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Random loads Fatigue. Experimental approach through thermoelasticity

Claudio Braccesi[a], Filippo Cianetti[a,]*, Michele Moretti[a], Gianluca Rossi[a]

[a]*Università degli Studi di Perugia, Department of Engineering, via G. Duranti, 93, Perugia, 06125 Italy*

Abstract

The aims achieved in this work have been the development of a software for the processing of thermal imaging films in frequency domain which makes the use of the thermovision useful to the assessment of the fatigue damageability of components loaded by random loads conditions. To this aim an experimental test rig and the relative measuring chain was built.

The measuring system allows to analyze a system or a mechanical component providing the user, in addition to the usual thermographic output, a whole series of results of both local and global type.

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Keywords: random loads; frequency domain fatigue evaluation; thermovision

1. Introduction

The evaluation of the fatigue behavior of mechanical components subjected to random loads [1] is traditionally carried out in industrial environment (i.e. automotive, aviation and aerospace) in time domain, by using both laboratory and field tests conducted on physical prototypes both numerical simulations conducted on virtual prototypes [2-4]. As an alternative to this consolidated approach, the assessment of fatigue damage in frequency domain is proving to be extremely accurate [5-8]. At the same time, it goes by completing the definition of a virtual evaluation scenario through which simulate with ever greater fidelity the behavior of mechanical systems rather complex and the stress and strain states of their components [9-14]. There are also the conditions that these assessment techniques can be translated into

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* Corresponding author. Tel.: +39 075 5853728; fax: +39 075 5853703.
E-mail address: filippo.cianetti@unipg.it

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experimental ones without losing their characteristics of speed and synthesis by the adoption of the most modern analysis techniques of thermal images, based on the principle of thermoelasticity [15, 16]. These measurement techniques are able to assess the stress state of a component when it is dynamically excited, through the analysis of the thermal signal emitted from the surface of the component itself. Previous activities [17] have demonstrated the possibility of translating the so-called frequency approach in these measurement techniques, by using random excitation signals, and providing the mapping of the surface stress state as function of frequency.

The aim of the research activity has been the development of a new procedure and relative software for the processing in frequency domain of thermal imaging films which makes the use of thermovision useful to assess the fatigue damageability of components and systems. The developed analysis procedure and relative software allow to analyze a system or a mechanical component, under random loads excitation, providing the user, in addition to the usual thermographic output, a whole series of results of both local and global type. These results are obtainable in form of images (maps), similar to the classical thermal ones, in which, pixel by pixel, the results, obtained either directly by the power spectral density (PSD) functions [1] (i.e. signal rms, spectral moments [1], damage) and by the related time histories (i.e. non-Gaussianity indices, correction factors of Gaussian damage [11]) are provided. For each pixel is then possible to deepen the analysis by asking the PSD function and the time history of the process (i.e. acceleration, strain, stress).

Accessory result of the activity has been the design, the implementation and experimental validation of a testing system (bench test) and of a thermographic, accelerometric and strain measurement chain, useful to obtain and validate the above aim. The whole system was subjected to an extensive campaign of numerical and experimental dynamic characterization.

2. Thermoelasticity and frequency domain

The thermoelastic stress analysis is based on the thermoelastic effect, introduced by Lord Kelvin in 1853 [15]. Under the hypothesis of adiabatic and reversible transformation the stress field and the relative variation of temperature are correlated. In case of homogenous and elastic continuum Kelvin formulated the following relation:

$$\frac{\Delta T}{T} = -\frac{\alpha}{\rho C_p}(\Delta\sigma_I) \tag{1}$$

Where T is absolute temperature, α is the thermal expansion coefficient [m/K], ρ is the density [Kg/m3], C_p is the specific heat at constant pressure [J/KgK], σ_I is the first stress invariant. The Kelvin equation clearly shows that a stress variation cause a temperature variation, so it's possible to measure stresses on a loaded component by analyze the variation of temperature. In measurement practice, for contactless temperature measurement an high speed and high thermal resolution thermocamera are used, necessary to collect the slight temperature variation, usually less than some hundredths of Kelvin degree. Usually the component is loaded with harmonic or random loads and the superficial stress pattern is obtained by a real time analysis of acquired temperature signals. When an harmonic load is applied, the temperature time history (TTH) is usually filtered with a *lock-in* signal processing [18]; a root mean square (RMS) or a convolution approach are applied in case of random loads.

