



Lorentz invariance relations between parton distributions and the Wandzura–Wilczek approximation

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ABSTRACT

The violation of the so-called Lorentz invariance relations between parton distribution functions is considered in a model independent way. It is shown that these relations are not violated in a generalized Wandzura–Wilczek approximation, indicating that numerically their violation may be small.

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1. Introduction

Parton distribution functions (PDFs) which are of higher twist and/or which are transverse momentum dependent (\mathbf{p}_T -dependent) contain important information on the partonic structure of the nucleon being complementary to that encoded in the usual twist-2 distributions. These PDFs also become more important because of the increasing accuracy of recent and planned high energy scattering experiments. The forward twist-3 PDFs are accessible through certain spin asymmetries in polarized inclusive deep inelastic lepton nucleon scattering (DIS) and Drell–Yan processes (integrated upon the transverse momentum of the dilepton pair) [1–8]. On the other hand the \mathbf{p}_T -dependent PDFs typically give rise to spin and azimuthal asymmetries in, for instance, semi-inclusive DIS [9–13] and Drell–Yan [14–17], and significant effort has already been devoted to measure such observables [18–30].

Several relations between (forward) twist-3 and (moments of) \mathbf{p}_T -dependent PDFs have been proposed in the literature [2,9,10]. The derivation of these relations is based upon the general, Lorentz invariant decomposition of the fully unintegrated correlator of two quark-fields, where the fields are located at arbitrary space-time positions. These so-called Lorentz invariance relations (LIRs) impose important constraints on the PDFs which may allow one to eliminate unknown PDFs in favor of the known ones whenever applicable. However, in Refs. [31,32] it was demonstrated by an explicit model calculation that two specific LIRs are actually violated. In [33] it was shown in a model independent way that the violation can be traced back to the path-ordered exponential in

the unintegrated quark–quark correlator. For completeness we will repeat the argument below.

In the present Letter we address the question to what extent the LIRs may be violated *numerically*. To this end it is outlined in a model independent way that the LIRs are actually not violated in a generalized Wandzura–Wilczek approximation where one systematically neglects certain quark–gluon–quark correlations as well as current quark mass terms. This result indicates that the numerical violation of the LIRs may be rather small and could even be neglected in special cases.

2. Lorentz invariance relations and their violation

In order to discuss the LIRs and their violation we start with the fully unintegrated quark–quark correlation function of a spin- $\frac{1}{2}$ hadron defined by¹

$$\begin{aligned} \Phi_{ij}(P, p, S|n_-) &= \int \frac{d^4\xi}{(2\pi)^4} e^{ip\cdot\xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n_-) \psi_i(\xi) | P, S \rangle. \end{aligned} \quad (1)$$

The target state is characterized by its four-momentum $P = P^+ n_+ + (M^2/2P^+) n_-$ and the covariant spin vector S ($P^2 = M^2$, $S^2 = -1$, $P \cdot S = 0$), with the two light-like vectors n_+ and n_- satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$. The momentum of the quark is denoted by p . The Wilson line $\mathcal{W}(0, \xi | n_-)$ ensures color gauge invariance of the correlator, where the specific path of the gauge link will be given below. The knowledge of the correlator in

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¹ Recent work on how the unintegrated quark–quark correlator may enter observables can be found in [34,35].

Eq. (1) is particularly useful for obtaining the general form of the \mathbf{p}_T -dependent correlator $\Phi(x, \mathbf{p}_T, S)$, which appears in the QCD-description of hard scattering processes like transverse momentum dependent semi-inclusive DIS and the Drell–Yan reaction. The connection between both objects is given by the relation

$$\Phi(x, \mathbf{p}_T, S) = \int dp^- \Phi(P, p, S|n_-), \quad (2)$$

with x defining the plus-momentum of the quark via $p^+ = xP^+$. The Wilson line in Eq. (1) can be fixed according to

$$\mathcal{W}(0, \xi|n_-) = [0, 0, \mathbf{0}_T; 0, \infty, \mathbf{0}_T] \times [0, \infty, \mathbf{0}_T; \xi^+, \infty, \xi_T] \\ \times [\xi^+, \infty, \xi_T; \xi^+, \xi^-, \xi_T], \quad (3)$$

