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The width of Θ^+ for large N_c in chiral quark soliton model

Michał Praszałowicz [1]

Nuclear Theory Group, Brookhaven National Laboratory, Upton, NY 19973-5000, USA

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Abstract

In the chiral quark soliton model the smallness of Θ^+ width is due to the cancellation of the coupling constants which are of different order in N_c. We show that taking properly into account the flavor structure of relevant SU(3) representations for arbitrary number of colors enhances the nonleading term by an additional factor of N_c, making the cancellation consistent with the N_c counting. Moreover, we show that, for the same reason, Θ^+ width is suppressed by a group-theoretical factor $\mathcal{O}(1/N_c)$ with respect to Δ and discuss the N_c dependence of the phase space factors for these two decays.

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1. Introduction

Recently five experiments announced discovery of a narrow, exotic baryonic state called Θ^+ [1]. Most probably this state belongs to the positive parity baryon antidecuplet, which naturally emerges in chiral soliton models [2–4]. Early prediction of its mass [3] obtained in the Skyrme model [5] extended to three flavors [6] is in surprising agreement with the present experimental findings. Moreover, if the discovery of the heaviest members of the antidecuplet, $\Xi_{3/2}$, announced by NA49 Collaboration [7] at 1860 MeV is confirmed, again the same model will be off only by 70 MeV [3].

However, the most striking experimental result is perhaps a very small width Γ_{Θ^+} which is estimated to be of the order of a few tens MeV or less [1]. Such a narrow width was predicted in a seminal paper by Diakonov, Petrov and Polyakov [4] within the SU(3) chiral quark soliton model (χQSM). The smallness of Γ_{Θ^+} in this model is due to the cancellation of the coupling constants which enter the collective decay operator. It is, however, at first sight to some extent unnatural that the cancellation occurs between the constants which are of different order in the number of colors, N_c. If only the leading term were retained, the width of Θ^+ would be of the same order as Γ_Δ [4,8].

In this Letter we show that in fact this cancellation occurs order by order in N_c. This is due to the fact that the additional factor of N_c appears when one properly takes into account the SU(3) flavor representation content of the lowest lying baryonic states. Indeed, for arbitrary number of colors, baryons do not fall into

E-mail address: michal@quark.phy.bnl.gov (M. Praszałowicz).

[1] Fulbright Fellow on leave from M. Smoluchowski Institute of Physics, Jagellonian University, Kraków, Poland.

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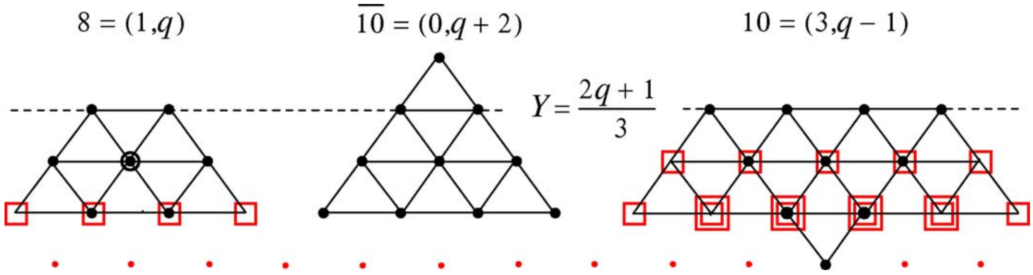

Fig. 1. Generalizations of octet, antidecuplet and decuplet for large N_c. Bold dots and circles denote physical states, squares and dots at the bottom denote spurious states which disappear in the limit $q \to 1$.

ordinary flavor octet, decuplet and antidecuplet, but are members of large SU(3) representations [9–11], which reduce to the standard ones only for $N_c = 3$. Taking this extra N_c dependence into account makes the SU(3) Clebsch–Gordan coefficients depend on N_c.

We also find that group-theoretical factors suppress Γ_{Θ^+} with respect to Γ_Δ. This suppression is, however, "undone" by the phase space factor, which scales differently with N_c in the chiral limit.