Previous studies [17] show that it's possible to apply the Fourier approach to the thermoelastic effect and so it's possible to obtain superficial stress pattern spectrums based on the superposition principle. The stress tensor $\sigma_{i,j}$ in a generic point P of the elastic, homogenous and isotropic solid at the generic time t can be expressed as the sum of components each on with a defined amplitude, frequency and phase:

$$\sigma_{i,j}(P,t) = \sum_{k=1}^{n} A_k\left|\sigma_{i,j,k}(P,t)\right|\sin(\omega_k t + \varphi_k) \tag{2}$$

Where k is the index of the n components, A_k is the frequency amplitude of the k-th component, ω_k and φ_k its pulse and phase. Introducing the thermoelastic effect and considering the temperature variation δT between two instants t and $t + dt$, equation (2) becomes:

$$\delta T(P,t+dt,t) = -\frac{\alpha T}{\rho C_p}\sum_{k=1}^{n} A_k\left|\sigma_{I,k}(P,t)\right| \left[\sin(\omega_k\,(t+dt)+\varphi_k)-\sin(\omega_k t+\varphi_k)\right] \tag{3}$$

where $\sigma_{I,k}$ is the k-*th* component of first stress invariant. The temperature in a generic point P at time t can be expressed as superposition of each frequency component of the first stress invariant (4).

$$T(P,t) = -\frac{\alpha T}{\rho C_p}\sum_{k=1}^{n} A_k\left|\sigma_{I,k}(P,t)\right| \sin(\omega_k t+\varphi_k) \tag{4}$$

The Fourier approach can be applied to temperature signal too and the following equation can easily be obtained:

$$T(P,t) = \sum_{k=1}^{n} T_k \sin\left(\omega_k t+\varphi'_k\right) \tag{5}$$

where T_k is the frequency amplitude of the k-th component of temperature ω_k and φ'_k its pulse and phase. By equating the second members of the equations (4) and (5), next expression can be obtained:

$$\frac{\Delta T_\omega}{T} = -\frac{\alpha}{\rho C_p}\left(\Delta\sigma_{I,\omega}\right) \tag{6}$$

where ΔT_ω is the amplitude of temperature at pulse ω and $\Delta\sigma_{I,\omega}$ is the amplitude of first stress invariant at the same frequency. Equation (6) shows that the temperature variation at a generic pulse ω is due to the first stress invariant variation $\Delta\sigma_{I,\omega}$ at the same frequency. The equation (6) allows to correlate the time domain temperature signals and frequency domain stresses. Applying the frequency analysis to temperature signals is possible to create maps of stress as frequency response functions or power spectrum density functions.

If a reference signal is provided it's possible to introduce the following complex ratio:

$$S(i\omega) = \frac{\sigma_I(i\omega)}{F(i\omega)} \tag{7}$$

where $\sigma_I(i\omega)$ is the Fourier transform of the first invariant of stress measured in a generic point of the component and $F(i\omega)$ is the Fourier transform of the excitation signal. The term $S(i\omega)$ represents the stress frequency transfer function in terms of first stress invariant.

