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Effect of magnetic field on peristaltic flow of a fourth grade fluid in a tapered asymmetric channel

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Abstract The problem of peristaltic transport of an incompressible non-Newtonian fluid in a tapered asymmetric channel is debated under long-wavelength and low-Reynolds number assumptions. The fluid is considered to be fourth order and electrically conducting by a transverse magnetic field. The tapered asymmetry in the flow is induced by taking peristaltic wave imposed on the non-uniform boundary walls to possess different amplitudes and phase. Series solutions for stream function, axial velocity and pressure gradient are given using regular perturbation technique, when Deborah number is small. Numerical computations have been performed for the pressure rise per wavelength. Influences of different physical parameters entering into the problem have also been discussed in detail.

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1. Introduction

Peristaltic transport is a form of material transport induced by a progressive wave of contraction or expansion along the length of a distensible tube, mixing and transporting the fluid in the direction of the wave propagation. This kind of phenomenon is termed as peristalsis. It plays an indispensable role in transporting many physiological fluids in the body under various situations as urine transport form kidney to bladder, the movement of chyme in the gastrointestinal tracts, transport of spermatozoa in the ductus efferentes of the male reproductive tract, movement of ovum in the fallopian tubes, swallowing of food through esophagus and the vasomotion of small blood vessels. Many modern mechanical devices have been designed on the principle of peristaltic pumping to transport the fluids without internal moving parts. For example, the blood pump in the heart-lung machine and peristaltic transport of noxious fluid in nuclear industry. The mechanism of peristaltic transport has attracted the attention of many investigators since its investigation by Latham (1966), a number of analytical, numerical and experimental studies (Burns and Pareks, 1967; Shapiro et al., 1969; Fung and Yih, 1968; Shapiro et al., 1969; Takabatake and Ayukawa, 1982; Akram and Nadeem, 2013; Mekheimer and El Kot, 2008;
Mishra and Ramachandra Rao (2003) have discussed the peristaltic flow for different fluids and have reported under various conditions with reference to physiological and mechanical situations. Most of these investigations are confined to the peristaltic flow only in a symmetric channel or tube.

Recently, physiologists observed that the intra-uterine fluid flow due to myometrial contractions is peristaltic type motion and the myometrial contractions may occur in both symmetric and asymmetric directions. Eytan and Elad (1999) have established a mathematical model of wall induced peristaltic fluid flow in a two-dimensional channel with wave strains having a phase difference, moving independently on the upper and lower walls to stimulate intra-uterine fluid motion in a sagittal cross-section of the uterus. In another work, Eytan et al. (1999) pointed out that the width of the sagittal cross-section of the uterine cavity increases toward the fundus and the cavity is not fully occluded during the contractions. Pozrikidis (1987) has extensively studied the streamline pattern and mean flow rate due to different amplitudes and phases of the wall deformation. Mishra and Ramachandra Rao (2003) have discussed the peristaltic motion of viscous fluid in a two-dimensional asymmetric channel under assumptions of long-wavelength and low-Reynolds number in a wave frame of reference. Followed by the above works exhaustive studies have been made by many authors in the domain (Kothandapani and Srinivas, 2008a,b; Siddiqui and Schwarz, 1993; Hayat et al., 2003; El Hakeem et al., 2006; Hayat and Ali, 2006; Srinivas and Kothandapani, 2008). Moreover, Srinivas and Pushparaj (2007) have investigated the peristaltic pumping of MHD gravity flow of a viscous incompressible fluid in a two-dimensional asymmetric channel having electrically insulated walls.