where $[a^+, a^-, \mathbf{a}_T; b^+, b^-, \mathbf{b}_T]$ denotes a gauge link connecting the points $a^\mu = (a^+, a^-, \mathbf{a}_T)$ and $b^\mu = (b^+, b^-, \mathbf{b}_T)$ along a straight line. It is important to note that the Wilson contour in Eq. (3) not only depends on the coordinates of the initial and final points but also on the light-cone direction n_- . The path is chosen such that, upon integration over the minus-momentum of the quark, it leads to a proper definition of the \mathbf{p}_T -dependent correlator in (2) [36–38]. The choice of the contour depends on the process under consideration [39]. Here we restrict ourselves to the case of semi-inclusive DIS, but all arguments hold as well for other processes like Drell–Yan. It is also worthwhile to mention that Wilson lines that are near the light-cone rather than exactly light-like (as those in (3)) are generally more appropriate in connection with unintegrated parton correlation functions. (More details on this important issue can be found in the recent work [40] and references therein.) Our general reasoning here remains valid if one uses a near light-cone direction instead of n_- .

The general structure of the correlator in (1) was derived in [41],² and the result is also given in Eq. (28) in Appendix A. One ends up with 32 matrix structures multiplied by scalar functions that were denoted by A_i and B_i in [41]. In turn, the \mathbf{p}_T -dependent PDFs can be defined through Dirac traces of the \mathbf{p}_T -dependent correlator in (2) given by [9,12,41]

$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) \equiv \frac{1}{2} \text{Tr}(\Phi(x, \mathbf{p}_T, S) \Gamma). \quad (4)$$

The results containing all the twist-2 and twist-3 PDFs are repeated in Appendix A, Eqs. (29)–(37). On the basis of the relation (2) one can now express the \mathbf{p}_T -dependent PDFs through p^- -integrals upon the scalar functions A_i and B_i . The results are listed in Appendix A, Eqs. (40)–(63).

In total there exist 32 \mathbf{p}_T -dependent PDFs which exactly agrees with the number of independent amplitudes A_i and B_i . If one neglects the dependence on the light-cone vector n_- , which is induced by the Wilson line, the correlator (1) merely consists of 12 matrix structures – those which are multiplied by the functions A_i . In that case the number of \mathbf{p}_T -dependent PDFs is larger than the number of the amplitudes A_i . This feature, in particular, gives rise to LIRs between certain \mathbf{p}_T -integrated PDFs and (moments of) \mathbf{p}_T -dependent PDFs [2,9,10]. Here we list the most important four LIRs on which we focus in this work:

$$g_T(x) \stackrel{\text{LIR}}{=} g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x), \quad (5)$$

$$h_L(x) \stackrel{\text{LIR}}{=} h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x), \quad (6)$$

$$f_T(x) \stackrel{\text{LIR}}{=} -\frac{d}{dx} f_{1T}^{\perp(1)}(x), \quad (7)$$

$$h(x) \stackrel{\text{LIR}}{=} -\frac{d}{dx} h_1^{\perp(1)}(x), \quad (8)$$

with

$$g_{1T}^{(1)}(x) = \int d^2 \mathbf{p}_T \frac{\mathbf{p}_T^2}{2M^2} g_{1T}(x, \mathbf{p}_T^2), \quad \text{etc.}, \quad (9)$$

specifying certain \mathbf{p}_T -moments of the PDFs [9]. All PDFs on the *lhs* of Eqs. (5)–(8) are twist-3 functions, while those on the *rhs* are not suppressed in observables. The PDFs in Eqs. (5) and (6) are time-reversal even (T-even), and the ones in Eqs. (7) and (8) are T-odd. The derivation of the LIR (8) is sketched in Appendix A in Eqs. (64)–(68).

If, however, one takes into account the Wilson line (n_- -dependence) in the correlator (1) the LIRs (5)–(8) are no longer fulfilled, i.e., they are spoiled by the presence of the amplitudes B_i . The reasoning shows in a model independent way that in the context of a gauge theory, for which the Wilson line is mandatory, the LIRs are violated. This is the general message of Ref. [33].

3. Lorentz invariance relations in a generalized Wandzura–Wilczek approximation

Knowing that the LIRs are violated it is now natural to ask to what extent they are violated *numerically*. If one gets an indication that the violation of the LIRs should be small, these relations can still serve as a useful tool – at least for qualitative studies of the partonic nucleon structure. In what follows we consider the LIRs and their violation in a model independent way using a special approximation. We start with the discussion of the LIRs (5) and (6) which contain T-even PDFs.

