Processes of heavy quark pair (lepton pair) and two gluon (two photon) production in the high energy quark (electron) proton peripheral collisions

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Abstract

We considered the three jet production processes in the region of the incident lepton, photon, quark or gluon fragmentation. The fourth jet is created by the recoil proton. The kinematics of jet production is discussed in jets production in the fragmentation region. The non-trivial relation between the momenta of the recoil proton and the polar angle of its emission was derived.

Based on this formalism the differential cross sections of QCD processes $gp \to (ggg)p$; $qp \to (q\bar{Q}Q)p$; $gp \to (gQ\bar{Q})p$ were obtained, including the distribution on transverse momentum component of jets fragments. It was shown that the role of the contribution of "non-Abelian" nature may become dominant in a particular kinematics of the final particles. The kinematics, in which the initial particle changes the direction of movement to the opposite one, was considered in the case of heavy quark–antiquark pair production.

Different distributions, including spectral, azimuthal and polar angle distribution on the fragments of jets can be arranged using our results. We present besides the behavior of the ratio of non-Abelian contribution to the cross section to the total contribution. We show that it dominates for large values of the transverse momenta of jets component (gluons or quarks). Some historical introduction to the cross-sections of peripheral processes, including 2γ creation mechanism production, including the result Brodsky–Kinoshita–Terazawa, is given.

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1. Introduction

It is known [1] that the differential cross sections of small angle elastic (and inelastic) scattering processes do not fall with increasing the center of the mass total energy $\sqrt{s}$, $s = 4E^2$. The reason for this is the contribution to the cross section from the photon exchange between charged particles. Similar phenomena take place as well in the strong interaction sector, where gluons take place instead of a photon.

The simplest processes of this kind are the scattering of a charged particle in the external field of nuclei and the elastic scattering of one sort of charged particles on the other one. The total cross sections of these processes do not exist due to contributions of large impact parameters, which correspond to small scattering angles. The momentum of the virtual photon in the scattering channel (t channel) tends to the mass shell. So the virtual photon in the t channel becomes a real one. In the case of inelastic processes $a + b \to a + b + X$, with the set of particles x belonging to one of the directions in the center of mass a or b, the cross sections are finite [2–9]. Besides, the square of 4-momentum of a virtual photon is negative and restricted from below by the magnitude of some quantity of the created set of particles, invariant mass square of (ax), (bx). The finiteness of the transfer momentum module caused the so-called Weizsacker–Williams (WW) enhancement [7]. Namely, the region of small momentum transfer is realized in the appearance of a large logarithmic factor $L = \ln(s^2/(m_1^2 m_2^2))$. For modern colliders this factor is of an order of 20. It often turns out that the consideration is restricted to the WW approximation. This means the accuracy of the order $1 + O\frac{1}{L}$. The cross sections of inelastic peripheral processes are as usually large.

The background caused by the events of the large-angle kinematics of produced particles determines the accuracy of peripheral cross sections

$$1 + O\left(\frac{\alpha}{\pi}, \frac{m^2}{s}\right). \tag{1}$$

So the total accuracy of theoretical estimates is better than 5%.

The cross sections of interaction of photons with the target will also not fall with energy when taking into account the contributions of higher orders of perturbation theory (PT).

The main attention in our paper is paid to the double gluon emission and production of the pair of heavy quarks with subsequent jet production, in the fragmentation region of the incident particle.

Our paper is organized as follows.

First, we give an estimation of the magnitudes of the cross sections of several processes in high energy $ep \to (eab)p$, $qp \to (qab)p$ collisions in the fragmentation region of a projectile e, q. In Section 2, we give a short historical introduction to the study of the processes of lepton pair production in high energy lepton–lepton, ion–ion collisions. In Section 3, the method of description of high energy processes based on the Sudakov parametrization of the four momenta of the problem is developed. The differential cross sections are expressed in terms of physically measurable energy fractions and the transverse component of the final particles. In Section 4, the simplest QCD processes with 2 jet production are presented. In Section 5, we consider the process of heavy quark–anti-quark pair production in collisions of projectile with the color-less target. In Section 6, the QED process of double bremsstrahlung is studied. In Section 7, a similar QCD process of emission of two gluons is considered. In Section 8, the specific details of jet production on a fixed target are considered.

In Conclusion, we discuss the results and pay attention to the relation of the contributions of Abelian (QED) and non-Abelian nature. It seems that in the case of large magnitudes of transversal quark momenta the role of non-Abelian contributions dominates [27]. In Conclusion, we also discuss the "jet reflection" phenomena. It consists in the change of the direction of motion of the light projectile to the opposite one in the case of heavy pair production.

In Appendices A–C, the explicit expressions beyond the WW approximation as well as in the WW one are presented. In Appendix D, this method is applied to the problem of transmission of the longitudinal polarization of the initial electron to the positron from the pair created.

The cross section of the heavy pair production in electron–proton and quark–proton collisions can be written as

$$\sigma^{ep \to (eQ\bar{Q})p} \approx \frac{\alpha^4 L_q}{\pi M_Q^2} \approx 4.8 \text{ pb};$$

$$\sigma^{qp \to (qQ\bar{Q})p} \approx \frac{\alpha^2 \alpha_s^2 L_q}{\pi M_Q^2} \approx 20 \text{ nb}. \tag{2}$$

We put here $\sqrt{s} = 3$ TeV, $M_Q = M = 1.5$ GeV. In this case $L_q \approx 30$. For a process of two photon and two gluon production we have

$$\sigma^{ep \to (e\gamma\gamma)p} \approx \frac{\alpha^4 L_q}{\pi k^2} \approx 10.8 \text{ pb};$$

$$\sigma^{qp \to (qgg)p} \approx \frac{\alpha^2 \alpha_s^2 L_q}{\pi k^2} \approx 50 \text{ nb}, \quad k^2 = 1 \text{ GeV}^2, \tag{3}$$

for typical transfer momentum squared $k^2 = 1 \text{ GeV}^2$.

2. QED peripheral process, pair production

In 1934, the cross section of pair production in high energy lepton collisions was calculated in the so-called double-WW approximation [8]

$$\sigma_{\bar{e}e \to \bar{e}e l l} = \frac{28\alpha^4}{27\pi m_l^2} \left(\ln\left(\frac{s}{m_e^2}\right) \right)^2 \ln\left(\frac{s}{m_l^2}\right), \quad l = e, \mu, \ s = 4E^2. \tag{4}$$

In 1937 G. Racah [9] published the total cross section of the process of pair creation in the collision of charged particles with the charges $Z_1 e$, $Z_2 e$, p_1, m_1, p_2, m_2 are the 4-momenta and masses of the initial particles

$$\sigma_{Z_1 Z_2 \to Z_1 Z_2 e^+ e^-} = \frac{28(Z_1 Z_2 \alpha^2)^2}{27\pi m_e^2} (l^3 - A l^2 + B l + C), \quad l = \ln \frac{2p_1 p_2}{m_1 m_2},$$

$$A = \frac{178}{28} \approx 6.36; \ B = \frac{1}{28} \left[7\pi^2 + 370 \right] \approx 15.7,$$

$$C = -\frac{1}{28} \left[348 + \frac{13}{2}\pi^2 - 21\xi(3) \right] \approx -13.8; \quad \xi(3) = 1.202. \tag{5}$$

In papers by Baier and Fadin [10] as well as Lipatov and Kuraev [11] the total cross section of the production process of an electron–positron pair in electron–positron collisions (only two exchanged photons) was obtained

$$\sigma_{e^\pm e_- \to e^\pm e_- e_+ e_-} = \frac{\alpha^4}{\pi m_e^2} \left[\frac{28}{27}\rho^3 - \frac{178}{27}\rho^2 - \left(\frac{164}{9}\frac{\pi^2}{6} - \frac{490}{27} \right)\rho \right.$$
$$\left. + \frac{401}{9}\xi(3) + \frac{52}{3}\frac{\pi^2}{6}\ln 2 + \frac{916}{27}\frac{\pi^2}{6} - \frac{676}{27} \right]$$
$$\approx \frac{\alpha^4}{\pi m_e^2} \left[1,03\rho^3 - 6.6\rho^2 - 11.7\rho + 104 \right], \quad \rho = \ln\frac{s}{m_e^2}. \tag{6}$$

In the case of production of a muon pair we obtain

$$\sigma_{e^\pm e_- \to e^\pm e_- \mu_+ \mu_-} = \frac{\alpha^4}{\pi m_\mu^2} \left[\frac{28}{27}\rho^3 - \frac{178}{27}\rho^2 - \left(\frac{535}{81} + \frac{14}{3}\frac{\pi^2}{6} \right) + \frac{28}{9}\rho^2 l + \frac{14}{9}\rho l^2 \rho \right.$$
$$\left. - \frac{562}{27}\rho l - \frac{64}{9}l^2 - \left(\frac{56}{9}\frac{\pi^2}{6} - \frac{5855}{162} \right)l - 7\xi(3) + \frac{214}{27}\frac{\pi^2}{6} + \frac{51\,403}{486} \right]$$
$$\approx \frac{\alpha^4}{\pi m_\mu^2} \left[1,03\rho^3 + 26.6\rho^2 - 56\rho - 342 \right],$$
$$\rho = \ln\frac{s}{m_\mu^2}, \; l = \ln\frac{m_\mu^2}{m_e^2} \approx 10.7, \; \xi(3) = 1.202. \tag{7}$$

These formulae are in agreement with ones obtained by G. Racah [9]. The method used to obtain the cross section consists in imposing some cuts on the transverse momenta and energy fractions, which in principle can be used in experiment. Adding separate contributions we obtain the results given above. In Appendix A, we give the sketch of derivation of the LL formula and discuss the experimental cuts.

Note that in the case of production of a heavy muon pair the corrections of the order $(m_\mu^2/s)L^n$ must be taken into account. Really, for $\sqrt{s} < 1$ GeV the cross section calculated theoretically is negative.

In 1970, in paper by Brodsky, Kinoshita and Terazawa (BKT) a special case of production of a heavy object by two virtual photons in electron–electron collisions was investigated [12]:

$$\sigma(s)^{ee \to eeF} = \left(\frac{\alpha}{\pi} \right)^2 \left(\ln\frac{E}{m_e} \right)^2 \int_{4M^2}^\infty \frac{ds_1}{s_1} \sigma^{\gamma\gamma \to F}(s_1) f\left(\frac{s_1}{s} \right), \tag{8}$$

with $2M$ being the mass of a created system and

$$f(z) = (2+z)^2 \ln\frac{1}{z} - 2(1-z)(3+z). \tag{9}$$

The BKT formula in the modern language describes the Drell–Yan process.

2.1. Derivation Brodsky–Kinoshita–Terazawa (BKT) result

In the case when the energies of the scattered electrons are essentially less then the energies of the initial ones, the formulae for total cross sections must be modified. We start from the usual expression for the matrix element

$$M = \frac{4\pi\alpha}{q_1^2 q_2^2} \bar{u}(p_1')\gamma_\mu u(p_1)\bar{u}(p_2')\gamma_\nu u(p_2)T^{\mu\nu}. \tag{10}$$

First we will use the four-momenta of virtual photons q_1 instead of the momenta of the scattered electron p_1'. Besides, we accept the Sudakov parametrization of four-momenta in the problem. For the phase volume of the scattered electron moving in direction close to the momentum of electron p_1

$$\frac{d^3 p_1'}{2\epsilon'} = d^4 q_1 \delta^4\big(p_1 - p_1' - q_1\big) d^4 p_1' \delta\big((p_1 - q_1)^2 - m^2\big)$$
$$= \frac{s}{2} d^2 \vec{q}_1 d\alpha_1 d\beta_1 \delta\big(-s\alpha_1(1 - \beta_1) - m^2 \beta_1 - \vec{q}_1^2\big). \tag{11}$$

Applying the Sudakov parametrization [18]

$$q_1 = \alpha_1 p_2 + \beta_1 \tilde{p}_1 + q_{1\perp}, \qquad q_{1\perp} p_2 = q_{1\perp} p_1 = 0;$$
$$\tilde{p}_1 = p_1 - p_2 \frac{m^2}{s}, \qquad p_1^2 = m^2; \qquad 2p_1 \tilde{p}_1 = m^2, \qquad q_{1\perp}^2 = -\vec{q}_1^2 < 0;$$
$$q_1^2 = -\frac{\vec{q}_1^2 + m^2 \beta_1^2}{1 - \beta_1}. \tag{12}$$

In such a way we obtain

$$\frac{d^3 p_1'}{2\epsilon'} = \frac{d\beta_1}{1 - \beta_1} \frac{1}{2} d\vec{q}_1^2 \frac{d\phi_1}{2\pi}. \tag{13}$$

The square of current, associated with electron $e(p_1)$ summed on spin states and averaged on the azimuthal angle ϕ_1 is

$$\Big\langle \sum [\bar{u}(p_1')\gamma_\mu u(p_1)][\bar{u}(p_1')\gamma_{\mu_1} u(p_1)]^* \Big\rangle = 4\Big(2p_{1\mu} p_{1\mu_1} + \frac{1}{2} q_1^2 g_{\mu\mu_1}\Big). \tag{14}$$

More convenient formulae can be obtained if one use the gauge condition $q_1^\mu T_{\mu\nu} = (\beta_1 p_1 + q_{1\perp}) T_{\mu\nu} = 0$. In such a way we obtain

$$\Big\langle \sum [\bar{u}(p_1')\gamma_\mu u(p_1)][\bar{u}(p_1')\gamma_{\mu_1} u(p_1)]^* \Big\rangle = -\frac{4\vec{q}_1^2 g_{\mu\mu_1}}{1 - \beta_1}\Big[\frac{1 - \beta_1}{\beta_1^2} + \frac{1}{2}\Big]. \tag{15}$$

Note now that the quantity

$$\int \frac{1}{8s_1} T_{\mu\nu}(T_{\mu_1\nu_1})^* g^{\mu\mu_1} g^{\nu\nu_1} d\gamma = \sigma^{\gamma\gamma \to F}(s_1), \tag{16}$$

coincide with the total cross section of production of the system F by two photons. In the similar way we obtain for the current associated with electron $e(p_2)$:

$$\Big\langle \sum \bar{u}(p_2')\gamma_\nu u(p_2)\big(\bar{u}(p_2')\gamma_{\nu_1} u(p_2)\big)^* \Big\rangle = -\frac{4\vec{q}_2^2 g_{\nu\nu_1}}{1 - \alpha_2}\Big[\frac{1 - \alpha_2}{\alpha_2^2} + \frac{1}{2}\Big]. \tag{17}$$

