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A mathematical modeling technique with network flows for social welfare maximization in deregulated electricity markets

Ryo Hase, Norihiko Shinomiya*

Graduate School of Engineering, Soka University Tangi 1–236, Hachioji, Tokyo, 192–8577, Japan

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ABSTRACT

This paper presents a sequential solution method to discover efficient trades in an electricity market model. The market model represents deregulated electricity market consisting of four types of participants: independent power producers, retailers, public utilities, and consumers. Our model is based on graph theory, and the market participants are denoted by a network composes of three types of agents including sellers, buyers, and traders. The market participants have different capacity and demand of electricity from each other, and each electricity trade should satisfy the capacity and demand. Our sequential solution method can discover efficient electricity trades satisfying the constraints regarding capacity and demand by utilizing network flow. Simulation results demonstrate the efficiency of electricity trades determined by our method by examining social welfare, which is the total of payoffs of all market participants. Furthermore, the simulation results also indicate the allocation of payoff to each market participant.

1. Introduction

Over decades, structural change from regulated electricity markets to competitive electricity markets has emerged in many countries. For ensuring safe electricity supply first, regulated electricity markets with centralized structure in which there are a few suppliers have been developed. However, some issues on electricity prices in the markets have arisen due to the centralized structure [1,2]. First, consumers cannot realize whether current electricity prices are appropriate or not. Second, electricity suppliers in the centralized markets do not seem to reduce electricity prices. Thus, these issues have pushed for the deregulation of electricity markets.

Deregulated electricity markets have various types of participants compared to the regulated markets. For example, the participants are classified as public utilities, independent power producers, electricity retailers, and consumers [3]. These participants make the structure of markets more complicated. Hence, the previous method to examine the characteristics of the centralized markets cannot be applied to the deregulated markets. Nevertheless, characteristics regarding the deregulated markets should be examined carefully before deregulation; in fact, insufficient experiments for the deregulation caused California electricity crisis in 2001 [4].

1.1. Related works

Numerous studies have been conducted to consider modeling techniques for deregulated electricity markets [5,6]. Grine et al. present a multi-layer model to consider electricity prices with another energy commodity [7]. Triki et al. consider an optimal capacity allocation problem to maximize profits of electricity sellers [8]. Hussein et al. formulate an optimization problem for forecasting prices in day-ahead electricity markets [9]. Corchero et al. present a stochastic programming model for the Spanish electricity market [10].

Moreover, there are various studies on *efficiency*, which means the optimal allocation of the electricity with appropriate prices [11]. Efficiency can be measured by *social welfare* that is the sum of payoffs of all market participants [12]. Stern et al. consider the relation between market clearing price mechanisms and the maximization of social welfare in deregulated electricity markets [13]. Mechanism designs to maximize social welfare in double-sided electricity markets are presented in [14,15]. Nicolaisen et al. propose a price setting problem in a double-price auction for wholesale markets by implementing a reinforcement learning algorithm [16]. Swami considers social welfare maximization with consider-

* Corresponding author.
E-mail addresses: ryo_hase.0428@icloud.com (R. Hase), shinomi@ieee.org (N. Shinomiya).

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ing congestion of transmission lines [17]. As one of the research topics on market efficiency, *network market* based on graph theory is proposed in [18]. This model is considered to be more realistic than anonymous networks because actual trades occur between participants that can interact with each other. Since electricity trades also take place between participants connected by transmission lines, we proposed an algorithm to find optimal matchings in an electricity market model based on network market [19].

However, the previous works have not focused on electricity retailers, because the previous market models contain only suppliers and consumers, not a retailer. In [20], Babic notes advantages of an agent-based modeling technique for electricity retail markets; however, no characteristics regarding retailers are demonstrated in the paper. To model activities of retailers in network markets, Kleinberg et al. considers optimal price setting on a tripartite graph by utilizing a game theoretical approach [21]. However, the method cannot deal with a multi-unit commodity such as electricity because it is assumed that participants trade only a single-unit commodity. Besides, Nava introduces the competition model utilizing network flows in oligopolistic markets [22]. Even though the model of Nava can cope with participants including retailers dealing with multi-unit commodities, the role of each participant is eventually determined by equilibrium in the model. Hence, the model cannot be applied to electricity markets because the roles of participants in electricity markets are determined before equilibrium prices are discovered.

1.2. Contribution

This paper proposes a sequential solution method to determine prices and efficient trades in an electricity market model with electricity retailers. In this paper, we formulated a determination problem for efficient electricity trades on a model with electricity as a multi-unit commodity, not a single-unit commodity. To solve the problem, we constructed the sequential solution method to choose electricity trades in the market model. In our solution method, a price setting algorithm extended from a price setting mechanism proposed in [21]. Moreover, to determine electricity trades on the model, a determination problem is formulated by utilizing integer programming and unsplittable flow [23]. Simulation results demonstrate the characteristics in deregulated electricity markets about efficiency in terms of social welfare and payoff allocation of each market participant. Although the simulation results about efficiency are similar to the result presented in [24], the parameter conditions about capacity of electricity sellers are different in this paper. Moreover, the results regarding payoff allocation indicate that important factors for the payoff allocation are the market structure and the period of time having elapsed since the deregulation.

1.3. Paper structure

This paper is structured as follows. Section 2 proposes our electricity market model with agents. In Section 3, a price setting game on the model is introduced. Section 4 presents overall procedure to determine electricity trades. Section 5 explains market participants assigned to our model. Section 6 demonstrates simulation results, and Section 7 concludes this paper.

2. Electricity market model with agents

This section introduces our electricity market model based on graph theory. The model denotes three types of agents and electricity trades conducted between them.

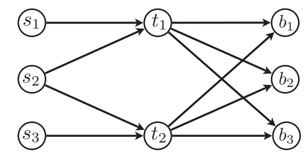


Fig. 1. Tripartite network G composed of agents.