First we recall the following relations between \mathbf{p}_T -integrated T-even PDFs [1,43]:

$$g_T(x) = \int_x^1 \frac{dy}{y} g_1(y) + \tilde{g}'_T(x), \quad (10)$$

$$h_L(x) = 2x \int_x^1 \frac{dy}{y^2} h_1(y) + \tilde{h}'_L(x), \quad (11)$$

where $\tilde{g}'_T(x)$ and $\tilde{h}'_L(x)$ denote (purely interaction dependent) quark–gluon–quark correlations and terms proportional to current quark masses. An explicit representation of these terms can be found, e.g., in [44] and partly also in [45]. Eqs. (10) and (11) isolate “pure twist-3 terms” in the PDFs $g_T(x)$ and $h_L(x)$. Here the underlying “working definition” of twist [46] (a PDF is of “twist t ” if its contribution to the cross section is suppressed, in addition to kinematic factors, by $1/Q^{t-2}$ with Q denoting the hard scale of the process) differs from the strict definition of twist (mass dimension of the operator minus its spin).

The remarkable experimental observation is that $\tilde{g}'_T(x)$ is consistent with zero within the error bars [3–8] and to good accuracy one has

$$g_T(x) \stackrel{\text{WW}}{\approx} \int_x^1 \frac{dy}{y} g_1(y), \quad (12)$$

which is the Wandzura–Wilczek (WW) approximation. Lattice QCD [47,48] and the instanton model of the QCD vacuum [49] support this observation. (Further discussions of the WW approximation in related and other contexts can be found in Refs. [50–56].) Interestingly, the latter predicts also $\tilde{h}'_L(x)$ to be small [57] such that

² For a spin-0 hadron see Refs. [33,42].

$$h_L(x) \stackrel{\text{WW}}{\approx} 2x \int_x^1 \frac{dy}{y^2} h_1(y). \quad (13)$$

An experimental test of this approximate relation does not yet exist. (In this context see also the recent theoretical study in Ref. [58].)

Now it is possible to show that the LIRs in Eqs. (5) and (6) are not violated if one generalizes the WW approximation. For this purpose we consider the following exact relations [9,12] originating from the QCD equations of motion (EOM):

$$g_{1T}^{(1)}(x) \stackrel{\text{EOM}}{=} x(g_T(x) - \tilde{g}_T(x)) - \frac{m}{M} h_1(x), \quad (14)$$

$$h_{1L}^{\perp(1)}(x) \stackrel{\text{EOM}}{=} -\frac{x}{2}(h_L(x) - \tilde{h}_L(x)) + \frac{m}{2M} g_{1L}(x), \quad (15)$$

with the (1)-moments of the PDFs defined in Eq. (9). The functions $\tilde{g}_T(x)$ and $\tilde{h}_L(x)$ denote twist-3 quark-gluon-quark correlations. In lightcone gauge these objects, like \tilde{g}'_T and \tilde{h}'_L , represent matrix elements of the type $\langle \bar{\Psi} A_T \Psi \rangle$. One therefore can assume that these functions are small as well, although the explicit form of $\tilde{g}_T(\tilde{h}_L)$ differs from the one of $\tilde{g}'_T(\tilde{h}'_L)$. In the following we denote as “WW-type approximation” the neglect of the tilde-functions (and quark mass terms) in the EOM-relations.³ Below we will also address the phenomenological justification of the WW-type approximation.

In order to proceed we introduce a measure $\Delta_g(x)$ and a measure $\Delta_h(x)$ for the violation of the LIRs (see for instance Refs. [44, 45] for explicit forms of these terms) according to

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) + \Delta_g(x), \quad (16)$$

$$h_L(x) = h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x) + \Delta_h(x). \quad (17)$$

If one substitutes the (1)-moments $g_{1T}^{(1)}$ and $h_{1L}^{\perp(1)}$ from (14) and (15) in Eqs. (16) and (17), and uses both the WW approximation and the WW-type approximation one finds

$$\Delta_g(x) \stackrel{\text{WW,WW-type}}{\approx} -g_1(x) - x \frac{d}{dx} \int_x^1 \frac{dy}{y} g_1(y) = 0, \quad (18)$$

$$\Delta_h(x) \stackrel{\text{WW,WW-type}}{\approx} -h_1(x) - x^2 \frac{d}{dx} \int_x^1 \frac{dy}{y^2} h_1(y) = 0. \quad (19)$$

Eqs. (18) and (19) show that the LIRs (5) and (6) are not violated if a generalized WW approximation is applied. This result is not entirely surprising keeping in mind that the violation of the LIRs is related to the amplitudes B_i which are associated with the gauge link of the fully unintegrated correlator in (1). Therefore, the B_i 's are necessarily related to quark-gluon-quark correlations which are neglected in the (generalized) WW approximation. This is also in line with the fact that in relativistic nucleon models without gluonic degrees of freedom the LIRs are typically fulfilled (see, e.g., Refs. [59,60]).