2. Large N_c limit

Both in the Skyrme model [5,6] and in the χQSM [12,13] baryons emerge as rotational states of the symmetric top which rotates in the SU(3) collective space [3,6]. This rotation is described in terms of a rotational SU(3) matrix $A(t)$. Baryonic wave functions [4,6,13,14] are given as SU(3) Wigner $D^{(\mathcal{R})}_{BJ}(A)$ matrices[2]:

$$\psi^{(\mathcal{R})}_{BS}(A) = \psi_{(\mathcal{R},B)(\mathcal{R}^*,S)}(A) = (-)^{Q_S}\sqrt{\dim(\mathcal{R})}\,D^{(\mathcal{R})*}_{BJ}(A). \tag{1}$$

In this notation $B = (Y, I, I_3)$ with Y being hypercharge, I and I_3 isospin and its third component. Left index $S = (Y_R = -N_c/3, S, S_3)$ and $J = (-Y_R = N_c/3, S, -S_3)$ is a state conjugate to S. The fact that the right index runs only over the SU(2) subspace of the SU(3) representation $\mathcal{R}$ follows from the form of the hedgehog ansatz for the static soliton field and the Wess–Zumino term [6,15]. Q_S is the charge of state S.

Note that under the action of left (flavor) generators $\hat{T}_\alpha = -D^{(8)}_{\alpha\beta}\hat{J}_\beta$ $\psi_{B\mathcal{R}}$ transforms like a tensor in representation $\mathcal{R}$, while under the right generators $\hat{J}_\alpha$ like a tensor in $\mathcal{R}^*$ rather than $\mathcal{R}$.

For $N_c = 3$ $Y_R = -1$ and the lowest allowed SU(3) representations are those of triality zero: 8, 10, $\overline{10}$, 27, etc. In large N_c limit, however, these representations are generalized in the following way [9,10] (other possible choices are discussed in Ref. [11]):

$$\text{“8”} = (1, q), \qquad \text{“10”} = (3, q-1), \qquad \text{“}\overline{10}\text{”} = (0, q+2) \tag{2}$$

where

$$q = \frac{N_c - 1}{2}. \tag{3}$$

Note that in this notation anti-ten is not a conjugate of decuplet, neither is an octet selfadjoint. Therefore we shall denote a complex conjugate by a $*$:

$$\mathcal{R}^* = (p, q)^* = (q, p). \tag{4}$$

For large representations like (2) physical flavor states have hypercharge different from the one in the real world, however isospin, charge and strangeness take the physical values. This is depicted in Fig. 1.

Baryon masses are given in terms of the effective Hamiltonian

$$M = M_{\rm sol} + \frac{1}{2I_1}S(S+1) + \frac{1}{2I_2}\left(C_2(\mathcal{R}) - S(S+1) - \frac{N_c^2}{12}\right),$$

where $C_2(\mathcal{R}) = (p^2 + q^2 + pq + 3(p+q))/3$ is the Casimir operator for representation $\mathcal{R} = (p, q)$ and $M_{\rm sol} = \mathcal{O}(N_c)$ is the classical soliton mass. It is easy

[2] The derivation leading to Eq. (1) can be found in Appendix A of Ref. [14] and follows closely the unpublished notes of P.V. Pobylitsa.

to convince oneself that in the chiral limit nonexotic splittings scale like $\mathcal{O}(1/N_c)$ while exotic–nonexotic like $\mathcal{O}(1)$, e.g.:

$$M_\Delta - M_N \sim \mathcal{O}\left(\frac{1}{N_c}\right),$$
$$M_\Theta - M_N \sim \mathcal{O}(1). \tag{5}$$

The fact that $\Theta - N$ mass difference is of the order of 1 was used to argue that the rigid-rotor quantization of the chiral soliton is not valid for exotic states [16]. The discussion of this point is beyond the scope of the present Letter, let us however note, that arguments have been also given in favor of the rigid-rotor quantization [17] despite Eq. (5)

3. Decay width

The baryon–meson (κ) coupling operator can be written in terms of the collective coordinates as [4]:

$$\hat{O}_\kappa = -i\frac{3}{2M_B}\left[G_0\hat{O}^{(0)}_{\kappa A} - G_1\hat{O}^{(1)}_{\kappa A} - G_2\hat{O}^{(2)}_{\kappa A}\right]p_A, \tag{6}$$

where

$$\hat{O}^{(0)}_{\kappa A} = D^{(8)}_{\kappa A}, \qquad \hat{O}^{(1)}_{\kappa A} = d_{Abc}D^{(8)}_{\kappa b}\hat{S}_c,$$
$$\hat{O}^{(2)}_{\kappa A} = \frac{1}{\sqrt{3}}D^{(8)}_{\kappa 8}\hat{S}_A. \tag{7}$$

