Computers and Mathematics with Applications 60 (2010) 1014–1025

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Designing a mathematical model for dynamic cellular manufacturing systems considering production planning and worker assignment

Iraj Mahdavi [a,*], Amin Aalaei [a], Mohammad Mahdi Paydar [a], Maghsud Solimanpur [b]

[a] *Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran*
[b] *Faculty of Engineering, Urmia University, Urmia, West Azerbaijan Province, Iran*

A R T I C L E I N F O

Keywords:
Dynamic cellular manufacturing system
Production planning
Worker assignment
Mathematical programming

A B S T R A C T

Since workers have an important role in doing jobs on machines, assignment of workers to cells becomes a crucial factor for full utilization of cellular manufacturing systems. This paper presents an integer mathematical programming model for the design of cellular manufacturing systems in a dynamic environment. The advantages of the proposed model are as follows: considering multi-period production planning, dynamic system reconfiguration, duplicate machines, machine capacity, available time of workers, and worker assignment. The aim of the proposed model is to minimize holding and backorder costs, inter-cell material handling cost, machine and reconfiguration costs and hiring, firing and salary costs. Computational results are presented by solving some examples.

1. Introduction

Cellular manufacturing (CM) involves a number of machine cells where each cell is responsible for the processing of families of similar parts. CM has emerged because of the need of manufacturing organizations to cope with the shorter product life-cycles, time-to-market and a shift to demands for mid-volume and mid-variety product mixes. Comprehensive summaries and taxonomies of studies devoted to cell formation problem (CFP) have been presented in [1–4]. Singh [5] has classified the approaches to cell formation into coding and classification systems, machine-components group analysis methods, graph theoretic methods, neural networks and heuristics, fuzzy clustering based methods, similarity coefficient based mathematical models, knowledge and pattern recognition methods, mathematical and heuristic methods. Selim et al. [4] have classified the CFP approaches into cluster analysis, graph partitioning, descriptive procedures, mathematical programming and artificial intelligence approaches. Recent works on CM design focus on the development of more integrated models and solution methodologies [6–8].

In most researches, CFP has been considered under static conditions in which cells are formed for a single time period with known and constant product mix and demand. The concept of dynamic cellular manufacturing system (DCMS) has been discussed in [9]. In a dynamic environment, a multi-period planning horizon is considered where each period has different product mix and demand requirements. Consequently, the formed cells in a period may not be optimal and efficient for the next period. Reconfiguration involves three aspects: (1) swapping of existing machines between cells called machine relocation, (2) adding new machines to cells, and (3) removing existing machines from cells. Most methods assume that the production quantity is equal to the demand in each planning period, meaning that production planning is ignored in these studies.

* Corresponding author. Tel.: +98 911 113 1380; fax: +98 111 229 0118.
E-mail address: irajarash@rediffmail.com (I. Mahdavi).

0898-1221/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2010.03.044

Several authors recently proposed models and solution procedures by considering dynamic cell reconfigurations over multiple time periods [10–15]. These methods assume that the production quantity is equal to demand in each planning period. In reality, however, production quantity may not be equal to the demand as it may be satisfied from inventory or backorders. Thus production quantity should be determined through production planning decisions in order to determine the number and type of machines to be installed in the system. However, in order to determine the production quantities in each planning period, the number and type of machines to be installed in manufacturing cells should in turn be known because of capacity consideration.

Defersha and Chen [16] addressed a dynamic cell formation problem incorporating several design factors such as cell reconfiguration, alternative routings, sequence of operations, duplicate machines, machine capacity, workload balancing, production cost as well as other realistic constraints. Ahkioon et al. [17] developed a preliminary CM model that integrates several manufacturing attributes considering multi-period planning, dynamic system reconfiguration, and production planning and alternate routings. Safaei and Tavakkoli-Moghaddam [18] extend the original model proposed by Safaei et al. [15] with a new contribution on the outsourcing by considering the carrying inventory, backorder, and partial subcontracting to cell formation (CF) and production planning in dynamic cellular manufacturing systems.

