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ORIGINAL ARTICLE

Solution of the boundary layer flow of an Eyring-Powell non-Newtonian fluid over a linear stretching sheet by collocation method

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KEYWORDS

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Abstract The aim of this article was to apply collocation method for boundary layer flow of an Eyring-Powell fluid over a stretching sheet in unbounded domain. The collocation method combined with a special technique, has been successfully applied for nonlinear equations of momentum with infinite boundary values. The solution for velocity is computed by applying the collocation method. The governing nonlinear differential equations are reduced to the ordinary differential equations by similarity transformations. The physical significance of different parameters on the velocity profile is discussed through graphical illustrations. It is noticed that the velocity increases by increasing the Eyring-Powell fluid material parameter (ϵ) whereas it decreases by increasing the fluid material parameter (δ).

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1. Introduction

The study of boundary layer flow over a stretching sheet has generated much interest in recent years in view of its numerous industrial applications such as the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also polymer industries. Further such flows have promising applications in the extrusion of a polymer sheet from a die or in the drawing of plastic films. During the manufacture of

these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The mechanical properties of the final product strictly depend on the stretching and cooling rates in the process. Crane [1] provided the classical solution for the boundary layer flow of viscous fluid over a sheet moving with velocity varying linearly with distance from a fixed point. Mukhopadhyay [2] investigated the slip effects on MHD boundary layer flow by an exponentially stretching sheet with suction/blowing and thermal radiation. Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet have been investigated by Bhat-tacharyya et al. [3]. Exact solutions for two-dimensional laminar flow over a continuously stretching or shrinking sheet in an electrically conducting quiescent couple stress fluid have been derived by Turkyilmazoglu [4]. Alsaedi et al. [5] discussed the

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Nomenclature

| | | | |
|------------------------------------|---|-----------------------------|---------------------------------|
| τ_{ij} (kg/m s ²) | shear-stress at the sheet | ρ (kg/m ³) | fluid density |
| μ (kg/m s) | dynamic viscosity of the fluid | U_ω (m/s) | stretching velocity |
| β | material parameter | ψ | stream function |
| C | material parameter | $f(\eta)$ | dimensionless stream function |
| u, v (m/s) | non-dimensional velocity components along x - and y -axes | η | similarity variable |
| x, y (m) | non-dimensional Cartesian coordinates | ε | non-dimensional fluid parameter |
| ν (m ² /s) | kinematic viscosity of the fluid | δ | non-dimensional fluid parameter |

effects of heat generation/absorption on stagnation point flow of nanofluid over a surface with convective boundary conditions. Ibrahim et al. [6] analyzed the MHD stagnation point flow of nanofluid toward a stretching sheet. Effect of slip on unsteady stagnation point flow of nanofluid over a stretching sheet was investigated by Malvandi et al. [7]. The mathematical formulation for flows of non-Newtonian fluids in general is complex. The most frequently used models of non-Newtonian models are the second grade, Maxwell, Oldroyd-B and power law. A broad description of the behavior in both steady and unsteady flow situations, together with mathematical models can be found in Refs. [8–14]. The Powell-Eyring model has certain advantages over the other non-Newtonian fluid models. Firstly, it is deduced from kinetic theory of liquids rather than the empirical relation. Secondly, it correctly reduces to Newtonian behavior for low and high shear rates. Patel and Timol [15] numerically examined the flow of Powell-Eyring model through asymptotic boundary conditions. Hayat et al. [16] studied the steady flow of an Powell-Eyring fluid over a moving surface with convective boundary conditions. Flow and heat transfer of Powell-Eyring fluid over shrinking surface in a parallel free stream is presented by Rosca and Pop [17]. Jalil et al. [18] studied the flow and heat transfer of Powell-Eyring fluid over a moving surface in a parallel free stream. Nonlinear problems play important roles in fluid mechanics and heat transfer. Except for a limited number of these problems, most of them do not have precise analytical solutions; therefore, these nonlinear equations should be solved using approximate analytical solutions, such as the Adomian's decomposition method [19,20], Difference transformation method [21,22], Homotopy perturbation method (HPM) [23,24], using numerical method (BVP) [25,26], and Control Volume based Finite Element Method (CVFEM) [27–36].