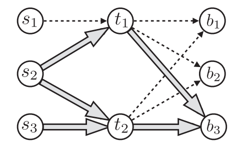


Fig. 2. Splittable flow.

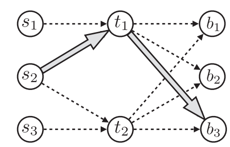


Fig. 3. Unsplittable flow.

2.1. Tripartite network representing electricity markets

Our electricity market model is represented by tripartite network $G = (S \cup B \cup T, A)$. Fig. 1 shows an example of G. G is composed of three types of agents: *buyer* $b_j \in B$, *seller* $s_i \in S$, and *trader* $t_k \in T$. Each arc indicates that agents at endpoints of the arc can conduct electricity trades between them. Arc set A contains arcs (s_i, t_k) or (t_k, b_j) due to following three constraints.

1. Each arc connects two agents not belonging to the same type of agents.
2. b_j must be provided electricity from t_k.
3. t_k must purchase electricity from s_i.

2.2. Notation of electricity flow on market model

In the model, each seller has a *capacity* of electricity, and each buyer has a *demand* of electricity. The capacity of s_i and the demand of b_j are denoted by c_i^s and d_j^b respectively. Let $c_{\min}$ be the minimum capacity of all c_i^s, and let $d_{\min}$ be the minimum demand of all b_j. Since a seller in our model can supply electricity to at least one buyer via a trader, $c_{\min}$ must satisfy $c_{\min} \geq d_{\min}$.

To denote electricity trades in the model, the notation of network flow is utilized. Integer $x(b, a)$ is the quantity of electricity flow on arc (b, a). Lower bound and upper bound of $x(b, a)$ are represented by $lb(b, a)$ and $ub(b, a)$, respectively. If $x(b, a) > 0$, electricity currents on (b, a); otherwise, there is no electricity flow on (b, a).

In addition, *unsplittable flow* is utilized to avoid determining complicated electricity trades satisfying the demand of a buyer. In Figs. 2 and 3, solid arcs represent electricity flow to satisfy d_3^b. Electricity flow in Fig. 2 is considered as *splittable flow*. The type of flow in Fig. 3 is called unsplittable flow. Only one $s_2 - b_3$ path is selected as unsplittable flow, and splittable flow adversely needs a larger number of arcs than unsplittable flow. Hence, our model uses unsplittable flow to determine electricity trades with a simple structure.

Unsplittable flow is realized by flow constraints on the model. s_i can supply electricity flow up to c_i^s. There is no electricity flow if s_i does not trade any electricity. Hence, $lb(s_i, t_k) = 0$, and $ub(s_i, t_k) = c_i^s$. Besides, b_j purchases d_j units of electricity, and flow constraints on (t_k, b_j) are denoted by $lb(t_k, b_j) = d_j^b$ and $ub(t_k, b_j) = d_j^b$. Fig. 4 indicates these capacity constraints.

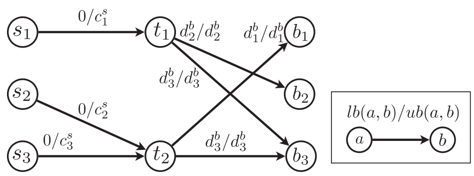


Fig. 4. Flow constraints on arcs in G.

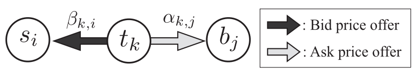


Fig. 5. Ask and bid prices.

3. Price setting game on market model

Agents in the model have property called valuation and payoff. The property is utilized in mechanisms for determining efficient electricity trades.

3.1. Valuation and trade value

Electricity prices offered by traders are determined by a price setting game. Each seller and buyer in the model has *valuation*, which describes the utility for trading one unit of electricity. v_j^b indicates the valuation of b_j for purchasing one unit of electricity. v_i^s denotes the valuation of s_i for supplying one unit of electricity. The sets of each valuation are defined by

$$\mathbf{v_s} = \left\{v_i^s \mid s_i \in S, v_i^s > 0\right\}, \mathbf{v_b} = \left\{v_j^b \mid b_j \in B, v_j^b > 0\right\}.$$

Electricity trades occur between seller s_i and buyer b_j via trader t_k. About the electricity trades among them, each agent obtains *payoff*. The total of the payoff regarding the trade is called *trade value*. Since we assume that costs for supplying electricity through arcs between s_i and b_j are zero regardless of t_k, trade value for d_j^b units of electricity is described by

$$w_{i,j} = \left(v_j^b - v_i^s\right)d_j^b. \tag{1}$$

When t_k conducts a trade between b_j and s_i, t_k has its own strategy denoted by $(\alpha_{k,\,j}, \beta_{k,\,i})$. This strategy consists of two types of prices called *ask price* $\alpha_{k,\,j}$ and *bid price* $\beta_{k,\,i}$. t_k offers $\alpha_{k,\,j}$ to b_j adjacent to t_k. Besides, $\beta_{k,\,i}$ is offered to s_i adjacent to t_k. In Fig. 5, t_k offers $\alpha_{k,\,j}$ to b_j and $\beta_{k,\,i}$ to s_i. Since there will be traders who lost money if $\beta_{k,\,i} > \alpha_{k,\,j}$, the strategy of t_k must be a *no-crossing strategy* [25] represented by $\beta_{k,\,i} \le \alpha_{k,\,j}$.