One of the main points in CM is the consideration of human issues since ignoring this factor can considerably reduce benefits of the cellular manufacturing. In some previous researches this issue is discussed. Nembhard [19] described a greedy heuristic approach based on individual learning rate for the improvement of productivity in organizations through targeted assignment of workers to tasks. Norman et al. [20] proposed a mixed-integer programming model for assigning workers to manufacturing cells in order to maximize the profit. Bidanda et al. [21] presented an overview and evaluation of the diverse range of human issues involved in CM based on an extensive literature review. In [22], a workforce planning model is presented that incorporates individual worker differences in ability to learn new skills and perform tasks. The model allows a number of different staffing decisions (i.e., hiring and firing) in order to minimize workforce-related and missed production costs. Aryanezhad et al. [23] presented a mathematical model to deal with dynamic cell formation and worker assignment problem with considering part routing flexibility and machine flexibility and also promotion of workers from one skill level. Solimanpur et al. [24] presented a fuzzy goal programming based approach for solving a multi-objective mathematical model of cell formation problem and production planning in dynamic virtual cellular manufacturing systems considering worker flexibility.

In this paper, an integrated mathematical model of the multi-period cell formation and production planning in a dynamic cellular manufacturing system considering the flexibility in worker assignment is proposed. The objective function is to minimize the summation of machine, reconfiguration, inter-cell material handling, inventory holding, backorder, worker hiring, firing and salary costs.

This paper is organized as follows. In Section 2, the proposed mathematical programming model is presented. Section 3 presents an example with computational results to validate and verify the proposed model. The paper ends with conclusions.

2. Problem formulation

In this section the mathematical model of cell formation problem is presented based on dynamic cellular manufacturing system with worker assignment. The objective is minimizing the sum of the penalty of deviation of production volume from the desirable value of the part demand (holding and backorder cost), inter-cell material handling, machine and reconfiguration, hiring, firing and salary worker costs. Main constraints are machine capacity, available time of workers, and production volume. The problem is formulated according to the following assumptions:

- The processing time of each operation of each part type on each machine type is known.
- The demand for each part type in each period is known.
- The capacity of each machine type is known.
- The available time of each worker type is known.
- The number of cells is given and constant through all periods.
- Only one worker is allotted for processing each part on each corresponding machine type.
- Inter-cell material handling cost is constant for all moves regardless of distances.
- Holding and backorder inventories are allowed between periods with known costs. Thus, the demand for a part in a given period can be satisfied in the preceding or succeeding periods.
- Maintenance and overhead costs of each machine type are known. These costs are considered for each machine in each cell and period irrespective of whether the machine is active or idle.
- System reconfiguration involves the addition and removal of machine to any cell and relocation from one cell to another between periods.
- Salary of each worker type is known. This cost is considered for each worker in each cell and period irrespective of whether the worker is active or idle.
- Reconfiguration involves the addition and removal of worker (hiring and firing) to any cell and relocation from one cell to another between periods.

2.1. Notations

2.1.1. Subscripts

Q Number of part types.
W Number of worker types.
M Number of machine types.
C Number of cells.
H Number of periods.
i Index for part type ($i = 1, 2, \ldots, Q$).
w Index for worker type ($w = 1, 2, \ldots, W$).
m Index for machine type ($m = 1, 2, \ldots, M$).
k Index for cell ($k = 1, 2, \ldots, C$).
h Index for period ($h = 1, 2, \ldots, H$).

2.1.2. Input parameters

r_{imw} 1 if machine type m is able to process part type i with worker w; =0 otherwise.
a_{im} 1 if part type i needs machine type m; =0 otherwise.
LM_k Minimum size of cell k in terms of the number of machine types.
UM_k Maximum size of cell k in terms of the number of machine types.
LW_k Minimum size of cell k in terms of the number of workers.
AM_m The number of available machines of type m.
AW_w The number of available workers of type w.
RW_{wh} Available time for worker type w in period h.
RM_{mh} Available time for machine type m in period h.
t_{imw} Processing time of part type i on machine type m with worker type w.
D_{ih} Demand of part type i in period h.
θ_i^{inter} Unit material handling cost between cells of each part type i.
γ_{ih} Unit holding cost of part type i in period h.
λ_{ih} Unit backorder cost of part type i in period h.
δ_m^{Ins} Installing cost of machine type m.
η_m^{Uns} Removing cost of machine type m.
α_m Maintenance and overhead costs of machine type m.
S_{wh} Salary cost of worker type w in period h.
HI_{wh} Hiring cost of worker type w within period h.
F_{wh} Firing cost of worker type w in period h.
A An arbitrary big positive number.