Here we use the similar Sudakov parametrization $q_2 = \beta_2 \tilde{p}_2 + \alpha_2 p_1 + q_{2\perp}$. Writing the phase volume as

$$d\Gamma = d\gamma \frac{1}{(2\pi)^6} \frac{\pi d\vec{q}_1^2 d\beta_1}{2(1 - \beta_1)} \frac{\pi d\vec{q}_2^2 d\alpha_2}{2(1 - \alpha_2)},$$
$$d\gamma = \frac{(2\pi)^4}{(2\pi)^6} \frac{d^3 q_+}{2E_+} \frac{d^3 q_-}{2E_-} \delta^4(q_1 + q_2 - q_+ - q_-). \tag{18}$$

Let us introduce as a new variable the invariant mass square of the created system $s_1 = (q_+ + q_-)^2 \approx s\alpha_2\beta_1$

$$\int d\alpha_2 d\beta_1 \theta (s\alpha_2\beta_1 - 4M^2) = \frac{1}{s} \int_{4M^2}^{\infty} ds_1 \int_{s_1/s}^{1} \frac{d\beta_1}{\beta_1}. \tag{19}$$

For the differential cross section we have

$$d\sigma = \frac{\alpha^2}{\pi^2} \frac{\vec{q}_1^2 d\vec{q}_1^2 \vec{q}_2^2 d\vec{q}_2^2}{(\vec{q}_1^2 + m^2\beta_1^2)^2 (\vec{q}_2^2 + m^2\alpha_2^2)^2} \cdot \int_{4M^2}^{\infty} ds_1 \sigma^{\gamma\gamma}(s_1) \frac{s_1}{s^2} I, \tag{20}$$

with

$$I = \int_{s_1/s}^{1} \frac{d\beta_1}{\beta_1} \left[\frac{1 - \beta_1}{\beta_1^2} + \frac{1}{2} \right] \left[\frac{1 - \alpha_2}{\alpha_2^2} + \frac{1}{2} \right], \quad \alpha_2 = \frac{s_1}{s\beta_1}. \tag{21}$$

The calculation leads to

$$I = \frac{s^2}{4s_1^2} f(z), \quad f(z) = (2 + z)^2 \ln \frac{1}{z} - 2(1 - z)(3 + z), \quad z = \frac{s_1}{s}. \tag{22}$$

Integration on the transversal momenta of virtual photons in the region $0 < \vec{q}_{1,2}^2 < E^2$ leads to the famous formulae of BKT

$$\sigma(s)^{ee \to eeF} = \left(\frac{\alpha}{\pi} \right)^2 \left(\ln \frac{E}{m_e} \right)^2 \int_{4M^2}^{\infty} \frac{ds_1}{s_1} \sigma^{\gamma\gamma \to F}(s_1) f\left(\frac{s_1}{s} \right). \tag{23}$$

Really, it consists in the probability P_e^γ to find the virtual photon in the electron:

$$dW_e^\gamma(\vec{k}_1, \beta_1) \sim 4\pi\alpha \frac{d\vec{k}_1^2 \cdot \vec{k}_1^2}{(\vec{k}_1^2 + m_e^2\beta_1^2)^2} \frac{d\beta_1}{1 - \beta_1} P_e^\gamma, \quad P_e^\gamma = \frac{1 - \beta_1}{\beta_1^2} + \frac{1}{2},$$

and the conversion of these probabilities with the cross section of a hard subprocess $\gamma\gamma \to F$. In this step it is useful keep in mind the following integrals,

$$\int_z^1 \frac{d\beta_1}{\beta_1} \left[\frac{1 - \beta_1}{\beta_1^2} + \frac{1}{2} \right] \left[\frac{1 - \alpha_2}{\alpha_2^2} + \frac{1}{2} \right] = \frac{1}{4z^2} f(z), \quad \beta_1\alpha_2 = z = \frac{s_1}{s} \tag{24}$$

The parton language can also be applied to describe the processes in the fragmentation region. In the case when the heavy object is created in the fragmentation region $e\bar{e} \to (eF)\bar{e}$ we have

$$\sigma(s)^{ee \to (eF)e} = \left(\frac{\alpha}{\pi} \right)^2 \left(\ln \frac{E}{m_e} \right)^2 \int_{4M^2}^{\infty} \frac{ds_1}{s_1} \sigma^{\gamma\gamma \to F}(s_1) \phi\left(\frac{s_1}{s} \right),$$

$$\phi(z) = 4 \ln \frac{1}{z} - (1 - z)(3 - z). \tag{25}$$

In Appendix D, this formalism is applied to the problem of transferring the longitudinal polarization of the initial electron to the positron.

Besides, the two-photon mechanism mentioned above, the so-called "bremsstrahlung" mechanism, must be taken into account. It consists in the emission of a light-like virtual photon by one of the initial particles with a subsequent conversion to the lepton pair. When calculating the differential and total cross sections the effect of the Fermi–Dirac statistic must be taken into account.

Other QED peripheral processes, single and double bremsstrahlung, take into account the radiative corrections as well as the details of calculation can be found in reviews [13–16,20].

It results in a non-leading contribution. Really, the contribution from the diagram corresponding to the single-photon production mechanism [15] is:

$$
\sigma_{br} = 2 \frac{\alpha^4}{\pi m^2} \left[\left(\frac{77}{54} \pi^2 - \frac{1099}{81} \right) \rho - \frac{223}{18} \xi(3) - \frac{17}{9} \pi^2 \ln 2 + \frac{163}{108} \pi^2 + \frac{5435}{486} \right]
$$
$$
= 2 \frac{\alpha^4}{\pi m^2} (0.5\rho - 1.7), \tag{26}
$$

where factor 2 takes into account both kinematic situations when a jet moves along both the initial directions.

The effect of identity of final particles taken into account, contributes to the total cross section (both directions are taken into account) [17]

$$
\sigma_{int} = 2 \frac{2\alpha^4 \rho}{105 \pi m^2} \left[-374\xi(3) - 120\pi^2 \ln 2 + \frac{13\,591}{90} \pi^2 - \frac{2729}{12} \right] = 2 \frac{\alpha^4}{\pi m^2} (-0.14)\rho; \tag{27}
$$

σ_{int} is the contribution from the interference term associated with the identity of particles in the final state.

For electron pair production and muon pair production the specific effect of the charge-odd contribution to spectral distributions takes place. It is caused by the interference of the "two-photon" mechanism and the bremsstrahlung one.

Similar effects take place in the process of bremsstrahlung and pair production by a gluon and a quark on the proton or the nuclei. We will restrict ourselves below only to the cases when a proton or nuclei remain to be a proton or nuclei. No excitation of the target is allowed.

In the case of large transverse momenta of the jet particle component the subtle effect of the double logarithmic contributions in the fragmentation region disappears.

3. Kinematic of peripheral processes, Sudakov parametrization

First, we remind the general Sudakov technique to study the peripheral kinematics of the QED process $e + p \to (e + l + \bar{l}) + p$ of creation of a heavy charged lepton pair in high-energy electron–proton collisions in the fragmentation region of the electron,

$$
e(p_1) + p(p_2) \to e(p_1') + l(q_-) + \bar{l}(q_+) + p(p_2'),
$$
$$
p_2^2 = p_2'^2 = m_p^2, \ p_1^2 = p_1'^2 = m^2, \ q_{\pm}^2 = M^2,
$$
$$
s = 2p_1 p_2 \gg M^2 \sim m_p^2 \gg m^2. \tag{28}
$$

The peripheral kinematics or the electron fragmentation region is defined as

$$
s \gg -q^2 = -(p_2 - p_2')^2 \sim M^2. \tag{29}
$$

It is convenient to use the Sudakov parametrization of momenta. For this aim we introduce two light-like 4 vectors constructed from the momenta of the initial particles $\tilde{p}_2 = p_2 - p_1(m_p^2/s)$, $\tilde{p}_1 = p_1 - p_2(m^2/s)$ [18]

$$
\begin{aligned}
q &= \alpha_q \tilde{p}_2 + \beta \tilde{p}_1 + q_\perp, \qquad q_\pm = \alpha_\pm \tilde{p}_2 + x_\pm \tilde{p}_1 + q_{\perp\pm}, \\
p_1' &= \alpha' \tilde{p}_2 + x \tilde{p}_1 + p_\perp, \\
a_\perp p_1 &= a_\perp p_2 = 0, \qquad a_\perp^2 = -\vec{a}^2 < 0, \\
\tilde{p}_1^2 &= \tilde{p}_2^2 = 0, \qquad 2p_1 \tilde{p}_1 = m^2,
\end{aligned} \tag{30}
$$

where $\vec{a}$ is the two-dimensional vector transversal to the beam axis (direction of $\vec{p}_1$, center of mass reference frame implied), and x, $x_\pm$ are the energy fractions of the scattered electron and the heavy lepton pair, and $x + x_- + x_+ = 1$. Below, we will omit the tilde sign. According to the energy–momentum conservation law, we also have

$$
\begin{aligned}
\vec{q} &= \vec{p} + \vec{q}_- + \vec{q}_+, \\
\alpha_q &= \alpha' + \alpha_+ + \alpha_- - \frac{m^2}{s}.
\end{aligned} \tag{31}
$$

The on mass shell condition for the scattered proton $p_2'^2 - m_p^2 = 0$, being written in terms of the Sudakov variables, reads (one must take into account the relation $2p_2 \tilde{p}_2 = m_p^2$)

$$
\begin{aligned}
(p_2 - q)^2 - m_p^2 &= s\alpha_q \beta - \vec{q}^2 - m_p^2 \alpha_q - s\beta = 0, \\
s\beta &= -\frac{\vec{q}^2 + m_p^2 \alpha_q}{1 - \alpha_q}.
\end{aligned} \tag{32}
$$

One finds for $q^2 = s\alpha_q \beta - \vec{q}^2$:

$$
q^2 = -\frac{\vec{q}^2 + \alpha_q^2 m_p^2}{1 - \alpha_q} \approx -\left(\vec{q}^2 + \frac{s_1^2}{s^2} m_p^2 \right). \tag{33}
$$

We conclude that in the case $s_1 \neq 0$ a virtual photon has a space-like 4-vector and, in addition $|q^2| > q_{min}^2 = m_p^2(s_1/s)^2$. The quantity $s_1 = 2qp_1 = (p_1' + q_+ + q_-)^2 - q^2 - m^2 = s\alpha$ in the WW approximation $\vec{q} = 0$ coincides with the square of the invariant mass of the jet moving in the initial quark momentum direction. Using the on mass shell conditions for momenta of the scattered muon and the created pair of heavy quarks

$$
p_1'^2 = s\alpha' x - \vec{p}^2 = m_q^2 = m^2, \qquad q_\pm^2 = s\alpha_\pm x_\pm - \vec{q}_\pm^2 = M^2, \qquad x + x_+ + x_- = 1, \tag{34}
$$

we find (in the WW approximation)

$$
\begin{aligned}
s_1 = s\alpha_q = \frac{1}{xx_+x_-} \Big[&x_-(1 - x_-)\vec{q}_+^2 + x_+(1 - x_+)\vec{q}_-^2 + 2x_-x_+\vec{q}_-\cdot\vec{q}_+ + m^2 x_+ x_- \\
&+ x(1 - x)M^2 \Big].
\end{aligned} \tag{35}
$$

The matrix element can be written as

$$
M = \frac{(4\pi\alpha)^2}{q^2} g^{\mu\nu} J_\mu^{(e)}(p_1) J_\nu^{(p)}(p_2), \tag{36}
$$

with $J^{(e,p)}$ being the currents associated with electron and proton blocks of the relevant Feynman diagram, and α is fine-structure constant. The main contribution arises from the longitudinal components of the tensor $g^{\mu\nu} = g_{\mu\nu\perp} + (2/s)(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu)$:

$$g^{\mu\nu} \approx \frac{2}{s} p_2^\mu p_1^\nu. \tag{37}$$

So we obtain for the squared module of the summed over spin states of the matrix element

$$\sum |M|^2 = (8\pi\alpha)^2 s^2 \frac{1}{(q^2)^2} \Phi^{(e)} \Phi^{(p)},$$

$$\Phi^{(e)} = \sum \left| \frac{1}{s} J_\lambda^{(e)} p_2^\lambda \right|^2, \quad \Phi^{(p)} = \sum \left| \frac{1}{s} J_\sigma^{(p)} p_1^\sigma \right|^2. \tag{38}$$

The quantities $\Phi^{(e,p)}$ (so-called impact factors) remain finite in the limit of high energies $s \to \infty$. In particular,

$$\Phi^{(p)} = \sum \left| \frac{1}{s} \bar{u}(p_2') \not{p}_1 u(p_2) \right|^2 = 2, \quad \not{p}_1 \equiv \gamma_\mu a^\mu. \tag{39}$$

The electron current obeys the gauge condition

$$q_\mu J^{(e)}(p_1)_\mu \approx (\alpha_q p_2 + q_\perp)_\mu J^{(e)}(p_1)_\mu = 0. \tag{40}$$

Using this relation we obtain for our process

$$\sum |M|^2 = 2 \frac{s^2}{s_1^2} \frac{(4\pi\alpha)^2 \vec{q}^2}{(q^2)^2} \sum \left| \frac{1}{s} J_\lambda^{(e)} e^\lambda \right|^2, \tag{41}$$

where $\vec{e} = \vec{q}/|\vec{q}|$ can be interpreted as a polarization vector of the virtual photon. To obtain the differential cross section

$$d\sigma^{ep\to(ejet_q)p} = \frac{1}{8s} \sum |M^{2\to 2+n}|^2 d\Gamma_{2+n}, \tag{42}$$

we must rearrange the phase volume of the final state (the electron remains to be a spectator, whereas the scattered muon is accompanied with n particles)

$$d\Gamma_{2+n} = (2\pi)^4 \delta^4 \left(p_1 + p_2 - p_1' - p_2' - \sum q_i \right) \frac{d^3 p_1'}{2E_1'(2\pi)^3} \frac{d^3 p_2'}{2E_2'(2\pi)^3} \Pi_i \frac{d^3 q_i}{2E_i(2\pi)^3}, \tag{43}$$

including the additional variable q as

$$d\Gamma_{2+n} \to d\Gamma_{2+n} d^4 q \delta^4 (p_2 - q - p_2'). \tag{44}$$

We use the Sudakov variables:

$$d^4 q = \frac{s}{2} d\alpha_q d\beta d^2\vec{q}; \qquad \frac{d^3 q_\pm}{2E_\pm} = \frac{s}{2} d\alpha_\pm dx_\pm d^2\vec{q}_\pm \delta(s\alpha_\pm x_\pm - \vec{q}_\pm^2 - M^2). \tag{45}$$

Performing the integrations over the "small" Sudakov variables α, $\alpha_\pm$ we obtain

$$d\Gamma_{2+n} = \frac{1}{sx} (2\pi)^4 (2\pi)^{-3(2+n)} 2^{-n-1} d^2\vec{q} \, \Pi_1^n \frac{dx_i}{x_i} d^2\vec{q}_i, \qquad x + \sum_1^n x_i = 1. \tag{46}$$

It can be noted that the cross section does not depend on s at large s and tends to zero in the limit of zero recoil momentum of the spectator electron $\vec{q} \to 0$. The last property is the consequence of gauge invariance of the theory. Once being integrated over the recoil momentum, the cross section reveals the so-called WW enhancement factor

$$L = \int_0^{Q^2} \frac{\vec{q}^2 d\vec{q}^2}{(\vec{q}^2 + m_2^2\alpha^2)^2} = \ln \frac{Q^2 s^2}{m_2^2 s_1^2} - 1 = L_q - 1, \tag{47}$$

where $Q^2 \sim M^2$ is the scale of transfer momentum squared in the process.