Equation (7) together with the use of classical high speed and high thermal resolution thermocamera allow to define the proposed measurement technique. This propose approach need random excitation of the component and the acquisition of component surface temperature time history. Moreover, stress or strain reference signal is necessary for stress calibration. In this case strain gauges are preferred instead of accelerometers or other devices, due to its light weight and easy use. In the hypothesis and, then, realized procedure, the acquired thermal images have not to be scaled in terms of temperature: the stored values come out from a 16 bit analog to digital conversion (ADC) performed by a *thermal images acquisition module.* It's not necessary to convert the images in a true temperature scale because a comprehensive scale factor will be applied during the signals processing. Each acquired thermal image is representable as a 2D data matrix and its size is defined by the spatial resolution of the array of the thermocamera. The ADC value of a generic point P on the surface of the component can be stored in the 2D data matrix, at the corresponding spatial coordinates (h,k). Stacking a sequence of thermal images, acquired with a constant frame rate f_s [Hz], it is possible to build up a 3D matrix where the ADC values, acquired at time $\tau = n/f_s$, $n = 1,2,3,\ldots$, in a generic point P of the surface of the component, is stored to the coordinates (h,k,n).

Considering the whole acquisition, a 3D data matrix M, with ($H\times K\times N_p$) dimensions, can be created, where H and

K are the spatial resolution or thermal array and N_p is the numerousness of acquired thermal images. This matrix is the thermal imaging film used as input for the thermal signal processing. The third coordinate of the M matrix clearly represent the time coordinate of ADC, i.e. of temperature. Along time coordinate it's possible to apply frequency analysis algorithms, such as FFT, PSD, etc. The output of a generic frequency analysis algorithm is a new matrix $W(h, k, n_\omega)$, with $(H\times K\times N_\omega)$ dimensions, with the same spatial size $(H\times K)$ of data matrix M, with N_ω that represents the frequency bands numerousness, used to perform the analysis and where n_ω represent the n-th indexed frequency used to perform the analysis. By extracting from W all the calculated values at the same frequency ω^*, a 2D data matrix W_{ω^*} can be obtained that represents a qualitative stress pattern at the selected frequency. The calculated values are proportional to the stress at frequency ω^*, and the real value can be easily assessed by a scale factor obtainable by the reference strain gauge measure. The scale function $K(\omega)$ can be easily calculated by the following equation [16], who consider the responses of differential thermography (TSA) and the strain gauge measurement.

$$K(\omega) = \left|\frac{FRF(S_{SG}, V_{exc})}{[FRF(avg(ADC), V_{exc})]_A}\right| \tag{8}$$

In equation (8) the numerator is the frequency response function between the strain gauge signal S_{SG} and the excitation one V_{exc}. The denominator is the frequency response function between the average values of ADC signals and the excitation signal V_{exc}. In order to correctly correlate thermal and strain measures, the average value $avg(ADC)$ is used. The average is calculated on the area A of the component surface where the strain gauge is stuck on. Once the scale function is calculated, it's possible to calculate the stress maps function:

$$\Theta(\omega) = \frac{W(\omega)}{K(\omega)} \tag{9}$$

$\Theta(\omega)$ is a $(I\times J\times N_\omega)$ matrix. A 2D sub-matrix, obtained by the extraction of all values at the same frequency ω^*, represents a differential thermography in stress scale at frequency ω^*.

An analogous procedure can be easily defined by substituting in the previous equations the FRF algorithm with the PSD one. This second approach allows to obtain stress maps in terms of PSD functions. From these it is possible to instantaneously obtain, in frequency domain, by the consolidated fatigue damage evaluation approaches [5-7], many results useful for the component fatigue behavior evaluation and, moreover, representable pixel per pixel in terms of color maps (see paragraphs 3 and 4).

3. Measurement chain, analysis software and test bench development

The proposed procedure has been implemented and validated by realizing a specific test chain. In particular a thermic-accelerometric measurement chain, a dedicated test rig (test bench) were designed and realized and an external analysis software for the parallel post-processing of the thermic measures was developed.