3.2. Payoff of each participant

b_j must purchase d_j^b units of electricity from one of the traders to satisfy its demand. The total payoff of b_j for purchasing electricity from t_k is represented by

$$P\left(b_j\right) = \left(v_j^b - \alpha_{k,j}\right)d_j^b. \tag{2}$$

About sellers, s_i is offered $\beta_{k,\,i}$ by t_k. To provide electricity, s_i will choose t_k offering $\beta_{k,\,i}$ that maximizes payoff of s_i. Payoff of s_i for supplying d_j^b units of electricity to b_j through t_k is denoted by

$$p\left(s_i, (k, j)\right) = \left(\beta_{k,i} - v_i^s\right)d_j^b. \tag{3}$$

s_i can provide electricity to one or more buyers if the total of demands do not exceed c_i^s. Therefore, s_i can obtain the total payoff represented by

$$P(s_i) = \sum_{(t_k, b_j)\in pair(s_i)} p\left(s_i, (k, j)\right), \tag{4}$$

where $pair(s_i)$ denotes the set of pairs of b_j and t_k provided electricity from s_i.

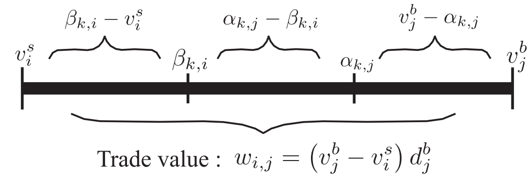


Fig. 6. Trade value and payoffs of agents.

t_k obtains payoff for trading d_j^b units of electricity between s_i and b_j, which is denoted by

$$p\left(t_k, (i, j)\right) = \left(\alpha_{k,j} - \beta_{k,i}\right)d_j^b. \tag{5}$$

Let $S(t_k)$ be the set of s_i adjacent to t_k, and let $B(t_k)$ be the set of b_j adjacent to t_k. The total payoff of t_k is represented by

$$P(t_k) = \sum_{s_i\in S(t_k), b_j\in B(t_k)} p\left(t_k, (i, j)\right) x_{k,i,j}. \tag{6}$$

Fig. 6 shows the relation between the trade value and payoffs of s_i, t_k, and b_j.

4. Procedure to determine electricity trades

To determine electricity trades on the model, we propose a sequential solution method. The method firstly calculates equilibrium electricity prices, and then the method determines electricity trades by using the prices.

4.1. Price setting algorithm

Algorithm 1 shows a price setting algorithm that calculates equilibrium prices. In this algorithm, each seller and buyer discovers its maximum payoff for trading electricity by considering the

Algorithm 1 Price setting $(G, \mathbf{v_s}, \mathbf{v_b}, \mu)$.

for $j \leftarrow 1$ to $|B|$ **do**
  if $|adj(b_j)| = 1$ **then**
    $q(b_j) = 0$.
  else
    Find $\hat{s}_i$ with the minimum valuation $\hat{v}_i^s$.
    $q(b_j) = (v_j^b - \hat{v}_i^s)\mu$.
    while $q(b_j) = (v_j^b - \acute{v}_i^s)\mu$ $(\acute{s}_i \ne \hat{s}_i, \acute{v}_i^s \ge \hat{v}_i^s)$ **do**
      Decrease $q(b_j)$.
    end while
  end if
end for
for $i \leftarrow 1$ to $|S|$ **do**
  if $|adj(s_i)| = 1$ **then**
    $q(s_i) = 0$.
  else
    Find $\hat{b}_j$ with the maximum valuation $\hat{v}_j^b$.
    $q(s_i) = (\hat{v}_j^b - v_i^s)\mu$.
    while $q(s_i) = (\acute{v}_j^b - v_i^s)\mu$ $(\acute{b}_j \ne \hat{b}_j, \acute{v}_j^b \le \hat{v}_j^b)$ **do**
      Increase $q(s_i)$.
    end while
  end if
end for
for $k \leftarrow 1$ to $|T|$ **do**
  return $\alpha_{k,j} = v_j^b - q(b_j)$ $(b_j \in B(t_k))$.
  return $\beta_{k,i} = v_i^s + q(s_i)$ $(s_i \in S(t_k))$.
end for

valuation of the other agents. This algorithm is based on price setting mechanism explained in [21]. We extend the algorithm to adjust payoffs of participants for ensuring no participant exclusively obtains larger payoff than other agents.

The process of Algorithm 1 is as follows. First, each buyer $b_j \in B$ calculates $q(b_j)$, which is the payoff of b_j for trading one unit of electricity. $q(b_j)$ is used to obtain ask prices $\alpha_{k,\,j}$. For all $a \in S \cup B$, let $adj(a)$ be the set of t_k connected to a. If $|adj(b_j) = 1|$, there is only one trader t_k adjacent to b_j. There is no competition between b_j and t_k in this case, and thus $q(b_j) = 0$. If $|adj(b_j) > 1|$, two or more traders t_k are connected to b_j. Let $\hat{s}_i$ be a seller adjacent to $t_k \in adj(b_j)$. $\hat{s}_i$ has the valuation $\hat{v}_i^s$ that is the minimum in the valuation of sellers adjacent to $t_k \in adj(b_j)$. In our price setting algorithm, a real number $\mu(0 < \mu \leq 0.5)$ is incorporated into the algorithm as a parameter that is used to adjust payoffs of sellers and buyers. The range of μ is set to realize the no-crossing strategy, explained in Section 3.1. With this notation, $q(b_j)$ is firstly set as $q(b_j) = (v_j^b - \hat{v}_i^s)\mu$. Then, $q(b_j)$ is decreased until $q(b_j)$ becomes equal to $(v_j^b - \acute{v}_i^s)\mu$, where $\acute{s}_i \neq \hat{s}_i$ is one of the sellers adjacent to $t_k \in adj(b_j)$ and has valuation $\acute{v}_i^s \geq \hat{v}_i^s$.

