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Essay Review

# What's in a word? Symmetry through the centuries

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**From *Summetria* to Symmetry: The Making of a Revolutionary Scientific Concept**
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Ask a friend to give an example of symmetry and she may very well cite the human body. Ask her then to say what that *means*, to give a *definition* of symmetry, and she may find that a precise formulation does not come easily. Ask, finally, that she capture in words the relationship—obvious similarity, subtle difference—between the left and right hands of that symmetrical human figure—and, when she flounders, console her by reporting that Immanuel Kant struggled with the same problem, in vain.

That little experiment offers a bare hint, a scratch (so to say) of the surface of the surface, of the issues surrounding the history of the idea of symmetry as set out in this flawed but fascinating book. Giora Hon teaches at the University of Haifa; his main current research interest (according to his website) is the "problem of experimental error". Bernard Goldstein, now at the University of Pittsburgh, is a well-known historian of ancient, medieval and early-modern astronomy and astrology.

At the heart of their brave enterprise is a tricky dilemma. One can write a history of the *concept* of symmetry, or one can write a history of the *word* "symmetry" (and its equivalents in other languages); and clearly these are not the same thing, though of course they overlap considerably and an ambitious account could hope to do justice to both. Hon and Goldstein adopt in this matter a policy which colours deeply their approach to their subject. They are certainly concerned with concepts, as their book's very title attests. Indeed they express (p. 9) the hope that their work will contribute to our understanding of the "making" of concepts in general. But on the other hand they declare (p. x) that they will banish "psychological" considerations from their study—though one might have thought that the "making of a concept" is very much a psychological activity. Instead the authors put their historiographical trust in *words*. They take the view that a writer who does not use a particular word cannot be supposed to have possessed the corresponding concept. They admit exceptions; thus they say (p. 207) that Euler recognized the "reverse relationship" in mirror-image symmetry though he had no term for it. But their general position is explicit. Using the word "actor" for any participant in their story, they assert (p. 31) that "one must find *symmetry* in an actor's text in order to claim that this is one of that actor's categories".

Of course one sees their point, and applauds it—up to a point. Hon and Goldstein inveigh strongly against the still-too-common sin of what they call (pp. 29ff) "anachronism", the practice of wrongly projecting modern ideas into the minds of our forebears. But I would argue that their demand for explicit textual evidence for a concept is deeply unrealistic as a matter of common experience and deeply limiting as a strategy for grasping intellectual history. The first five editions of *The Origin of Species* do not contain the word "evolution"; must we infer that Darwin lacked

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the concept? The stance taken by the authors seems particularly ironic and deplorable in the context of symmetry, which is one of those phenomena that one can often "see" very easily, in both senses of the word, with the eye and with the mind, without (perhaps) the need, or the desire, or the capacity, for verbal description. Hon and Goldstein want to rule out of the historical record what they call (p. 127) the "perceptual mode of symmetry", namely human beings' direct visual apprehension of bilateral and of rotational symmetry. They criticize (p. 8) John Ruskin and Ernst Mach—an odd couple, on the face of it!—for having apparently believed that symmetry is a concept without a history because of their view that this "perceptual mode" is innate. Hon and Goldstein's own attitude to that last question remains regrettably unclear. At the very outset of their book (p. vii) they ask whether "the ancients" perceived the "symmetrical elements" that we see, and they reply that "the answer is negative", whereas later (p. 8) they concede that Ruskin and Mach's assumption of an "innate faculty" may be right after all. But in any case they propose (p. 9) to exclude from their narrative all reference to the psychology of perception. This decision, and their general insistence on verbal evidence, ensures that their book is in its main thrust a chronicle of the meanings and uses of words.

Their argument for the sidelining of "perceptual modes of symmetry" kicks off a substantial introductory section intended to pave the way for the historical survey which follows. Next up is a discussion whose goal is to "clarify philosophical issues involved in symmetry as a scientific concept". Two conclusions seem to be stressed. Drawing on Hermann Weyl's classic book [Weyl, 1952], Hon and Goldstein affirm (p. 15) that "the modern concept of symmetry must be formulated in group theoretic terms". This sounds promising; but the surrounding exposition suggests that the authors' grasp of the relevant mathematics is—shall we say—a bit tenuous. Although on page 14 they quote from Weyl the phrase "the [ . . . ] *transformations* form a group" (my italics), they go on to say, just one page later, that "Symmetry is a property of objects or elements that form a group, and, to form this group, there must be a transformation [sic] with an invariance". Unhappily, so basic a misunderstanding seems likely to cripple a reader's confidence in anything else that the authors say in this sphere. The other philosophical conclusion reached here is that (p. 26) symmetry is unique among scientific concepts in having both ontological and (via the structure of arguments) epistemological bearing.