Here $\hat{S}_a$ are generalized spin right SU(3) generators related to the known "isospin", V-spin and U-spin operators in the following way

$$\hat{I}_3 = \hat{S}_3, \qquad \hat{I}_\pm = \hat{S}_1 \pm i\hat{S}_2, \qquad \hat{V}_\pm = \hat{S}_4 \pm i\hat{S}_5,$$
$$\hat{U}_\pm = \hat{S}_6 \pm i\hat{S}_7, \qquad \hat{Y} = \frac{2}{\sqrt{3}}\hat{S}_8. \tag{8}$$

Note that these operators act on the right index of the wave function (1), for which "isospin" is related to the physical spin. We have adopted here a convention that Greek indices run over all possible values: $\alpha, \beta, \ldots,$ $\kappa = 1, \ldots, 8$, capital Latin indices over the SU(2) subgroup: $A, B, \ldots = 1, 2, 3$ and small Latin indices $a, b, c, \ldots = 4, 5, 6, 7$. In order to calculate the width for the decay $B \to B' + \kappa$ we have to evaluate the matrix element of $\hat{O}_\kappa$ between the baryon wave functions, square it, average over initial and sum over final spin and isospin [4]:

$$\bar{\mathcal{M}}^2_B = \frac{1}{(2I_B+1)(2S_B+1)} \times \sum_{I_{B3},S_{B3}}\ \sum_{I_{B'3},S_{B'3}} \left|\langle B', S_{B'3}|\hat{O}_\kappa|B, S_{B3}\rangle\right|^2. \tag{9}$$

Coupling constants G_i are related to the axial-vector couplings by a Goldberger–Treiman relation and scale differently with N_c:

$$G_0 \sim N_c^{3/2}, \qquad G_1, G_2 \sim N_c^{1/2}. \tag{10}$$

Finally, in order to get the width one has to multiply (9) by the phase space volume and the final result reads

$$\Gamma_B = \frac{1}{2\pi}\bar{\mathcal{M}}^2_B\, p, \tag{11}$$

where

$$p = |\vec{p}_\kappa| = \frac{\sqrt{(M^2-(M'+m_\kappa)^2)(M^2-(M'-m_\kappa)^2)}}{2M} \tag{12}$$

is the momentum of meson κ. In Ref. [4] Eq. (11) was multiplied by a ratio of the baryon masses which is important for the numerical results, which, however, scales as $\mathcal{O}(1)$ with N_c and therefore is irrelevant for further discussion.

The action of the D functions entering the collective operators can be calculated with the help of the SU(3) Clebsch–Gordan coefficients [18,19]:

$$\dim(\mathcal{R}_3)\int dA\, D^{(\mathcal{R}_3)*}_{B_3J_3}(A)D^{(\mathcal{R}_2)}_{B_2J_2}(A)D^{(\mathcal{R}_1)}_{B_1J_1}(A) = \sum_\gamma \left(\begin{array}{cc|c} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3^\gamma \\ B_1 & B_2 & B_3 \end{array}\right)\left(\begin{array}{cc|c} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3^\gamma \\ J_1 & J_2 & J_3 \end{array}\right), \tag{13}$$

where γ is the degeneracy index. In order to calculate matrix elements of (7) between wave functions (1) we shall also use the action of the operators $\hat{V}_\pm$ and $\hat{U}_\pm$ on the spin states [18]:

$$\begin{array}{cccc} \hat{U}_+ & \nwarrow & \nearrow & \hat{V}_+ \\ \hat{V}_- & \swarrow & \searrow & \hat{U}_- \end{array} \tag{14}$$

Note that the spin states belong to $\mathcal{R}^*$. The relevant action is depicted in Fig. 2.