Second, each seller $s_i \in S$ determines $q(s_i)$, which is the payoff of s_i for trading one unit of electricity. By setting $q(s_i)$, bid prices $\beta_{k,\,i}$ can be determined. The process to set $q(s_i)$ is similar to the process to calculate $q(b_j)$. If $|adj(s_i) = 1|$, there is no competition between s_i and t_k, and $q(s_i) = 0$. If $|adj(s_i) > 1|$, let $\hat{b_j}$ be a seller adjacent to $t_k \in adj(s_i)$, and $\hat{b_j}$ has the valuation $\hat{v}_j^b$ that is the maximum in the valuation of buyers adjacent to $t_k \in adj(s_i)$. $q(s_i)$ is firstly set as $q(s_i) = (\hat{v}_j^b - v_i^s)\mu$. Then, $q(s_i)$ is increased until $q(s_i)$ becomes equal to $(\acute{v}_j^b - v_i^s)\mu$, where $\acute{b_j} \neq \hat{b_j}$ is one of the buyers adjacent to $t_k \in adj(s_i)$ and has valuation $\acute{v}_j^b \leq \hat{v}_j^b$.

Finally, ask price $\alpha_{k,\,j}$ and bid price $\beta_{k,\,i}$ of each trader t_k are set based on $q(b_j)$ and $q(s_i)$ for all b_j and s_i. These ask and bid prices finally determined are equilibrium prices on the market model.

4.2. Optimization problem for trade determination

4.2.1. Maximization of payoff for each trader

To determine electricity trades, each trader solves a maximization problem of payoff by using prices calculated by Algorithm 1. In this problem, trades are greedily chosen by each trader to maximize its payoff. This problem is formulated as an integer program similar to the generalized assignment problem [26].

The maximization problem is formulated as follows. Let $x_{k,\,i,\,j} \in [0,\,1]$ denote the electricity trades on $s_i - b_j$ path via t_k. $x_{k,i,j} = 1$ means electricity trades are conducted on the $s_i - b_j$ path. Adversely, trades on the $s_i - b_j$ path is not conducted if $x_{k,i,j} = 0$. The condition of network flow on the model determines $x_{k,\,i,\,j}$, and the condition is represented by

$$x_{k,i,j} = \begin{cases} 1 & (x(s_i, t_k) > 0 \ \cap \ x(t_k, b_j) > 0), \\ 0 & (x(s_i, t_k) = 0 \ \cup \ x(t_k, b_j) = 0). \end{cases}$$

By using $x_{k,\,i,\,j}$ and $p(t_k,\,(i,\,j))$, the maximization problem of t_k is described as the following integer program.

$$\max P(t_k) = \sum_{\substack{s_i \in S(t_k),\, b_j \in B(t_k) \\ 0 \leq x_{k,i,j} \leq 1, \\ p(t_k,\,(i,\,j)) \geq 0, \\ \sum_{b_j \in B(t_k)} x_{k,i,j}\ d_j^b \leq c_i^s.}} p(t_k,\,(i,\,j)) x_{k,i,j}. \tag{7}$$

In (7), $x_{k,i,j} = 1$ means trades on $s_i - b_j$ path are selected to maximize payoff of t_k.

4.2.2. Trades satisfying all capacity and demand

Even though every trader determines all trades by (7), demands for some sellers might exceed their own capacity. This situation means some consumers cannot purchase electricity from a seller who does not have enough capacity to satisfy all demands. Hence, electricity trades satisfying all capacity and demand should be independently chosen from the electricity trades which each trader selected by solving (7). To choose the electricity trades considering capacity, a maximization problem of social welfare is utilized in this paper.

To describe the maximization problem, a maximum unsplittable flow problem is utilized. In the problem, bipartite network $G_{\rm bi} = (S \cup B, A_{\rm bi})$ is constructed. Arc set $A_{\rm bi}$ corresponds to the set of possible trades x_t. Thus, for all $t_k \in T$, $A_{\rm bi}$ contains $(s_i,\, b_j)$ if $x_{k,i,j} = 1$ in (7). The capacity of flow on $(s_i,\, b_j)$ is denoted by $0 \leq x_{i,\,j} \leq 1.s_i$ can supply flow up to c_i^s, and demand of flow of b_j is d_j^b. Finally, the following integer program gives $W(x_t)$ that is the maximum social welfare on $G_{\rm bi}$.

$$\max\ W(x_t) = \sum_{\substack{(s_i,\, b_j) \in A_{\rm bi} \\ x_{i,j} \leq 0, \\ \sum_{s_i \in adj(b_j)} x_{i,j} \leq 1, \\ \sum_{b_j \in adj(s_i)} x_{i,j}\ d_j^b \leq c_i^s.}} x_{i,j} w_{i,j}. \tag{8}$$

4.3. Overall procedure for trade determination

The overall procedure of our solution method to determine efficient trades is described in Procedure 1. First, equilibrium prices are calculated by Algorithm 1. Then, every trader discovers trades maximizing payoff of the trader. After that, efficient trades x_t will be determined in all trades that the traders want to conduct. Finally, social welfare $W(x_t)$ can be calculated.

Procedure 1 trade determination

Input: $G, \mathbf{v_s}, \mathbf{v_b}, \mu$.
Output: $W(x_t), x_t,$.

Four steps of the procedure:
1. Price setting $(G, \mathbf{v_s}, \mathbf{v_b}, \mu)$.
2. For all t_k, determine $x_{k,i,j}(s_i \in S(t_k), b_j \in B(t_k))$ by (7).
3. Construct $G_{\rm bi}$ by using $x_{k,i,j}(t_k \in T, s_i \in S, b_j \in B)$.
4. Obtain $W(x_t)$ and x_t by solving (8) with $G_{\rm bi}$.

5. Four types of participants assigned to model

By setting conditions of the network and agents in the model, four types of market participants can be considered. Those participants can be utilized to reveal characteristics of deregulated electricity markets.

5.1. Participants in deregulated electricity markets

In this paper, a day-ahead electricity market is focused on as deregulated electricity markets. Prices in the day-ahead markets are determined hourly or half hourly [27]. Participants supposed in this paper are classified into four types: the public utility, independent power producers, retailers, and consumers. The participants are described by agents explained in Section 2.1.