It is well-known that the flow phenomena of non-Newtonian fluids arise in various Engineering, industrial and technological applications. In view of different physical structure and behavior of non-Newtonian fluids, there is no single mathematical expression which describes all the characteristics of non-Newtonian fluids. Hence, several mathematical models of non-Newtonian fluids such as micropolar fluid, power-law fluids, viscoelastic fluids etc. were developed by researchers. Moreover, the formulation of a constitutive equation of non-Newtonian fluids is greatly higher-order nonlinear and complex in nature. Among the differential-type non-Newtonian fluid models, the most generalized model is fourth grade fluid model which represents most of the non-Newtonian fluid properties at one time. One of the important features of fourth grade fluid is capable to exhibit normal stress differences in simple shear flows, leading to characteristic phenomena such as rod-climbing or die-swell. The main aim of this work is to analyze a tapered asymmetric wall-induced peristaltic motion of a fourth grade fluid with the presence of a uniform magnetic field. Mention in this contest to some recent interesting analytical, experimental and review studies pertaining to non-Newtonian fluids which may give valuable insights into their behaviour (Haynes, 1960; Joseph, 1980; Srivastava et al., 1983; Renardy, 1988; Vajravelu et al., 2013; Akram et al., 2013; Nadeem et al., 2014a,b; Riaz et al., 2014a,b; Umavathi and Mohite, 2014; Aohimere and Olajuwon, 2014; Kothandapani and Prakash, 2015b). Further, Haroun (2007) has also analyzed the problem of non-linear peristaltic flow of a fourth-grade fluid in an inclined channel. Moreover, Ali and Hayat (2007) have developed a mathematical model for the flow of an incompressible Carreau fluid in an asymmetric channel using a perturbation technique for a small Weissenberg number. Radhakrishnamacharya and Radhakrishna Murty (1993) discussed the problem of Heat transfer to peristaltic transport in a Non-uniform channel. Vajravelu et al. (2012) have developed a peristaltic transport of a Williamson fluid in asymmetric channels with permeable walls. They have reported a perturbation analysis with a Weissenberg number and elucidated that the pressure gradient decreases with increasing Weissenberg number.

More recently, Akram (2014) has scrutinized the effects of nanofluid on peristaltic flow of a Carreau fluid model in the presence of an inclined channel and magnetic field. The effects of permeable walls and magnetic field on the peristaltic flow of a Carreau fluid in a tapered asymmetric channel have been studied by Kothandapani et al. (2015b). The mathematical observations for the peristaltic flow of a Williamson fluid model in a cross-section of a rectangular duct having compliant walls was reported by Ellahi et al. (2013). The effects of magnetohydrodynamics on the peristaltic flow of Jeffrey fluid in a rectangular duct under the constraints of low Reynolds number and long wavelength have been discussed by Bhatti et al. (2014). Khan et al. (2014) have analyzed the peristaltic motion of Oldroyd fluid in an inclined asymmetric channel by regular perturbation method. Akram et al. (2014) studied the peristaltic flow of a couple stress fluid in a non-uniform rectangular duct. Akbar et al. (2015) considered the peristaltic flow in asymmetric channel with permeable wall. In their study, the influences of induced magnetic field, heat generation, heat flux water and CNTs nanofluid have been taken into account. The influence of walls attributes on the peristaltic transport in a three dimensional rectangular channel has been analyzed by Riaz et al. (2014). Kothandapani and Prakash (2015a) have studied the influences of inclined magnetic field, heat source, thermal radiation, and chemical reactions on peristaltic flow of a Newtonian nanofluid in a vertical generalized channel. They have also noted that the temperature profile increases with an increase of the non-uniform parameter, while it decreases when the thermal radiation parameter is increased. Akram and Nadeem (2014) have discussed the influence of nanofluid on peristaltic transport of a hyperbolic tangent fluid model under the effects of inclined magnetic field.

Further, it is worthwhile to mention here that the intrauterine fluid flow in a sagittal cross-section of the uterus discloses a narrow channel enclosed by two fairly parallel walls with wave trains having different amplitudes and phase difference (Eytan et al., 2001). With the aid of sufficient literature support, a theoretical investigation on the peristaltic motion of a fourth-grade fluid in a tapered channel or non-uniform asymmetric channel is carried out. To the best of our familiarity, so far no attempt has been made to examine the MHD Peristaltic transport of the fourth-grade in the tapered asymmetric channel which may help to imitate intra-uterine fluid motion in a sagittal cross-section of the uterus. The governing equations are solved under the long-wavelength and low-Reynolds number approximations. The stream function and the pressure gradient could be expanded in perturbation series in a small material parameter. The effects of emerging parameters on the axial velocity and pressure drop could be studied and the phenomenon of trapping is also discussable.
2. Mathematical formulation

