowing the nature of the dark energy, it is important to investigate the curvature of the universe with various dark energy models.

In the present Letter we have developed, the reconstruction program for the expansion history of the universe, by using a single scalar field in the non-flat universe. At first in the non-interacting case we have presented a number of examples which prove that it is actually possible to unify early-time inflation with late-time acceleration.

Studying the interaction between the dark energy and ordinary matter will open a possibility of detecting the dark energy. Being a dynamical component, the scalar field dark energy is expected to interact with the ordinary matters. For example, Carroll [24] (see also [25]) has considered an interaction of form $Q F_{\mu\nu} \tilde{F^{\mu\nu}}$ with $F_{\mu\nu}$ being the electromagnetic field strength tensor which has interesting implication on the rotation of the plane of polarization of light coming from distant sources. Recent data on the possible variation of the electromagnetic fine structure constant has triggered interests in studies related to the interactions between quintessence and the matter fields. Due to this we have generalized the technique of first part of paper to the non-interacting case. Again, various explicit examples of unification of early-time inflation and late-time acceleration have been presented in those formulations.

References

[1] A.G. Riess, et al., Supernova Search Team Collaboration, Astron. J. 116 (1998) 1009, astro-ph/9805201;
S. Perlmutter, et al., Supernova Cosmology Project Collaboration, Astrophys. J. 517 (1999) 565, astro-ph/9812133;
P. Astier, et al., Astron. Astrophys. 447 (2006) 31, astro-ph/0510447.

[2] K. Abazajian, et al., SDSS Collaboration, Astron. J. 128 (2004) 502, astro-ph/0403325;
K. Abazajian, et al., SDSS Collaboration Astron. J. 129 (2005) 1755, astro-ph/0410239.

[3] D.N. Spergel, et al., WMAP Collaboration, Astrophys. J. Suppl. 148 (2003) 175, astro-ph/0302209;
D.N. Spergel, et al., astro-ph/0603449.

[4] A. Einstein, Sitzungsber. K. Preuss. Akad. Wiss. 142 (1917), The Principle of Relativity, Dover, New York, 1952, p. 177.

[5] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1;
V. Sahni, A.A. Starobinsky, Int. J. Mod. Phys. D 9 (2000) 373, astro-ph/9904398;
S.M. Carroll, Living Rev. Rel. 4 (2001) 1, astro-ph/0004075;
P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 559, astro-ph/0207347;
T. Padmanabhan, Phys. Rep. 380 (2003) 235, hep-th/0212290.

[6] P.J. Steinhardt, in: V.L. Fitch, D.R. Marlow (Eds.), Critical Problems in Physics, Princeton Univ. Press, Princeton, NJ, 1997.

[7] P.J.E. Peebles, B. Ratra, Astrophys. J. 325 (1988) L17;
B. Ratra, P.J.E. Peebles, Phys. Rev. D 37 (1988) 3406;
C. Wetterich, Nucl. Phys. B 302 (1988) 668;
R.R. Caldwell, R. Dave, P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582, astro-ph/9708069;
A.R. Liddle, R.J. Scherrer, Phys. Rev. D 59 (1999) 023509, astro-ph/9809272;
V. Faraoni, Phys. Rev. D 62 (2000) 023504.

[8] C. Armendariz-Picon, V.F. Mukhanov, P.J. Steinhardt, Phys. Rev. Lett. 85 (2000) 4438, astro-ph/0004134;
C. Armendariz-Picon, V.F. Mukhanov, P.J. Steinhardt, Phys. Rev. D 63 (2001) 103510, astro-ph/0006373.

[9] A. Sen, JHEP 0207 (2002) 065, hep-th/0203265;
T. Padmanabhan, Phys. Rev. D 66 (2002) 021301, hep-th/0204150.