The next subsection offers a review of previous histories of symmetry. Hon and Goldstein are not ungenerous toward their predecessors, but they find two persistent faults. One (pp. 29ff) is the "anachronism" that I have already mentioned, the wrong ascription of later views to earlier thinkers. The second criticism of existing accounts recalls the authors' demand for explicit textual evidence of possession of a concept. They deplore (pp. 34ff) earlier scholars' willingness to believe, from psychological plausibility, that an historical figure possessed and used a concept which he or she did not articulate. In their minds these two failings are linked: "by using the false technique of implicit reading" an unwary historian will "open the gates to a flood" of fatal anachronisms (p. 37).

The book's long introductory section concludes with a quite detailed outline of the historical account which will occupy the remainder. Here in particular the authors offer a first delineation of their central thesis, that Legendre "revolutionized the concept of symmetry" (p. 49) through a dramatic definition in his *Éléments de géométrie* (1794). They announce that they will follow subsequent events only to 1815, leaving later developments to other pens.

Their splendid survey—which, as I said, is essentially a history of the meanings of words—begins with the Greek *summetria*, which signified "proportionality". In the ancient and early-modern history of this word and of its translations Hon and Goldstein identify two strands, which they label respectively "mathematical" and "aesthetic". The mathematical use is straightforward. The locus classicus in Greece is Euclid's *Elements*, where (X, Def. 1) *summetria* connotes, in exact accordance with the meanings of its two components, "co-measure", commensurability of geometric magnitudes. The Latin translation *commensurabiles* was coined by Boethius, and passed with unchanged meaning down the centuries, ultimately fading from use.

The "aesthetic" thread in the history of *summetria* presents a much more tangled tale. In ancient Greece the word signified proper proportions, as for example in Plato's assertion [*Timaeus* 87c] that to be both good and beautiful a creature must be "well-proportioned" (Cornford's translation of *summetron* [Cornford, 1987]). The canonical statement of this connotation is by Vitruvius (*De architectura*, 1st century B.C.). According to Vitruvius symmetry (*symmetria*) is obtained when the parts of some object are in "appropriate agreement" (*conveniens consensus*) with the whole. The tradition thus established had (say Hon and Goldstein) a very long run, down to the Renaissance. Then Leon Battista Alberti (1485) shifted attention, in the context of architecture, from the parts-whole relation to the practice of contriving that elements on either side of a central feature "answer each other" [*responderent*], wrongly crediting the idea already to Vitruvius. Thus two meanings of "symmetry" became current, namely "proportion" and "correspondence". In 1673 Claude Perrault separated these, laying it down that the correct meaning was "proportion" in Vitruvius but "correspondence" in contemporary usage (p. 153)—the latter being the more useful concept in monumental architecture. Augustin Charles Davilier coined (1691) the term "respective symmetry" (*Simmetrie respective*) for architecture in which opposite sides of a central feature are like (*pareils*) one another, but Hon and Goldstein are at pains to insist (p. 155) that this is not yet "bilateral symmetry" in its full modern sense, which (they urge) connotes "mirror" imaging that reverses left and right. Such are the highlights of a long development of which my précis here is the merest sketch. The chapter describing the Italian and French developments is the longest in the book, and for my money one of the best. The authors' command of the sources seems total, and their scholarship is superb.

They turn next to occurrences of "symmetry" in the giants of the Scientific Revolution, who—they insist—do *not* deal in modern connotations. For example they cite (p. 158) Copernicus's familiar complaint—a powerful motivation for him—that previous astronomers, cobbling together models of the solar system without the unifying benefit of heliocentricity, were like someone trying to assemble a human figure from random parts and producing "a monster rather than a man". These predecessors failed, says Copernicus, to "deduce the design of the universe and the true symmetry [*symmetriam*] of its parts". Hon and Goldstein assign this declaration to the legacy of Vitruvius, in which, as we saw, "symmetry" meant the proper proportions of parts in a whole. They go on to find several uses of the term in Galileo, and these too they place—convincingly, in my opinion—in the Vitruvian tradition. Commentators have pictured Kepler as invoking symmetry in his study of snowflakes, but in fact (say the authors) he uses neither the term nor the concept.