Let us consider the MHD flow of an incompressible and electrically conducting fourth order-fluid through a two-dimensional tapered asymmetric channel. We assume that infinite wave train traveling with velocity $c$ along the non uniform walls. We choose a rectangular coordinate system for the channel with $X$ along the direction of wave propagation and parallel to the centerline and $Y$ transverse to it. The wall of the tapered asymmetric channel are given Fig. 1 by the equations

\[ \Phi(X, t) = \frac{2n}{a}(X - ct) \quad \text{upper wall}, \]

\[ \Phi(X, t) = -\frac{2n}{a}(X - ct) + \phi \quad \text{lower wall}, \]

where $a_1, a_2$ are the amplitudes of the waves, $\lambda$ is the wave length, $2d$ is the width of the channel at the inlet, $m'(m' < 1)$ is the non-uniform parameter, the phase difference $\phi$ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ represents to symmetric channel in which waves are out of phase and when $\phi = \pi$ the waves are in phase, and further $a_1, h_1, d$ and $\phi$ satisfies the condition

\[ a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \leq (2d)^2. \]

The Cauchy $\phi$ and extra $(S)$ stress tensors (Haynes, 1960) are given by

\[ \mathbf{T} = -\mathbf{P} + \mathbf{S}, \]

\[ \mathbf{S} = \mu \mathbf{A} + 2\mu \mathbf{S} + \beta_1 \mathbf{A} + \beta_3 (\mathbf{A} \mathbf{A} + \mathbf{A} \mathbf{A}^T) + \gamma_1 \mathbf{A} + \gamma_3 (\mathbf{A} \mathbf{A} + \mathbf{A} \mathbf{A}^T), \]

where $\mu$ is the constant viscosity and $\beta_1, \beta_3, \gamma_1 \text{ and } \gamma_3$ are the material constants. The Rivlin – Ericksen tensors $(\mathbf{A}_n)$ are given by

\[ \mathbf{A}_n = \frac{d^2 \mathbf{V}}{dt^2} + \frac{d \mathbf{V}}{dt} \mathbf{V} + \frac{d \mathbf{V}}{dt} \mathbf{V}^T, \]

\[ \mathbf{A}_0 = \frac{d \mathbf{V}}{dt} + \mathbf{V} \mathbf{V}^T, \quad n > 1. \]

The governing equations of the continuity and momentum equations for two-dimensional case are

\[ \frac{\partial \mathbf{U}}{\partial X} + \frac{\partial \mathbf{V}}{\partial Y} = 0, \]

\[ \rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} + \nabla \mathbf{V} \right) = -\frac{\partial \mathbf{P}}{\partial X} + \frac{\partial \mathbf{S}_{XX}}{\partial X} + \frac{\partial \mathbf{S}_{YY}}{\partial Y}, \]

\[ \rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{V} + \nabla \mathbf{U} \right) = -\frac{\partial \mathbf{P}}{\partial Y} + \frac{\partial \mathbf{S}_{XX}}{\partial X} + \frac{\partial \mathbf{S}_{YY}}{\partial Y}, \]

in which $\rho$ is the fluid density, $(\mathbf{U}, \mathbf{V})$ are velocity components in the direction of the laboratory frame $(X, Y)$, $\sigma$ is the electrical conductivity and $B_0$ is the constant magnetic field.

We employ the following dimensionless variables in the governing equations of motion

\[ \begin{align*}
  x &= \frac{X}{\lambda}, & y &= \frac{Y}{d}, & t &= ct, & u &= \frac{U}{c}, & v &= \frac{V}{c}, & h_1 &= \frac{h_1}{d}, \\
  h_2 &= \frac{h_2}{d}, & p &= \frac{d^2 \mathbf{P}}{c^2 d^2}, & a_1 &= \frac{a_1}{d}, & b &= \frac{a_2}{d}, \\
  S_{ij} &= \frac{d}{\mu} S_{ij}(\Phi); & (i,j) &= (1,2,3). 
\end{align*} \]

The stream function $\psi(x, y)$ is defined by

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \]

