The relation between the problem posing and problem solving skills of prospective elementary mathematics teachers

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Abstract

The present study investigates the relation between the success of prospective teachers in problem posing and solving in relation to the problems raised by the topic of series and sequences. The sample of the study is composed of 76 senior students from the Department of Elementary Mathematics Teaching in Erzurum Atatürk University. The participants were administered problem posing-solving tests which encompassed the topic of series and sequences and the tests were developed in line with the sub-problems. According to the analysis of the data, there was a significant relation between problem posing and problem solving skills. Furthermore, it was established that there was a parallelism between the number of problem posed and success in problem solving.

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Keywords: Mathematics teaching; problem posing; problem solving; elementary prospective mathematics teachers; problem posing-solving.

1. Introduction

It can be indicated that recent studies in the field of mathematics education have concentrated on structuring the information, and the solution of non-routine problems. The importance of knowing how the students solve these problems and which strategies they use has been emphasized in the studies. Also, the studies perceive the problem solving activity as a transitional phase for problem posing. Crespo, (2003) accepts the development of problem posing skills as one of the important aims of mathematics learning and teaching and emphasizes that it should occupy the center space in mathematical activities. According to Silver (1994) problem posing is important for the achievement of many results such as development of problem solving skills and positive contributions to attitudes towards mathematics.


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In the light of the above, problem posing can be expressed in two ways:
a) Problem posing for discovering a situation or an experience.
For instance, mathematicians pose problems about mathematics and contribute to mathematics by solving the problems. From this perspective, one of the important contributions of problem posing is the provision of posing sub-problems for overcoming difficulties encountered during problem solving.
b) Problem posing based on problem solutions.
At this point, problem solving can be considered as a transition phase for problem posing. Gonzalez (1998) describes problem posing as the fifth phase of Polya’s problem solving, it can be considered as an indicator of the relation between problems and problem solving. Students should comprehend the solution of the problem in order to pose problems by investigating the problem and shaping their thoughts which form the model for new problems. According to English (1997), the same situation forms via students’ questioning the central thought in a solved problem or enlarging or changing the data in the solved problem.

Problem posing is an information source in itself in terms of detection of comprehension level of students in mathematical operations, problem solving skills and attitudes towards mathematics (Van den Heuvel-Panhuizen, Middleton & Streefland, 1995; English, 1996). It is suggested that students should have more opportunities for developing the skills of problem posing (NCTM, 2000). Giving opportunities to students for posing their problems enables students to think broadly and differently and develop problem solving skills (Brown & Walter, 1993; English, 1996). Problem posing is not independent from problem solving (Silver, 1994; Silver & Cai, 1996; Cai & Hwang, 2002; English, 2003).

According to Albayrak, İpek & İşık (2006), difficulties encountered during the delivery of key operations skills problem posing skills are associated with a failure to deal with this issue during the pre-service period of teachers. Teachers should have frequently been required to deal with problem posing during the pre-service period, it is vitally important for the in-service period. In this respect, establishing the relation between problem posing and problem solving skills and transferring these skills to different conditions are matters of great concern. For this purpose, the present study investigated the relation between the success of students in problem posing and solving in relation to the problems posed in the topic of series and sequences. The following sub-problems were investigated in relation to this problem:

1. Can prospective teachers pose problems about the established pattern?
2. Can prospective teachers solve the established problems on the topic of series and sequences?
3. Which solving strategies (concrete, semiabstract, abstract) do the prospective teachers prioritize?
4. Is there a relation between the number of posed problem and the success of problem solving in prospective teachers?

2. Method

2.1. Research group

The sample of the study was composed of 33 female and 43 male, a total of 76 prospective teachers who received training at the Kazım Karabekir Faculty of Education, Ataturk University during the spring term of 2008-2009 academic year. The participants were chosen from volunteer students.

2.2. Data collection tool

Problem posing and solving tests which encompassed the sub-problems were developed. The tests were composed of items which includes symbolic or figural representations about the pattern of series and sequences. The opinions of three experts and relevant literature were used to ascertain the reliability of the tests. Prospective teachers were asked to pose three different problems which included models in each open-ended item of problem posing tests. Each one of problem posing-solving tests was composed of three items. Furthermore, each item in the problem solving test had two choices. Each item in the problem solving test included patterns in problem posing test.
2.3. Analysis of the data

Elementary prospective mathematics teachers were administered problem posing in the first lesson and they were administered problem a solving test in the second lesson. Data analysis schema of Cai and Hwang (2003) was taken as the basis for the analysis of the data acquired from the problem posing and solving tests. Posed problems were firstly classified according to their status of forming a new situation beyond the given models in the question. If the problem asked a situation beyond the models in the problem posing test, it was described as problem extension. The example of problem extension posed by the prospective teacher for the model in the second question of the problem posing test was as follows:

"A man earned 1 million in a game of chance for the first day, he lost 4 million in the second game, and he earned 9 million in the third game. He earned or lost in this way for the following games. Accordingly, what is the profit-loss condition of the man at the end of the 11th month?"