Passing to the 18th century, the tireless authors find "symmetry" in Linnaeus, for whom it signifies (in plants) the arrangement of parts in a whole for the purpose of reproduction; here again the ultimate source is Vitruvius. In the developing science of crystallography symmetry is characterized, naturally enough, in geometric terms, for example by Jean-Baptiste Louis de Romé de L'Isle (1783) and by René-Just Haüy (1790s). Hon and Goldstein report (p. 191) that Haüy gives no *definition* of "symmetry"—he "invokes the term casually, as if the concept is well understood". I quote this ostensibly unremarkable statement because it will bear on my criticism (below) of the authors' main thesis, that one aspect of the work of Legendre was revolutionary.

In these surveys of early-modern science, as in their chapter on Renaissance architecture, Hon and Goldstein have touched many more bases than a cursory summary can even suggest. I pass with regret over those rich and informative pages, for I want instead to respond in detail to the discussion of Legendre which constitutes their book's principal thesis. Their final prelude to that thesis is a chapter linking the great names of Euler and of Kant as having (they say) failed where Legendre would succeed, in investigations of three-dimensional mirror-image symmetry. In 1750 a correspondent asked Euler which of the two possible representations of the constellations—from inside and from outside the celestial sphere, respectively—is to be preferred. Euler's reply addressed the question as posed but did not try to generalize. For his part Kant (1768) sought to wring from various examples—human hands, snails, screws—an intrinsic distinction of "left" from "right". But (say Hon and Goldstein, p. 218) this quest was for Kant merely a means to an end, namely an attempt to discern an absolute directionality of space, and when this ultimately failed the great philosopher abandoned his studies of symmetry without realizing their significance.

And so the authors come at last, more than two-thirds of the way through their book, to the bold and original claim that forms its climax. They now set out in full detail their argument that Legendre, the "hero" of their saga, gave a "definition" of "symmetry" which was at once a drastic break with the past and a seminal influence on the future. Earlier sections have already tossed out a series of teasers, describing Legendre's move variously as "radical" (p. vii), "revolutionary" (p. ix), "far-reaching" (p. 62), "pathbreaking" (p. 64), and, headiest of all, a breakthrough that "changed our perception and understanding of the world" (p. 50). That kind of build-up can drive a reader's anticipation sky-high.

Or perhaps the most tempting first reaction is to fall back on one of the trendy locutions of our time: "Who knew?" I am aware of no *hint* of this extraordinary achievement in *any* earlier exposition where it might seem relevant. To cite just two much-consulted accounts, the article on Legendre in the *Dictionary of Scientific Biography* [Itard, 1981] does not even *mention* symmetry, and Hermann Weyl's mathematically authoritative book on symmetry does not even *mention* Legendre. If Hon and Goldstein are on solid ground then a lot of rewrite desks will have to swing into early action.

But *are* they on solid ground? To put the matter more strikingly: have *all* previous expositors of Legendre managed to miss an achievement of huge significance? For my part I have serious doubts. Let us look at the textual evidence. In the *Éléments de géométrie* Legendre states and proves the theorem (V, 23) that

> If two solid angles are composed of three plane angles that are equal to each other, respectively, the planes in which the angles are equal will be equally inclined to one another.

There follows a scholium [Legendre, 1817, 155; quoted and translated by Hon and Goldstein on their p. 233], that is the key text for their claim that Legendre here wrought a revolution. About the coincidence of two solid angles, Legendre says:

> This coincidence, however, only takes place, on the supposition that the equal plane angles are *arranged in the same way* for both solid angles; for if the equal plane angles were *arranged in the inverse order* ... the two solid angles would be equal in all their constituent parts without however it being possible to superpose one on the other. This kind of equality which is neither absolute nor of superposition is worthy of being distinguished by a special expression: we will call it *equality by symmetry* [*égalité par symmétrie*].

In Note I of the *Éléments* [Legendre, 1817, 276–277; p. 236 of the present work] Legendre rephrases this slightly: he points out that in earlier books on solid geometry certain proofs "that depend on the coincidence of figures are not correct" because they fail to take into account the fact that "two solids, two solid angles, two spherical triangles or polygons may be equal in all their constituent parts without, nevertheless, coinciding by superposition", and therefore

> We have [...] thought it necessary to give a special name to this equality that does not entail coincidence; we have called it *equality by symmetry*; and the figures to which it applies we call *symmetrical figures*.