Eq. (6) is satisfied automatically and Eqs. (7) and (8) become

\[ \begin{align*}
  \delta R \left( \frac{\partial \psi_x}{\partial y} \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_y}{\partial x} \frac{\partial \psi_x}{\partial y} \right) &= -\frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} S_{xx} + \frac{\partial P}{\partial y} S_{yy} - M^2 \frac{\partial \psi_x}{\partial y}, \\
  -\delta^2 R \left( \frac{\partial \psi_x}{\partial y} \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_x}{\partial x} \frac{\partial \psi_y}{\partial y} \right) &= -\frac{\partial P}{\partial y} + \frac{\partial P}{\partial x} S_{xx} + \frac{\partial P}{\partial x} S_{yy},
\end{align*} \]

where $\rho$ is the pressure, $\mu$ is the viscosity and $S$ is the extra stress. In the above, the wave number $\delta$, the Hartmann number $M$, the Reynolds number $R$ and the material coefficients are defined respectively as

\[ \delta = \frac{2n \lambda}{a}, \quad M = \sqrt{\frac{\sigma B_0 d \lambda}{\mu}, \quad R = \frac{\rho c d^2}{\mu}, \quad \lambda_1 = \frac{2 \lambda c}{\mu d}, \quad \lambda_2 = \frac{2 \lambda c}{\mu d}, \quad \xi_1 = \frac{\beta_1 c^2}{\mu d}, \quad \xi_2 = \frac{\beta_3 c^2}{\mu d}, \]

\[ \zeta_3 = \frac{\gamma_1 c^3}{\mu d}, \quad \eta_1 = \frac{\gamma_3 c^3}{\mu d}, \quad \eta_2 = \frac{\gamma_3 c^3}{\mu d}, \quad \eta_3 = \frac{\gamma_3 c^3}{\mu d}, \quad \eta_4 = \frac{\gamma_5 c^3}{\mu d}, \quad \eta_5 = \frac{\gamma_5 c^3}{\mu d}, \quad \eta_6 = \frac{\gamma_5 c^3}{\mu d}. \]

We obtain, under the long wavelength and low Reynolds number approximations, the following expressions
\[
\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( S_{yy} - M^2 \frac{\partial^2 \psi}{\partial y^2} \right) = 0, \quad (14)
\]
\[
\frac{\partial p}{\partial y} = 0, \quad (15)
\]
\[
S_{yy} = \frac{\partial^2 \psi}{\partial y^2} + 2F \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3, \quad (16)
\]
in which \( \Gamma (= \xi_2 + \xi_3) \) is the Deborah number, \( \psi \) the stream function, \( \rho \) the fluid density, \( \mu \) the constant viscosity and continuity equation is always satisfied. Further, Eq. (15) indicates \( p \) is independent of \( y \). From Eqs. (14) and (15), we have
\[
\frac{\partial^2}{\partial y^2} \left( S_{yy} \right) - M^2 \frac{\partial^2 \psi}{\partial y^2} = 0. \quad (17)
\]

The appropriate boundary conditions in dimensionless form are
\[
\psi = \frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \quad \text{at} \quad y = h_2, \quad (18)
\]
\[
\psi = -\frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \quad \text{at} \quad y = h_1. \quad (19)
\]

We note that \( h_1(x, t) \) and \( h_2(x, t) \) represent the dimensionless form of the surfaces of the peristaltic walls
\[
h_2 = 1 + mx + b \sin(2\pi(x - t)) \quad \text{and} \quad h_1 = -1 - mx - a \sin(2\pi(x - t) + \phi).
\]

### 3. Method of solution

It is clear that the resulting equation of motion Eq. (17) is non-linear. It seems to be impossible to obtain the general solution

\[
\psi_0 = -\frac{F_0(M/2)(h_1 + h_2 - 2y) \sinh(M(h_1 - h_2))}{4 \sinh^2(M(h_1 - h_2))} \quad (20)
\]

\[
-\frac{M^2 }{16} \left( \cosh(3M(y))(A_2^1 + 3A_3 A_2^0) \right) + \frac{M^2 \sinh(3M(y))(B_2^1 + 3A_3 A_2^0)}{16} - \frac{3M^2 (A_1^1 - A_2^2 A_0^1) y \sinh(My)}{4} + \frac{3M^2 (A_0^1 - A_2^2 A_0^2) y \cosh(My)}{4},
\]