4. 2-jet QCD processes in quark (gluon) proton collisions

The differential cross sections of the processes

$$q(p_1) + p(p_2) \to q(p_1') + g(k) + p(p_2'); \tag{48}$$

$$e(p_1) + p(p) \to e(p_1') + \gamma(k) + p(p_2'); \tag{49}$$

differs only by the color factors from similar expressions in QED.

$$d\sigma^{qp\to(g,q)p} = \frac{N^2 - 1}{2N} d\sigma^{ep\to(e\gamma)p}, \tag{50}$$

with

$$d\sigma^{ep\to e\gamma p} = \frac{2\alpha^3 d^2 q d^2 p' dx \bar{X}}{\pi^2 (q^2)^2 (DD')^2} R^\gamma [1 + \xi_3 B_3 + \xi_1 B_1],$$

$$R^\gamma = DD' q^2 (1 + x^2) - 2xm^2 (D - D')^2, \tag{51}$$

with

$$B_3 = \frac{2x}{R_\gamma} [A^2 q^2 \cos(2\varphi_q) + B^2 p^2 \cos(2\varphi_p) + 2AB|\vec{q}||p|\cos(\varphi_q + \varphi_p)],$$

$$B_1 = \frac{2x}{R_\gamma} [A^2 q^2 \sin(2\varphi_q) + B^2 p^2 \sin(2\varphi_p) + 2AB|\vec{q}||p|\sin(\varphi_q + \varphi_p)], \tag{52}$$

and

$$A = \frac{1}{\bar{x}}(D' - xD), \qquad B = \frac{1}{x}(D - D'), \tag{53}$$

and $B_{1,3}$ are the effective Stokes parameters of a gluon. Besides

$$D = m^2 \bar{x}^2 + (\vec{p} - \vec{q})^2; \qquad D' = m^2 \bar{x}^2 + (\vec{p} - \vec{q}x)^2, \quad \bar{x} = 1 - x, \tag{54}$$

$\vec{p}$ is the transverse component of the scattered electron momentum; $\vec{q}$ is the same value for the recoil proton; φ_p, φ_q are the azimuthal angles between the transverse component of a gluon and $\vec{p}, \vec{q}$.

For the process of quark–anti-quark pair production by a gluon on a proton we have

$$d\sigma^{gp\to(Q\bar{Q})p} = \frac{1}{2} \frac{2\alpha^3}{\pi^2 (q^2)^2} \Phi^\gamma d^2 q_+ d^2 q dx_+,$$

$$\Phi^\gamma = \frac{1}{(D_+ D_-)^2} \{2m^2 x_+ x_- (D_+ - D_-)^2 + \vec{q}^2 (x_+^2 + x_-^2) D_+ D_-\}, \quad D_\pm = \vec{q}_\pm^2 + m^2;$$

$$\vec{q}_+ + \vec{q}_- = \vec{q}. \tag{55}$$

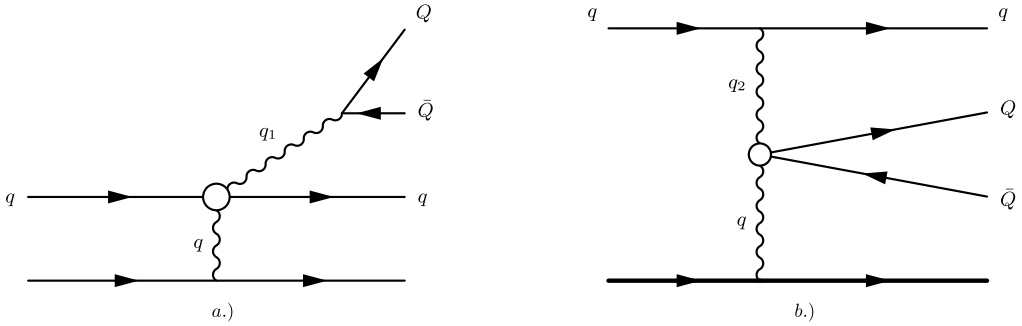The first factor is the color factor $\frac{1}{N^2-1} \frac{N^2-1}{2} = \frac{1}{2}$.

Fig. 1. Production of heavy quark pair.

5. Production of heavy charged lepton (quark) pairs in electron (quark) proton collisions

We will distinguish two mechanisms of the heavy fermion pair creation, the so-called "bremsstrahlung mechanism" (see Fig. 1a) and the "two-photon" one (Fig. 1b). The matrix element of the process

$$
\begin{aligned}
&e(p_1) + p(p_2) \rightarrow e\big(p_1'\big) + Q(q_-) + \bar{Q}(q_+) + p\big(p_2'\big), \\
&p_1^2 = p_1'^2 = m^2; \ p_2^2 = p_2'^2 = m_p^2, \ q_\pm^2 = M^2
\end{aligned}
\tag{56}
$$

in the kinematic region of e particle fragmentation can be written as

$$
\begin{aligned}
M^{ep \rightarrow (eQ\bar{Q})p} = (4\pi\alpha)^2 \frac{2s}{q^2} N_2 \bigg[&\frac{1}{q_1^2 s} \bar{u}\big(p_1'\big) Q_\mu u(p_1) \bar{u}(q_-) \gamma_\mu v(q_+) \\
&+ \frac{1}{q_2^2 s} \bar{u}(q_-) R_\lambda v(q_+) \bar{u}\big(p_1'\big) \gamma_\lambda u(p_1) \bigg], \\
q_1^2 = (q_- + q_+)^2, \ &q_2^2 = \big(p_1 - p_1'\big)^2, \\
N_2 = \frac{1}{s} \bar{l}_2\big(p_2'\big) \hat{p}_1 l_2(p_2). &
\end{aligned}
\tag{57}
$$

Here we adopt Sudakov's parametrization of the four-vectors

$$
p_1' = \alpha' p_2 + x p_1 + p_\perp; \qquad q_\pm = \alpha_\pm p_2 + x_\pm p_1 + q_{\pm\perp}; \qquad q = \alpha_q p_2 + q_\perp.
\tag{58}
$$

The first term in the square brackets contains the Compton sub-process $e(p_1) + \gamma^*(q) \rightarrow e(p_1') + \gamma(q_1)$ amplitude $\bar{u}(p_1') Q_\mu u(p_1)$ with (we use here the on-mass shell conditions for the initial and final electrons (quarks))

$$
\begin{aligned}
Q_\mu &= \frac{1}{D'} \gamma_\mu (\not{p}_1 + \not{q} + m) \hat{p}_2 - \frac{1}{D} \not{p}_2 (\not{p}_1' - \not{q} + m) \gamma_\mu \\
&= s \left(\frac{1}{D'} - \frac{x}{D} \right) \gamma_\mu + \frac{\gamma_\mu \not{q} \not{p}_2}{D'} + \frac{\not{p}_2 \not{q} \gamma_\mu}{D},
\end{aligned}
$$

where we use the notation

$$
D' = (p_1 + q)^2 - m^2 = \frac{1}{x x_+ x_-} d', \qquad D = -\big[\big(p_1' - q\big)^2 - m^2\big] = \frac{1}{x_+ x_-} d,
$$
$$
d' = d + \bar{x} x_+ x_- \vec{q}^{\,2} - 2 x_+ x_- \vec{q}(\vec{q}_+ + \vec{q}_-);
$$

$$d = m^2 x_+ x_- \bar{x} + M^2 x \bar{x} + \vec{q}_+^2 x_- \bar{x}_- + \vec{q}_-^2 x_+ \bar{x}_+ + 2(\vec{q}_+ \vec{q}_-) x_- x_+. \tag{59}$$

So we obtain

$$Q_\mu = \frac{x_+ x_-}{dd'} \big[sx\rho\gamma_\mu + xd\gamma_\mu \hat{q} \hat{p}_2 + d' \hat{p}_2 \hat{q} \gamma_\mu \big], \quad \rho = d - d'. \tag{60}$$

The two-photon amplitude contains a Dirac sub-process $\gamma^*(q_1) + \gamma^*(q) \to Q(q_-) + \bar{Q}(q_+)$ with the amplitude $\bar{u}(q_-) R_\lambda v(q_+)$

$$R_\lambda = -\gamma_\lambda \frac{\hat{q} - \hat{q}_+ + M}{D_+} \hat{p}_2 - \hat{p}_2 \frac{\hat{q}_- - \hat{q} + M}{D_-} \gamma_\lambda. \tag{61}$$

Again, with on-mass shell conditions for the heavy fermion pair it can be written as

$$R_\lambda = s\gamma_\lambda r_1 - \frac{\gamma_\lambda \hat{q} \hat{p}_2}{D_+} + \frac{\hat{p}_2 \hat{q} \gamma_\lambda}{D_-},$$
$$r_1 = \frac{x_+}{D_+} - \frac{x_-}{D_-}, \tag{62}$$

with the definitions

$$D_+ = -\big[(q - q_+)^2 - M^2\big] = \frac{1}{xx_-} d_+, \qquad D_- = -\big[(q - q_-)^2 - M^2\big] = \frac{1}{xx_+} d_-;$$
$$d_+ = d + xx_+ \big(\vec{q}^2 - 2\vec{q}\vec{q}_-\big) + x_+ x_- \big(\vec{q}^2 - 2\vec{q}(\vec{q}_+ + \vec{q}_-)\big);$$
$$d_- = d + xx_- \big(\vec{q}^2 - 2\vec{q}\vec{q}_+\big) + x_+ x_- \big(\vec{q}^2 - 2\vec{q}(\vec{q}_+ + \vec{q}_-)\big);$$
$$q_2^2 = (p_1 - p_1')^2 = -\frac{1}{x}\big[\vec{p}^2 + \bar{x}^2 m^2\big], \quad \vec{p} = \vec{q} - \vec{q}_- - \vec{q}_+. \tag{63}$$

With this notation we have

$$R_\lambda = \frac{x}{d_+ d_-} [sx_+ x_- \rho_1 \gamma_\lambda - x_- d_- \gamma_\lambda \hat{q} \hat{p}_2 + x_+ d_+ \hat{p}_2 \hat{q} \gamma_\lambda], \quad \rho_1 = d_- - d_+. \tag{64}$$

The square of the matrix element summed over spin states has the form

$$\sum |M^{ep \to (eQ\bar{Q})p}|^2 = \frac{8s^2}{(q^2)^2} 16(4\pi\alpha)^4 R^{Q\bar{Q}}, \quad R^{Q\bar{Q}} = R_{br} + R_{2\gamma} + R_{odd},$$
$$R_{br} = \frac{1}{s^2 (q_1^2)^2} \frac{1}{4} Tr(\hat{q}_- + M)\gamma_\mu (\hat{q}_+ - M)\gamma_\nu \frac{1}{4} Tr \hat{p}_1' Q_\mu \hat{p}_1 Q_\nu^+;$$
$$R_{2\gamma} = \frac{1}{s^2 (q_2^2)^2} \frac{1}{4} Tr(\hat{q}_- + M) R_\mu (\hat{q}_+ - M) R_\nu^+ \frac{1}{4} Tr(\hat{p}_1' + m)\gamma_\mu (\hat{p}_1 + m)\gamma_\nu;$$
$$R_{odd} = \frac{2}{s^2 q_1^2 q_2^2} \frac{1}{4} Tr(\hat{q}_- + M)\gamma_\mu (\hat{q}_+ - M) R_\lambda^+ \frac{1}{4} Tr \hat{p}_1' Q_\mu \hat{p}_1 \gamma_\lambda. \tag{65}$$

It is important to know that all the quantities entering into $R^{Q\bar{Q}}$ do not depend on s and are proportional to $\vec{q}^2$ in the WW limit $\vec{q} \to 0$.