The developed test chain is shown in figure (1). It is composed by an "*excitation module*" characterized by an electrodynamic shaker (**A**, LDS V650) driven by a signal processing unit (**B**, Controller SignalStar Vector). The drive signal is constantly controlled in feed-back in order to apply the desired PSD input. It is characterized by a "*thermal images acquisition module*" composed by a thermocamera (**C**, Deltatherm 1650 Stress Photonics), that acquires the thermal images with a user defined frame rate and send them to the frame grabber (**F**) and then stored into a PC hard drive. The last module is "*the data acquisition module*" that by a data logger (**D**, Prosig P8004) collects a reference signal from the shaker and a strain signal from a strain gauge stuck on the component. This acquisition of reference and strain signals is synchronous with the temperature acquisition rate thanks to a TTL signal (**E**) provided by the thermocamera.

The designed specimen is an aluminum clepsydra specimen, asymmetric, and asymmetrically bounded in a configuration of beam fixed and hinged at the ends (fig. 2). The specimen allowed to place an additional mass at an

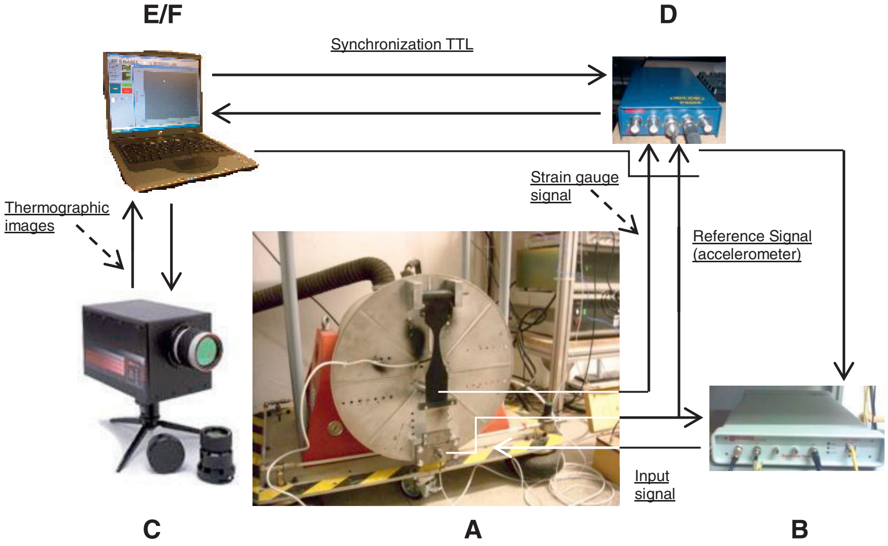


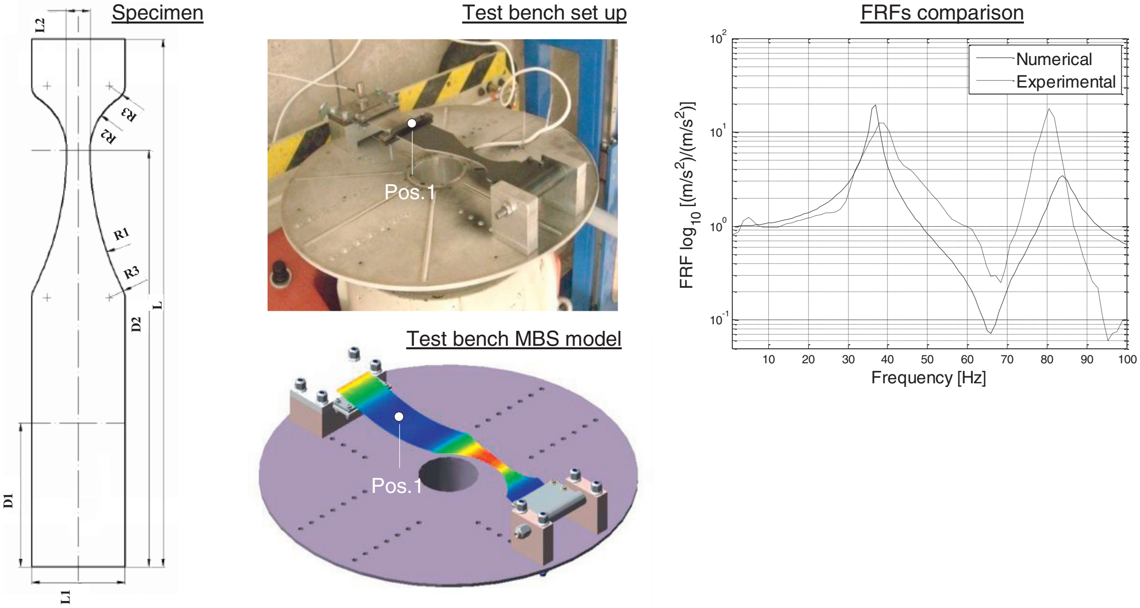
Fig. 1 – Measurement chain for the frequency domain damage experimental assessment