2. Mathematical model

We consider a steady, laminar, two-dimensional flow of an incompressible, non-Newtonian Eyring Powell fluid over a linear stretching sheet. It is assumed that the sheet is being stretched with a linear velocity $U_\omega x$, where $U_\omega = b$ is the linear stretching velocity; x is the distance from the slit. The theory of rate processes is used to derive the Eyring-Powell model (1994) for describing the shear of non-Newtonian flow. The shear tensor in an Eyring Powell model is given by Hayat et al. [37] (see Fig. 1).

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1} \left(\frac{1}{C} \frac{\partial u_i}{\partial x_j} \right) \quad (1)$$

where μ is the dynamic viscosity, β and C are the fluid parameters of the Eyring-Powell model. We take the second order approximation of function as [37]:

$$\sinh^{-1} \left(\frac{1}{C} \frac{\partial u_i}{\partial x_j} \right) \cong \frac{1}{C} \frac{\partial u_i}{\partial x_j} - \frac{1}{6} \left(\frac{1}{C} \frac{\partial u_i}{\partial x_j} \right)^3, \quad \left| \frac{1}{C} \frac{\partial u_i}{\partial x_j} \right| \ll 1 \quad (2)$$

For an incompressible fluid obeying Eyring-Powell model, the equation of continuity and the x -momentum equation can be simplified using the boundary layer approximation as [37]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho \beta C} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2 \rho \beta C^3} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} \quad (4)$$

where u and v are the velocity components along x and y -directions, respectively, $\nu = \mu/\rho$ is the kinematic viscosity and ρ is the fluid density. The boundary conditions set for Eqs. (3) and (4) are as follows:

$$\begin{cases} u = U_\omega x = bx, & v = 0 & \text{at } y = 0 \\ u \rightarrow 0 & \text{as } y \rightarrow \infty \end{cases} \quad (5)$$

We proceed for solutions through stream function satisfying the following:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

And

$$\psi = (bv)^{\frac{1}{2}} x f(\eta), \quad \eta = \left(\frac{b}{\nu} \right)^{\frac{1}{2}} y \quad (7)$$

Here, $f(\eta)$ is the dimensionless stream function and η is the similarity variable. Now Eq. (3) is clearly satisfied and Eqs. (4)–(7) give

$$(1 + \varepsilon) f''' - \varepsilon \delta f''^2 f''' - f'^2 + f f'' = 0 \quad (8)$$

where ε and δ are the material fluid parameters. These quantities have following definitions as follows:

$$\varepsilon = \frac{1}{\mu \beta C}, \quad \delta = \frac{b^3 x^2}{2 C^2 \nu} \quad (9)$$

The boundary conditions set Eq. (5) become the following:

$$\begin{cases} f(\eta) = 0, & f'(\eta) = 1 & \text{at } \eta = 0 \\ f'(\eta) = 0 & \text{as } \eta \rightarrow \infty \end{cases} \quad (10)$$

Therefore, we have to solve Eq. (8) along with the boundary conditions of Eq. (10).

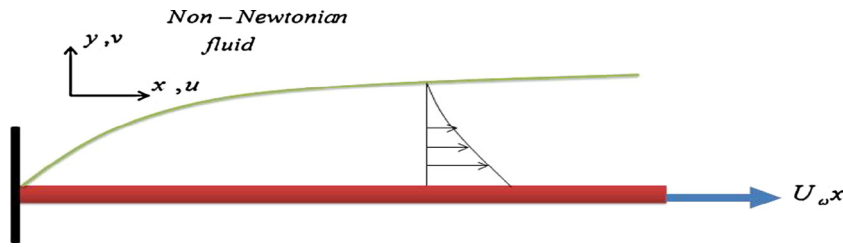


Figure 1 Velocity boundary layers on a stretching sheet.