1. *Public utility* (*PU*) conducts electricity generation and supply. The PU is a large firm that has conducted generation and supply since the market was regulated. In the deregulated market, the PU can purchase electricity from another generator. The PU is denoted by a pair of seller and trader.

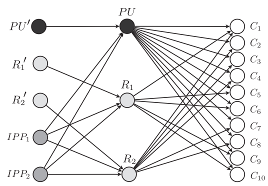


Fig. 7. $\text{prob}(\text{R}, \text{C}) = 0.2$.

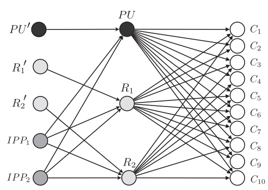


Fig. 8. $\text{prob}(\text{R}, \text{C}) = 0.8$.

2. *Retailer* (R) conducts electricity trades with consumers. A retailer is described by one pair of a trader and a seller if the retailer has its own generator. Otherwise, one trader denotes a retailer.
3. *Independent power producer* (*IPP*) has its own generator to sell its electricity to customers. An IPP is assigned to one of the sellers, and it can supply electricity to PU and retailers.
4. *Consumer* (C) is an end-user of electricity. A consumer purchases electricity from one of the best suppliers connected to the consumer. The consumer is assigned to one of the buyers.

5.2. Constraints on network structure

To describe market participants, network G is constructed with following constraints. G has only one PU, and PU is connected to all consumers in G since PU existed in an electricity market before deregulation. The market model contains some IPP and retailers that have newly joined the market after deregulation. For simplicity, the model contains the same number of IPP and retailers. Each IPP is connected to all retailers and PU.

After the deregulation, a consumer cannot choose a retailer if the consumer does not know the retailer. To describe this situation, R is connected to C with the probability represented by $prob(R, C) \in (0, 1]$. If $prob(R, C) = 1$, all arcs between retailers and a consumer are constructed. As the time has elapsed since the deregulation, each consumer will increase the number of retailers which the consumer know. Hence, long time has passed since the deregulation if $prob(R, C)$ is high. Figs. 7 and 8 show example models with $prob(R, C) = 0.2$ and $prob(R, C) = 0.8$, respectively. In these figures, R' and R denote seller s_i and trader t_k of a retailer respectively. Besides, PU' and PU represent s_i and t_k of PU respectively in the figures.

5.3. Constraints on parameters of participants

The capacity of newly joining participants, such as IPP and retailers, is relatively lower than the capacity of PU in deregulated electricity markets. The capacity of PU is enough to supply electricity to all consumers since PU was responsible for electricity supply before deregulation. In this paper, capacity of sellers is set according to the following constraint.

$$c_R \leq c_{IPP} \leq c_{PU}. \tag{9}$$

With regard to valuation of sellers, newly joining participants have lower valuation than PU in this paper. This condition means newly joining participants can offer low-cost electricity than PU. Besides, valuation of all buyers is fixed to the same value for simplicity. The valuation of buyers is enough to purchase electricity from any sellers. Following constraint indicates the conditions described above.

$$v_R < v_{IPP} < v_{PU} < v_C. \tag{10}$$

Table 1
Payoff of market participants and agents.

Participant	Agent	Payoff of participant
PU	s_i and t_k	$P(PU) = P(s_i) + P(t_k)$.
R	s_i and t_k	$P(R) = P(s_i) + P(t_k)$.
IPP	s_i	$P(IPP) = P(s_i)$.
C	b_j	$P(C) = P(b_j)$.

5.4. Metrics indicating characteristics of markets

To evaluate trades determined by our method, two kinds of the metrics are considered in this paper.

5.4.1. Efficiency rate

Although $W(x_t)$ is the maximum social welfare on G_{bi}, $W(x_t)$ does not necessarily correspond to $W(x)$ that is the upper bound of social welfare on G. $W(x)$ can be obtained by constructing bipartite network $G'_{\text{bi}} = (S \cup B, A'_{\text{bi}})$ from G. A'_{bi} is the set of possible trades x on G'_{bi}. If s_i can trade with b_j through at least one trader in G, arc set A'_{bi} contains an arc (s_i, b_j). With G'_{bi}, the following integer program gives $W(x)$.

$$\max \; W(x) = \sum_{\substack{(s_i, b_j) \in A'_{\text{bi}} \\ 0 \leq x_{ij} \leq 1, \\ w_{i,j} \geq 0, \\ \sum_{b_j \in B} x_{i,j} \; d^b_j \leq c^s_i.}} x_{i,j} w_{i,j}. \tag{11}$$

$W(x_t)$ is not necessarily the same as $W(x)$ since the calculation of $W(x)$ does not consider electricity price setting. However, the difference between $W(x_t)$ and $W(x)$ should be small to keep high efficiency. Hence, the performance regarding efficiency of our sequential method can be evaluated by comparing $W(x_t)$ with $W(x)$. The comparison between $W(x_t)$ and $W(x)$ can be conducted by examining *Efficiency Rate* (ER), such that

$$ER(x_t, x) = \{W(x_t)/W(x)\} \times 100 \ [\%]. \tag{12}$$

5.4.2. Payoff rate

In regulated electricity markets, PU is responsible for providing electricity to consumers. Hence, PU exclusively obtains payoff for supplying electricity. On the other hand, payoff for providing electricity is also allocated to newly joining market participants in deregulated electricity markets. Therefore, it is important to analyze who can acquire how much payoff to trade electricity in deregulated electricity markets.

In this paper, *Payoff Rate* (PR) is utilized to indicate the rate of payoff of each participant in social welfare. The payoff of each participant can be described by using the payoff of agents. Table 1 shows the relation between the payoff of each participant and agents. In Table 1, $P(a)$ denotes payoff of market participant $a \in (PU \cup R \cup IPP \cup C)$. PR for a is denoted by

$$PR(a) = \left\{P(a)/W(x_t)\right\} \times 100 \ [\%]. \tag{13}$$

6. Experimental results

This section demonstrates simulation results of our sequential solution method. After simulation conditions are introduced, results regarding efficiency rate and payoff rate are presented.