appropriate distance and with an assigned value in order to influence the amplitude of the response and the location of the central frequencies. The main features are the following: material aluminum alloy Al 5754 H32, thickness S 2 mm, length L 360 mm, width L1 60 mm, narrow section width L2 15 mm, distance of minimum cross-section area D2 280 mm , additional mass distance D1 105 mm, curvature radius R1 200 mm, curvature radius R2 40 mm, fillet radius R3 10 mm, additional mass 150 g. As concern analysis software, it actually works in series with time domain acquisition of thermal images (film). It post-processes thermographic films to obtain numerical (i.e. time histories, PSDs, PDFs, cumulatives) and graphical (maps) reports. This phase is characterized by the user's choice of the image portion to be analyzed (a pixels matrix of h rows and k columns). The acquisition of the images is done pixel by pixel, instant by instant so as to define a time domain information database from which obtain results both in time and in frequency domain. The data are post-processed in a numerical code (Matlab®), and the procedure generates results in various forms but, in particular, in a graphics mode (color map), entirely analogous to thermographic one. They are definable as local value or color map results (afterwards respectively identified with L and M): temperature time history (L), stress power spectral density function (L), probability density function (L), stress amplitude cumulative (L), stress signal root mean square (M), and Gaussian indices (skewness, kurtosis) (M). Correction factors for Gaussian damageability [6], PSD spectral moments, till 4*th* order, equivalent alternating stress and fatigue damage per unit time (i.e. obtained by using linear cumulation rule of *Palmgren-Miner* [19]) can be all expressed as maps (M). Concerning the stress cycles counting (Probability Density Function an Cumulative) the *Dirlik* [5, 7] approach and *Rainflow* counting method [19] were respectively adopted for frequency and time domain.

This test chain envisages an integrated measuring and analyzing system which would allow the user to perform the usual thermographic measure obtaining directly on screen an image that represents, for example, the time unit damage at which "that" component is subjected in "those" load conditions.

4. Numerical/experimental bench validation

In order to validate the input and outputs design process the dynamic behavior of the test bench and the specimen were numerically and experimentally tested. Finite element models (FEM) of the specimen and of the fixture, developed in Ansys®, were validated by a parallel numerical and experimental dynamic characterization. As concern the whole test bench a multibody model (MBS) of the system, developed in Adams/View®, was also validated by performing analyses in time and frequency domain [10, 12]. Figure 2 shows the numerical/experimental comparison conducted on the test bench by adopting the MBS model, obtained by assembling the fixture as a rigid body and the sample as a flexible component, modeled by modal approach [20, 21]. MBS modelling and simulation were used to demonstrate how it is possible to adopt the frequency domain approach also for systems characterized by complex kinematics and that need this modelling approach [10, 12]. The figure represents the FRF function between shaker acceleration as input and acceleration, measured at the added mass location, as output.


