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The DEA game cross efficiency model applied to the Brazilian football championship

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Abstract

In this paper, we use DEA to measure the Brazilian football teams efficiency in the season 2014. In this context, each team is a DMU, where we select three inputs: the number of home matches, the average attendance and the average points obtained at the last four seasons. The total points obtained at the season 2014 is the output. We evaluate the teams cross-efficiency by DEA game, which is an approach suitable when there is no cooperation among DMUs. This procedure also improves the efficiencies discrimination.

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1. Introduction

Football is one of the most popular sports in the world, especially in Brazil, where that sport attracts thousands of fans during all the year.

The mainly football tournament in Brazil is the Brazilian football championship, which is a classical double round robin tournament, composed by twenty teams. In this context, the main objective of this paper is to investigate the efficiency of the teams in that tournament.

In the literature, we found some researches that also study the efficiency of Brazilian football teams. [1] applied the classical BCC DEA model on the season 2008. We are selecting in this paper the same set of inputs and outpust used in that paper. [2] applied a DEA bootstrap procedure and considered several financial variables such as attendance, receipt, operational costs, assets and payroll. [3] also used financial variables and a DEA model to measure Brazilian football teams efficiency. [4] used a variable selection method for choosing which aspects inside the game field are more relevant for three different DEA models, evaluating teams on three different goals: Defense, Connection and Attack. [5] analyzed the cost efficiency of Brazilian first league football teams using a bayesian varying efficiency distribution model.

Other leagues around the world are also studied, such as the German [6], Spanish [7], Iranian [8] and English [9], [10], [11], among others.

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In this paper, we also use DEA to measure the Brazilian football teams efficiency in the season 2014. In this context, each team is a DMU, where we select three inputs: the number of home matches, the average attendance and the average points obtained at the last four seasons. We choose the first two inputs in order to consider the home advantage [12,13] and the third input was choosen in order to consider the teams tradition. The total points obtained at the season 2014 is the output.

Besides the classical DEA efficiency, we also evaluate the DEA game cross-efficiency [14]. This approach is suitable when there is no cooperation among DMUs. This approach also improves the efficiencies discrimination.

This paper is divided as follows: Section 2 describes the DEA models used and its results. Section 3 presents and discusses the results obtained in this paper while Section 4 shows the conclusions and the perspectives for future works.

2. Methodology

To calculate the efficiency of each team in the Brazilian football championship in the season 2014, we initially use the classical output-oriented DEA BCC. For each DMU k (observed DMU), we solve the model (I) to find its classical DEA BCC efficiency output oriented. The mathematical notation used in the model (I) is as follows:

- $n \rightarrow$ number of DMUs
- $r \rightarrow$ number of inputs
- $s \rightarrow$ number of outputs
- $x_i^j \rightarrow$ value of the input i for the DMU j.
- $y_i^j \rightarrow$ value of the output i for the DMU j.
- $u_i \rightarrow$ nonnegative variable indicating the weight related to output i
- $v_i \rightarrow$ nonnegative variable indicating the weight related to input i
- $v^* \rightarrow$ unsigned variable

$$(I) \qquad \min \quad \sum_{i=1}^{r} v_i x_i^k - v^* \tag{1}$$

$$\text{Subject to} \quad \sum_{i=1}^{s} u_i y_i^k = 1 \tag{2}$$

$$-\sum_{i=1}^{r} v_i x_i^j + \sum_{i=1}^{s} u_i y_i^j - v_k^* \leq 0, \ \forall j \in \{1, ..., n\} \tag{3}$$

For each pair of DMUs k and d, we also calculate the DMU k efficiency using the weights from DMU d (E_{dk}). Thus, E_{dk} is calculated as follows:

$$E_{dk} = \frac{\sum_{i=1}^{r} v_i^d x_i^k - v_d^*}{\sum_{i=1}^{s} u_i^d y_i^k} \tag{4}$$

The classical average cross efficiency of DMU k is calculated by the following way:

$$E_k = \frac{\sum_{d=1}^{n} E_{dk}}{n}, \tag{5}$$

To avoid $E_{dk} < 0$ [15,16], we add the following group of constraints to model (I):

$$\sum_{i=1}^{r} v_i x_i^j - v_k^* \geq 0, \ \forall j \in \{1, ..., n\} \tag{6}$$

It should be noted that with constraint (6), the model is no longer BCC, being less benevolent. The efficient frontier may be changed. Some papers propose secondary objectives in cross efficiency because different weight sets can generate the same efficiency value [17,18].

In this paper, we apply a cross efficiency method named DEA Game that is not affected by the previous situation. In DEA game, proposed by [14], each DMU is seen as a competitor in an uncooperative environment. So, to calculate the cross-efficiency of DMU k related to DMU d, a set of weights is found in order to maximize the efficiency of DMU k with the additional constraint that d efficiency α_d does not decrease. In this context, to calculate the efficiency of each DMU, it is necessary to know the efficiencies of the others, and vice-versa. This problem is solved through an iterative process, where the DMUs efficiencies are found, and these values represent a Nash equilibrium.

Model (II) calculates the cross-efficiency of DMU k related to d using DEA game.

$$(II) \qquad \min \qquad E'_{k,d} = \sum_{i=1}^{r} v_i x_i^k - v^* \tag{7}$$

$$\text{subject to} \quad \sum_{i=1}^{s} u_i y_i^k = 1 \tag{8}$$

$$-\sum_{i=1}^{r} v_i x_i^j + \sum_{i=1}^{s} u_i y_i^j - v_k^* \leq 0, \ \ \forall j \in \{1, ..., n\} \tag{9}$$

$$-\sum_{i=1}^{r} v_i x_i^d + \alpha_d \sum_{i=1}^{s} u_i y_i^d + v^* \geq 0 \tag{10}$$

We have to remeber that the efficiency α_d is greater or equal to 1. Thus, the new constraint (10) indicates that the efficiency of the DMU d does not decrease. Iterative algorithm 1 describes the steps to find the efficiency of DMUs that represent a Nash equilibrium solution. In this algorithm, α_j^t represents the efficiency of DMU j at iteration t.