[10] R.R. Caldwell, Phys. Lett. B 545 (2002) 23, astro-ph/9908168;
R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301, astro-ph/0302506;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 562 (2003) 147, hep-th/0303117;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 565 (2003) 1, hep-th/0304131;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 595 (2004) 1;

S. Nojiri, S.D. Odintsov, Phys. Rev. D 72 (2005) 023003;
V. Faraoni, Class. Quantum Grav. 22 (2005) 3235;
M.R. Setare, Eur. Phys. J. C 50 (2007) 991.
[11] F. Piazza, S. Tsujikawa, JCAP 0407 (2004) 004, hep-th/0405054.
[12] B. Feng, X.L. Wang, X.M. Zhang, Phys. Lett. B 607 (2005) 35, astro-ph/0404224;
A. Anisimov, E. Babichev, A. Vikman, JCAP 0506 (2005) 006, astro-ph/0504560;
M.R. Setare, Phys. Lett. B 641 (2006) 130, hep-th/0611165;
E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D 70 (2004) 043539, hep-th/0405034;
S. Nojiri, S.D. Odintsov, S. Tsujikawa, Phys. Rev. D 71 (2005) 063004, hep-th/0501025;
Y.-f. Cai, H. Li, Y.-S. Piao, X.-m. Zhang, Phys. Lett. B 646 (2007) 141.
[13] L. Amendola, Phys. Rev. D 62 (2000) 043511, astro-ph/9908023;
D. Comelli, M. Pietroni, A. Riotto, Phys. Lett. B 571 (2003) 115, hep-ph/0302080;
M.R. Setare, Phys. Lett. B 642 (2006) 1;
M.R. Setare, JCAP 0701 (2007) 023;
D. Saez-Gomez, arXiv: 0804.4586 [hep-th].
[14] C. Deffayet, G.R. Dvali, G. Gabadadze, Phys. Rev. D 65 (2002) 044023, astro-ph/0105068;
V. Sahni, Y. Shtanov, JCAP 0311 (2003) 014, astro-ph/0202346;
M.R. Setare, Phys. Lett. B 642 (2006) 421.
[15] A.Y. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B 511 (2001) 265, gr-qc/0103004.
[16] M. Li, Phys. Lett. B 603 (2004) 1;
K. Enqvist, M.S. Sloth, Phys. Rev. Lett. 93 (2004) 221302, hep-th/0406019;
K. Ke, M. Li, Phys. Lett. B 606 (2005) 173, hep-th/0407056;
D. Pavon, W. Zimdahl, Phys. Lett. B 628 (2005) 206, gr-qc/0505020;
E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D 71 (2005) 103504, hep-th/0502082;
B. Hu, Y. Ling, Phys. Rev. D 73 (2006) 123510, hep-th/0601093;
H. Li, Z.K. Guo, Y.Z. Zhang, Int. J. Mod. Phys. D 15 (2006) 869, astro-ph/0602521;
M.R. Setare, Phys. Lett. B 644 (2007) 99, hep-th/0610190;
M.R. Setare, J. Zhang, X. Zhang, JCAP 0703 (2007) 007, gr-qc/0611084;
M.R. Setare, Phys. Lett. B 648 (2007) 329;
M.R. Setare, Phys. Lett. B 653 (2007) 116.
[17] C.L. Bennet, et al., Astrophys. J. Suppl. 148 (2003) 1;
D.N. Spergel, Astrophys. J. Suppl. 148 (2003) 175;
K. Ichikawa, et al., astro-ph/0605481.
[18] Q.G. Huang, M. Li, JCAP 0408 (2004) 013.
[19] M. Tegmark, et al., Astrophys. 606 (2004) 702.
[20] D. Polarski, A. Ranquet, Phys. Lett. B 627 (2005) 1;
U. Franca, Phys. Lett. B 641 (2006) 351;
K. Ichikawa, T. Takahashi, Phys. Rev. D 73 (2006) 083526;
Y. Gong, A. Wang, Phys. Rev. D 75 (2007) 043520;
K. Ichikawa, T. Takahashi, JCAP 0702 (2007) 001;
E.L. Wright, astro-ph/0701584;
G. Zhao, et al., astro-ph/0612728.
[21] C. Clarkson, M. Cortes, B.A. Bassett, JCAP 0708 (2007) 011.
[22] E. Elizalde, S. Nojiri, S.D. Odintsov, D. Saez-Gomez, V. Faraoni, Phys. Rev. D 77 (2008) 106005.
[23] S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B 632 (2006) 597;
S. Nojiri, S.D. Odintsov, Gen. Relativ. Gravit. 38 (2006) 1285;
S. Capozziello, V.F. Cardone, E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D 73 (2006) 043212;
S. Nojiri, S.D. Odintsov, H. Stefancic, Phys. Rev. D 74 (2006) 086009.
[24] S.M. Carroll, Phys. Rev. Lett. 81 (1998) 3067.
[25] X.M. Zhang, hep-ph/0410292.