We also point out that in the generalized WW approximation instead of having the three functions g_1 , g_T , and $g_{1T}^{(1)}$, there is only one independent PDF. The same applies to the h -functions. In particular, one can immediately write

$$g_{1T}^{(1)}(x) \stackrel{\text{WW,WW-type}}{\approx} x \int_x^1 \frac{dy}{y} g_1(y), \quad (20)$$

$$h_{1L}^{\perp(1)}(x) \stackrel{\text{WW,WW-type}}{\approx} -x^2 \int_x^1 \frac{dy}{y^2} h_1(y). \quad (21)$$

Phenomenological work on the basis of those relations was, for instance, carried out in [61–63]. In Ref. [63], Eq. (21) was used in order to describe data for a certain longitudinal single spin asymmetry in semi-inclusive DIS. This investigation shows that the approximation (21) is not excluded, although more precise data would be helpful for having a stronger test. Further support for the generalized WW approximation – and therefore also justification for the WW-type approximation – comes from studies [64] of the azimuthal $\cos\phi$ -dependence of the unpolarized cross section for semi-inclusive DIS on the basis of the Cahn effect [65,66]. This approach, though neglecting tilde-functions from the QCD equations of motion (and the Boer–Mulders effect, see Ref. [12] for more details), provides a quite satisfactory description of the available data [64].

One has to keep in mind that neglecting several tilde-functions as done in the last example, even if each by itself is small, may naturally lead to a poor approximation. Two examples from semi-inclusive DIS at subleading twist illustrate this. In the longitudinal target spin asymmetry proportional to $\sin\phi$ of the produced hadron, the ‘overeager’ use of the WW-approximation would imply that the observable is due to the Collins effect only, but data [18] do not follow the flavor pattern observed for the pure Collins effect [20,22,24]. The use of the WW-approximation is also not recommended in the case of the beam spin asymmetry, in which the entire effect is due to tilde-functions. The data on this small but clearly non-zero asymmetry [19] provide direct access to tilde-functions and hence quark-gluon-quark correlations. In both cases the asymmetries are small, which means that there does not need to be a general conflict with the WW-approximation.

In case of the LIRs (7) and (8) which contain T -odd PDFs the situation is slightly different and in principle even simpler. Due to time-reversal invariance the \mathbf{p}_T -integrated T -odd PDFs $f_T(x)$ and $h(x)$ vanish [12,41],

$$f_T(x) = \int d^2\mathbf{p}_T f_T(x, \mathbf{p}_T^2) = 0, \quad (22)$$

$$h(x) = \int d^2\mathbf{p}_T h(x, \mathbf{p}_T^2) = 0, \quad (23)$$

which implies, considering the LIRs in Eqs. (7) and (8), that

$$\frac{d}{dx} f_{1T}^{\perp(1)}(x) \stackrel{\text{LIR}}{=} 0, \quad (24)$$

$$\frac{d}{dx} h_1^{\perp(1)}(x) \stackrel{\text{LIR}}{=} 0. \quad (25)$$

This means that $f_{1T}^{\perp(1)}$ and $h_1^{\perp(1)}$ are constants. In fact, since these moments have to vanish for $x = 1$, one can conclude that they should vanish for the entire x -range. So far we did not use any approximation and only assumed that the LIRs (7) and (8) are not violated. Now let us explore the EOMs [12] which, keeping in mind (22) and (23), imply (see also Refs. [38,67,68])

$$f_{1T}^{\perp(1)}(x) \stackrel{\text{EOM}}{=} x \tilde{f}_T(x), \quad (26)$$

$$h_1^{\perp(1)}(x) \stackrel{\text{EOM}}{=} \frac{x}{2} \tilde{h}(x). \quad (27)$$

In the WW-type approximation the tilde-functions are set to zero. It then follows directly from Eqs. (26) and (27) that $f_{1T}^{\perp(1)}$ and $h_1^{\perp(1)}$

³ Note that in Ref. [12] for instance this approximation (for brevity) was just called “WW approximation” because, like Eqs. (12) and (13), it also corresponds to neglecting purely interaction dependent terms.

are zero (see also [12]). This is consistent with the results following from the LIRs (24) and (25). So the LIRs (7) and (8) are also not violated in the WW-type approximation.