Now Hon and Goldstein stake their entire argument on the view that in these passages Legendre is giving a *definition* of "symmetry". But my own reservations begin right there—with the stubborn fact that that is *not* what Legendre *says* he is doing. His words leap off the page: he is actually defining "a new kind of equality". His underlying strategy is surely clear, because it is so familiar in mathematics: he will find it possible, and fruitful, to identify, through a notion of pseudo-"equality", things which are nearly alike but not quite—here, two equal-angled but nonsuperposable solids—if their similarities are useful, or their differences inessential, for the investigation in hand. One of the texts quoted above sets out the sphere of relevance and application of the definition: the different *ordering* of the two sets of angles.

But what then of symmetry, the heart of the authors' case that this passage is revolutionary? Legendre needed a label for his new version of equality, and—I would argue—"symmetry" provided an obvious choice. Symmetry is manifestly present in the relationship of the two solids, and (on my interpretation) Legendre took that for granted, and further took for granted that his readers could "see" it too. We recall that the crystallographer Haüy did exactly the same thing, at almost exactly the same time, in *his* context. Much later in the *Éléments* [Legendre, 1817, 305; pp. 247–248 of the present volume] Legendre points out that one can "get a very correct idea of the set-up" in such cases by picturing the two solids as mirror images, but clearly this gloss is intended only as an aid to the struggling. Hon and Goldstein may well be right to credit Legendre with the first *explicit* use of "symmetry" in this context of reverse ordering, but if the symmetrical character of the solids' relationship was by now a matter of general recognition then the magnitude of Legendre's innovation would seem much reduced.

At the end of their book the authors devote a five-page "coda" to the question of *why* Legendre adopted the term "symmetry" in this—to them—supremely important definition. They concede (p. 295) that he gives no reason, and they lament that no suggestion that they can offer inspires confidence. To my mind their strangest remark about Legendre's possible motivation is their assertion, much earlier (p. 62), that he "sought a term for a new reciprocal relation in three dimensions *without any aesthetic sense*, and chose to adopt a term that had already been used in mathematics for a relation" (my italics). (The last clause recalls the Greek meaning of *summetria* as commensurability.) In ascribing to Legendre a wish to avoid aesthetic associations Hon and Goldstein, scrupulous scholars though they are, make a claim for which there is no shred of textual support—and no psychological plausibility either. Actually the claim is strange twice over—for of course symmetry *does* have aesthetic resonance. I suspect that if we could ask Legendre whether *he* thought so, he would say "Yes, of course—in the arts; but I was doing mathematics". In any case, if I am right that the term's relevance in this context was a matter of general agreement, then the need to seek motives for its use disappears.

It may be worth adding that two well-informed contemporaries of Legendre, on evidence supplied by Hon and Goldstein themselves, seem to support the interpretation that I am suggesting. Thus Lacroix wrote in 1798 that (p. 251)

> Legendre, to whom one owes the remark and the clarification of the difficulty which is presented by the equality of inverse trihedral angles, names them *symmetrical*, because he considers them as constructed on different sides of the same plane.

Lacroix considered the idea "ingenious", but his tone is hardly that of someone contemplating a change in "our perception and understanding of the world", and his explanation of Legendre's terminology presents it as perfectly natural. Similarly an obituary of Legendre by Jean-Frédéric-Théodore Maurice (p. 223) says merely that "he considered, for the first time, *equality by symmetry*"—another case (note the flatness of the verb "considered") of the rhetoric of "revolution" under firm control.

I conclude that the use of "symmetry" in Legendre's new sense of "equality" does not warrant Hon and Goldstein's picture of a radical break with the past; on the contrary, I argue, that use presumes an established communal understanding. But what of our authors' perhaps more dramatic claim, of that definition's enormous impact on the future? Can we join them in seeing Legendre's move (p. 2) as "revolutionary in its implications for scientific advances after 1794"? To answer these questions we must of course see what claims are made for it. Here I should perhaps stress that I shall take for granted the usefulness of Legendre's definition *within geometry*—surely no one doubts that. The crucial issue is the wider reverberations.