Specimen

Test bench set up

FRFs comparison

Test bench MBS model

Fig. 2 – Specimen geometry (left image). In the center of figure the test chain set up vs its multibody model. At the right the experimental/numerical comparison in terms of FRF function between shaker acceleration as input and pos.1 acceleration as output

5. Experimental tests campaign

After the experimental validation of the design assumptions a series of tests were conducted in order to verify the entire measurement and analysis methodology. By using the measurement chain and the software described in paragraph 3, the behavior of the specimen when subjected to random loads was analyzed. After a calibration phase by a thermographic/strain gauge comparison, an acceleration input was defined in frequency domain (PSD function) to not cause, at least theoretically, a "premature" specimen failure and to generate in this case a narrow band dynamic behavior (monomodal with a center frequency around 35 Hz). In figure 3, as a flow chart, the information flow and the obtained results are represented. In ascending order, the needed information (Fig.3 no. 1-2) and those obtained as processing results (Fig.3 no. 3-6) are described. The images no. 1 and 2 of figure 3 refer to the input, in this case accelerometric, defined in frequency domain (PSD) and reconstructed by the controller in time domain. The images from no. 3 to no. 6 represent local results (L) obtained by the procedure in a point of the choose image portion. In particular, the interest is focused at position h=18, k=87 of the pixel matrix (Fig. 3). In Figure 4, as an example, some of the maps (M) associated with some of the main results are represented, obtained either from time or frequency domain. The time unit damage is expressed in logarithmic scale and, to evaluate it, the following formulation of the *Wöhler* curve [19] $S = \alpha N^{\beta}$ was adopted, in which S represents the alternating stress component and N the number of cycles to failure. The material's constants considered are $\alpha = 262.43$ MPa, and $\beta = -0.065$.

6. Conclusions

The frequency domain approach to fatigue damage evaluation allows to quickly obtain statistically stabilized results, allowing to assess in a short time (tens of seconds) stress PSD functions, statistically stabilized, without the need to acquire endless time histories to obtain equally stabilized results, as requested, by definition, by time domain