3. Basic idea of collocation method

In collocation method, a trial family of approximate solution T , containing a finite number of undetermined coefficient c_1, c_2, \dots and c_n can be constructed by the superposition of some basic functions such as polynomials, trigonometric functions, and the trial solution is so selected that it satisfies the essential boundary conditions for the problem. Therefore the problem of constructing an approximate solution becomes one of determining the unknown coefficients c_1, c_2, \dots and c_n . So the residual stays close to zero throughout the domain of the solution.

3.1. Application

For some sufficiently large M , the condition $\eta = \infty$ in Eq. (10) can be replaced by the condition:

$$f'(M) = 0 \tag{11}$$

Under the transformation $z = \frac{\eta}{M}$, equation of momentum transformed to the following:

$$(1 + \varepsilon)M^2 g'''(z) - \varepsilon \delta (g''(z))^2 (g'''(z)) - M^4 (g'(z))^2 + M^4 g(z) (g''(z)) = 0 \tag{12}$$

where $g(z) = \frac{f(\eta)}{M}$, and the ‘‘prime’’ denotes the derivatives with respect to $z \in [0, 1]$. The boundary conditions (10) are transformed to the following:

$$\begin{cases} g(z) = 0, & g'(z) = 1 \text{ at } z = 0 \\ g'(z) = 0 \text{ at } z = 1 \end{cases} \tag{13}$$

We wish to obtain an approximate solution for this problem in the interval $0 < z < 1$. To construct a trial solution $g(z)$, we choose the basic function to polynomial in z . The trial solution contains seven undetermined coefficients and satisfies the condition for all values of c as follows:

| Table 1 The results of coefficients for C. | |
|--|---------------|
| C_0 | 0.1239326461 |
| C_1 | -0.7559681898 |
| C_2 | 2.160377031 |
| C_3 | -3.919879135 |
| C_4 | 5.157155263 |
| C_5 | -5.250603287 |
| C_6 | 4.185825100 |

$$g(z) = c_0 z^9 + c_1 z^8 + c_2 z^7 + c_3 z^6 + c_4 z^5 + c_5 z^4 + c_6 z^3 - (4.5c_0 + 4c_1 + 3.5c_2 + 3c_3 + 2.5c_4 + 2c_5 + 1.5c_6 + 0.5)z^2 + z \tag{14}$$

When $g(z)$ are introduced into differential equations it yields residual $R(c, z)$ as follows:

$$\begin{aligned} R(c_0, c_1, c_2, \dots, z) = & (1 + \varepsilon)M^2 (504c_0 z^6 + 336c_1 z^5 + 210c_2 z^4 \\ & + 120c_3 z^3 + 60c_4 z^2 + 24c_5 z + 6c_6 \\ & - \varepsilon \delta (72c_0 z^7 + 56c_1 z^6 + 42c_2 z^5 \\ & + 30c_3 z^4 + 20c_4 z^3 + 12c_5 z^2 + 6c_6 z \\ & - 9c_0 - 8c_1 - 7c_2 - 6c_3 - 5c_4 \\ & - 4c_5 - 3c_6 - 1)^2 (504c_0 z^6 + 336c_1 z^5 \\ & + 210c_2 z^4 + 120c_3 z^3 + 60c_4 z^2 \\ & + 24c_5 z + 6c_6) - M^4 (9c_0 z^8 + 8c_1 z^7 \\ & + 7c_2 z^6 + 6c_3 z^5 + 5c_4 z^4 + 4c_5 z^3 + 3c_6 z^2 \\ & - 2(4.5c_0 + 4c_1 + 3.5c_2 + 3c_3 + 2.5c_4 \\ & + 2c_5 + 1.5c_6 + 0.5)z + 1)^2 \\ & + M^4 (c_0 z^9 + c_1 z^8 + c_2 z^7 + c_3 z^6 + c_4 z^5 \\ & + c_5 z^4 + c_6 z^3 - (4.5c_0 + 4c_1 + 3.5c_2 \\ & + 3c_3 + 2.5c_4 + 2c_5 + 1.5c_6 \\ & + 0.5)z^2 + z) (72c_0 z^7 + 56c_1 z^6 + 42c_2 z^5 \\ & + 30c_3 z^4 + 20c_4 z^3 + 12c_5 z^2 + 6c_6 z \\ & - 9c_0 - 8c_1 - 7c_2 - 6c_3 \\ & - 5c_4 - 4c_5 - 3c_6 - 1) \end{aligned} \tag{15}$$