2.1.3. Decision variables

Y_{ikh} 1 if part type i is processed in cell k in period h; =0 otherwise.
X_{imwkh} 1 if part type i is to be processed on machine type m with worker w in cell k in period h; =0 otherwise.
NM_{mkh} Number of machines of type m allotted to cell k in period h.
NW_{wkh} Number of workers of type w allotted to cell k in period h.
P_{ih} Production volume of part type i to be produced in period h.
I_{ih} Inventory of part type i at the end of period h; $I_{i0} = 0$.
B_{ih} Backorder of part type i in period h; $B_{i0} = 0$.
K_{mch}^{+} Number of machines of type m added to cell c during period h.
K_{mch}^{-} Number of machines of type m removed from cell c during period h.
L_{wch}^{+} Number of workers of type w added to cell c during period h.
L_{wch}^{-} Number of workers of type w removed from cell c during period h.

2.2. Mathematical model

$$\begin{aligned}\text{Min } = {} & \sum_{h=1}^{H}\sum_{i=1}^{Q}\gamma_{ih}I_{ih} + \sum_{h=1}^{H}\sum_{i=1}^{Q}\lambda_{ih}B_{ih} + \sum_{h=1}^{H}\sum_{i=1}^{Q}\left[\left(\sum_{k=1}^{C}Y_{ikh}\right) - 1\right]\theta_i^{\text{inter}}P_{ih} \\ & + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{m=1}^{M}\alpha_m NM_{mkh} + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{m=1}^{M}\delta_m^{Ins}K_{mch}^{+} + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{m=1}^{M}\eta_m^{Uns}K_{mch}^{-} \\ & + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{w=1}^{W}S_{wh}NW_{wkh} + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{w=1}^{W}HI_{wh}L_{wch}^{+} + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{w=1}^{W}F_{wh}L_{wch}^{-}. \end{aligned} \quad (1)$$

S.t.:

$$\sum_{m=1}^{M}\sum_{i=1}^{Q} X_{imwkh} t_{imw} P_{ih} \leq NW_{wkh} RW_{wh} \quad \forall w, h, k; \tag{2}$$

$$\sum_{w=1}^{W}\sum_{i=1}^{Q} X_{imwkh} t_{imw} P_{ih} \leq NM_{mkh} RM_{mh} \quad \forall m, h, k; \tag{3}$$

$$Y_{ikh} = \min\left(1, \sum_{m=1}^{M}\sum_{w=1}^{W} X_{imwkh}\right) \quad \forall i, k, h; \tag{4}$$

$$D_{ih} = P_{ih} + I_{ih-1} - B_{ih-1} - I_{ih} + B_{ih} \quad \forall i, h; \tag{5}$$

$$\sum_{k=1}^{C} X_{imwkh} \leq r_{imw} \quad \forall i, m, w, h; \tag{6}$$

$$\sum_{k=1}^{C}\sum_{w=1}^{W} X_{imwkh} = a_{im} \quad \forall i, m, h; \tag{7}$$

$$\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W} X_{imwkh} \leq A \times P_{ih} \quad \forall i, h; \tag{8}$$

$$NM_{mkh-1} + K^{+}_{mkh} - K^{-}_{mkh} = NM_{mkh} \quad \forall m, k, h; \tag{9}$$

$$\sum_{k=1}^{C} NM_{mkh} \leq AM_{m} \quad \forall m, h; \tag{10}$$

$$\sum_{m=1}^{M} NM_{mkh} \geq LM_{k} \quad \forall k, h; \tag{11}$$

$$\sum_{m=1}^{M} NM_{mkh} \leq UM_{k} \quad \forall k, h; \tag{12}$$

$$NW_{wkh-1} + L^{+}_{wkh} - L^{-}_{wkh} = NW_{wkh} \quad \forall w, k, h; \tag{13}$$

$$\sum_{k=1}^{C} NW_{wkh} \leq AW_{k} \quad \forall k, h; \tag{14}$$

$$\sum_{w=1}^{W} NW_{wkh} \geq LW_{k} \quad \forall k, h; \tag{15}$$

$$Y_{ikh} \in \{0, 1\} \quad \forall i, k, h; \tag{16}$$

$$X_{imwkh} \in \{0, 1\} \quad \forall i, m, w, k, h; \tag{17}$$

$$NM_{mkh}, K^{+}_{mkh}, K^{-}_{mkh} \geq 0 \text{ and are integer} \quad \forall m, k, h; \tag{18}$$

$$NW_{wkh}, L^{+}_{wkh}, L^{-}_{wkh} \geq 0 \text{ and are integer} \quad \forall w, k, h; \tag{19}$$

$$P_{ih}, B_{ih}, I_{ih} \geq 0 \text{ and are integer} \quad \forall i, h. \tag{20}$$

The objective function consists of several cost items as follows:

(The first term) Holding cost: The holding cost of inventories of all parts over all the periods in the planning horizon.