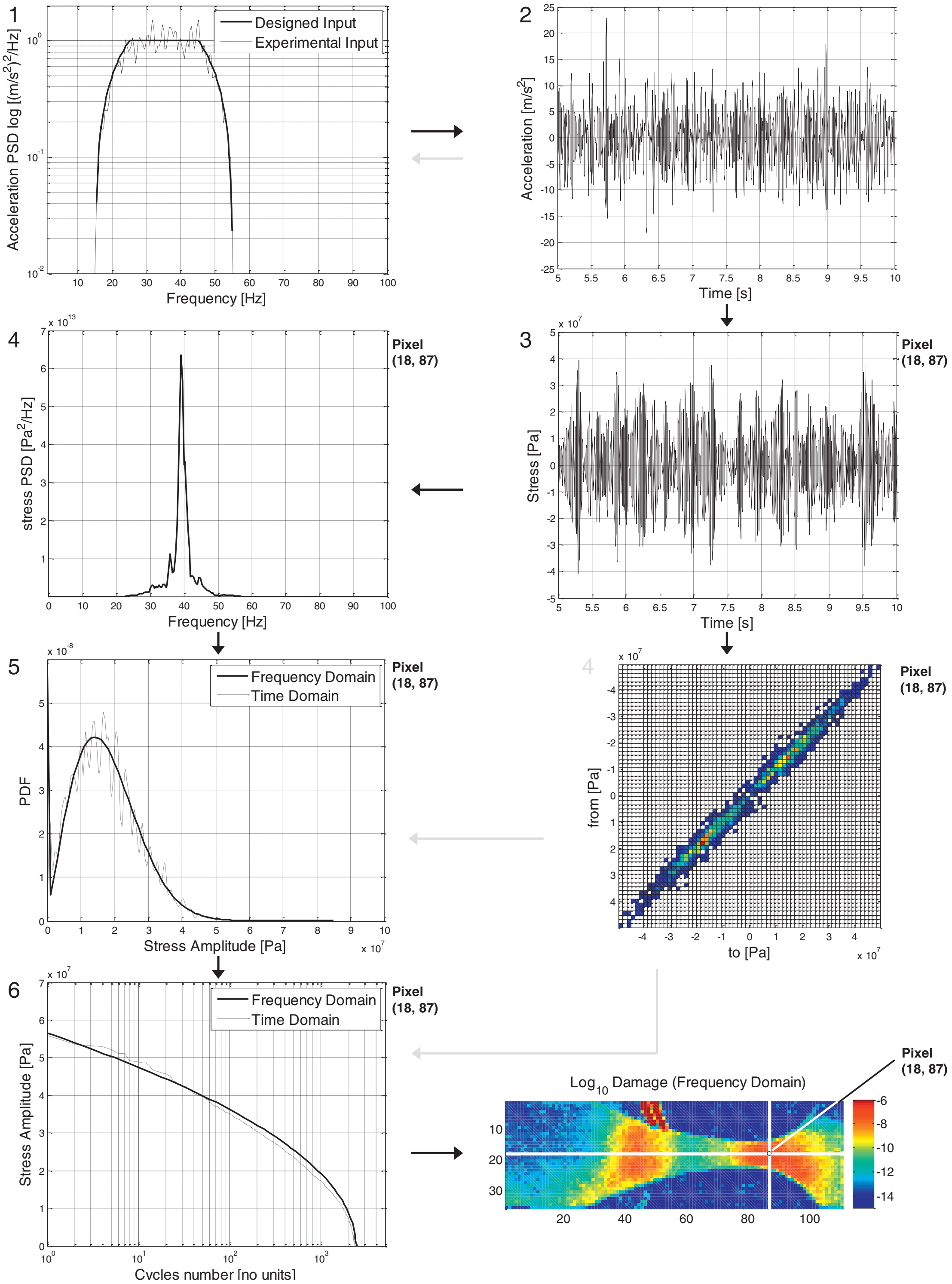


Fig. 3 – Graphical representation of the proposed procedure and of the obtainable results, local (L) and global (M). In this case the results for the (h, k)-th pixel with $h = 18$ and $k = 87$, together with the per time unit damage map are represented

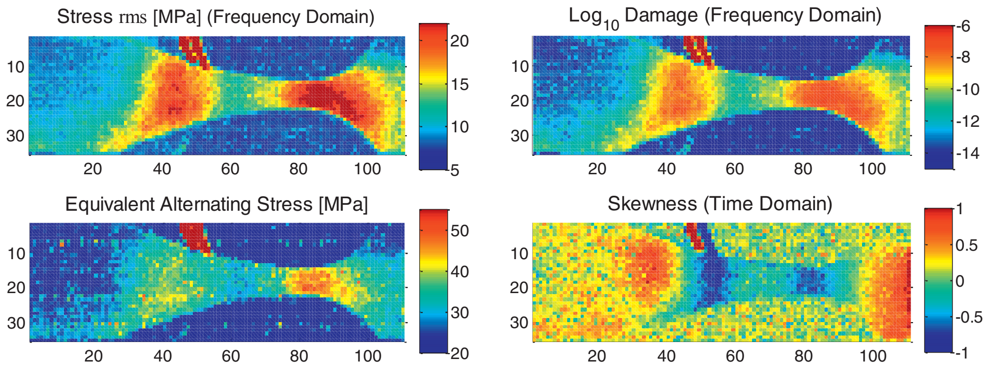


Fig. 4 – Example of global results (M), obtained in the experimental test campaign

approach to random process. The principal aim reached in this work has been the development of a software for the frequency domain processing of thermal imaging films which makes the use of the thermovision useful to the assessment of the fatigue damageability of components loaded by random loads conditions if combined with the classical frequency domain approach for the fatigue damage evaluation.

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