Now finding approximate solution for the problem in the interval $0 < z < 1$ becomes one adjusting the values of c_0, c_1, \dots . So that residual stays close to zero throughout the interval $0 < z < 1$. The basic assumption is that the residual does not deviate much from zero between collocation locations:

$$R\left(\frac{1}{8}\right) = 0, R\left(\frac{2}{8}\right) = 0, R\left(\frac{3}{8}\right) = 0, R\left(\frac{4}{8}\right) = 0, \dots \tag{16}$$

By using Ref. [26], we consider $M = 5.69$. According to the M , we can obtain coefficients. Results are shown in Table 1. After substituting coefficients into Eq. (14) we have the following:

$$g(z) = 0.1239326461z^9 - 0.7559681898z^8 + 2.160377031z^7 - 3.919879135z^6 + 5.157155263z^5 - 5.250603287z^4 + 4.185825100z^3 - 2.505925596z^2 + z \tag{17}$$

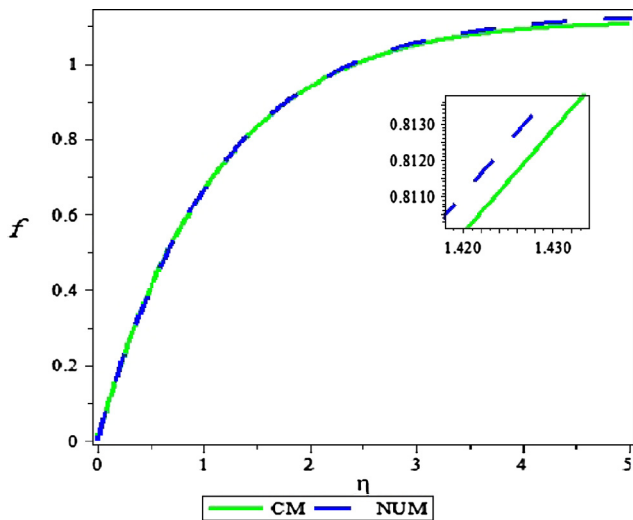


Figure 2 The comparison of answers obtained by Collocation method and numerical method for $f(\eta)$ when $\varepsilon = 0.3, \delta = 0.1$.

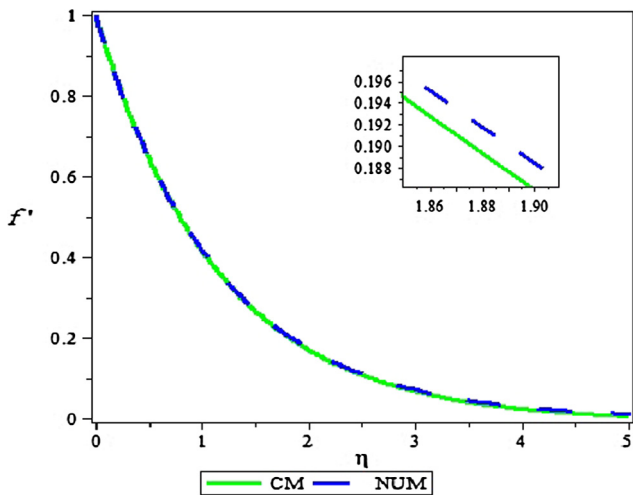


Figure 3 The comparison of answers obtained by Collocation method and numerical method for $f'(\eta)$ when $\varepsilon = 0.3, \delta = 0.1$.