Finally we shall need Clebsch–Gordan coefficients for large SU(3) representations (2). Here we list the Clebsch–Gordan series for the highest weights in two

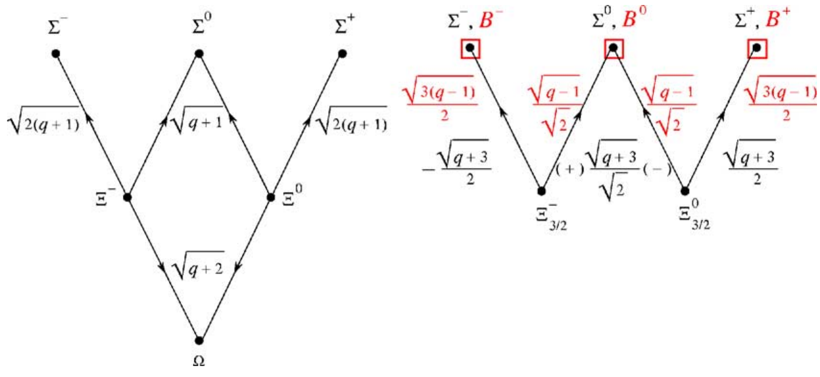

Fig. 2. Action of $\hat{U}_\pm$ and $\hat{V}_\pm$ operators on spin $S_3 = \pm 1/2$ states belonging to antidecuplet $\mathcal{R}^* = (q+2, 0)$ and decuplet $\mathcal{R}^* = (q-1, 3)$. In the latter case the upper entries refer to the transitions to the spurious states B.

cases relevant to the present calculation [9,10]:

$$|\text{“}10\text{”}, \Delta^{++}\rangle = \sqrt{\frac{q}{q+1}}|8, \pi^+\rangle \otimes |\text{“}8\text{”}, p\rangle - \sqrt{\frac{1}{q+1}}|8, K^+\rangle \otimes |\text{“}8\text{”}, \Sigma^+\rangle,$$

$$|\text{“}\overline{10}\text{”}, \Theta^+\rangle = \sqrt{\frac{1}{2}}|8, K^0\rangle \otimes |\text{“}8\text{”}, p\rangle - \sqrt{\frac{1}{2}}|8, K^+\rangle \otimes |\text{“}8\text{”}, n\rangle. \tag{15}$$

Remaining Clebsch–Gordan coefficients can be found by applying lowering operators to (15).

The result of the calculations are as follows (for spin down states and $\vec{p} = (0, 0, p)$):

- $\Delta \to \pi N$

$$\langle N|\hat{O}_\pi|\Delta\rangle = -i\frac{3}{M_N + M_\Delta}\left(\begin{array}{cc|c} 8 & \text{“}8\text{”} & \text{“}10\text{”} \\ \pi & N & \Delta \end{array}\right) \times \sqrt{\frac{q+3}{3(q+4)}}\left[G_0 + \frac{1}{2}G_1\right]p, \tag{16}$$

- $\Theta^+ \to KN$

$$\langle N|\hat{O}_K|\Theta^+\rangle = -i\frac{3}{M_N + M_{\Theta^+}}\left(\begin{array}{cc|c} 8 & \text{“}8\text{”} & \text{“}10\text{”} \\ K & N & \Theta^+ \end{array}\right) \times \sqrt{\frac{q+1}{2(q+2)(q+4)}} \times \left[G_0 - \frac{q+1}{2}G_1 - \frac{1}{2}G_2\right]p. \tag{17}$$

For Δ decay we get:

$$\bar{\mathcal{M}}_\Delta^2 = \frac{3}{(M_N + M_\Delta)^2}\frac{q(q+3)}{2(q+1)(q+4)} \times \left[G_0 + \frac{1}{2}G_1\right]^2 p^2 \frac{3}{(M_N + M_\Delta)^2} \times \frac{(N_c-1)(N_c+5)}{2(N_c+1)(N_c+7)}\left[G_0 + \frac{1}{2}G_1\right]^2 p^2, \tag{18}$$

whereas for Θ^+ we have:

$$\bar{\mathcal{M}}_\Theta^2 = \frac{3}{(M_N + M_\Theta)^2}\frac{3(q+1)}{2(q+2)(q+4)} \times \left[G_0 - \frac{q+1}{2}G_1 - \frac{1}{2}G_2\right]^2 p^2 \times \frac{3}{(M_N + M_\Theta)^2}\frac{3(N_c+1)}{(N_c+3)(N_c+7)} \times \left[G_0 - \frac{N_c+1}{4}G_1 - \frac{1}{2}G_2\right]^2 p^2. \tag{19}$$

Two important remarks are here in order. First of all, and this is our main result announced in the Introduction, for Θ^+ decay constant G_1 is enhanced by a factor of N_c and therefore the second term in Eq. (19) is of the same order as G_0. These two terms cancel against each other yielding numerically Θ^+ width much smaller than the width of Δ. This cancellation is therefore consistent with N_c counting and justifies the use of nonleading terms in the decay operator (6).