Keeping in mind that in the combinations $\hat{p}_2 \hat{q}$ and $\hat{q} \hat{p}_2$ one can replace $\hat{q} \to \hat{q}_\perp$ one may use the relations needed in calculating the traces (we neglect the contributions of an order of m^2/M^2 compared to the ones of an order of unity)

$$p_1^2 = p_1'^2 = p_2^2 = 0; \qquad q_+^2 = q_-^2 = M^2; \qquad q^2 = -\vec{q}^2;$$

$$2p_2p_1 = s; \qquad 2p_2p_1' = sx; \qquad 2p_2q_+ = sx_+; \qquad 2p_2q_- = sx_-; \qquad 2p_2q = 0;$$

$$2p_1q = 0; \qquad qq_- = -\vec{q}\vec{q}_-; \qquad qq_+ = -\vec{q}\vec{q}_+; \qquad qp_1' = -\vec{q}\vec{p};$$

$$2q_+p_1' = \frac{1}{xx_+}\left[x^2M^2 + (x_+\vec{p} - x\vec{q}_+)^2\right]; \qquad 2q_-p_1' = \frac{1}{xx_-}\left[x^2M^2 + (x_-\vec{p} - x\vec{q}_-)^2\right];$$

$$2p_1p_1' = -q_2^2 = \frac{1}{x}\vec{p}^2; \qquad 2p_1q_+ = \frac{1}{x_+}\left[M^2 + \vec{q}_+^2\right]; \qquad 2p_1q_- = \frac{1}{x_-}\left[M^2 + \vec{q}_-^2\right];$$

$$q_1^2 = \frac{1}{x_+x_-}\left[\bar{x}^2M^2 + \vec{r}^2\right], \qquad \vec{r} = x_-\vec{q}_+ - x_+\vec{q}_-, \qquad \vec{p} = \vec{q} - \vec{q}_+ - \vec{q}_-. \tag{66}$$

The differential cross sections have the form

$$d\sigma^{ep \to (eQ\bar{Q})p} = \frac{2\alpha^4}{\pi}\frac{R^{Q\bar{Q}}}{(q^2)^2}d\gamma_4,$$

$$d\gamma_4 = \frac{dx_+dx_-}{xx_+x_-}\frac{d^2\vec{q}}{\pi}\frac{d^2\vec{q}_+}{\pi}\frac{d^2\vec{q}_-}{\pi}. \tag{67}$$

In the case of processes with a quark instead a muon, we must take into account the quark color degrees of freedom

$$d\sigma^{qp \to (qQ\bar{Q})p} = C_{col}\frac{\alpha_s^2}{\alpha^2}d\sigma^{ep \to (eQ\bar{Q})p}, \tag{68}$$

with $C_{col} = (N^2 - 1)/(4N^2)$, where we also take into account the averaging over the color of quarks. The explicit expressions for R_{br}, $R_{2\gamma}$, R_{odd} are given in Appendix A. In the double WW approximation the contribution to the differential cross section has the form

$$\frac{d\sigma^{ep \to (Q\bar{Q}e)p}}{dx_+dx_-dz} = \frac{\alpha^4 L^2}{\pi M^2}\Phi(z, x_+, x_-), \qquad z = \frac{\vec{q}_+^2}{M^2}. \tag{69}$$

Exact formulae are given in Appendix B (see (B.8)).

The function

$$\Phi(z, x_+ = x_- = 1/3) = \Phi(z) = \frac{3}{8}\frac{7 + 268z + 243z^2}{(1 + z)^4}$$

is presented in Fig. 4.

In Table 1 the charge asymmetry defined as

$$A_{+-}(y_+, y_-, x_+, x_-) = \left[\frac{2}{q_1^2q_2^2}R_{WW}^{odd}\right] \Big/ \left[\frac{1}{(q_1^2)^2}R_{WW}^{br} + \frac{1}{(q_2^2)^2}R_{WW}^{2g}\right], \qquad y_\pm = \frac{\vec{q}_\pm^2}{M^2}, \tag{70}$$

is presented at the symmetric point $x = x_- = x_+ = 1/3$, $\phi = \pi/2$ for several typical values $y_+ < y_-$. This quantity has a value of an order of $A_{+-} \sim 10^{-2}$. For the use of $y_+ > y_-$ the quantity A_{+-} changes the sign.

6. Double bremsstrahlung in electron–proton collisions

In the lowest order of perturbation QED theory there are 20 Feynman diagrams describing the double bremsstrahlung process (see Fig. 2, two first diagrams)
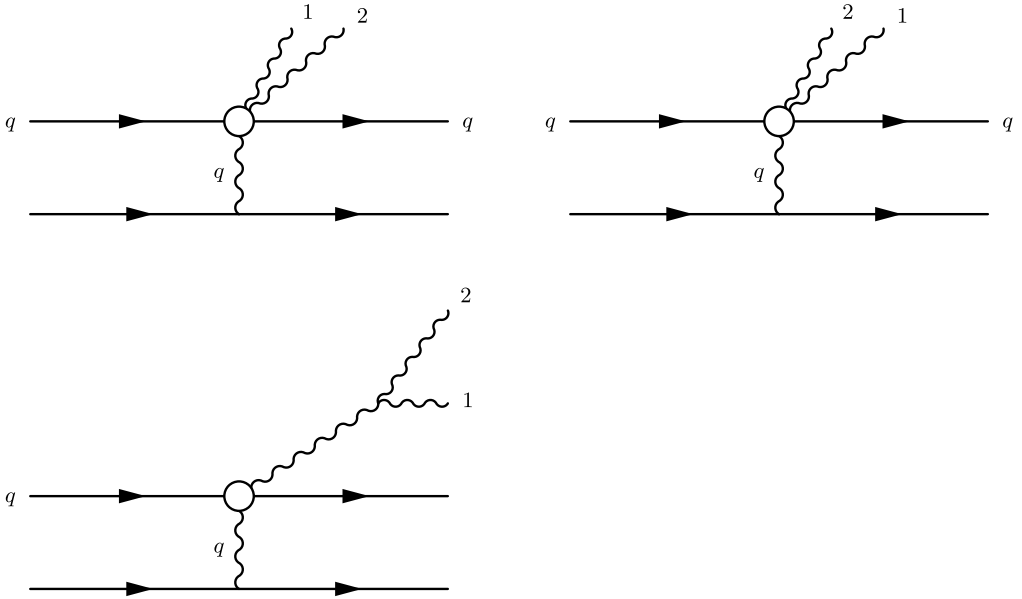
Fig. 2. Emission of two gluons.

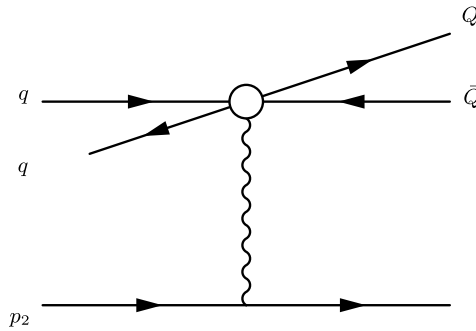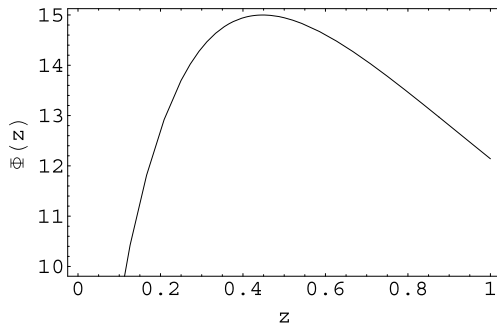
Fig. 3. To "cumulation effect".

Fig. 4. The $\Phi(z)$ (defined in (69)) as a function of z.

Table 1
The function $A_{+-}(y_+, y_-)$ (defined in (70)) is presented for different values of the final quark (lepton) transverse momenta y_+ and y_- for the quark pair (lepton pair) production (in units of M).

y_+	4	3	3	2	2	2	1	1	1	1
y_-	5	4	5	3	4	5	2	3	4	5
A_{+-}	0.0279	0.0295	0.0314	0.0326	0.0348	0.0377	0.0405	0.0421	0.0462	0.0513

$$e(p_1) + p(p_2) \rightarrow e(p_1') + \gamma(k_1) + \gamma(k_2) + p(p_2'), \tag{71}$$

i.e. emission of two hard photons in collisions of the high energy electron with a charged heavy target (heavy lepton). We will restrict ourselves to the consideration of the emission from the electron line only. The set of six Feynman diagrams provides the gauge invariant set (see Fig. 2, two first diagrams). With respect to the exchanged photon they split into two independent subsets of Feynman amplitudes, both gauge invariant (see Fig. 2, two first diagrams). The relevant matrix element is

$$M^{ep \rightarrow (e\gamma\gamma)p} = \frac{2s(4\pi\alpha)^2}{q^2} N \frac{1}{s} \bar{u}(p_1')[O_{12} + O_{21}]u(p_1), \tag{72}$$

with (see details in Appendices C, D),

$$O_{12} = \frac{1}{(1)D}N_1 - \frac{1}{(1)(2')}N_2 + \frac{1}{(2')D'}N_3,$$
$$N_1 = \not{p}_2(\not{p}_1' - \not{q} + m)\not{e}_2(\not{p}_1 - \not{k}_1 + m)\not{e}_1;$$
$$N_2 = \not{e}_2(\not{p}_1' + \not{k}_2 + m)\not{p}_2(\not{p}_1 - \not{k}_1 + m)\not{e}_1;$$
$$N_3 = \not{e}_2(\not{p}_1' + \not{k}_2 + m)\not{e}_1(\not{p}_1 + \not{q} + m)\not{p}_2, \tag{73}$$

and

$$O_{21} = \frac{1}{(2)D}N_1' - \frac{1}{(2)(1')}N_2' + \frac{1}{(1')D'}N_3',$$
$$N_1' = \not{p}_2(\not{p}_1' - \not{q} + m)\not{e}_1(\not{p}_1 - \not{k}_2 + m)\not{e}_2;$$
$$N_2' = \not{e}_1(\not{p}_1' + \not{k}_1 + m)\not{p}_2(\not{p}_1 - \not{k}_2 + m)\not{e}_2;$$
$$N_3' = \not{e}_1(\not{p}_1' + \not{k}_1 + m)\not{e}_2(\not{p}_1 + \not{q} + m)\not{p}_2;$$
$$N = \frac{1}{s}\bar{Y}(p_2')\not{p}_1 Y(p_2). \tag{74}$$

Here $e_i = e_i(k_i)$ are the polarization vectors of hard photons. It can be checked that the expression for $M^{2Y \rightarrow (2\gamma\gamma)Y}$ turns to zero in replacing $p_2 \rightarrow q$ as well as $e_i \rightarrow k_i$ which is the consequence of gauge invariance.

We adopt below the Sudakov parametrization of the relevant 4-vectors

$$q = \alpha_q p_2 + q_\perp; \qquad k_i = \alpha_i p_2 + x_i p_1 + k_{i\perp};$$
$$p_1' = \alpha' p_2 + x p_1 + p_\perp, \tag{75}$$

and use the notation and relations (different compared to the previous section)

$$(1) = 2p_1 k_1 = \frac{1}{x_1}\left[m^2 x_1^2 + \vec{k}_1^2\right] = \frac{y_1}{x_1}; \qquad (2') = 2p_1' k_2 = \frac{1}{xx_2}\left[m^2 x_2^2 + \vec{r}_2^2\right] = \frac{z_2}{xx_2};$$
$$(2) = 2p_1 k_2 = \frac{1}{x_2}\left[m^2 x_2^2 + \vec{k}_2^2\right] = \frac{y_2}{x_2}; \qquad (1') = 2p_1' k_1 = \frac{1}{xx_1}\left[m^2 x_1^2 + \vec{r}_1^2\right] = \frac{z_1}{xx_1};$$

$$D = -[(p_1' - q)^2 - m^2] = \frac{d}{x_1 x_2}; \qquad D' = (p_1 + q)^2 - m^2 = \frac{1}{x x_1 x_2} d';$$

$$\vec{r}_2 = \bar{x}_1 \vec{k}_2 + x_2(\vec{k}_1 + \vec{q}); \quad \vec{r}_1 = \bar{x}_2 \vec{k}_1 + x_1(\vec{k}_2 + \vec{q});$$

$$d' = d + \vec{q}^2 \bar{x} x_1 x_2 - 2x_1 x_2 \vec{q}(\vec{k}_1 + \vec{k}_2);$$

$$d = m^2 x_1 x_2 \bar{x} + x_1 \bar{x}_1 \vec{k}_2^2 + x_2 \bar{x}_2 \vec{k}_1^2 + 2x_1 x_2 \vec{k}_1 \vec{k}_2;$$

$$2k_1 k_2 = \frac{1}{x_1 x_2}(x_2 \vec{k}_1 - x_1 \vec{k}_2)^2; \quad 2p_1 p_1' = \frac{1}{x}[\vec{p}^2 + m^2 x^2];$$

$$(1) + (2) + (1') + (2') = D + D'. \tag{76}$$

The expression for the matrix element given above can be written in a form to display the explicit gauge invariance, which is suitable especially for investigation in the (WW) [7] approximation. For this aim we note that the combinations

$$R_1 = \frac{x}{(1)D} - \frac{\bar{x}_1}{(1)(2')} + \frac{1}{(2')D'}; \qquad R_2 = \frac{x}{(2)D} - \frac{\bar{x}_2}{(2)(1')} + \frac{1}{(1')D'};$$

$$r = \frac{1}{D'} - \frac{x}{D}, \tag{77}$$

turn to zero in the limit $\vec{q} \to 0$. Excluding the term containing the denominator $(1)(2')$ we can rewrite the expression for O_{12} in the form

$$O_{12} = \frac{1}{\bar{x}_1}\big[R_1 \not{e}_2(\not{p}_1' + \not{k}_2 + m)\not{p}_2(\not{p}_1 - \not{k}_1 + m)\not{e}_1 + r\not{e}_2\not{p}_2\not{e}_1$$

$$+ c_1 \not{e}_2(\not{p}_1' + \not{k}_2 + m)[\bar{x}_1 \not{e}_1 \not{q}\not{p}_2 + \not{p}_2 \not{q}\not{e}_1]$$

$$+ d_1[\bar{x}_1 \not{p}_2 \not{q}\not{e}_2 + x\not{e}_2 \not{q}\not{p}_2](\not{p}_1 - \not{k}_1 + m)\not{e}_1\big], \tag{78}$$

with

$$r = \frac{x x_1 x_2}{d d'}\rho, \quad \rho = d - d' = x_1 x_2[-\bar{x}\vec{q}^2 + 2\vec{q}(\vec{k}_1 + \vec{k}_2)]; \qquad c_1 = \frac{x_1(x x_2)^2}{z_2 d'},$$

$$d_1 = -\frac{x_2 x_1^2}{y_1 d}. \tag{79}$$

A similar expression for the set of other Feynman diagrams (can be obtained from the first one by the replacement $k_1, e_1 \leftrightarrow k_2, e_2$) is

$$O_{21} = \frac{1}{\bar{x}_2}\big[R_2 \not{e}_1(\not{p}_1' + \not{k}_1 + m)\not{p}_2(\not{p}_1 - \not{k}_2 + m)\not{e}_2 + r\not{e}_1\not{p}_2\not{e}_2$$

$$+ c_2 \not{e}_1(\not{p}_1' + \not{k}_1 + m)[\bar{x}_2 \not{e}_2 \not{q}\not{p}_2 + \not{p}_2 \not{q}\not{e}_2]$$

$$+ d_2[\bar{x}_2 \not{p}_2 \not{q}\not{e}_1 + x\not{e}_1 \not{q}\not{p}_2](\not{p}_1 - \not{k}_2 + m)\not{e}_2\big], \tag{80}$$

with

$$c_2 = \frac{x_2(x x_1)^2}{z_1 d'}, \qquad d_2 = -\frac{x_1 x_2^2}{y_2 d}. \tag{81}$$

The matrix element squared summed over spin states is

$$\sum |M^{ep \to (e\gamma\gamma)p}|^2 = \frac{32(4\pi\alpha)^4 s^2}{\pi(q^2)^2} R^{\gamma\gamma},$$

$$R^{\gamma\gamma} = (1 + \mathcal{P}_{12})\frac{1}{4s^2} Tr[\not{p}_1' O_{12}\not{p}_1 O_{12}^+ + \not{p}_1' O_{12}\not{p}_1 O_{21}^+]. \tag{82}$$

From a topological point of view it is convenient to write down $R_{\gamma\gamma}$ as a sum of planar and non-planar Feynman diagrams for the cross section

$$R^{\gamma\gamma} = (1 + \mathcal{P}_{12})[R_{pl} + R_{npl}]. \tag{83}$$

The differential cross section has the form

$$d\sigma^{ep\to(e\gamma\gamma)p} = \frac{1}{2!}\frac{\alpha^4}{2\pi}\frac{R_{\gamma\gamma}}{(q^2)^2}d\gamma_4;$$

$$d\gamma_4 = \frac{dx_1 dx_2}{x x_1 x_2}\frac{d^2\vec{q}}{\pi}\frac{d^2\vec{k}_1}{\pi}\frac{d^2\vec{k}_2}{\pi}. \tag{84}$$

Factor $1/2!$ takes into account the identity of photons in the final state.