Then, under the transformation $g(z) = \frac{f(\eta)}{5.69}$ we get the following:

$$\begin{aligned}
 f(\eta) = & 1.127944136 \times 10^{-7} \eta^9 - 0.3914872802 \times 10^{-5} \eta^8 \\
 & + 0.6365843423 \times 10^{-4} \eta^7 - 0.6572208667 \\
 & \times 10^{-3} \eta^6 + 0.4919955116 \times 10^{-2} \eta^5 \\
 & - 0.2850180777 \times 10^{-1} \eta^4 + 0.1292875022 \eta^3 \\
 & - 0.4404087162 \eta^2 + 0.9999999998 \eta
 \end{aligned} \tag{18}$$

4. Description of the fourth-order Runge–Kutta method (BVP)

Runge–Kutta 4th order method is a numerical technique applied in solving differential equations of the form:

$$\frac{dy}{dx} = f(x, y), y(0) = y_0 \tag{19}$$

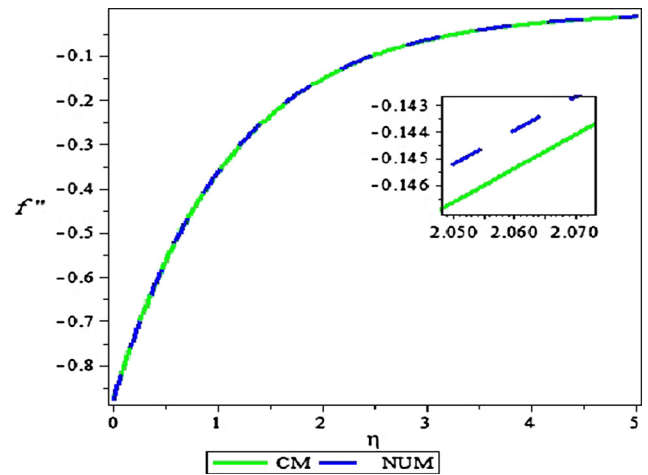


Figure 4 The comparison of answers obtained by Collocation method and numerical method for $f''(\eta)$ when $\varepsilon = 0.3, \delta = 0.1$.

Table 2 Obtained results, in comparison with numerical method for $f'(\eta)$.

| η | NUM | CM | Error of CM |
|--------|--------|----------|-------------|
| 0 | 1 | 0.846224 | 0 |
| 0.2 | 0.846 | 0.716626 | 5.25E-04 |
| 0.4 | 0.716 | 0.607213 | 5.40E-04 |
| 0.6 | 0.607 | 0.514687 | 4.15E-04 |
| 0.8 | 0.514 | 0.436322 | 2.85E-04 |
| 1 | 0.436 | 0.36986 | 1.84E-04 |
| 1.2 | 0.370 | 0.313426 | 1.12E-04 |
| 1.4 | 0.313 | 0.265463 | 5.92E-05 |
| 1.6 | 0.265 | 0.224669 | 1.64E-05 |
| 1.8 | 0.225 | 0.18995 | 2.08E-05 |
| 2 | 0.190 | 0.16039 | 5.36E-05 |
| 2.2 | 0.160 | 0.135211 | 8.20E-05 |
| 2.4 | 0.135 | 0.113756 | 1.06E-04 |
| 2.6 | 0.114 | 0.095469 | 1.25E-04 |
| 2.8 | 0.0956 | 0.079877 | 1.40E-04 |
| 3 | 0.0800 | 0.066577 | 1.54E-04 |
| 3.2 | 0.0667 | 0.055229 | 1.66E-04 |
| 3.4 | 0.0554 | 0.045544 | 1.77E-04 |
| 3.6 | 0.0457 | 0.037277 | 1.87E-04 |
| 3.8 | 0.0375 | 0.030219 | 1.96E-04 |
| 4 | 0.0304 | 0.024193 | 2.03E-04 |
| 4.2 | 0.0244 | 0.019046 | 2.07E-04 |
| 4.4 | 0.0193 | 0.014647 | 2.11E-04 |
| 4.6 | 0.0149 | 0.010882 | 2.16E-04 |
| 4.8 | 0.0111 | 0.00766 | 2.25E-04 |
| 5 | 0.0790 | 0.846224 | 2.38E-04 |