Secondly, the overall factor in front of $[\cdots] \times p^2$ is $\mathcal{O}(1)$ for $\Delta \to \pi N$ and $\mathcal{O}(1/N_c)$ for $\Theta^+ \to KN$. This effect, as can be seen from Eqs. (16), (17), is entirely due to the "spin" part of the matrix elements $\langle B'|\hat{O}_\kappa|B\rangle$. Indeed, flavor Clebsch–Gordan coefficients in Eqs. (16), (17) scale as $\mathcal{O}(1)$ with N_c, as can be inferred from Eq. (15). This is, however, not a complete N_c dependence, since momentum p also depends on N_c. We shall come back to this dependence in a moment.

After multiplying by the phase factors we get:

$$\begin{aligned}\varGamma_\Delta &= \frac{3}{2\pi(M_N+M_\Delta)^2}\frac{(N_c-1)(N_c+5)}{2(N_c+1)(N_c+7)}\\ &\quad\times\left[G_0+\frac{1}{2}G_1\right]^2 p^3,\\ \varGamma_\Theta &= \frac{3}{2\pi(M_N+M_\Theta)^2}\frac{3(N_c+1)}{(N_c+3)(N_c+7)}\\ &\quad\times\left[G_0-\frac{N_c+1}{4}G_1-\frac{1}{2}G_2\right]^2 p^3,\end{aligned}\tag{20}$$

where p is given by Eq. (12). In the chiral limit $m_\kappa=0$

$$p=\frac{(M-M')(M+M')}{2M}.\tag{21}$$

Since the difference $M-M'$ scales differently with N_c for Δ and Θ^+ decays (5):

$$\begin{aligned}&\Delta\to\pi N, \qquad p_\pi\sim\mathcal{O}\left(\frac{1}{N_c}\right),\\ &\Theta\to KN, \qquad p_K\sim\mathcal{O}(1)\end{aligned}\tag{22}$$

and the overall scaling for the widths reads

$$\varGamma_\Delta\sim\mathcal{O}\left(\frac{1}{N_c^2}\right), \qquad \varGamma_\Theta\sim\mathcal{O}(1).\tag{23}$$

It is interesting to ask at this point how well the N_c scaling arguments work numerically in Nature. For the mass differences we get (assuming $M_{\Theta^+}=1540$, $M_{\Xi_{3/2}}=1860$ MeV, which gives $\bar{M}_{\overline{10}}=1752$ MeV for the average antidecuplet mass):

$$\bar{M}_{10}-\bar{M}_8=234, \qquad \bar{M}_{\overline{10}}-\bar{M}_8=601\tag{24}$$

(in MeV) which is in a reasonable agreement with an expected factor of $N_c=3$, see Eq. (5). As far as the phase-space factor is concerned, the physical value of the meson momentum reads (in MeV)

$$p_\pi=225, \qquad p_K=268\tag{25}$$

and it is hard to argue that the scaling of Eq. (22) really holds. Formally for $m_\kappa\neq0$ meson momentum p scales in both cases[3] as $\mathcal{O}(1)$. In that case:

$$\varGamma_\Delta\sim\mathcal{O}(N_c), \qquad \varGamma_\Theta\sim\mathcal{O}(1)\tag{26}$$

which would explain parametrically the narrowness of Θ^+ with respect to Δ.

4. Nonrelativistic limit

Coupling constants $G_{0,1,2}$ are related to the axial couplings through Goldberger–Treiman relation. On the other hand we know explicit model formulae for these couplings [14,20]:

$$\begin{aligned}&G_0\sim A_0-\frac{A_1^{(-)}}{I_1^{(+)}}, \qquad G_1\sim2\frac{A_2^{(+)}}{I_2^{(+)}},\\ &G_2\sim2\frac{A_1^{(+)}}{I_1^{(+)}}\end{aligned}\tag{27}$$

up to the same proportionality factor of the order of $M_B/F_\pi\sim\mathcal{O}(\sqrt{N_c})$. Explicit formulae and numerical values of the inertia parameters A and I can be found in Ref. [14]. If in the χQSM one artificially sets the soliton size $r_0\to0$, then the model reduces to the free valence quarks which, however, "remember" the soliton structure [21]. In this limit, many quantities, like the axial-vector couplings, are given as ratios of the group-theoretical factors [20]:

$$\begin{aligned}&A_0\to-N_c, \qquad \frac{A_1^{(+)}}{I_1^{(+)}}\to-1\,,\\ &\frac{A_1^{(-)}}{I_1^{(+)}}\to2, \qquad \frac{A_2^{(+)}}{I_2^{(+)}}\to-2.\end{aligned}\tag{28}$$