Also for the T -odd functions we already have some phenomenological input on the status of the WW-type approximation. Since a nonzero asymmetry, typically attributed to the Sivvers effect, was found in the HERMES experiment [20,22,24], in the case of T -odd PDFs this approximation seems to be violated. On the other hand the observed effect is not very large (of the order of few percent), and one should not expect WW-type approximations to work to a much better accuracy than that. Moreover, the Sivvers effect studied at COMPASS is compatible with zero both for a deuteron as well as a proton target [21,23,25,26]. Therefore, the current experimental situation is not in conflict with a rather small \tilde{f}_T in Eq. (26).

4. Summary

We have studied LIRs between parton distributions, known to be violated in general, with the aim to understand how strong this violation might be. It was found that LIRs are *satisfied* in a generalized WW approximation in which one systematically neglects certain quark–gluon–quark correlations as well as quark mass terms. That would mean that LIRs could provide useful approximations for unknown PDFs whenever applicable. Our approximation goes beyond the successful “standard WW approximation” quoted in Eqs. (12) and (13). In particular, we also neglected purely interaction dependent terms which show up in relations originating from the QCD equations of motion (see also Ref. [12]). We argued that there exists experimental evidence for the validity of the generalized WW approximation. On the other hand more (precise) data and tests are needed before a final conclusion can be reached. Only forthcoming data analyses and experiments at COMPASS, HERMES, and Jefferson Lab can ultimately reveal to what extent the generalized WW approximation (and the LIRs) provide useful approximations. Eventually, it is likely that the quality of the approximation depends on the particular case (function) under consideration.

Note added

After completion of our work the manuscript [45] appeared where, on the basis of the present data for the DIS structure function g_2 , a violation of the WW-relation in Eq. (12) of the order 15–40% has been reported. However, the authors of [45] also point out that more data are needed to ultimately settle the situation. In any case, if generalized WW-relations were valid within a similar accuracy, they would constitute helpful tools at the present stage for phenomenological studies of first data.

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Appendix A. Quark–quark correlators and the Lorentz invariance relations

For completeness we give here the general structure of the fully unintegrated quark–quark correlator in Eq. (1), the \mathbf{p}_T -dependent quark–quark correlator (expressed in terms of \mathbf{p}_T -dependent PDFs) in Eq. (2), the relations between PDFs and the amplitudes A_i , B_i

which parameterize the correlator in (1), as well as a brief account on how to derive the LIRs in the case that the amplitudes B_i are absent (like in a non-gauge theory).

The fully unintegrated quark–quark correlator (1) can be decomposed according to [41]

$$\begin{aligned}
\Phi(P, p, S|n_-) &= MA_1 + \not{p}A_2 + \not{p}A_3 + \frac{i}{2M}[\not{p}, \not{p}]A_4 + i(p \cdot S)\gamma_5 A_5 \\
&+ M\not{S}\gamma_5 A_6 + \frac{p \cdot S}{M}\not{p}\gamma_5 A_7 + \frac{p \cdot S}{M}\not{p}\gamma_5 A_8 + \frac{[\not{p}, \not{S}]}{2}\gamma_5 A_9 \\
&+ \frac{[\not{p}, \not{S}]}{2}\gamma_5 A_{10} + \frac{p \cdot S}{2M^2}[\not{p}, \not{p}]\gamma_5 A_{11} \\
&+ \frac{1}{M}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho S_\sigma A_{12} + \frac{M^2}{P \cdot n_-}\not{n}_- B_1 \\
&+ \frac{iM}{2P \cdot n_-}[\not{p}, \not{n}_-]B_2 + \frac{iM}{2P \cdot n_-}[\not{p}, \not{n}_-]B_3 \\
&+ \frac{1}{P \cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu \gamma_5 P_\nu p_\rho n_{-\sigma} B_4 \\
&+ \frac{1}{P \cdot n_-}\varepsilon^{\mu\nu\rho\sigma}P_\mu p_\nu n_{-\rho} S_\sigma B_5 + \frac{iM^2}{P \cdot n_-}(n_- \cdot S)\gamma_5 B_6 \\
&+ \frac{M}{P \cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu n_{-\rho} S_\sigma B_7 \\
&+ \frac{M}{P \cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu p_\nu n_{-\rho} S_\sigma B_8 \\
&+ \frac{p \cdot S}{M(P \cdot n_-)}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho n_{-\sigma} B_9 \\
&+ \frac{M(n_- \cdot S)}{(P \cdot n_-)^2}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho n_{-\sigma} B_{10} \\
&+ \frac{M}{P \cdot n_-}(n_- \cdot S)\not{p}\gamma_5 B_{11} + \frac{M}{P \cdot n_-}(n_- \cdot S)\not{p}\gamma_5 B_{12} \\
&+ \frac{M}{P \cdot n_-}(p \cdot S)\not{n}_-\gamma_5 B_{13} + \frac{M^3}{(P \cdot n_-)^2}(n_- \cdot S)\not{n}_-\gamma_5 B_{14} \\
&+ \frac{M^2}{2P \cdot n_-}[\not{n}_-, \not{S}]\gamma_5 B_{15} + \frac{p \cdot S}{2P \cdot n_-}[\not{p}, \not{n}_-]\gamma_5 B_{16} \\
&+ \frac{p \cdot S}{2P \cdot n_-}[\not{p}, \not{n}_-]\gamma_5 B_{17} + \frac{n_- \cdot S}{2P \cdot n_-}[\not{p}, \not{p}]\gamma_5 B_{18} \\
&+ \frac{M^2(n_- \cdot S)}{2(P \cdot n_-)^2}[\not{p}, \not{n}_-]\gamma_5 B_{19} \\
&+ \frac{M^2(n_- \cdot S)}{2(P \cdot n_-)^2}[\not{p}, \not{n}_-]\gamma_5 B_{20}, \tag{28}
\end{aligned}$$