Hon and Goldstein actually cast Legendre's supposed legacy in, so to say, several different lights. One of these seems at first sight impressive and important. Legendre, we are told (p. 241), "has recast symmetry in terms of an abstract mathematical relation"—which does sound (however vaguely) like the wave of the group-theoretic future. This characterization is said to follow from the fact that the two solids in his definition of "equality by symmetry" need not be viewed as parts of a single object—their symmetrical relationship is independent of their relative positions in space. In my opinion this grabs the wrong end of the stick: that the two solids *need not* be spatially related is unimportant, but that they *can* be viewed as mirror images is the key to Legendre's ingenious idea and appropriate terminology. But let us see what consequences follow from this move by Legendre toward "abstraction". So far as I can tell, Hon and Goldstein never do spell out its significance for the future; instead they gloss their use of the term—in a way that ends by astonishing. They quote at length D'Alembert's *Encyclopédie* article on geometry, written of course for a lay audience, which explains that mathematicians deal in abstractions from, idealizations of, the physical world. Legendre, they go on to say (p. 242), "worked in this framework": he applied his new concept of symmetry *only* within geometry, not to natural or to man-made objects. Thus his supposed breakthrough consisted in doing this piece of his mathematics in precisely the way that everybody had done mathematics since antiquity—such at least is my reading of the authors' conclusion here. But perhaps I am missing something.

Another formulation of Legendre's achievement and influence, which indeed the authors say (p. 48) is their "principal claim" for him, may also raise some readers' eyebrows. "Legendre's definition of symmetry", they say, "is unprecedented: the concept is defined as a relation, not as a property". I read this statement with a double sense of surprise. As the stuff of "revolution" it seems pretty tame; but, more to the point, surely it is not even *true*—have the authors not told us that in antiquity "symmetry" meant (in mathematics) commensurability, a relation between two magnitudes? The importance of this distinction between property and relation is a recurrent theme of the book, but at the risk of seeming terminally obtuse I have to confess that, try as I may, I cannot see the point of it. Cannot the bilateral symmetry of the human body be viewed under either light—as a property of the whole or as a relation between the two sides? But however that may be, the supposed importance of Legendre's viewing symmetry as a relation turns out to sit uneasily with the authors' other (and seemingly much more significant) proclamation of his legacy, namely that (p. 2) his definition "marks the watershed in the history of the scientific concept of symmetry". As I reported earlier, Hon and Goldstein take this concept to be characterized by the application of group theory—and they urge repeatedly (pp. 2, 51, 59, 65), that *there* the distinction between property and relation is unimportant. Given that denouement, the assigning of lasting value to Legendre's stance would seem a tough sell.

So perhaps his "far-reaching" influence on the future lay in another direction entirely. The authors say (pp. 48–49) that his "definition" appears "revolutionary" in the light of the facts that (i) in its immediate wake "symmetry" was used in "a variety of scientific domains", and (ii) in each of these a definition of symmetry was given. They offer a number of examples, including the work of Monge on naval engineering (1788), Biot on analytic geometry (1802), astronomy (1805), and optics (1806), Poinsot on statics (1803), and more. Setting aside the fact that Monge's work actually preceded Legendre's *Éléments* by six years, what evidence buttresses Hon and Goldstein's conjecture of influence on the others? I can detect none. Legendre's definition, as we saw, bears on three-dimensional mirror-image symmetry, whereas in all the cases just cited the symmetries are two-dimensional and therefore much simpler. But perhaps Legendre was an example, an inspiration? No hint of that is cited by the authors—no text that they quote mentions Legendre. In the end they are reduced (p. 299) to the "conjecture [!] that Legendre's usage may [!] have made it easier for others to invoke symmetry in scientific contexts". The bold pronouncement of "pathbreaking" impact on the future seems here rather muted.

Indeed if we ask where, in fact, the likes of Monge, Biot, and Poinsot got their impulse to invoke symmetry in their respective investigations the answer seems clear—and it has nothing at all to do with Legendre. Hon and Goldstein themselves provide the clue. They say of Biot (p. 275) that in discussing elliptical orbits he "introduced the term, *symmetry*, without explanation, expecting the reader to be acquainted with this usage"—precisely, it turns out, as all his colleagues did, each in his own domain. In other words they proceeded exactly as I have argued that Legendre proceeded in the definition of which the authors make so much. Viewing matters in this light has the magical effect of banishing at one stroke several apparent puzzles over which Hon and Goldstein furrow their brows (pp. 60–62, 292): that (i) despite their earlier claim that definitions of symmetry proliferated immediately after Legendre, people were in fact "reluctant" to give such definitions; (ii) nobody seems to have tried to *compare* the definitions at work in different contexts; and (iii) although these mathematicians knew one another, history records no "personal exchanges" about symmetry among them. It seems to me that motivations for all these activities disappear if we merely suppose that each of the scientists cited by Hon and Goldstein *saw* symmetry in the configuration he was studying, took it as wholly unproblematic, and exploited it accordingly. Surely symmetry was for these people just a means to an end, used at informal, intuitive levels that cared nothing for the history and nuances of words—and needed not the example set by Legendre. The authors carry out for each of them an elaborate textual analysis, valuable in its own right; but perhaps their obsession with words and their explicit banning of psychological considerations costs them some perspective. Thus (p. 267) they reproduce from Pierre Bouguer's 1746 treatise on naval architecture a schematic horizontal section of a ship, visibly symmetric about a central line from bow to stern, and they comment that