With these values we get that the nucleon axial coupling [20,21]

$$g_A\to\frac{N_c+2}{3}=\frac{5}{3}\tag{29}$$

which is the well known naive quark model result [22].

For the antidecuplet decay strength we get:

$$G_{\overline{10}}=G_0-\frac{N_c+1}{4}G_1-\frac{1}{2}G_2$$

[3] In the case of Δ decay p_π is imaginary since, strictly speaking, in the large N_c limit $M_\Delta=M_N$ and the decay does not occur.

$$\sim \left(A_0 - \frac{A_1^{(-)}}{I_1^{(+)}}\right) - \frac{N_c+1}{2}\frac{A_2^{(+)}}{I_2^{(+)}} - \frac{A_1^{(+)}}{I_1^{(+)}}$$
$$= \left(A_0 - \frac{N_c}{2}\frac{A_2^{(+)}}{I_2^{(+)}}\right) + \left(-\frac{A_1^{(-)}}{I_1^{(+)}} - \frac{1}{2}\frac{A_2^{(+)}}{I_2^{(+)}} - \frac{A_1^{(+)}}{I_1^{(+)}}\right)$$
$$\to \left(-N_c + \frac{N_c}{2}2\right) + \left(-2 + \frac{1}{2}2 + 1\right) = 0. \quad (30)$$

We see that the cancellation is exact in each order in N_c.[4]

Let us speculate that the cancellation of the leading order terms in (30) is exact. Then $G_{\overline{10}} \sim \mathcal{O}(\sqrt{N_c})$ while $G_{10} \sim \mathcal{O}(N_c^{3/2})$. In this case we would get

$$\Gamma_\Delta \sim \mathcal{O}(N_c) \times p^3 \to \begin{cases} \mathcal{O}(1/N_c^2), & m_\kappa = 0, \\ \mathcal{O}(N_c), & m_\kappa \neq 0, \end{cases}$$
$$\Gamma_\Theta \sim \mathcal{O}\left(\frac{1}{N_c^2}\right) \times p^3 \to \begin{cases} \mathcal{O}(1/N_c^2), & m_\kappa = 0, \\ \mathcal{O}(1/N_c^2), & m_\kappa \neq 0, \end{cases} \quad (31)$$

which would mean that in the chiral limit both decay widths scale as $\mathcal{O}(1/N_c^2)$ while for $m_\kappa \neq 0$ Θ^+ decay is damped by a factor $\mathcal{O}(N_c^3)$with respect to Δ.

5. Summary

Our primary goal was to show that the cancellation which takes place in the case of the Θ^+ width is consistent with the N_c counting. Indeed, by employing correct generalizations of standard SU(3) representations for arbitrary number of colors, we have shown that there is additional N_c enhancement of the constant G_1 which is formally one power of N_c less than G_0. This enhancement comes entirely from the spin part of the matrix elements $\langle B'|\hat{O}_\kappa|B\rangle$ and carries over to the decays of all particles in antidecuplet.

We have also found that there is $\mathcal{O}(1/N_c)$ suppression factor in the Θ^+ width with respect to Δ, coming from the same source, namely from the N_c dependence of the $SU(3)_{\text{flavor}}$ Glebsch–Gordan coefficients. Unfortunately, this suppression is “undone” by the phase space factor p^3, which in the chiral limit scales differently for Θ^+ and Δ decays (22). If we assume that meson masses are nonzero, then the suppression survives (26). This kind of “noncommutativity” of the chiral limit and large N_c expansion is well known and there are many examples where it creates problems.

Finally, we have shown that in the nonrelativistic quark model limit, i.e., in the limit where we artificially squeeze the soliton, the cancellation in the decay strength for Θ^+ is exact and occurs independently at each order of N_c. In this limit Θ^+ width vanishes identically.

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