It gives y_{i+1} in the form $y_{i+1} = y_i + (a_1k_1 + a_2k_2 + a_3k_3 + a_4k_4)h$ where constants are easily obtained as follows:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{20}$$

and we also have the following:

$$k_1 = f(x_i, y_i) \tag{21}$$

$$k_2 = f\left(x_i + \frac{1}{2}, y_i + \frac{1}{2}k_1h\right) \tag{22}$$

$$k_3 = f\left(x_i + \frac{1}{2}, y_i + \frac{1}{2}k_2h\right) \tag{23}$$

$$k_4 = f(x_i + h, y_i + k_3h) \tag{24}$$

These are also derived from the first four terms of Taylor series as below:

$$y_{i+1} = y_i + f(x_i, y_i)h + \frac{1}{2!}f'(x_i, y_i)h^2 + \frac{1}{3!}f''(x_i, y_i)h^3 + \frac{1}{4!}f'''(x_i, y_i)h^4 \tag{25}$$

5. Results and discussion

In this manuscript the collocation method is employed for boundary layer equations. Figs. 2-4 show the profiles of $f(\eta)$, $f'(\eta)$ and $f''(\eta)$ obtained by the collocation method for different values of η . In comparison with the numerical solution we can see a very good agreement between the collocation and numerical results. Numerical comparison between collocation method and numerical solution and absolute error $f'(\eta)$ is tabulated in Table 2. The effect of ε on the non-dimensional velocity field f' is presented in Fig. 5. It is noticed that when we increase the value of ε , f' increases. Fig. 6 depicts the influence of parameter δ on f' . Here, the non-dimensional velocity field f' is a decreasing function of δ . Figs. 7 and 8 show the effect of ε and δ on the stress profiles $f''(\eta)$. According to figures when

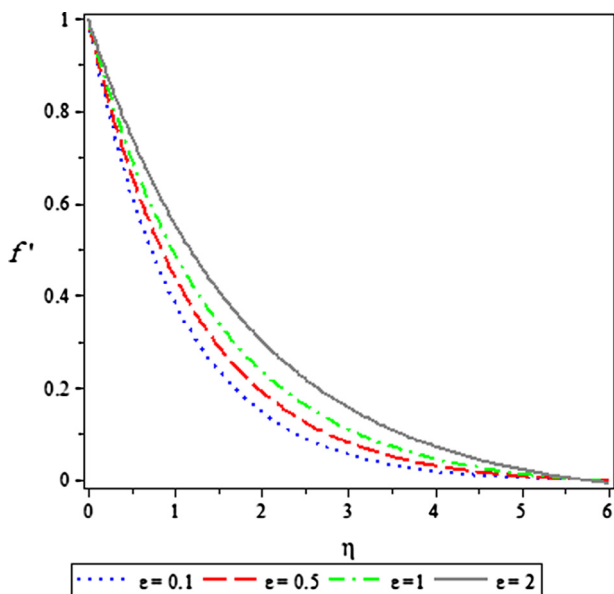


Figure 5 Effect of ε on $f'(\eta)$: $\delta = 0.2$.