Problems without problem extension were described as the problems which were limited to models in the problem posing test. The example of problem without problem extension posed by the prospective teacher for the model in the second question of the problem posing test was as follows;

"According to concerned model or pattern, how many block are there in the 4th figure?"

In line with this classification, the analysis of the problem solving success was made according to the condition of more than two or less than two cases of problems with problem extension.

Data acquired from the problem solving test was classified according to rightness of the answers and the type of solution strategies. The solution strategies of the prospective teachers were determined according to answers given to the second question of problem solving. This question was prepared to suit different problem solving strategies. Concrete strategies require making lists, drawing figures, guessing and controlling. One example of a concrete strategy used by the prospective teachers is given in Figure 3. Although the semi-abstract strategies are composed of multiple operation orders, they do not include comprehensive algorithm in the problem solving. The strategy “In 7th month 6.6=36, 36+91=127” used by prospective teachers in the problem about the number of ants which would be born in 7th month can be considered as a semi-abstract strategy. Abstract strategies require the identification of general rule given in the problem. The example of abstract strategy used by the by prospective teachers in the problem about the number of ants which would be born in 7th month is given in Figure 2.

Each problem item was coded with 1 if it was solved correctly and if it was solved wrongly the item was coded with 0 in problem solving test. In parallel with this, if the problem was problem extension it was coded with 1, if it was not problem extension it was coded with 0 in the problem posing test. Furthermore, total test scores of the teachers were determined. Dependent t-test and correlation analysis were used for investigating the relation between
problem posing and problem solving skills; descriptive statistical techniques were used for analysis of problem solving success according to the number of problem posing with problem extensions.

3. Findings and interpretation

3.1. Findings acquired from the problem posing test

Table 1. The percentages of different problem posing (problem extensions) in prospective teachers

<table>
<thead>
<tr>
<th>n</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 1 2 3</td>
<td>0 1 2 3</td>
<td>0 1 2 3</td>
</tr>
<tr>
<td>76</td>
<td>22 13 20 45 9 11 26 54 30 13 27 30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The percentages for condition of 0 (not posing any problems), 1 (posing one problem), 2 (posing two problems) or 3 (posing three problems) problem posing in problem posing test were given in Table 1. According to the data in the Table 1, it can be indicated that the level of skill of posing three problems for a situation was generally low.

3.2. Findings acquired from the problem solving test

Table 2. Problem solving success of prospective teachers

<table>
<thead>
<tr>
<th>n</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a b</td>
<td>a b</td>
<td>a b</td>
</tr>
<tr>
<td>76</td>
<td>55 18</td>
<td>88 83</td>
<td>55 34</td>
</tr>
</tbody>
</table>

Correctness percentages of problem solutions in prospective teachers were given in Table 2. According to the findings, it can be declared that the correctness percentage of problem 2 which was suitable for using different solution strategies was higher than the other problems.

Table 3. Solution strategies used in problem 2

<table>
<thead>
<tr>
<th>Problem 2</th>
<th>Abstract</th>
<th>Semi Abstract</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>36</td>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>b</td>
<td>29</td>
<td>21</td>
<td>33</td>
</tr>
</tbody>
</table>

Findings on the solution strategies of prospective teachers were presented in Table 3 as percentages. According to data acquired from problem 2, it can be indicated that prospective teachers used abstract and concrete strategies more than semi-abstract strategies.

3.3. The relation between problem posing and solving

Table 4. The problem solving success according to the different number of problem posing (problem extensions)

<table>
<thead>
<tr>
<th></th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a b</td>
<td>60 21</td>
<td>94 87</td>
<td>60 38</td>
</tr>
<tr>
<td>a b</td>
<td>48 7</td>
<td>50 71</td>
<td>46 17</td>
</tr>
</tbody>
</table>

Posing problem with at least two problem extensions
Posing problem with less than two problem extensions
Findings on the problem solving success of prospective teachers according to their level of problem posing beyond the given model are given in Table 4 as percentages. According to Table 4, the problem solving success of the prospective teachers rose in parallel with the increase in the number of problem posing beyond the given model.