Table 2
Conditions of parameters in simulations.

Parameter	Assigned value
# of agents	$\|S\|$=5 or 7, $\|T\|$=3 or 5, $\|B\|$=10,15, or 30.
# of participants ($\|S\|$=5)	PU: 1, R: 2, IPP: 2, C : 10, 15, or 30.
# of participants ($\|S\|$=7)	PU: 1, R: 4, IPP: 4, C : 10, 15, or 30.
v_j^b	20 for all b_j.
v_i^s ($\|S\| = 5$)	$v_1^s = 10, v_2^s = 4, v_3^s = 3, v_4^s = 9, v_5^s = 8.$
v_i^s ($\|S\| = 7$)	$v_1^s = 10, v_2^s = 4, v_3^s = 3, v_4^s = 2, v_5^s = 9, v_6^s = 8, v_7^s = 7.$
d_j^b	$d_j^b = j.$
μ	0.25
$prob(R, C)$	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.
Iteration	100

Table 3
Capacity pattern ($|B| = 10, 15, 30$).

$\|B\|$	10				
CP	1	2	3	4	5
PU	55	55	55	55	55
IPP	3	6	13	27	31
R	1	2	6	18	69
$\|B\|$	15				
CP	1	2	3	4	5
PU	120	120	120	120	120
IPP	7	15	30	60	84
R	1	4	13	40	69
$\|B\|$	30				
CP	1	2	3	4	5
PU	465	465	465	465	465
IPP	29	58	116	232	328
R	5	17	51	155	268

6.1. Conditions

For conducting simulations of trade determination, a simulation software for our model was developed with Java and lp_solve, which is an integer programming solver. Simulation experiments were conducted with the condition described in Table 2. In the simulations, we assumed that each consumer has the unique demand of electricity. Hence, the index of each consumer is set as the fixed demand of the consumer since the index is unique to each consumer. Since the model structure depends on $prob(R, C)$, 100 times of iterations for every $prob(R, C)$ and CP were conducted.

In deregulated electricity markets, newly joining participants expands their capacity for electricity supply as the time elapses. Hence, different types of capacity of newly joining participants were set in the simulations. Table 3 shows Capacity Patterns (CP), which are the conditions of capacity of each seller. CP 1 indicates newly joining participants do not have large capacity because not a long period has elapsed since the start of deregulation. In CP 2, more periods of time have passed after the deregulation than CP 1, and the difference of capacity between market participants became smaller than CP 1. In CP 3, 4, and 5, newly joining participants got more capacity as the index of CP increases. For all CP, capacity of PU is equal to the total of all demands, and all consumer can purchase electricity from PU as the worst choice.

6.2. Results and discussion

6.2.1. Determined electricity trades

First, the result of one of the iterations is focused on to examine determined electricity trades. In this result, $prob(R, C) = 0.3$, and CP 3 was selected. Table 4 represents ask and bid prices determined by Algorithm 1. Regarding bid price $\beta_{k,i}$, all bid prices were more than valuation v_i^s. Hence, any trade with these bid prices gives $p_{s_i} \geq 0$ to s_i. Moreover, all ask price $\alpha_{k,j}$ were less than valuation v_j^b. Hence, any trade with these ask prices gives $p_{b_j} \geq 0$ to b_j.

Table 4
Ask and bid prices obtained in a simulation.

(s_i, t_k)	v_i^s	$\beta_{k,i}$	(t_k, b_j)	v_j^b	$\alpha_{k,j}$
(PU', PU)	10.00	10.00	(PU, C_1)	20.00	14.67
(R_1', R_1)	4.00	4.00	(PU, C_2)	20.00	16.00
(R_2', R_2)	3.00	3.00	(PU, C_3)	20.00	16.00
(IPP_1, PU)	9.00	12.67	(PU, C_4)	20.00	16.00
(IPP_1, R_1)	9.00	12.67	(PU, C_5)	20.00	16.00
(IPP_1, R_2)	9.00	12.67	(PU, C_6)	20.00	16.00
(IPP_2, PU)	8.00	12.00	(PU, C_7)	20.00	14.67
(IPP_2, R_1)	8.00	12.00	(PU, C_8)	20.00	16.00
(IPP_2, R_2)	8.00	12.00	(PU, C_9)	20.00	16.00
			(PU, C_{10})	20.00	16.00
(t_k, b_j)	v_j^b	$\alpha_{k,j}$	(t_k, b_j)	v_j^b	$\alpha_{k,j}$
(R_1, C_1)	20.00	14.67	(R_2, C_1)	20.00	14.67
(R_1, C_2)	20.00	16.00	(R_2, C_3)	20.00	16.00
(R_1, C_4)	20.00	16.00	(R_2, C_5)	20.00	16.00
(R_1, C_7)	20.00	14.67	(R_2, C_6)	20.00	16.00
(R_1, C_9)	20.00	16.00	(R_2, C_7)	20.00	14.67
(R_1, C_{10})	20.00	16.00	(R_2, C_8)	20.00	16.00

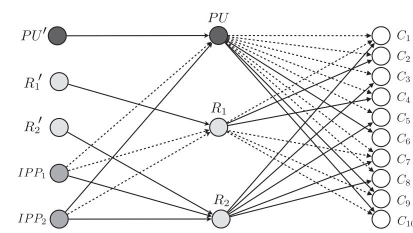


Fig. 9. Determined trades on *G*.

Table 5
$p(t_k, (i, j))$ and $w_{i,j}$ of each determined trade.