3.1. For the system of order zero

\[
\frac{\partial^2 \psi_0}{\partial y^2} - M^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0. \quad (21)
\]

\[
\frac{\partial}{\partial x} \left( \frac{\partial^2 \psi_0}{\partial y^2} - M^2 \frac{\partial^2 \psi_0}{\partial y^2} \right) = \frac{\partial^2}{\partial y^2} \left( \frac{\partial^2 \psi_0}{\partial y^2} - M^2 \frac{\partial^2 \psi_0}{\partial y^2} \right) = -2 \frac{\partial^2}{\partial y^2} \left( \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^3 \right). \quad (22)
\]

\[
\psi_0 = \frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0, \quad \text{at} \quad y = h_2, \quad (23)
\]

\[
\psi_0 = -\frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0, \quad \text{at} \quad y = h_1. \quad (24)
\]

3.2. For the system of order one

\[
\frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} = 0. \quad (25)
\]

\[
\frac{\partial}{\partial x} \left[ \frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} \right] = \frac{\partial}{\partial y} \left[ \frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} \right] + 2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 - M^2 \frac{\partial^2 \psi}{\partial y^2}, \quad (26)
\]

\[
\psi_1 = \frac{F_0}{2} \frac{\partial \psi_1}{\partial y} = 0, \quad \text{at} \quad y = h_2, \quad (27)
\]

\[
\psi_1 = -\frac{F_0}{2} \frac{\partial \psi_1}{\partial y} = 0, \quad \text{at} \quad y = h_1. \quad (28)
\]

3.3. Solution for system of order \( \Gamma^{(0)} \)

\[
3.4. \text{Solution for system of order } \Gamma^{(1)}
\]

\[
\psi_1 = A_1 + A_2 y + A_3 \cosh(My) + A_4 \sinh(My)
\]

\[
\frac{\partial}{\partial x} \left( \frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} \right) = \frac{\partial}{\partial y} \left( \frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{\partial^2}{\partial y^2} \left( \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 \right), \quad (29)
\]
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\[
\frac{dp_x}{dx} = A_1 M^3 \sinh(Mh_2) + A_4 M^2 \cosh(Mh_3) + A_9 M^2,
\]  
where

\[A_1 = \frac{-2\psi_b(Mh_2)}{4 \sinh^2(Mh_2/2) - \sinh^2(Mh_2)} + \frac{A_4 M^2 \cosh(Mh_3)}{A_9 M^2},\]

\[A_9 = \frac{A_1}{2} - \frac{M^4(3A^2 + 3A^2) \sinh(3Mh_3)}{16} + \frac{M^4(3A^2 + 3A^2) \sinh(3Mh_3)}{4} - \frac{M^3(3A^2 - A^2) \sinh(Mh_3)}{4} - \frac{M^3(3A^2 - A^2) \sinh(Mh_3)}{4},\]

\[A_{10} = \frac{-A_1}{2} - \frac{3M^2(B^2 + 3A^2) \sinh(3Mh_3)}{16} + \frac{3M^2(A^2 + 3A^2) \sinh(3Mh_3)}{4} + \frac{M^2(3A^2 - A^2) \sinh(Mh_3)}{16} - \frac{M^2(3A^2 - A^2) \sinh(Mh_3)}{4},\]

\[A_{11} = -M(Mh_1 - \sinh(Mh_2)),\]

\[A_{12} = -M(Mh_1 - \cosh(Mh_2)),\]

\[A_{13} = \cosh(Mh_3) - \cosh(Mh_2) + M(h_1 - h_2) \sinh(Mh_2),\]

\[A_{14} = \sinh(Mh_3) - \sinh(Mh_2) + M(h_1 - h_2) \cosh(Mh_2),\]

\[A_4 = \frac{k_{11}(A_{10} - A_9) - A_{11} A_2 + A_{11} A_3 - A_{11} A_4}{A_{12} A_{11} - A_{11} A_{12}},\]

\[A_5 = \frac{A_{13} + A_{10} - A_9}{A_{14} A_{13} - A_{11} A_{14}} - A_{11},\]

\[A_6 = -A_9 h_2 - A_4 \cosh(Mh_3) - A_9 \sinh(Mh_3) - A_5.\]