In the WW approximation we have

$$2\pi\frac{d\sigma^{ep\to(e\gamma\gamma)p}}{dx_1 dx_2 dy_1 dy_2 d\phi} = \frac{\alpha^4(L_p - 1)}{4\pi}\frac{1}{M^2}R^{\gamma\gamma}(y_1, y_2, x_1, x_2, \phi),$$

$$y_i = \frac{k_i^2}{M^2}, \tag{85}$$

with M being the mass of a heavy quark in the scale parameter for the values of the transverse momenta of photons (gluons).

7. Quark–proton collision: emission of two gluon jets

The matrix element of the process of two gluon jets production in a peripheral quark–colorless fermion target collision (see Fig. 2)

$$q(p_1) + Y(p_2) \to q(p_1') + Y(p_2') + g(k_1) + g(k_2), \tag{86}$$

has the form

$$M = \frac{32s\alpha\alpha_s}{q^2}J^q N, \quad J^q = \bar{u}(p_1')Ru(p_1), \quad N = \frac{1}{s}\bar{u}(p_2')\gamma_\mu u(p_2)p_1^\mu,$$

$$R = O_{12}R_2 + O_{21}R_1 + (R_2 - R_1)O_3, \tag{87}$$

where $R_1 = (t^a t^b)_{r_2 r_1}$, $R_2 = (t^b t^a)_{r_2 r_1}$, with $r_2(r_1)$ describing the color states of the scattered (initial) quark. Here the quantities O_{12}, O_{21} were obtained above (see (78), (80)), with the replacement $k_1 \to q_+$, $k_2 \to q_-$, where we imply $e_1 \to e^a$, $e_2 \to e^b$ and

$$O_3 = -\frac{2}{q_1^2}\left[-\frac{1}{D}\not{p}_2(\not{p}_1' - \not{q} + m)\not{V}^{ab} + \frac{1}{D'}\not{V}^{ab}(\not{p}_1 + \not{q} + m)\not{p}_2\right],$$

$$V^{ab} = (k_1 e^b)\not{e}^a - \not{e}^b(k_2 e^a) + \not{k}_2(e^a e^b), \quad q_1^2 = (k_1 + k_2)^2. \tag{88}$$

The quantity q_1^2 is presented (57), (66). It can be checked that the matrix element obeys gauge invariance, namely, it turns to zero if one replaces $p_2 \to q$ and $e_i(k_i) \to k_i$. The expression for the matrix element at $R_1 = R_2 = 1$, coincides with the QED result [21]. Below we will use the expression for O_3 in the form

$$O_3 = -\frac{2}{q_1^2}\frac{x_1 x_2}{dd'}\left[xs\rho\not{V} + d'\not{p}_2\not{q}\not{V} + xd\not{V}\not{q}\not{p}_2\right],$$

$$V = \not{e}^a(k_1 e^b) + \not{k}_2(e^a e^b) - \not{e}^b(k_2 e^a), \quad \rho = 2x_1 x_2 \vec{q}\,\vec{Q}. \tag{89}$$

To work with the irreducible color states, we use the projectors in color space

$$C_1 = \frac{1}{\sqrt{N(N^2 - 1)}} \delta^{ab} \delta_{r_2 r_1};$$

$$C_2 = \sqrt{\frac{2N}{(N^2 - 1)(N^2 - 4)}} d^{abc} (t^c)_{r_2 r_1};$$

$$C_3 = i \sqrt{\frac{2}{N(N^2 - 1)}} f^{abc} (t^c)_{r_2 r_1}. \tag{90}$$

These projectors obey the equations

$$C_i \tilde{C}_j = \begin{pmatrix} (c_i^{ab})_{r_1 r_2} \\ ((c_i^{ab})_{r_1 r_2})^+ = (c_i^{ab})_{r_2 r_1} \end{pmatrix} \tag{91}$$

$$C_i \tilde{C}_j = \delta_{ij}, \quad i, j = 1, 2, 3. \tag{92}$$

Here $(\tilde{A})_{r_1 r_2} = (A)_{r_2 r_1}$ and summation over a, b is implied.

In our case [1]

$$R_1 = \sqrt{\frac{N^2 - 1}{4N}} \left[C_1 + \sqrt{\frac{N^2 - 4}{2}} C_2 + \frac{N}{\sqrt{2}} C_3 \right],$$

$$R_2 = \sqrt{\frac{N^2 - 1}{4N}} \left[C_1 + \sqrt{\frac{N^2 - 4}{2}} C_2 - \frac{N}{\sqrt{2}} C_3 \right]. \tag{93}$$

The expansion on irreducible color representations is

$$R = C_1(R\tilde{C}_1) + C_2(R\tilde{C}_2) + C_3(R\tilde{C}_3)$$

$$= \sqrt{\frac{N^2 - 1}{4N}} \left[\left(C_1 + \sqrt{\frac{N^2 - 4}{2}} C_2 \right)(O_{12} + O_{21}) + \frac{N}{\sqrt{2}}(O_{21} - O_{12} - 2O_3) \right]. \tag{94}$$

So the matrix element squared summed over color and spin states can be written as

$$\sum |M|^2 = \frac{32s^2(16\pi^2\alpha\alpha_s)^2}{(q^2)^2} \frac{N^2 - 1}{4N} F,$$

$$F = F^{Abel} + F^{non\text{-}Abel},$$

$$F^{Abel} = \frac{N^2 - 2}{2}(1 + \mathcal{P}_{12})(R_{pl} + R_{npl}) + \frac{N^2}{2}[(1 + \mathcal{P}_{12})(R_{pl} - R_{npl})],$$

$$F^{non\text{-}Abel} = 2N^2[R_{33} - R_{321} + R_{312}], \tag{95}$$

with

$$R_{pl} = \frac{1}{4s^2} Tr \not{p}'_1 O_{12} \not{p}_1 O_{12}^+;$$

$$R_{npl} = \frac{1}{4s^2} Tr \not{p}'_1 O_{12} \not{p}_1 O_{21}^+;$$

---

[1] As a check we have $(R_1 \tilde{R}_1) = (R_2 \tilde{R}_2) = Tr t^a t^b t^b t^a = N C_F^2$; $(R_1 \tilde{R}_2) = Tr t^a t^b t^a t^b = -\frac{1}{2} C_F$ [19,28]. These relations are fulfilled.

Table 2

The function $F^{non\text{-}Abel}(y_1, y_2)$ and $F(y_1, y_2)$ (defined in (95)) is presented for different values of the final gluon transverse momenta y_1 and y_2.

y_1	4	3	3	2	2	2	1	1	1	1
y_2	5	4	5	3	4	5	2	3	4	5
$F^{non\text{-}Abel}$	0.0398	0.0845	0.0561	0.2309	0.1315	0.0812	1.0521	0.4257	0.2074	0.1135
F	0.0415	0.0882	0.0591	0.2422	0.1396	0.0872	1.117	0.4625	0.2306	0.1294

Table 3

The function $A^{gg}(y_1, y_2)$ (defined in (98)) is presented for different values of the gluon transverse momenta y_1 and y_2.

y_1	4	3	3	2	2	2	1	1	1	1
y_2	5	4	5	3	4	5	2	3	4	5
A^{gg}	0.8396	0.8318	0.8024	0.8167	0.7755	0.7382	0.7755	0.7034	0.6393	0.5805

$$R_{33} = \frac{1}{4s^2} Tr \not{p}_1' O_3 \not{p}_1 O_3^+;$$

$$R_{312} = \frac{1}{4s^2} Tr \not{p}_1' O_3 \not{p}_1 O_{12}^+;$$

$$R_{321} = \frac{1}{4s^2} Tr \not{p}_1' O_3 \not{p}_1 O_{21}^+. \tag{96}$$

In the case $x_1 = x_2 = x = 1/3$ and $\phi = \pi/2$, $F^{non\text{-}Abel}$ and F are presented (see Table 2) for typical values of y_1, y_2.

The differential cross section is

$$d\sigma^{qY \to (qgg)Y} = \frac{(\alpha \alpha_s)^2}{8\pi} \frac{N^2 - 1}{N^2} \frac{F}{(q^2)^2} d\gamma_4. \tag{97}$$

The explicit expressions for F as well as for $R_{\gamma\gamma}$ are too cumbersome. Nevertheless, they are suitable for further analytic and numerical integration when obtaining different distributions.

As some probe of QCD the quantity

$$A^{gg} = \frac{F^{non\text{-}Abel}}{F} \tag{98}$$

can be investigated since it reveals a specific QCD deviation from the QED process of the double bremsstrahlung. It is presented in Table 3 in the WW approximation for $x = x_1 = x_2 = 1/3$, $\phi = \pi/2$, for different values y_1, y_2. For values $q_1^2 \sim M^2$ the non-Abelian contribution dominates in A^{gg} (see Table 3). For large q_1^2, $A^{gg} \sim \frac{M^2}{q_1^2}$.

8. Three-jet structure in peripheral collisions on a fixed target

In the case of jet production with a projectile on a fixed target (nucleon), an additional wide-angle jet can be created by the recoil target particle.

Consider for simplicity the photo-production of a pair $q\bar{q}$ process on a nucleon.

$$\gamma(k) + p(p) \to q(q_-) + \bar{q}(q_+) + p'(p'), \tag{99}$$

$s = 2pk$, $k^2 = 0$, $p = M(1, 0, 0, 0)$.

As the Sudakov expansion basis we use $k = \omega(1, 1, 0, 0)$ and $p_1 = p - \frac{M^2}{s}k = \frac{M}{2}(1, -1, 0, 0)$. For the transferred 4-momentum $q = p - p'$ we have

$$q = \alpha_q k + \beta p_1 + q_\perp. \tag{100}$$

Solving the on-mass shell condition of a recoil proton we find

$$p_1'^2 - M^2 = s\alpha_q\beta - \vec{q}^2 - M^2\beta - s\alpha_q = 0, \quad s\alpha_q = \vec{q}^2. \tag{101}$$

Considering its longitudinal and transversal component we have

$$\vec{p}'^2 = \vec{q}^2 + \frac{1}{4M^2}|\vec{q}^2|^2. \tag{102}$$

So we have

$$\sin^2\theta = \frac{\vec{q}^2}{\vec{p}'^2}, \tag{103}$$

with $\theta(\vec{p}', \vec{k})$, and

$$\left|\frac{\vec{p}'}{M}\right| = \frac{2\cos\theta}{\sin^2\theta}. \tag{104}$$

The recoil jet penetrates in a rather wide cone $\theta \sim 60°$. Relation (104) was first obtained in [26].