Determined trade	$p(t_k, (i, j))$	$w_{i,j}$	d_j^b	$w_{i,j}\ d_j^b$
$R_2' - R_2 - C_1$	11.67	17	1	17
$R_1' - R_1 - C_2$	12.00	16	2	32
$R_2' - R_2 - C_3$	13.00	17	3	51
$R_1' - R_1 - C_4$	12.00	16	4	64
$IPP_1 - R_2 - C_5$	3.33	11	5	55
$IPP_2 - PU - C_6$	4.0	12	6	72
$IPP_2 - R_2 - C_7$	2.67	12	7	84
$IPP_1 - R_2 - C_8$	3.33	11	8	88
$PU' - PU - C_9$	6.00	10	9	90
$PU' - PU - C_{10}$	6.00	10	10	100
	$W(x_t)$			653

Based on the prices shown in Table 4, Procedure 1 determined electricity trades x_t. In Fig. 9, solid arcs denote trades x_t, and dotted arcs show no electricity trades are conducted on the arcs. Table 5 shows $p(t_k, (i, j))$ and trade value $w_{i,j}$ for each determined trade. The first column at Table 5 represents determined trades described by the notation such as $s_i - t_k - b_j$. Since $p(t_k, (i, j)) \geq 0$ for all t_k, no-crossing trades were conducted in this simulation. In terms of social welfare, $W(x_t) = 653.0$, and $W(x) = 667.0$. Therefore, $ER(x_t, x) = 97.9\%$ in this example. More detailed analysis on $ER(x_t, x)\%$ is presented in Section 6.2.2.

Table 6 shows PR of each participant in the simulation. As shown in Table 6, no participant obtains zero payoffs and acquires a lot of payoffs exclusively. Allocation of PR of PU, IPP, and retailers varied in each iteration since the number of trades that each participant involves in each iteration is not the same as other iterations. In Section 6.2.3, PR is examined in more detail with other simulation conditions.

Table 6
Payoff rate of each participant.

Participant	PR [%]	Participant	PR [%]
C_1	0.82	C_6	3.68
C_2	1.23	C_7	5.72
C_3	1.84	C_8	4.90
C_4	2.45	C_9	5.51
C_5	3.06	C_{10}	6.13

Participant	PR [%]
PU	21.13
R_1	11.03
R_2	17.26
IPP_1	7.30
IPP_2	7.96

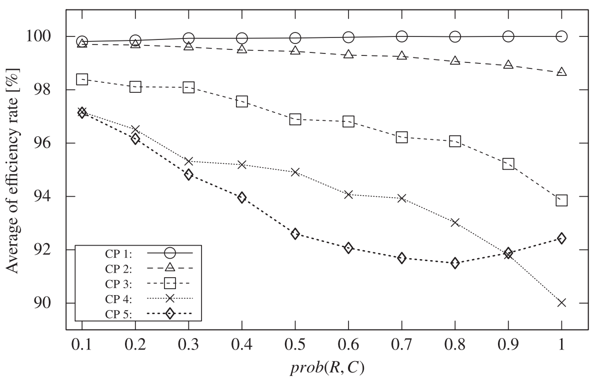


Fig. 10. Average of efficiency rate ($|S| = 5, |B| = 10$).

6.2.2. Analysis regarding efficiency rate

Our sequential solution method demonstrated high ER in the simulation. Fig. 10 shows the average of $ER(x_t, x)$ in the simulation with the model in which $|S| = 5$ and $|B| = 10$. In Fig. 10, the horizontal axis denotes $prob(R, C)$, and the vertical axis corresponds to the average of $ER(x_t, x)$ in the simulation. The average of $ER(x_t, x)$ was larger than 90% for all $prob(R, C)$ and CP. This result shows our solution method can be used to determine efficient trades of a multi-unit commodity such as electricity. Since the model in [21] dealing with only a single-unit commodity, our model can be used for other types of markets compared to [21].

For all CP, the average of ER became small as $prob(R, C)$ increased. The reason for this decline might relate to the number of possible trades on G. If $prob(R, C)$ is high, each trader has a larger number of possible trades on G. In this condition, however, demands of traders concentrate on participants providing inexpensive electricity. Hence, our method determines trades satisfying all capacity and demand, and this determined trades demonstrates relatively lower ER.

In terms of CP except for CP 5, high ER was obtained when the capacity of IPP and retailers is relatively lower than PU. On the other hand, ER became worse when the difference of capacity of participants became smaller. For CP 5, ER increased when $prob(R, C) \geq 0.9$. In CP 5, newly joining participants also have large capacity similar to PU. Hence, many consumers could purchase inexpensive electricity from newly joining participants in this condition.

6.2.3. Analysis regarding payoff rate

The relation between payoff allocation and structure of market network were examined by investigating PR of market participants. First, Fig. 11 shows simulation result about PR of each market participant for each CP. In Fig. 11, the horizontal axis corresponds to the name of each participant, and the vertical axis indicates PR of each participant. The parameters of G in this simulation were set to $|S| = 5, |B| = 10,$ and $prob(R, C) = 0.5$. Large part of PR was exclusively allocated to PU for CP 1. For CP 2, PR of the PU decreased, and PR of newly joining participants increased. Furthermore, for CP 3, PR was fairly allocated to market participants compared to other CP. However, large part of PR was adversely allocated to retailers for CP 4 and 5. Hence, if the valuation of market participants has been fixed to the same value, large part of PR is exclusively allocated to retailers obtaining enough capacity to supply electricity.

For examining the characteristics of PR with various structures of G, the standard deviation of PR was analyzed. The standard deviation of PR indicates whether payoff allocation is fair or not. For instance, large standard deviation means payoff is not allocated to participants evenly. Fig. 12 indicates the standard deviation of PR with $|S| = 5$. Each of Fig. 12 (a), (b), and (c) show the result with the different setting of $|B|$. The horizontal axis denotes $prob(R, C)$, and the vertical axis shows the average of the standard deviation of PR in the 100 iterations. In terms of CP, Fig. 12 indicates similar characteristics found in Fig. 11. The smallest standard deviation was obtained with CP 3, and the standard deviation became high with CP 1 and CP 5.