where the convention $\varepsilon^{0123} = 1$ is understood. The scalar amplitudes depend on the available kinematical invariants. Note that all the B_i 's are associated with matrix structures containing the lightlike vector n_- .

The \mathbf{p}_T -dependent correlator in Eq. (2) can be specified by all possible Dirac traces $\Phi^{[T]}$ defined in Eq. (4), which in turn are parameterized through \mathbf{p}_T -dependent PDFs. A list of all traces was given in Refs. [12,41]. Limiting oneself to twist-2 and twist-3 effects one has (in the conventions of [12]):

$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \mathbf{p}_T^2), \tag{29}$$

$$\Phi^{[\gamma^+ \gamma_5]}(x, \mathbf{p}_T, S) = \lambda g_{1L}(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{p}_T^2), \tag{30}$$

$$\begin{aligned} \Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{p}_T, S) &= S_T^i h_1(x, \mathbf{p}_T^2) + \lambda \frac{p_T^i}{M} h_{1L}^\perp(x, \mathbf{p}_T^2) \\ &\quad - \frac{p_T^i p_T^j + \frac{1}{2} \mathbf{p}_T^2 g_T^{ij}}{M^2} S_{Tj} h_{1T}^\perp(x, \mathbf{p}_T^2) - \frac{\varepsilon_T^{ij} p_{Tj}}{M} h_1^\perp(x, \mathbf{p}_T^2), \end{aligned} \quad (31)$$

$$\Phi^{[1]}(x, \mathbf{p}_T, S) = \frac{M}{P^+} \left[e(x, \mathbf{p}_T^2) - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} e_T^\perp(x, \mathbf{p}_T^2) \right], \quad (32)$$

$$\Phi^{[i\gamma_5]}(x, \mathbf{p}_T, S) = \frac{M}{P^+} \left[\lambda e_L(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} e_T(x, \mathbf{p}_T^2) \right], \quad (33)$$

$$\begin{aligned} \Phi^{[\gamma^i]}(x, \mathbf{p}_T, S) &= \frac{M}{P^+} \left[-\varepsilon_T^{ij} S_{Tj} f_T(x, \mathbf{p}_T^2) - \lambda \frac{\varepsilon_T^{ij} p_{Tj}}{M} f_L^\perp(x, \mathbf{p}_T^2) \right. \\ &\quad \left. - \frac{p_T^i p_T^j + \frac{1}{2} \mathbf{p}_T^2 g_T^{ij}}{M^2} \varepsilon_{Tjk} S_T^k f_T^\perp(x, \mathbf{p}_T^2) + \frac{p_T^i}{M} f^\perp(x, \mathbf{p}_T^2) \right], \end{aligned} \quad (34)$$

$$\begin{aligned} \Phi^{[\gamma^i \gamma_5]}(x, \mathbf{p}_T, S) &= \frac{M}{P^+} \left[S_T^i g_T(x, \mathbf{p}_T^2) + \lambda \frac{p_T^i}{M} g_L^\perp(x, \mathbf{p}_T^2) \right. \\ &\quad \left. - \frac{p_T^i p_T^j + \frac{1}{2} \mathbf{p}_T^2 g_T^{ij}}{M^2} S_{Tj} g_T^\perp(x, \mathbf{p}_T^2) - \frac{\varepsilon_T^{ij} p_{Tj}}{M} g^\perp(x, \mathbf{p}_T^2) \right], \end{aligned} \quad (35)$$