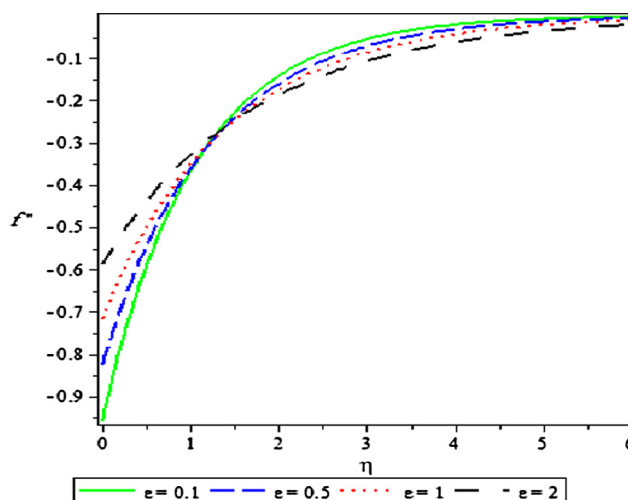


Figure 7 Effect of ε on $f''(\eta)$: $\delta = 0.2$.

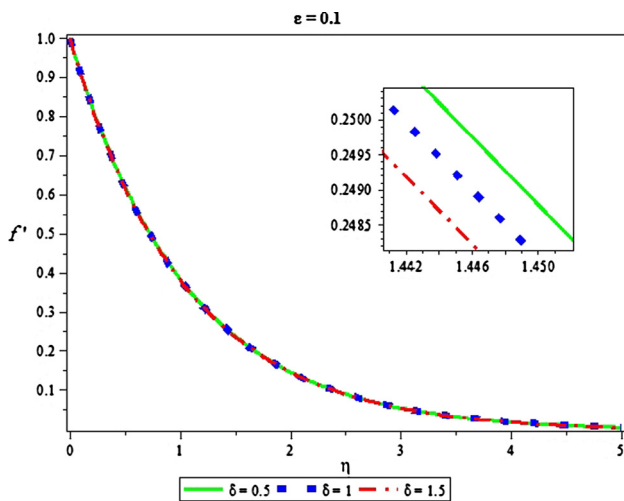


Figure 6 Effect of δ on $f'(\eta)$: $\varepsilon = 0.1$.

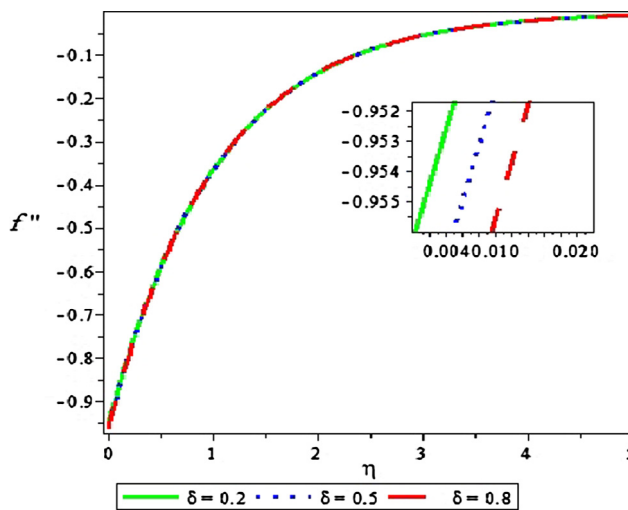


Figure 8 Effect of δ on $f''(\eta)$: $\varepsilon = 0.1$.

we increase the value of ε , f' decreases and with increasing δ , f'' increases.

6. Conclusions

In this paper, the collocation method combined with a special technique has been successfully applied for nonlinear equations of momentum with infinite boundary values. The effects of non-Newtonian fluid parameter investigated the velocity and stress profiles of a non-Newtonian Eyring-Powell fluid over a linear stretching sheet. Effects of Powell-Eyring fluid parameters on velocity and stress profiles are quite opposite. It should be noted that the approach used in the present work has a general meaning and therefore can be used widely when applying collocation method to different nonlinear problems in unbounded domain.

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