Solving the resulting zeroth and first order systems and then invoking

\[F = F_0 + \Gamma F_1,\]  

Summarizing the perturbation solutions up to first order for \(\psi, dp_x/dx\) and \(\Delta p\) as

\[\psi = \psi_0 + \Gamma \psi_1, \quad \frac{dp_x}{dx} = \frac{dp_0}{dx} + \Gamma \frac{dp_1}{dx}, \quad \Delta p = \Delta p_0 + \Gamma \Delta p_1.\]  

Using \(F_0 = F - o(\Gamma F_1)\) and then neglecting the terms greater than \(o(\Gamma)\) the results given by Eq. (32) can be explicitly calculated.

The non-dimensional expression for the average rise in pressure \(\Delta p\) is given as follows:

\[\Delta p = \int_0^1 \int_0^1 \frac{dp}{dx} dx dt.\]  

4. Results and discussion

In order to discuss the results quantitatively, we assume the instantaneous volume rate of the flow \(F(x, t)\), periodic in \((x - t)\) (Kothandapani and Prakash, 2015a; Kothandapani et al., 2015c; Srivastava et al., 1983) as

\[F(x, t) = \theta + a \sin [2\pi(x - t) + \phi] + b \sin 2\pi(x - t),\]  
in which \(\theta\) is the time-average of the flow over one period of the wave and

\[F = \int_{h_1}^{h_2} u dy.\]

To study the behavior of solutions, numerical calculations for several values of material parameter \((\Gamma)\), Hartmann number \((M)\), the phase difference \((\phi)\), the non uniform parameter \((m)\) and the amplitudes of upper and lower walls \((a_{dhb})\) have been carried out. The effect of these parameters on \(\Delta p\), the integral appearing in Eq. (46) has been evaluated numerically using mathematica and the results are presented graphically. Fig. 2 shows the variation of \(\Delta p\) against time mean flow rate \(\theta\). The whole region is considered into five parts: (i) Peristaltic pumping region where \(\Delta p > 0\) and \(\theta > 0\). (ii) augmented pumping region where \(\Delta p < 0\) and \(\theta > 0\). (iii) When \(\Delta p > 0\) and \(\theta < 0\), then it is a retrograde pumping region. (iv) There is a co-pumping region where \(\Delta p < 0\) and \(\theta < 0\). (v) \(\Delta p = 0\) corresponds to the free pumping region.

Fig. 2(a) elucidates the variation of average rise in pressure with mean flow rate \(\theta\) for different values of Hartmann number \((M)\). Due to the influence of magnetic parameter, the relation between \(\theta\) and \(\Delta p\) is a non-linear in retrograde and peristaltic pumping. The pumping rate found to increase with an increase in magnetic parameter. Fig. 2(b) shows the variation of \(\Delta p\) against dimensionless flow rate \(\theta\) for various values of Deborah number \((\Gamma)\). It indicates in respect of Newtonian fluid \((\Gamma = 0)\), the relation between average rise in pressure and mean flow rate is linear in character whereas for non-Newtonian fluid, it is non-linear and further peristaltic pumping increases with increases \((\Gamma)\). It has been observed from Fig. 2(c) that average rise in pressure increases with the increase in amplitude of the upper wall \((b)\). Fig. 2(d) is plotted to see the effect of average rise in pressure \(\Delta p\) against dimensionless flow rate \(\theta\) for various values of non-uniform parameter \((m)\). One could perceive that with an increase in non-uniform parameter \((m)\), the pumping decreases in the regions of peristaltic pumping, retrograde pumping and while in the co-pumping region the pumping found to increase. Fig. 2(f) reveals the average rise in pressure decreases for all the regions of study with an increase in phase difference.