$$\begin{aligned} \Phi^{[i\sigma^{ij} \gamma_5]}(x, \mathbf{p}_T, S) &= \frac{M}{P^+} \left[-\varepsilon_T^{ij} h(x, \mathbf{p}_T^2) + \frac{S_T^i p_T^j - p_T^i S_T^j}{M} h_T^\perp(x, \mathbf{p}_T^2) \right], \end{aligned} \quad (36)$$

$$\Phi^{[i\sigma^{+-} \gamma_5]}(x, \mathbf{p}_T, S) = \frac{M}{P^+} \left[\lambda h_L(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} h_T(x, \mathbf{p}_T^2) \right]. \quad (37)$$

Here use has been made of the Sudakov decompositions

$$S = \lambda \frac{P^+}{M} n_+ - \lambda \frac{M}{2P^+} n_- + S_T, \quad (38)$$

$$p = xP^+ n_+ + p^- n_- + p_T, \quad (39)$$

and of $\varepsilon_T^{ij} = \varepsilon^{-+ij}$.

On the basis of the relation in (2) it is straightforward to express the \mathbf{p}_T -dependent PDFs through the amplitudes A_i and B_i . We again restrict ourselves to the twist-2 and twist-3 case for which we obtain

$$f_1(x, \mathbf{p}_T^2) = 2P^+ \int dp^- (A_2 + xA_3), \quad (40)$$

$$f_{1T}^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- A_{12}, \quad (41)$$

$$\begin{aligned} g_{1L}(x, \mathbf{p}_T^2) &= 2P^+ \int dp^- \left(-A_6 - \frac{P \cdot p - M^2 x}{M^2} (A_7 + xA_8) \right. \\ &\quad \left. - B_{11} - xB_{12} \right), \end{aligned} \quad (42)$$

$$g_{1T}(x, \mathbf{p}_T^2) = 2P^+ \int dp^- (A_7 + xA_8), \quad (43)$$

$$h_1(x, \mathbf{p}_T^2) = 2P^+ \int dp^- \left(-A_9 - xA_{10} + \frac{\mathbf{p}_T^2}{2M^2} A_{11} \right), \quad (44)$$

$$h_{1L}^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- \left(A_{10} - \frac{P \cdot p - M^2 x}{M^2} A_{11} - B_{18} \right), \quad (45)$$

$$h_{1T}^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- A_{11}, \quad (46)$$

$$h_1^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- (-A_4), \quad (47)$$

$$e(x, \mathbf{p}_T^2) = 2P^+ \int dp^- A_1, \quad (48)$$

$$e_T^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- B_5, \quad (49)$$

$$e_L(x, \mathbf{p}_T^2) = 2P^+ \int dp^- \left(-\frac{P \cdot p - M^2 x}{M^2} A_5 - B_6 \right), \quad (50)$$

$$e_T(x, \mathbf{p}_T^2) = 2P^+ \int dp^- A_5, \quad (51)$$

$$\begin{aligned} f_T(x, \mathbf{p}_T^2) &= 2P^+ \int dp^- \left(-\frac{P \cdot p - M^2 x}{M^2} A_{12} - B_7 \right. \\ &\quad \left. - xB_8 - \frac{\mathbf{p}_T^2}{2M^2} B_9 \right), \end{aligned} \quad (52)$$

$$\begin{aligned} f_L^\perp(x, \mathbf{p}_T^2) &= 2P^+ \int dp^- \left(-A_{12} + B_8 + \frac{P \cdot p - M^2 x}{M^2} B_9 + B_{10} \right), \end{aligned} \quad (53)$$

$$f_T^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- (-B_9), \quad (54)$$

$$f^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- A_3, \quad (55)$$

$$g_T(x, \mathbf{p}_T^2) = 2P^+ \int dp^- \left(-A_6 + \frac{\mathbf{p}_T^2}{2M^2} A_8 \right), \quad (56)$$

$$g_L^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- \left(-\frac{P \cdot p - M^2 x}{M^2} A_8 - B_{12} \right), \quad (57)$$

$$g_T^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- A_8, \quad (58)$$

$$g^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- (-B_4), \quad (59)$$

$$h(x, \mathbf{p}_T^2) = 2P^+ \int dp^- \left(\frac{P \cdot p - M^2 x}{M^2} A_4 + B_2 + xB_3 \right), \quad (60)$$