Algorithm 1 DEA game

Require: ϵ

Step 1: Set t=1. For each DMU k, calculate the classical average cross-efficiency E_k and set $\alpha_k^t = E_k$, $\forall k \in \{1, ..., n\}$.

Step 2: For each pair of DMUs k and d, solve model (II) and obtain $E'_{k,d}$.

Step 3: Set $\alpha_k^{t+1} = \frac{\sum_{d=1}^{n} E'_{dk}}{n}$.

Step 4: If for some k, $|\alpha_k^{t+1} - \alpha_k^t| > \epsilon$, then return to step 2. Otherwise, the algorithm ends and α_k^{t+1} is the optimum DEA game cross-efficiency of DMU k.

[14] proved that the algorithm converges and that the final solution represents a Nash equilibrium. The author also observed that the algorithm converges to the same efficiency even using different cross efficiencies at the first step.

3. Results

We select, as inputs, the number of home matches, the average attendance and the average points obtained at the last four seasons. As output, we have the total points obtained at the season 2014. That variables was also used by [1] on the season 2008 and aim to explain the impact of the home advantage and the teams tradition on the final classification.

Now we cite some explanations to the fact of the the number of home matches is not the same:

- Some stadiums were unavailable due to the 2014 FIFA World Cup.

- Some teams were punished and therefore could not play in their stadiums.
- Some teams choosed better stadiums to play in order to obtain more profit.

Table 1 shows the values of the inputs and outputs used in this paper and the official final position of each team in the end of the season 2014.

Table 1. Values of the inputs and outputs

Team	Inputs: home matches	Inputs: average attendance	Inputs: average points 2010-2013	Output: Total points 2014	Official position (2014)
Atlético Mineiro	16	14132	54.75	62	5
Atlético Paranaense	14	12237	55.00	54	8
Bahia	15	12579	47.00	37	18
Botafogo	9	11362	57.75	34	19
Chapecoense	19	10021	0.00	43	15
Corinthians	15	28960	61.50	69	4
Coritiba	17	12329	51.00	47	14
Criciúma	19	9029	46.00	32	20
Cruzeiro	16	29678	60.00	80	1
Figueirense	17	8378	58.00	47	13
Flamengo	11	26411	50.00	52	10
Fluminense	14	18490	64.25	61	6
Goiás	15	6942	46.00	47	12
Grêmio	16	21028	61.75	61	7
Internacional	16	22318	54.50	69	3
Palmeiras	13	19755	44.67	40	16
Santos	12	9243	54.75	53	9
São Paulo	15	28544	57.50	70	2
Sport	12	18220	41.00	52	11
Vitória	14	10267	50.50	38	17

In Table 1, note that the average points obtained by the Chapecoense is 0.00. It happened because this team did not participate in the first division tournament during the seasons 2010, 2011, 2012 and 2013.

Table 2 shows the results obtained by the application of the methods described in Section 2.

Table 2. Results

Team	BCC efficiency	DEA Game cross efficiency	Ranking: Classical BCC	Ranking: DEA Game	Dif.
Atlético Mineiro	1.0000	0.9782	1	3	-2
Atlético Paranaense	0.9289	0.9185	12	10	2
Bahia	0.6501	0.6389	19	19	0
Botafogo	1.0000	0.7956	2	15	-13
Chapecoense	1.0000	0.9731	3	4	-1
Corinthians	0.9276	0.8995	13	11	2
Coritiba	0.8116	0.7734	16	16	0
Criciúma	0.6297	0.5657	20	20	0
Cruzeiro	1.0000	0.9980	4	2	2
Figueirense	0.9262	0.8348	14	14	0
Flamengo	1.0000	0.8654	5	12	-7
Fluminense	0.9331	0.9224	11	9	2
Goiás	1.0000	0.9467	6	7	-1
Grêmio	0.8716	0.8535	15	13	2
Internacional	0.9786	0.9683	9	5	4
Palmeiras	0.6954	0.6696	18	18	0
Santos	1.0000	1.0000	7	1	6
São Paulo	0.9444	0.9249	10	8	2
Sport	1.0000	0.9473	8	6	2
Vitória	0.7036	0.6894	17	17	0

Results show that the DEA game cross efficiency helps differentiating the DMUs, creating a ranking that is not affected by the optimal weights multiplicity. The Santos team was efficient for both methods since this team has lower values of inputs. The champion of this tournament was Cruzeiro, that obtained efficiency greater than 0.99 for both methods. The Botafogo is in the postion 19 in the official rank but it was efficient for the classical BCC model. It happened due to the benevolence of this method since that team has the small number of home matches. On the other hand, that team lost more positions with the creation of a new ranking. Chapecoense obtained good results for both methods since it has an input with value 0.00.

4. Conclusions

In this paper, we show through DEA methodology, classical and cross-efficiency of Brazilian first division teams in season 2014. In this context, each team is a DMU, where we select three inputs: the number of home matches, the average attendance and the average points obtained at the last four seasons. The total points obtained at the season 2014 is the output. For the classical efficiency, we use the BCC method while we use the DEA Game for the cross efficiency.

The DEA Game cross efficiency model was suitable for our case study since there is noncooperation between DMUs. Besides, this model generates a ranking that is not affected by the multiplicity of optimal weights and increases the discrimination between DMUS.

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