9. The process $gp \rightarrow (g\bar{Q}Q)p$

Another process where the non-Abelian structure of QCD manifests itself is the crossing process to one considered in IV

$$g(k) + p(p_2) \rightarrow (\bar{Q}(q_+)Q(q_-)g(k_1))p(p_2'). \tag{105}$$

The matrix element can be written as

$$
\begin{aligned}
M^{(1)} &= \left(M_1^{(1)} + M_2^{(1)} + M_3^{(1)} + M_7^{(1)} + M_8^{(1)}\right)_{\mu\nu}, \\
M^{(2)} &= \left(M_4^{(2)} + M_5^{(2)} + M_6^{(2)} + M_7^{(2)} + M_8^{(2)}\right)_{\mu\nu}.
\end{aligned} \tag{106}
$$

$$M = \bar{u}(q_-)\left[t^c t^a M^{(1)} + t^a t^c M^{(2)}\right]_{\mu\nu} v(q_+)e_\mu^a(k)e_\nu^b(k_1),$$

$$M_1^{(1)} = \frac{\gamma_\nu(\slashed{q}_- + \slashed{k}_1 + m)\gamma_\mu(-\slashed{q}_+ + \slashed{q} + m)\slashed{p}_2}{[(\slashed{q}_- + \slashed{k}_1)^2 - m^2][(-\slashed{q}_+ + \slashed{q})^2 - m^2]} = R_1;$$

$$M_2^{(1)} = \frac{\gamma_\nu(\slashed{q}_- + \slashed{k}_1 + m)\slashed{p}_2(-\slashed{q}_+ + \slashed{k} + m)\gamma_\mu}{[(\slashed{q}_- + \slashed{k}_1)^2 - m^2][(-\slashed{q}_+ + \slashed{k})^2 - m^2]} = R_2;$$

$$M_3^{(1)} = \frac{\slashed{p}_2(\slashed{q}_- - \slashed{q} + m)\gamma_\nu(-\slashed{q}_+ + \slashed{k} + m)\gamma_\mu}{[(-\slashed{q}_+ + \slashed{k})^2 - m^2][(\slashed{q}_- - \slashed{q})^2 - m^2]} = R_3;$$

$$M_7^{(1)} + M_8^{(1)} = \frac{V_{\mu\lambda\nu}}{(k_1 - k)^2}\left[\frac{\gamma_\lambda(-\slashed{q}_+ + \slashed{q} + m)\slashed{p}_2}{(-q_+ + q)^2 - m^2} + \frac{\slashed{p}_2(\slashed{q}_+ - \slashed{q} + m)\gamma_\lambda}{(q_+ - q)^2 - m^2}\right] = R_4; \tag{107}$$

$$M_4^{(2)} = \frac{\gamma_\mu(\slashed{q}_- - \slashed{k}_1 + m)\gamma_\nu(-\slashed{q}_+ + \slashed{q} + m)\slashed{p}_2}{[(-q_- - k)^2 - m^2][(-q_+ + q)^2 - m^2]} = Q_1;$$

$$M_5^{(2)} = \frac{\gamma_\mu(\slashed{q}_- - \slashed{k} + m)\slashed{p}_2(-\slashed{q}_+ - \slashed{k}_1 + m)\gamma_\nu}{[(q_- - k)^2 - m^2][(-q_+ - k_1)^2 - m^2]} = Q_2;$$

$$M_6^{(2)} = \frac{\slashed{p}_2(\slashed{q}_- - \slashed{q} + m)\gamma_\mu(-\slashed{q}_+ - \slashed{k}_1 + m)\gamma_\nu}{[(q_- - q)^2 - m^2][(-q_+ - k_1)^2 - m^2]} = Q_3;$$

$$M_7^{(2)} + M_8^{(2)} = \frac{-V_{\mu\lambda\nu}}{(k_1 - k)^2}\left[\frac{\gamma_\lambda(-\slashed{q}_+ + \slashed{q} + m)\slashed{p}_2}{(-q_+ + q)^2 - m^2} + \frac{\slashed{p}_2(\slashed{q}_- - \slashed{q} + m)\gamma_\lambda}{(q_- - q)^2 - m^2}\right] = Q_4;$$

$$V_{\mu\lambda\nu} = -(k_1 + k)_\lambda g_{\mu\nu} + (2k - k_1)_\nu g_{\mu\lambda} + (2k_1 - k)_\mu g_{\lambda\nu},$$

$$M_{\mu\nu}^{(1,2)}k_1^\nu = M_{\mu\nu}^{(1,2)}k^\mu = 0. \tag{108}$$

It can be checked that both contributions $M^{(1)}$ and $M^{(2)}$ obey the gauge condition.

Let as combine the complete amplitude

$$M \sim t^c t^a M^{(1)} + t^a t^c M^{(2)} = \sqrt{\frac{N^2-1}{4N}}\left\{\left[c_1 + \sqrt{\frac{N^2-1}{2}}c_2\right] \cdot \left(M^{(1)} + M^{(2)}\right) \right.$$
$$\left. + \frac{N}{\sqrt{2}}c_3\left[M^{(1)} - M^{(2)}\right]\right\}, \tag{109}$$

where

$$M^{(1)} + M^{(2)} = R_1 + R_2 + R_3 + R_4 + Q_1 + Q_2 + Q_3 + Q_4,$$
$$M^{(1)} - M^{(2)} = (R_1 + R_2 + R_3) - (Q_1 + Q_2 + Q_3) + 2R_4. \tag{110}$$

Then the amplitude square read as:

$$\left|M^{gP\to(gQ\bar{Q})P}\right|^2 \approx \frac{N^2-2}{2}Tr\slashed{q}_-(M_1 + M_2)\slashed{q}_+(M_1 + M_2)^*$$
$$+ \frac{N^2}{2}Tr\slashed{q}_-(M_1 - M_2)\slashed{q}_+(M_1 - M_2)^* = A^{total}, \tag{111}$$

where

$$M_1 + M_2 = R_1 + R_2 + R_3 + Q_1 + Q_2 + Q_3;$$
$$M_1 - M_2 = R_1 + R_2 + R_3 - Q_1 - Q_2 - Q_3 + 2R_4. \tag{112}$$

Again one can define the asymmetry A, which appears because of non-Abelian nature of QCD. This asymmetry is defined as (see Table 4):

$$A = \frac{A^{non\text{-}Abel}}{A^{total}}; \tag{113}$$

where

$$A^{non\text{-}Abel} = 2N^2\{Tr\slashed{q}_-R_4\slashed{q}_+R_4^+ + Tr\slashed{q}_-(R_1 + R_2 + R_3 - Q_1 - Q_2 - Q_3)\slashed{q}_+R_4^+\}. \tag{114}$$

$$R_1 = \frac{\gamma_\nu(\slashed{q}_- + \slashed{k}_1)\gamma_\mu(-\slashed{q}_+ + \slashed{q})\slashed{p}_2}{d_{-1}d_{+q}}; \qquad R_2 = \frac{\gamma_\nu(\slashed{q}_- + \slashed{k}_1)\slashed{p}_2(-\slashed{q}_+ + \slashed{k})\gamma_\mu}{d_{-1}d_{+k}};$$

$$R_3 = \frac{\slashed{p}_2(\slashed{q}_- - \slashed{q})\gamma_\nu(-\slashed{q}_+ + \slashed{k})\gamma_\mu}{d_{-q}d_{+k}}; \qquad Q_1 = \frac{\gamma_\mu(\slashed{q}_- - \slashed{k})\gamma_\nu(-\slashed{q}_+ + \slashed{q})\slashed{p}_2}{d_{-k}d_{-q}};$$

Table 4
The function $A(y_+, y_-)$ (defined in (113)) is presented for different values of the final quark (lepton) transverse momenta y_+ and y_- for the quark pair (lepton pair) production (in units of M).

y_+	4	3	3	2	2	2	1	1	1	1
y_-	5	4	5	3	4	5	6	3	4	5
A	0.0428	0.117	0.0652	0.4489	0.193	0.0925	0.0574	0.84	0.275	0.1167

$$Q_2 = \frac{\gamma_\mu (\slashed{q}_- - \slashed{k}) \slashed{p}_2 (-\slashed{q}_+ - \slashed{k}_1) \gamma_\nu}{d_{-k} d_{+(-k_1)}}; \qquad Q_3 = \frac{\slashed{p}_2 (\slashed{q}_- - \slashed{q}) \gamma_\mu (-\slashed{q}_+ - \slashed{k}_1) \gamma_\nu}{d_{-q} d_{+(-k_1)}};$$

$$R_4 = \frac{V_{\mu\lambda\nu}}{d} \left[\frac{\gamma_\lambda (-\slashed{q}_+ + \slashed{q}) \slashed{p}_2}{d_{+q}} + \frac{\slashed{p}_2 (\slashed{q}_- - \slashed{q}) \gamma_\lambda}{d_{-q}} \right]. \tag{115}$$

10. Conclusion

Heavy charged lepton or quark pairs production in interactions of electron (quark) proton $ep \to (eQ\bar{Q})p$ were considered. We obtain master formula for the differential cross section and the charge asymmetry of this processes. We investigate characteristic of differential cross section and the charge asymmetry of this processes.

We considered production of two gluon jets in quark proton collisions. We study the function which is characterized the cross section. We also introduced the asymmetry A which arises because of the non-Abelian nature of QCD in heavy quark pair production.

Moreover, we considered $gp \to (gQ\bar{Q})p$ process and obtained the differential cross section, including the distribution on transverse momentum components of jets fragments. We study the asymmetry function which characterizes the process.

We study transfer of circular polarization of the initial electron to the positron in the fragmentation regime. The investigation of the degree of polarization transferred from the final lepton transverse momenta for the lepton pair production was performed.

In conclusion, we remind a remarkable property of the kinematics of processes in the fragmentation region. It is known as a "cumulation" phenomenon. It consists of events with production of a heavy quark–anti-quark pair, accompanied by the "reflected" scattered parent light particle (see Fig. 3). It was known in processes of production of a muon–anti-muon pair in the fragmentation region of an electron in electron–positron collisions [24]. It turns out that the electron "accompanying" the pair created in the kinematic region near the threshold moves in the direction opposite to the initial electron direction. In the case of production of a heavy quark–anti-quark pair by one of the valence quarks from the initial proton, the parent (light) quark is effectively reflected. So the jet created by this quark corresponds in fact to two jets, one consists of the pair created and two spectator quarks from the initial proton and the other moving in the opposite direction created by the "reflected" quark. To see it, let us consider the kinematics of a peripheral process $q(p_1) + q(p_2) \to Q(p_a) + \bar{Q}(p_b) + q(p_1') + q(p_2')$. Using the Sudakov parametrization (3) with

$$\tilde{p}_1 = E(1, 1, 0, 0), \qquad \tilde{p}_2 = E(1, -1, 0, 0), \qquad p_\perp = (0, 0, \vec{p}), \tag{116}$$

we obtain for 4-momentum of the scattered quark

$$\frac{1}{E} p_1' = \frac{m^2 + \vec{p}^2}{xs} (1, -1, 0, 0) + x(1, 1, 0, 0) + (0, 0, \vec{p}). \tag{117}$$

Comparing its component along the z axis from the first and second terms we find that for

$$x - \frac{m^2 + \vec{p}^2}{4E^2 x} < 0, \quad \vec{p}^2 = (\vec{p}_a + \vec{p}_b)^2, \tag{118}$$

the "reflection" phenomenon takes place. For instance, assuming $\vec{p}^2 \sim M^2 \gg m^2$ we have $x < (M/2E) \sim 1$. This situation can be realized near the threshold of the heavy pair production.

The expressions for the differential cross section of two hard photon emission were obtained in the WW approximation in [22] by using the explicit expression of the double Compton scattering cross section obtained in [23].

Using the formulae given above, the energy–energy correlations of the jets in the final state can be constructed. It consists in the construction of an average of the product of the energy fractions of the heavy quarks. As well, the azimuthal angle correlation, which is the average of $2(\vec{q}_{+i})(\vec{q}_{-j})/\sqrt{s}$, can be investigated. Energy spectra, total cross sections and sum rules for different processes in the fragmentation region can be investigated in full analogy with the QED program for colliding $eY = (eX)Y$ [13].

The approach developed here can be used for description of jets in the fragmentation region with creation of K, $\bar{K}$ states when the heavy strange quark and anti-quark are created. The jets originated from D, $\bar{D}$ and B, $\bar{B}$ can be considered as well.

In the plot (see Fig. 4), the dependence of the $Q\bar{Q}$ pair production cross section from the so-called "two-photon" mechanism is presented. It has rather a large cross section and can be measured in experiment.

In Table 1, the charge-asymmetry effect due to interference of the "bremsstrahlung" and "two-photon" mechanisms is presented as a ratio of the corresponding contributions to the differential cross section. This quantity can also be measured in spite of its rather small value $|A_{+-}| \sim 0.02$–0.03.

In Table 2, the contributions from the so-called "planar" and "non-planar" contribution to the differential cross section of the double bremsstrahlung process in electron–target collisions are presented.

Table 3 gives the ratio of contributions with the QED-type gluon splitting contribution to the process of two gluon jet production in the quark–target collisions. Its rather large expected value A^{gg} can also be measured in experiment.

Acknowledgements

Appendix A. Heavy quark pair production beyond the WW approximation

The explicit expression for R_{br} is

$$R_{br} = \left(\frac{x_+ x_-}{dd'}\right)^2 \big[dd' \big[\vec{q}^2 \big[2xx_+ x_- (p_1 p_1') - xx_+ (p_1' q_-) - xx_- (p_1' q_+) - 2xx_+ (\vec{q}\vec{q}_-) \\ - 2xx_- (\vec{q}\vec{q}_+) - x^2 x_+ (p_1 q_-) - x^2 x_- (p_1 q_+)$$

$$
\begin{aligned}
&+ 2x^2(q_+q_-)\big] + 2x(1+x)(\vec{q}\,\vec{q}_+)(\vec{q}\,\vec{q}_-) + 2xx_+(\vec{q}\,\vec{q}_-)^2 + 2xx_-(\vec{q}\,\vec{q}_+)^2\big]\big] \\
&+ 2\rho^2 x^2\big[M^2(p_1 p_1') + (p_1 q_+)(p_1' q_-) + (p_1 q_-)(p_1' q_+)\big] \\
&+ xd'^2\vec{q}^2\big[M^2 + x_+(p_1 q_-) + x_-(p_1 q_+)\big] + x^2 d^2\vec{q}^2\big[M^2 x + x_+(p_1' q_-) \\
&+ x_-(p_1' q_+)\big] + 2x^2 d\rho\big[-M^2\vec{q}^2 + (\vec{q}\,\vec{q}_+)(M^2 - 2(p_1' q_-)) \\
&+ (\vec{q}\,\vec{q}_-)(M^2 - (p_1' q_+))\big] + 2xd'\rho\big[(M^2 + x_+(p_1 q_-) + x_-(p_1 q_+))(\vec{q}\,\vec{p}) \\
&- x\big((p_1 q_+)(\vec{q}\,\vec{q}_-) + (p_1 q_-)(\vec{q}\,\vec{q}_+)\big)\big],
\end{aligned}
\tag{A.1}
$$

with the notation given above (see (59), (60), (66)).