Influences of geometric parameters on the velocity distribution have been illustrated in Fig. 3. The change in values of \((m)\) on the axial velocity \(u\) is shown in Fig. 3(a). It is interesting to note that an increase in \(m\) causes an increase in the
magnitude of $u$ at the boundaries. However, at the center of the channel the magnitude of $u$ gets decreased. A similar behavior is seen for the case of Hartmann parameter and it is projected in Fig. 3(b). From Fig. 3(c) it appears that the velocity profile traces a parabolic path. It increases with an increase of $C$ in the upper wall and converse in behavior is observed at the lower part of channel. Fig. 3(d) shows that with an increase in mean volume flow rate $h$, the axial velocity increases. The influence of the amplitude of the upper wall $b$ on the velocity is depicted in Fig. 3(e) for a fixed value of other parameters. It could be observed that an increase in the amplitude of upper wall $b$ increases the magnitude of the velocity in the upstream. The axial velocity for the phase angle $\phi$ is shown in Fig. 3(f). It has been observed that an increase in $\phi$ values causes as decrease in the magnitude of axial velocity $u$ at the lower wall.

The phenomenon of trapping is another interesting topic in peristaltic transport. The formation of an internally circulating bolus of fluid through closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave. The streamlines for different $a$ are shown in Fig. 4. It has been noticed that the bolus increases in the lower and upper of the tapered channel with increasing $a$. The impact of non-uniform parameter $m$ on trapping is shown in Fig. 5. It is examined that the size of the bolus increases with an increase in $m$. The streamlines for the different values of Hartmann number $M$ are plotted in Fig. 6 for the fixed values of all other parameters. One could observe that the volume of the bolus decreases with increasing $M$. Fig. 7 illustrates the variation of mean flow rate $\theta$ on trapping. The size of the trapping bolus appears to be increasing with an increase in $\theta$ and it is complementary when $\phi$ is increased as shown Fig. 8.

**Fig. 2** Variation of average rise in pressure $\Delta p$ versus $\theta$. 

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Fig. 3 Axial velocity profile $u(y)$.

Fig. 4 Streamlines for $b = 0.3, m = 0.2, \phi = \pi/2, \theta = 1.5, M = 1, \Gamma = 0.1, t = 0.5$, (a) $a = 0$, (b) $a = 0.3$. 

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Fig. 5  Streamlines for $a = 0.2, b = 0.1, \phi = \pi/3, \theta = 1.6, M = 0.5, \Gamma = 0.2, t = 0.5$, (a) $m = 0$, (b) $m = 0.3$.

Fig. 6  Streamlines for $a = 0.2, b = 0.1, \phi = \pi/4, \theta = 1.6, m = 0.2, \Gamma = 0.3, t = 0.5$, (a) $M = 0.1$, (b) $M = 3$.

Fig. 7  Streamlines for $a = 0.4, b = 0.2, \phi = \pi/3, M = 2.0, m = 0.3, \Gamma = 0.2, t = 0.5$, (a) $\theta = 1.6$, (b) $\theta = 1.7$. 

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5. Conclusion

A mathematical model has been developed to study the peristaltic transport of a fourth-grade fluid in the tapered asymmetric channel under the influence of magnetic field. The channel asymmetry is produced by choosing the peristaltic wave train on the non-uniform walls to have different amplitudes and phase. A long-wavelength and low-Reynolds number approximations are adopted. A regular perturbation method is employed to obtain the expression for stream function, axial velocity and pressure gradient. Numerical study has been conducted for the average rise in pressure over a wavelength. The effects of material parameter ($\Gamma$), Hartmann number ($M$), wave amplitudes, channel width and phase angle on the pressure rise, axial velocity and stream lines are also investigated in detail. The following observations are made in the present study.

- The average rise in pressure increases with increase of $M$, $\Gamma$ and $b$ while it decreases by increasing $m$ and $\phi$.
- The average rise in pressure per wavelength in respect of fourth-grade fluid is higher as compared with that of Newtonian fluid.
- The axial velocity decreases with increase of $M$ and $m$.
- The velocity profile increases with an increasing $\Gamma$ in the upper part of the channel. However, the converse behavior is noticed in the case of lower part of the tapered asymmetric channel.
- The size of trapped bolus decreases with an increase in Hartmann number.
- Finally, the results of Haroun (2007) in the absence inclined parameter can be captured as a special case of our analysis by taking $M = 0$ and $m = 0$.

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References