> The figure is striking, for we can immediately discern the bilaterally symmetrical arrangement of the two sides of the figure. But Bouguer, as noted, did not invoke the term, *symmetry*. Hence, the usage of *symmetry* in its bilateral sense cannot be ascribed to Bouguer.

To this one wants to respond, "No doubt; and no doubt the issue is important; but in the last analysis he just *looked at* the diagram and *saw* what he needed to see".

One further aspect of Hon and Goldstein's discussion of the aftermath of Legendre's "definition" adds perhaps an element of poignancy to their story. They expound (pp. 286ff) Lacroix's definition (1797) of "symmetric functions" (*functions symétriques*) as one more piece of evidence that applications of symmetry took off after Legendre. They quote (p. 287) Lacroix's assertion that a symmetric function remains invariant under permutation of its roots— apparently without realizing that here they stand on the edge of the group-theoretic treatment of symmetry that elsewhere looms so large for them. The hero's mantle that they reserve for Legendre would fit Lacroix much better.

If I am right in some of these reservations about their main thesis then Hon and Goldstein may wish to tinker accordingly should their book enjoy a second edition. And given that opportunity the publisher would do well to try to rectify some failings of another kind, blemishes in the nitty-gritty aspects of book-making. The authors' prose, to start with that, is generally competent, but it never soars to the sky, and sometimes it bogs down badly in the mud. The sentences on Bouguer which I quoted two paragraphs back can serve as a prime example; for another, take the passage (p. 9) in which the authors say, "The reason that motivates this exclusion concerns our belief ..." where they could have said, "We make this exclusion because we believe ...". At such places one fears that Springer's editors must simply have nodded off. The book is well organized, and each chapter ends with a very useful "Conclusion" (which however would be better called a "Summary"). Unfortunately this has the consequence that most themes are discussed in at least three places (the long introductory section, the relevant chapter, and that chapter's "Conclusion"), and often a reader who wants the authors' full view of a particular issue must piece together fragments from all of these sources. This multiplicity also means that the text can seem very repetitive—key points seem to be made over and over again. An extreme case of repetition occurs on page 153, where an entire paragraph resurfaces almost verbatim after already gracing page 54. The scattering of discussions of a given topic makes more regrettable the fact that the book's index, while reasonably good, has limitations. It failed the only two specific tests that I put to it: although "Legendre, influence of" and "Symmetry, modern scientific concept of" are central among the authors' concerns, the index provides no steer to either. Perhaps less pardonable than any of these flaws are editorial lapses—some careless, some apparently stemming from ignorance. Two consecutive lines on page 296 exhibit respectively the non-use and the correct use of the convention for distinguishing in print between a word and the thing which the word signifies. Commas are missing at points where they should appear, and (more commonly) present where they should not be. In this last respect one sin stands out: the book's theme ensures that the construction "the term, *symmetry*, ..." appears very often, and eventually those unnecessary commas exasperate. Amusingly, several occurrences of the construction *drop* the commas—and somehow the sky never falls.

A final judgment on this worthy, curious volume must balance two sharply contrasting perspectives: substantial virtues on the one hand, grave weaknesses on the other. I want to stress a third time that facts, ideas, people—giants of the stature of Archimedes, Newton, Leibniz, Montesquieu, Diderot, and more—crowd these pages in a profusion to which I have here done painfully scant justice. I believe, for reasons that I have tried to convey, that the authors' cherished core thesis—that Legendre gave a new "definition" of symmetry which had a "revolutionary" impact on later thought—is profoundly mistaken. But I also believe that, flaws notwithstanding, much in the authors' tale, especially its earlier chapters, will remain of great value, a lasting contribution to our understanding of the history of a concept (or rather, a word!) as elusive as it is important.

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