With regard to the axis denoting $prob(R, C)$, the number of buyers $|B|$ affected variation of the standard deviation. In Fig. 12(a), the standard deviation varies widely as $prob(R, C)$ increases. The variation of the standard deviation in Fig. 12(b) is smaller than that of Fig. 12(a). Besides, the standard deviation of Fig. 12(c) almost remains stable. Hence, the dispersion of the standard deviation of PR will become stable if the number of buyers increases.

The characteristics shown in Fig. 12 can also be found in Fig. 13. Since $|S| = 7$ in Fig. 13, the number of sellers is larger than that of Fig. 12. The standard deviation in Fig. 13 is smaller than that in Fig. 12. These results also indicate that the variation of PR will decrease if there is a large number of sellers in the markets.

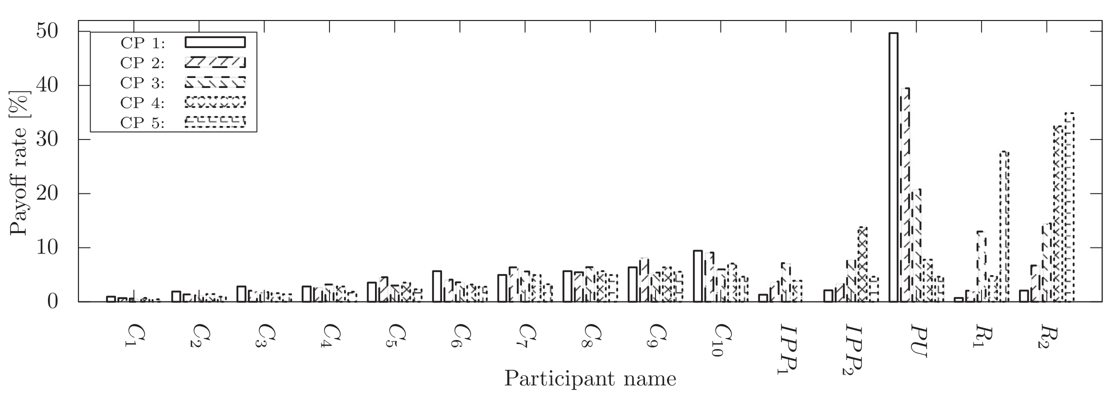
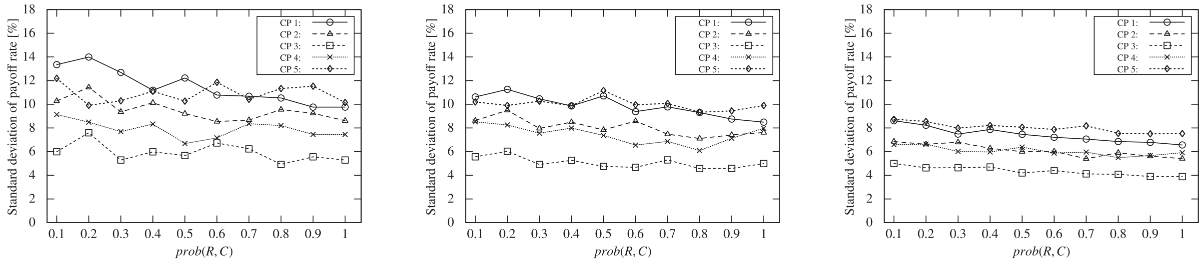
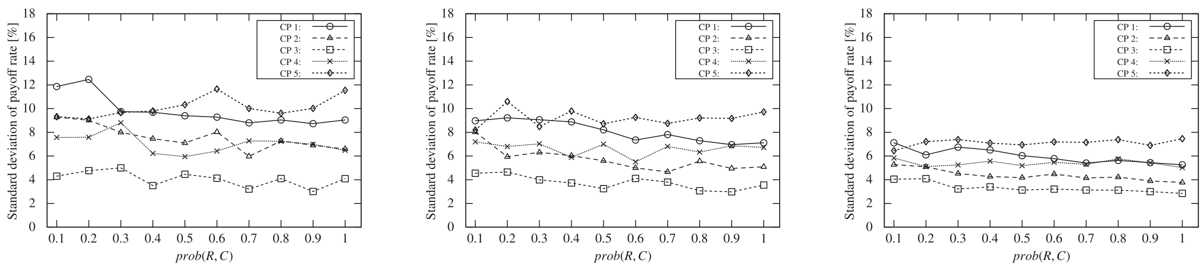


Fig. 11. Payoff rate of each participants with each CP.



(a) $|B| = 10$. (b) $|B| = 15$. (c) $|B| = 30$.

Fig. 12. Standard deviation of payoff rate ($|S| = 5$).



(a) $|B| = 10$. (b) $|B| = 15$. (c) $|B| = 30$.

Fig. 13. Standard deviation of payoff rate ($|S| = 7$).

7. Conclusion and future works

This paper proposed a market model and a sequential solution method to determine efficient electricity trades in deregulated electricity markets. By conducting simulation experiments, social welfare and payoff allocation on the electricity market model were investigated. As a result, our sequential solution method determined efficient electricity trades even though the efficiency depends on the structure of the markets and capacity of participants. Furthermore, the results also indicated that the payoff allocation for each market participant was affected by the period that have elapsed since the start of the deregulation.

To construct more realistic electricity market models, two kinds of tasks are left as future works. First, simulation experiments with various parameter conditions will reveal more interesting characteristics of deregulated electricity markets. Experiments with models containing a larger number of participants are especially needed since this paper only dealt with small models. Second, market participants are likely to decide their action in electricity markets by considering payoff obtained in previous experiences. Since our method cannot currently describe such kind of behavior of participants, a method to deal with the dynamic behavior of market participants should be integrated into our model.

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