The quantity $R_{2\gamma}$ enters into the differential cross section in combination $R_{2\gamma}/((q_2^2)^2)$ with $q_2^2 = -(m^2\bar{x}^2 + \vec{p}^2)/x$. To see rather delicate compensations in the region of small q_2^2, $\vec{p}^2$, we must rearrange the electron tensor as (here we use the gauge condition $q_{2\mu}\bar{u}(q_-)Q_\mu v(q_+) = 0$ and $q_2 \approx \bar{x}p_1 - p_\perp$)

$$
\frac{1}{4}\mathrm{Tr}(\not{p}_1' + m)\gamma_\mu(\not{p}_1 + m)\gamma_\nu = 2p_{1\mu}p_{1\nu} + \frac{q_2^2}{2}g_{\mu\nu} = \frac{2}{\bar{x}^2}p_{\perp\mu}p_{\perp\nu} + \frac{1}{2}q_2^2 g_{\mu\nu}.
\tag{A.2}
$$

In this form the compensation is clearly seen. So we have

$$
R_{2\gamma} = (1 + \mathcal{P}_{+-})\left(\frac{x}{d_+d_-}\right)^2\left[\frac{2}{\bar{x}^2}R_{2ga} + \frac{q_2^2}{2}R_{2gb}\right],
\tag{A.3}
$$

with

$$
\begin{aligned}
R_{2ga} &= \frac{1}{2}(x_+x_- - \rho_1)^2\left[2(\vec{p}\,\vec{q}_-)(\vec{p}\,\vec{q}_+) + \frac{1}{2}q_1^2\vec{p}^2\right] - x_+x_-^2\rho_1 d_-\big[2x_+(\vec{p}\,\vec{q}_-)(\vec{p}\,\vec{q}) + \vec{p}^2(\vec{q}\,\vec{r})\big] \\
&+ \frac{1}{2}x_-^3 x_+ d_-^2\vec{p}^2\vec{q}^2 - \frac{1}{4}(x_+x_-)^2 d_+ d_-\big(2(\vec{p}\,\vec{q})^2 - \vec{q}^2\vec{p}^2\big); \\
R_{2gb} &= -\rho_1^2 q_1^2(x_+x_-)^2 + 2x_+x_-^2\rho_1 d_-(\vec{q}\,\vec{r}) - x_-^3 x_+ d_-^2\vec{q}^2,
\end{aligned}
\tag{A.4}
$$

where we remind

$$
\begin{aligned}
&\rho_1 = -2x(\vec{q}\,\vec{r}); \quad \vec{r} = x_-\vec{q}_+ - x_+\vec{q}_-; \qquad \vec{p} = \vec{q} - \vec{Q}, \quad \vec{Q} = \vec{q}_+ + \vec{q}_-; \\
&q_1^2 = \frac{1}{x_+x_-}\big[M^2\bar{x}^2 + \vec{r}^2\big].
\end{aligned}
\tag{A.5}
$$

The quantity R_{odd} has a following form

$$
\begin{aligned}
R_{odd} = \frac{xx_+x_-}{dd'd_+d_-}\bigg[&(1 - \mathcal{P}_{+-})\bigg[xx_+d_+\rho\big[x_-(p_1 q_+)(\vec{q}\,\vec{q}_+) + (\vec{q}\,\vec{q}_-)(\bar{x}_+(p_1 q_+) + (p_1' q_+))\big] \\
&- x_-(p_1 q_+)\vec{q}^2\big] + x_+xd_+d\bigg[\frac{1}{2}\vec{q}^2\big[-M^2 x - x_-(p_1 q_+) + 2x_-(\vec{q}\,\vec{q}_+) \\
&- x_+x_-(p_1 p_1') + xx_+(p_1 q_-) - x(\vec{q}\,\vec{q}_-)\big] - \bar{x}_+(\vec{q}\,\vec{q}_-)(\vec{q}\,\vec{q}_+) - x_-(\vec{q}\,\vec{q}_+)^2\bigg] \\
&+ x_+d_+d'\bigg[\frac{1}{2}\vec{q}^2\big[-M^2 x - xx_-(p_1 q_+) - x(q_+q_-) + 2x_+(\vec{q}\,\vec{q}_-) + x_+(p_1 q_-) \\
&- x_+x_-(p_1 p_1')\big] - \bar{x}_-(\vec{q}\,\vec{q}_-)(\vec{q}\,\vec{q}_+) - x_+(\vec{q}\,\vec{q}_-)^2\bigg]\bigg]
\end{aligned}
$$

$$+ 2xx_+x_-\rho\rho_1\big[M^2(p_1p_1') + (p_1q_+)(p_1'q_-) + (p_1q_-)(p_1'q_+)\big]$$

$$+ xx_+x_-d\rho_1\big[(\vec{q}\vec{q}_+)(M^2 - (p_1'q_-)) + (\vec{q}\vec{q}_-)(M^2 - (p_1'q_+)) - \vec{q}^2M^2\big]$$

$$+ d'\rho_1 x_+x_-\big[-(\vec{q}\vec{q}_+)(M^2 + \bar{x}_-(p_1q_-) + x_-(p_1q_+))$$

$$- (\vec{q}\vec{q}_-)(M^2 + \bar{x}_+(p_1q_+) + x_- + x_+(p_1q_-)) + \vec{q}^2(M^2 + x_-(p_1q_+)$$

$$+ x_+(p_1q_-))\big]\big], \tag{A.6}$$

with $\rho = x_+x_-[-\bar{x}\vec{q}^2 + 2(\vec{q}\vec{Q})]$. The operator $\mathcal{P}_{+-}$ acts as $\mathcal{P}_{+-}f(x_+, \vec{q}_+; x_-, \vec{q}_-) \to f(x_-, \vec{q}_-; x_+, \vec{q}_+)$.

Appendix B. Distributions in the WW approximation, heavy fermion pair production

Differential distributions in the WW approximation are

$$d\sigma^{ep \to (eQ\bar{Q})p} = \frac{2\alpha^4}{\pi}(L_q - 1)R_{WW}^{Q\bar{Q}}\frac{dx_+dx_-}{xx_+x_-}dq_+^2 dq_-^2\frac{d\phi}{2\pi},$$

$$R_{WW}^{Q\bar{Q}} = \frac{1}{(q_1^2)^2}R_{WW}^{br} + \frac{1}{(q_2^2)^2}R_{WW}^{2g} + \frac{2}{q_1^2 q_2^2}R_{WW}^{odd}, \tag{B.1}$$

where we imply $q_\pm^2 = \vec{q}_\pm^2$; ϕ is the azimuthal angle between two-dimensional vectors $\vec{q}_+, \vec{q}_-$,

$$q_1^2 = \frac{1}{x_+x_-}\big[\bar{x}^2M^2 + r^2\big]; \qquad q_2^2 = -\frac{1}{x}\big[m^2\bar{x}^2 + Q^2\big],$$

$$r^2 = \vec{r}^2, \quad \vec{r} = x_-\vec{q}_+ - x_+\vec{q}_-; \quad Q^2 = \vec{Q}^2, \quad \vec{Q} = \vec{q}_+ + \vec{q}_-, \tag{B.2}$$

and

$$R_{WW}^{br} = (1 + \mathcal{P}_{+-})\left(\frac{x_+x_-}{d^2}\right)^2\bigg[d^2\big[x_+x_-Q^2 - x_+(p_1'q_-) - x^2x_+(p_1q_-) + x^2(q_+q_-)$$

$$+ \frac{1}{2}x(1 + x)(\vec{q}_+\vec{q}_-) + x_+xq_+^2 + \frac{1}{2}x(1 + x^2)M^2 + xx_+(p_1q_-)$$

$$+ 2x^2x_+(p_1'q_-)\big] + x^2Q^2\big[M^2(p_1p_1') + 4(p_1q_+)(p_1'q_-)\big]$$

$$+ 2x^2x_+x_-d(\vec{Q}\vec{q}_+)\big[M^2 - 2(p_1'q_-)\big]$$

$$- x_+x_-xdQ^2\big[M^2 + 2x_+(p_1q_-) + 2(p_1q_+)\big]$$

$$- 2x^2x_+x_-d(\vec{Q}\vec{q}_+)(p_1q_-)\bigg]; \tag{B.3}$$

$$R_{WW}^{2g} = (1 + \mathcal{P}_{+-})\left(\frac{x}{d^2}\right)^2\bigg[\frac{2}{\bar{x}^2}p^2R_a + \frac{1}{2}q_2^2R_b\bigg],$$

$$R_a = (xx_+x_-)^2r^2\left((\vec{q}_+\vec{q}_-) + \frac{1}{2}q_1^2\right) + xx_+x_-^2d\big[x_+(\vec{r}\vec{q}_-) + r^2\big] + \frac{1}{2}d^2x_-^3x_+;$$

$$R_b = -2(xx_+x_-)^2r^2q_1^2 - 2xx_+x_-^2dr^2 - x_-^3x_+d^2; \tag{B.4}$$

$$
\begin{aligned}
R_{WW}^{odd} = {} & (1 - \mathcal{P}_{+-}) \frac{x x_+ x_-}{d^4} \Bigg[\frac{1}{2} d^2 \big[-M^2 (1+x) x x_+ - x x_+ x_- \big(p_1' q_- \big) + x_+^2 \big(p_1' q_+ \big) \\
& - x[x_+ \bar{x}_+ + \bar{x}_-](\vec{q}_+ \vec{q}_-) - x x_+ x_- \vec{q}_+^2 - x_+^2 \vec{q}_-^2 - x_+^2 x_- (1+x) \big(p_1 p_1' \big) \\
& - x x_+ (1+x)(q_+ q_-) + (x x_+)^2 (p_1 q_-) - x x_+ x_- \big(p_1 q_+ \big) \big] \\
& + x_+ x_- d \vec{Q} \big[x x_+ x_- (p_1 q_+) \vec{q}_+ + \vec{q}_- \big[x x_+ \bar{x}_+ (p_1 q_+) + x x_+ \big(p_1' q_+ \big) \big] \big] \\
& - \frac{1}{2} x d \vec{r} \big[-\vec{q}_+ \big[M^2 x_+ x_- \bar{x} + x x_+ x_- \big(p_1' q_- \big) + x_+ x_- \bar{x}_- (p_1 q_-) + x_+ x_-^2 (p_1 q_+) \big] \\
& - \vec{q}_- \big[M^2 x_+ x_- \bar{x} + x x_+ x_- \big(p_1' q_+ \big) + x_+ x_- \bar{x}_+ (p_1 q_+) + x_- x_+^2 (p_1 q_-) \big] \big] \\
& - 2 (x_+ x_- x)^2 (\vec{Q} \vec{r}) \big[M^2 \big(p_1 p_1' \big) + (p_1 q_-) \big(p_1' q_+ \big) + (p_1 q_+) \big(p_1' q_- \big) \big] \Bigg].
\end{aligned}
\tag{B.5}
$$

Here we use the notation

$$
\big(p_1 p_1' \big) = \frac{1}{2x} Q^2, \qquad (p_1 q_+) = \frac{1}{2x_+} \big[M^2 + q_+^2 \big]; \qquad (p_1 q_-) = \frac{1}{2x_-} \big[M^2 + q_-^2 \big];
$$

$$
\big(p_1' q_+ \big) = \frac{1}{2 x x_+} \big[M^2 + r_+^2 \big]; \qquad \big(p_1' q_- \big) = \frac{1}{2 x x_+} \big[M^2 + r_-^2 \big];
$$

$$
(q_+ q_-) = \frac{1}{2 x_+ x_-} \big[M^2 \big(x_+^2 + x_-^2 \big) + r^2 \big]; \qquad r^2 = (x_- \vec{q}_+ - x_+ \vec{q}_-)^2;
$$

$$
q_1^2 = \frac{1}{x_+ x_-} \big[M^2 \bar{x}^2 + r^2 \big], \qquad q_2^2 = -\frac{1}{x} \big[m^2 \bar{x}^2 + Q^2 \big];
$$

$$
d = m^2 x_+ x_- \bar{x} + M^2 x \bar{x} + q_+^2 x_- \bar{x}_- + q_-^2 x_+ \bar{x}_+ + 2 x_+ x_- q_+ q_- \cos \phi;
$$

$$
r_-^2 = (\bar{x}_+ \vec{q}_- + x_- \vec{q}_+)^2; \qquad r_+^2 = (\bar{x}_- \vec{q}_+ + x_+ \vec{q}_-)^2; \qquad \big(p_1 p_1' \big) = \frac{1}{2x} Q^2,
$$

$$
(\vec{q}_+ \vec{q}_-) = q_+ q_- \cos \phi.
\tag{B.6}
$$

The differential distribution in the WW approximation is

$$
d\sigma^{ep \to (e Q \bar{Q}) p} = \frac{2 \alpha^4}{\pi} (L_q - 1) \Bigg[\frac{1}{(q_1^2)^2} R_{WW}^{br} + \frac{1}{(q_2^2)^2} R_{WW}^{2g} + \frac{2}{q_1^2 q_2^2} R_{WW}^{odd} \Bigg]
$$

$$
\times \frac{dx_+ dx_-}{x x_+ x_-} dq_+^2 dq_-^2 \frac{d\phi}{2\pi}.
\tag{B.7}
$$

We put as well the contribution to the differential cross section from two gamma mechanisms integrated over both virtual photon transfer momentum variables

$$
\frac{d\sigma_{2g}^{ep \to (e Q \bar{Q}) p}}{dx_+ dx_- dq_+^2} = \frac{2 \alpha^4}{\pi} \frac{L_q - 1}{x x_+ x_-} (1 + \mathcal{P}_{+-}) \frac{x^2}{d_0^4} \Bigg[\frac{2 x^2}{\bar{x}^2} (L_p - 1) R_{a0} + \frac{1}{2} x L_p R_{b0} \Bigg],
$$

$$
R_{a0} = (x x_+ x_-)^2 q_+^2 \bar{x}^2 \big[q_{10}^2 - q_+^2 \big] + \bar{x} x_+ x_- d_0 q_+^2 + \frac{1}{2} d_0^2 x_+ x_-^3;
$$

$$
R_{b0} = 2 (x x_+ x_- \bar{x})^2 q_+^2 q_{10}^2 + 2 (\bar{x} x_-)^2 x x_+ d_0 q_+^2 + d_0^2 x_+ x_-^3,
\tag{B.8}
$$

where

$$q_{10}^2 = \frac{\bar{x}^2}{x_+ x_-}\left[M^2 + \vec{q}_+^2\right]; \qquad d_0 = \bar{x}\left[m^2 x_+ x_- + x\left(M^2 + \vec{q}_+^2\right)\right];$$

$$L_p = \ln\frac{M^2}{m^2 \bar{x}^2}, \qquad L_q = \ln\frac{M^2 s^2}{M_Y^2 d_0^2}. \tag{B.9}$$

The restrictions

$$x_+ + x_- = \bar{x} = 1 - x, \qquad \frac{2M}{\sqrt{s}} < x_\pm < 1 - \frac{2M}{\sqrt{s}},$$

$$\frac{\bar{x}^2}{x_+ x_-} > \frac{4M^2}{M^2 + \vec{q}_+^2}, \tag{B.10}$$

are implied.

The distribution as a function of the two gluon invariant mass square was considered in [25].