Table 5. The relation between problem posing and solving

<table>
<thead>
<tr>
<th>Variables</th>
<th>N</th>
<th>p</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving Test Score</td>
<td>76</td>
<td>.000</td>
<td>.420</td>
</tr>
<tr>
<td>Problem Posing Test Score</td>
<td>76</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The data about the relation between the problem posing scores and problem solving scores of prospective teachers is given in Table 5. There was a significant relation between these two test scores (r=.420, p<0.01).

4. Discussion

In the present study, the relation between the success of elementary mathematics prospective teachers in problem posing and solving the posed problems about the topic of series and sequences was investigated. According to data in Table 1, it can be stated that the level of problem posing skills of prospective teachers in a given situation was low. Furthermore, it is evident that there are serious difficulties in problem posing taking the prospective teachers who could not even pose a problem suitable for the given situation; the voluntary participation of prospective teachers was taken into consideration.

The success percentage was higher in the second question of problem posing; this may be associated with the semi-abstract quality of the pattern and convenience of the pattern for correlating with many situations in daily life. According to the results in Table 2, the conviction is in the direction of low problem solving success “except for problem 2”. The high level of success in the solving of the second question in the problem solving test results from the convenience of the question for using different problem solving strategies. According to the results in Table 3, prospective teachers use abstract strategies for conditions requiring forming formula and they use concrete strategies for conditions requiring forming algorithm. According to data in Table 4, prospective teachers who posed two or more problems have higher levels of problem solving than the prospective teachers who posed less than two problems. This finding is consistent with the statement “there is a significant relation between problem posing and solving taking the cognitive ways into consideration” of Lowrie (2002).

The present study concluded that there is a significant relation between problem posing and solving scores of prospective teachers. This situation is consistent with result of the studies which emphasize that problem posing is not independent from problem solving (Leung, 1993; Silver & Cai, 1993; Gonzales, 1994; Silver, 1994; Silver & Cai, 1996; Cai & Hwang, 2002; English, 2003). The positive relation between problem posing and problem solving is an indicator of the acceptance of problem posing skills as a phase in the development of problem solving skills.

5. Conclusion and Recommendations

The present study has assessed that prospective teachers have difficulty in posing a problem for a given pattern and solving the posed problem. During the lessons, sample problems set in the books are used. Taking into account the routine problems of printed books, which do not attract the attention of the students, they are not convenient for their capacity, students do not encounter similar events in the real life, and the importance of problem posing by the teachers is evident. Problem posing and solving via the experiences of the students are important elements in a constructivist approach. Thus, the use and permanence of the learnt knowledge was contributed.

Prospective teachers should be compared according to their status of problem posing skills. Especially in branch teaching lessons, the correlation of knowledge with daily life problems should be encouraged. Students should be encouraged to relate problems to those encountered in their daily lives and this should not be neglected. Furthermore, the relation to real life of problems should be used in the preparation of the curriculum and in the student selection examinations.
References


Appendixes

Appendix 1. Problem posing test

1. Consider a pattern which continue as $1 - 2^2 + 3^2 - 4^2 + \ldots$, write down three different problem sentences including this pattern.

2. The number of balls in the under mentioned tables are organized according to a rule. Write down three different problem sentences including the pattern in this model.
3. Write down three different problem sentences convenient for the pattern of $1 + 3 + 5 + \ldots + (2n - 1) + (2n + 1) + (2n - 1) + \ldots 5 + 3 + 1$.

Appendix 2. Problem solving test

1. “A man earned 1 lira in a game of chance for the first day, he lost 4 liras in the second game, he earned 9 liras in the third game. He continued to play the game every month” Accordingly; he gained or lost in this way for the following games. Accordingly:
   a. Indicate the profit-loss condition of the man at the end of the two years.
   b. Find out how many months it would take for his profit to be 2850 liras.

2. “The number of young ants which are born in an ant colony is as follows according to months;
   1\textsuperscript{st} month, 1 ant
   2\textsuperscript{nd} month, 7 ants
   3\textsuperscript{rd} month, 19 ants
   4\textsuperscript{th} month, 37 ants
   5\textsuperscript{th} month, 61 ants

   are born. The ant colony continues to breed in this way. Accordingly;
   a. Find out the number of ants which will be born in the 7\textsuperscript{th} month?
   b. As the difference in the numbers of ants which are born in any two months is 48, find out these months.

3. “Two vehicles on A and B points are moving towards each other in a straight direction. The vehicle on the A point started to move an hour ago. These vehicles cover 1 km stretch of road in the first hour, they increase the covered road by 2 km in each hour. Accordingly;
   a. Find out the total km of the road covered by these vehicles after 10 hours.
   b. As the distance between these two vehicles is 1105 km, find out how much later they meet