$$h_T^\perp(x, \mathbf{p}_T^2) = 2P^+ \int dp^- (-A_{10}), \quad (61)$$

$$\begin{aligned} h_L(x, \mathbf{p}_T^2) &= 2P^+ \int dp^- \left(-A_9 - \frac{P \cdot p}{M^2} A_{10} + \left(\frac{P \cdot p - M^2 x}{M^2} \right)^2 A_{11} \right. \\ &\quad \left. - B_{15} + \frac{P \cdot p - M^2 x}{M^2} B_{16} + \frac{P \cdot p - M^2 x}{M^2} xB_{17} \right. \\ &\quad \left. + \frac{P \cdot p - M^2 x}{M^2} B_{18} + B_{19} + xB_{20} \right), \end{aligned} \quad (62)$$

$$\begin{aligned} h_T(x, \mathbf{p}_T^2) &= 2P^+ \int dp^- \left(-\frac{P \cdot p - M^2 x}{M^2} A_{11} - B_{16} - xB_{17} \right). \end{aligned} \quad (63)$$

In the following we give a brief account on how to derive the LIRs in the case that the amplitudes B_i are absent using as an example the LIR in Eq. (8). (See also the original references [2,9,10].) Starting from (60) one gets

$$\begin{aligned} h(x) &= 2P^+ \int dp^- d^2 \mathbf{p}_T \left(\frac{P \cdot p - M^2 x}{M^2} A_4 + B_2 + xB_3 \right) \\ &= \int d\sigma d\tau d^2 \mathbf{p}_T \delta(\tau - x\sigma + M^2 x^2 + \mathbf{p}_T^2) \end{aligned}$$

$$\begin{aligned} & \times \left(\frac{\sigma - 2M^2x}{2M^2} A_4 + B_2 + xB_3 \right) \\ & = \pi \int d\sigma d\tau \Theta(\tau - x\sigma + M^2x^2) \\ & \times \left(\frac{\sigma - 2M^2x}{2M^2} A_4 + B_2 + xB_3 \right), \end{aligned} \quad (64)$$

where $\sigma = 2p \cdot P$, $\tau = p^2$, $A_i = A_i(\sigma, \tau)$ and $B_i = B_i(\sigma, \tau)$. Using (9) and (47) allows one to write

$$\begin{aligned} h_1^{\perp(1)}(x) & = 2P^+ \int dp^- d^2\mathbf{p}_T \frac{\mathbf{p}_T^2}{2M^2} (-A_4) \\ & = \int d\sigma d\tau d^2\mathbf{p}_T \frac{\mathbf{p}_T^2}{2M^2} \delta(\tau - x\sigma + M^2x^2 + \mathbf{p}_T^2) (-A_4). \end{aligned} \quad (65)$$

To solve the integral in (65) in the same way as in (64) one can use the relation [2]

$$\begin{aligned} & \int d\sigma d\tau d^2\mathbf{p}_T \frac{\mathbf{p}_T^2}{2M^2} \delta(\tau - x\sigma + M^2x^2 + \mathbf{p}_T^2) \mathcal{F}(x, \sigma, \tau) \\ & = -\pi \int_x^1 dy \int d\sigma d\tau \Theta(\tau - y\sigma + M^2y^2) \\ & \times \left(\frac{\sigma - 2M^2y}{2M^2} \mathcal{F}(y, \sigma, \tau) \right. \\ & \left. - \frac{\tau - y\sigma + M^2y^2}{2M^2} \frac{\partial \mathcal{F}}{\partial y}(y, \sigma, \tau) \right), \end{aligned} \quad (66)$$

where $\mathcal{F}(x, \sigma, \tau)$ is a generic function representing a linear combination of the amplitudes A_i , B_i . This relation can be proved by differentiating both sides with respect to x and using the fact that the integration area vanishes at $x = 1$. Applying relation (66) to (65) one obtains

$$\begin{aligned} h_1^{\perp(1)}(x) & = -\pi \int_x^1 dy \int d\sigma d\tau \Theta(\tau - y\sigma + M^2y^2) \\ & \times \left(\frac{\sigma - 2M^2y}{2M^2} (-A_4) \right), \end{aligned} \quad (67)$$

$$\begin{aligned} \frac{d}{dx} h_1^{\perp(1)}(x) & = -\pi \int d\sigma d\tau \Theta(\tau - x\sigma + M^2x^2) \\ & \times \left(\frac{\sigma - 2M^2x}{2M^2} A_4 \right). \end{aligned} \quad (68)$$

Comparing now (64) and (68) it is clear that the LIR in (8) is fulfilled if the amplitudes B_i (here B_2 and B_3) are absent.

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