Appendix C. Distributions in the WW approximation, two photon and two gluon emission

Differential distribution for the process $eY \to (e\gamma\gamma)Y$ in the WW approximation is

$$d\sigma_{WW}^{ep\to(e\gamma\gamma)p} = \frac{2\alpha^4}{\pi}(L_q - 1)(1 + \mathcal{P}_{12})[R_{pl} + R_{npl}]dk_1^2 dk_2^2 \frac{d\phi}{2\pi};$$

$$d\sigma_{WW}^{qp\to(qgg)p} = \frac{\alpha^2\alpha_s^2}{\pi}\frac{N^2-1}{N^3}(L_q-1)\Bigg[(1+\mathcal{P}_{12})[R_{pl}+R_{npl}]\frac{N^2-2}{2} +$$

$$+ \frac{N^2}{2}\Big[(1+\mathcal{P}_{12})(R_{pl}-R_{npl}) + 4(R_{33}+R_{321}-R_{312})\Big]\Bigg]dk_1^2 dk_2^2 \frac{d\phi}{2\pi}. \tag{C.1}$$

The expressions for R_i are

$$R_{pl} = \frac{1}{\bar{x}_1^2}I_{pl}; \qquad R_{npl} = \frac{1}{\bar{x}_1\bar{x}_2}I_{npl};$$

$$R_{33} = I_{33}; \qquad R_{321} = -\frac{1}{\bar{x}_2}I_{321}; \qquad R_{312} = -\frac{1}{\bar{x}_1}I_{312}, \tag{C.2}$$

and

$$I_{pl} = \frac{1}{4s^2}Tr\not{p}_1' O_{12}\not{p}_1 O_{12}^+;$$

$$I_{npl} = \frac{1}{4s^2}Tr\not{p}_1' O_{12}\not{p}_1 O_{21}^+;$$

$$I_{33} = \frac{1}{4s^2}Tr\not{p}_1' O_3\not{p}_1 O_3^+;$$

$$I_{321} = \frac{1}{4s^2}Tr\not{p}_1' O_3\not{p}_1 O_{21}^+;$$

$$I_{312} = \frac{1}{4s^2}Tr\not{p}_1' O_3\not{p}_1 O_{12}^+. \tag{C.3}$$

The simplified expressions for effective vertices are

$$
\begin{aligned}
O_{12} =\ & R_1 \not{e}_2 (\not{p}'_1 + \not{k}_2) \not{p}_2 (\not{p}_1 - \not{k}_1) \not{e}_1 + r \not{e}_2 \not{p}_2 \not{e}_1 \\
& + c_1 \not{e}_2 (\not{p}'_1 + \not{k}_2)[\not{k}_1 \not{e}_1 \not{q} \not{p}_2 + \not{p}_2 \not{q} \not{e}_1] \\
& + d_1 [\bar{x}_1 \not{p}_2 \not{q} \not{e}_2 + x \not{e}_2 \not{q} \not{p}_2](\not{p}_1 - \not{k}_1) \not{e}_1,
\end{aligned}
\tag{C.4}
$$

$$
\begin{aligned}
O_{21} =\ & R_2 \not{e}_1 (\not{p}'_1 + \not{k}_1) \not{p}_2 (\not{p}_1 - \not{k}_2) \not{e}_2 + r \not{e}_1 \not{p}_2 \not{e}_2 \\
& + c_2 \not{e}_1 (\not{p}'_1 + \not{k}_1)[\bar{x}_2 \not{e}_2 \not{q} \not{p}_2 + \not{p}_2 \not{q} \not{e}_2] \\
& + d_2 [\bar{x}_2 \not{p}_2 \not{q} \not{e}_1 + x \not{e}_1 \not{q} \not{p}_2](\not{p}_1 - \not{k}_2) \not{e}_2,
\end{aligned}
\tag{C.5}
$$

$$
O_3 = -\frac{2}{q_1^2} \frac{x_1 x_2}{d^2} \big[x s \rho \not{V} + d' \not{p}_2 \not{q} \not{V} + x d \not{V} \not{q} \not{p}_2 \big],
$$

$$
V = \not{e}_1 (k_1 e_2) + \not{k}_2 (e_1 e_2) - \not{e}_2 (k_2 e_1),
\tag{C.6}
$$

where

$$
R_1 = \frac{2(xx_1^2 x_2^2)^2}{k_2^2 r_2^2 d^2} \vec{q} \big[d\vec{r}_2 - xx_2 k_1^2 \vec{Q} \big]; \qquad
R_2 = \frac{2(xx_1^2 x_2^2)^2}{k_2^2 r_1^2 d^2} \vec{q} \big[d\vec{r}_1 - xx_1 k_2^2 \vec{Q} \big];
$$

$$
r = \frac{2xx_1^2 x_2^2}{d^2} \vec{q} \vec{Q}, \quad d = m^2 \bar{x} x_1 x_2 + k_1^2 x_2 \bar{x}_2 + k_2^2 x_1 \bar{x}_1 + 2x_1 x_2 k_{12},
$$

$$
\rho = 2x_1 x_2 \vec{q} \vec{Q},
\tag{C.7}
$$

where we use the notation $k_i^2 = \vec{k}_i^2$, $k_{12} = \vec{k}_1 \vec{k}_2$,

$$
\begin{aligned}
\vec{r} &= x_2 \vec{k}_1 - x_1 \vec{k}_2, & \vec{Q} &= \vec{k}_1 + \vec{k}_2; \\
\vec{r}_1 &= \bar{x}_2 \vec{k}_1 + x_1 \vec{k}_2, & \vec{r}_2 &= \bar{x}_1 \vec{k}_2 + x_2 \vec{k}_1,
\end{aligned}
\tag{C.8}
$$

and use, besides,

$$
\begin{aligned}
c_1 &= \frac{x_1 (xx_2)^2}{r_2^2 d}, & d_1 &= -\frac{x_2 x_1^2}{k_1^2 d}, \\
c_2 &= \frac{x_2 (xx_1)^2}{r_1^2 d}, & d_2 &= -\frac{x_1 x_2^2}{k_2^2 d}, \\
(p_1 p'_1) &= \frac{1}{2x} \vec{Q}^2, & (p_1 k_1) &= \frac{1}{x_1} k_1^2, \quad (p_1 k_2) = \frac{1}{x_2} k_2^2, \\
(p'_1 k_1) &= \frac{1}{xx_1} r_1^2, & (p'_1 k_2) &= \frac{1}{xx_2} r_2^2, \quad (k_1 k_2) = \frac{1}{2x_1 x_2} r^2.
\end{aligned}
\tag{C.9}
$$

Appendix D. Transfer of circular polarization of the initial electron to the positron in the fragmentation region

This phenomenon is similar to "handedness" when the initial polarized particle causes the polarization of the fermionic fragments of the jet created by this projectile, or reveals itself in kinematical correlations of momenta of different pions from the jet.

The matrix element of the process $e(p_1, \lambda)\bar{e}(p_2) \to [e(p'_1)e(q_-)\bar{e}(q_+, \lambda_1)]\bar{e}(p'_2)$ has the form

$$
\frac{4\pi\alpha^2}{q_1^2 q^2} \frac{1}{2} \bar{u}(p'_1) \gamma_\mu (1 + \lambda\gamma_5) u(p_1) \cdot \frac{1}{2} \bar{u}(q_-) O_{\mu\nu} (1 - \gamma_5) v(q_+) \cdot \bar{u}(p'_2) \gamma_\nu u(p_2).
\tag{D.1}
$$

For the summed on the spin states square of the matrix element we have

$$
\sum |M|^2 = (4\pi\alpha)^4 8 \left\{ \frac{1}{4} Tr(\slashed{q}_- + m) O_\mu (\slashed{q}_+ + m) O_\nu^* \cdot \frac{\vec{q}_1^2 g_{\mu\nu}}{1 - \beta_1} \left(\frac{1 - \beta_1}{\beta_1^2} + \frac{1}{2} \right) \right.
$$
$$
\left. + \lambda \frac{1}{4} Tr \slashed{q}_- Q_\mu \slashed{q}_+ Q_\nu^+ \gamma_5 \cdot \frac{1}{4} Tr \slashed{p}_1 \slashed{q}_1 \gamma_\mu \gamma_\nu \gamma_5 \right\} \frac{q_1^2 q^2 (1 - \beta_1)^2}{(\vec{q}_1^2 + \beta_1^2 m^2)^2 (\vec{q}^2 + m^2 \alpha_2^2)^2},
$$
(D.2)

with

$$
Q_\mu = \frac{1}{s_1^2 x_+ x_-} [2\vec{q}\vec{r}\gamma_\mu - s_1 x_- \gamma_\mu \slashed{q} \slashed{p}_2 + s_1 x_+ \slashed{p}_2 \slashed{q} \gamma_\mu], \quad s_1 = \frac{\bar{x}}{x_+ x_-} [\vec{q}_-^2 + am^2],
$$
(D.3)

with $a = \frac{\bar{x}_+ \bar{x}_-}{x}$, $\bar{x} = 1 - x = x_+ + x_-$.

Averaging over the azimuthal angle $d^2\vec{q}$ permits one to extract the general factor $s^2\vec{q}^2$, which will be absorbed in the total expression for spectral distributions on the energy fractions of fermions in a jet. Using the expression for phase volume in the fragmentation region

$$
d\Gamma_4 = \frac{d^3 p_1' d^3 p_2' d^3 q_+ d^3 q_-}{2\varepsilon_1' 2\varepsilon_2' 2\varepsilon_+ 2\varepsilon_-} \frac{(2\pi)^4}{(2\pi)^2} \delta^4(p_1 + p_2 - p_1' - p_2' - q_+ - q_-)
$$
$$
= \pi^3 (2\pi)^{-8} \frac{dx_- d\beta_1}{8sx x_+ x_-} \frac{d^2 q}{\pi} \frac{d^2 q_1}{\pi} \frac{d^2 q_-}{\pi},
$$
(D.4)

we first extract the leading logarithmic factor L_1 and L_q with

$$
L_1 = \int_0^s d\vec{q}_1^2 \vec{q}_1^2 / (\vec{q}_1^2 + m_e^2 \beta_1^2)^2, \qquad L_q = \int_0^s d\vec{q}^2 \vec{q}^2 / (\vec{q}^2 + m_e^2 \alpha_2^2)^2.
$$

With the logarithmic accuracy we have

$$
L_1 = L_q = L = \ln \frac{s}{m_e^2}.
$$
(D.5)

For the unpolarized part of the cross section we obtain

$$
d\sigma_{unp} = \frac{\alpha^4}{2\pi} L^2 \frac{d\vec{q}_-^2}{s_1^4 (x_+ x_-)^2} \frac{dx_- dx(1 + x^2)}{2\bar{x}^2} \left[x_+ x_- (x_+^2 + x_-^2) s_1^2 \right.
$$
$$
\left. - 2\bar{x}^3 s_1^2 \vec{q}_-^2 + 2 \frac{\bar{x}^2}{x_+ x_-} \vec{q}_-^2 [\bar{x}^2 \vec{q}^2 + m^2 (\bar{x}^2 + 2x_+ x_-)] \right].
$$
(D.6)

Integration over $d\vec{q}_-^2$ leads to

$$
d\sigma_{unp} = \frac{\alpha^4}{2\pi m_e^2} L^2 \frac{x dx_- dx(1 + x^2)}{\bar{x}^4 \bar{x}_+ \bar{x}_-} \left[-2x_+ x_- + \frac{2}{3} \bar{x}^2 + \frac{x}{3\bar{x}_+ \bar{x}_-} [\bar{x}^2 + 2x_+ x_-] \right].
$$
(D.7)

Here we imply the threshold restriction $(4m_e^2/s < x_+ + x_-)$.

Consider now the contribution to the cross section associated with the polarized part of the matrix element. Performing the extraction of factor $\vec{q}_1^2$ and the relevant azimuthal averaging procedure we must do a shift transformation $\vec{q}_- = \tilde{\vec{q}}_- + (x_-/\bar{x})\vec{q}_1$ and $\vec{q}_+ = -\tilde{\vec{q}}_- + (x_-/\bar{x})\vec{q}_1$.

Table 5
The function $F(x_-, x)$ (defined in (D.10)) is presented for different values of the final lepton transverse momenta x_+ and x_- for the lepton pair production (in units of M).

$x_-\backslash x_+$	0.25	0.3	0.35	0.4	0.45	0.5
0.25		0.853				
0.3	−0.853		0.705			
0.35		−0.705		0.6315		
0.4			−0.6315		0.6308	
0.45				−0.6303		0.786
0.5					−0.786	

In terms of the shifted variables the quadratic form s_1 is $s_1 = (\bar{x}/(x_+x_-))[\vec{\tilde{q}}_-^2 + am^2]$. In a similar way we obtain

$$d\sigma_{pol} = \frac{\alpha^4}{2\pi}L^2\frac{d\vec{q}_-^2}{s_1^4(x_+x_-)^3}xdx_-dx\lambda(x_+ - x_-)$$

$$\times\left[-x_+x_-s_1^2 + \frac{1}{\bar{x}}\left(2\bar{x}^2 - x_+x_-\right)s_1\vec{q}_-^2 - \frac{2(x_+ - x_-)^2}{x_+x_-}(\vec{q}_-^2)^2\right]. \quad (D.8)$$

We note that compared with the unpolarized case the terms proportional to the electron mass squared do not contribute. Further integration leads to the final result

$$d\sigma_{pol} = \lambda\frac{\alpha^4}{12\pi m_e^2}L^2\frac{x^2dx_-dx}{\bar{x}^4\bar{x}_+\bar{x}_-x_+x_-}(x_+ - x_-)\left[4\bar{x}^2 - 5x_+x_-\right]. \quad (D.9)$$

The degree of polarization transferred from the initial electron to the final positron can be found as (see Table 5)

$$\frac{\langle\lambda\rangle_{positron}}{\lambda} = \frac{d\sigma_{pol}}{d\sigma_{unp}} = F(x_-, x),$$

$$F(x_-, x) = (x_+ - x_-)\frac{4\bar{x}^2 - 5x_+x_-}{(x^2 + 1)x_+x_-[-6x_+x_- + 2\bar{x}^2 + \frac{x}{\bar{x}_+\bar{x}_-}(\bar{x}^2 + 2x_+x_-)]}. \quad (D.10)$$

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