(The second term) Backorder cost: The cost of delay in the delivery of parts over all periods in the planning horizon.

(The third term) Inter-cell material handling: The cost of moving parts between cells when parts cannot be produced completely in a dedicated cell. The inter-cell material handling cost happens when parts are moved between cells. This occurs when parts need to be processed in multiple cells, because all machine types required to process the parts are either not available in the cell to which the parts are allocated or because the cell does not have enough capacity. Inter-cell moves decrease the efficiency in the CFP by increasing material handling requirements and flow time, and complicating production control.

(The fourth term) Machine cost: This cost refers to the maintenance and overhead costs of machines and is calculated based on the number of machines used in the CF for a specific period.

(The fifth and sixth terms) Machine relocation cost: The cost of relocating machines from one cell to another between periods. In dynamic and deterministic production conditions, the best CF design for one period may not be an efficient design for other periods. By rearranging the manufacturing cells, the CF can continue operating efficiently as the product mix and demand change. In this paper, it is assumed that when a machine is relocated from one cell to another, it is removed from its current position and is moved to another place. It is then installed in a new cell or stored somewhere out of manufacturing system. When a machine is removed from one cell, a removal cost and relocation cost is incurred. If a removed machine is installed in another cell, an installation cost is incurred as well.

(The seventh term) Salary cost: The salary paid for workers in the planning horizon.

(The eighth term) Hiring cost: This cost is incurred, when some workers have to be hired and assigned to a cell due to lack of personnel in this cell.

(The ninth term) Firing cost: This cost is incurred when some workers are to be fired since no longer are required.

Constraints (2) and (3) ensure that the available time for workers and capacity of machines are not exceeded, respectively. Constraint (4) is to determine whether part type i is processed within cell k in period h or not. Constraint (5) balances the amount of part type i between two consecutive periods. In other words, if $I_{ih} > 0$ then we have surplus inventory which results in holding cost, and if $B_{ih} > 0$ it implies shortage inventory and backorder cost. Constraints (6) and (7) imply that only one worker is allotted for processing each part on each machine type. This model is flexible to enable a worker to work on several machines. This means that, if one part is required to one machine type to be processed, more than one worker will be able to service this machine type. Constraint (8) ensures that if $P_{ih} = 0$, no machine, worker and cell should be considered for part i in period h. Eq. (9) guarantees that the number of machines in the current period is equal to the number of machines in the previous period plus the number of machines moved in, minus the number of machines moved out. This equation balances the number of each machine type in each period and cell. Constraint (10) guarantees that the total number of machines of each type assigned to different cells in each period will not exceed the total number of available machines of that type. Constraints (11) and (12) specify the lower and upper bounds for number of machines allocated to each cell in each period. Similar to Eq. (9), Eq. (13) balances the number of workers between consecutive time periods. Constraint (14) guarantees that the total number of workers of each type assigned to different cells in each period will not exceed total available number of workers of that type. Constraint (15) ensures that at least LW_k workers will be assigned to cell k in each period.

2.3. Linearization of the proposed model

The proposed model is a nonlinear integer programming model because of the nonlinear term (1.3) in the objective function and also constraints (2), (3) and (4). Some auxiliary variables are to be defined to linearize these nonlinear terms of the model. Thus, the following new variables are defined.

$$F_{ikh} = Y_{ikh}P_{ih}$$
$$J_{imwkh} = X_{imwkh}P_{ih}.$$

By considering these equations, following constraints should be added to the mathematical model:

$$F_{ikh} \geq P_{ih} - A(1 - Y_{ikh}) \quad \forall i, k, h; \tag{21}$$
$$F_{ikh} \leq P_{ih} + A(1 - Y_{ikh}) \quad \forall i, k, h; \tag{22}$$
$$J_{imwkh} \geq P_{ih} - A(1 - X_{imwkh}) \quad \forall i, m, w, k, h; \tag{23}$$
$$J_{imwkh} \leq P_{ih} + A(1 - X_{imwkh}) \quad \forall i, m, w, k, h; \tag{24}$$
$$F_{ikh} \geq 0 \text{ and is integer} \quad \forall i, k, h; \tag{25}$$
$$J_{imwkh} \geq 0 \text{ and is integer} \quad \forall i, m, w, k, h. \tag{26}$$

Also to linearize the proposed model, we replace constraint (15) with two constraints as below:

$$\sum_{m=1}^{M}\sum_{w=1}^{W} X_{imwkh} \leq A \times Y_{ikh} \quad \forall i, k, h; \tag{27}$$
$$\sum_{m=1}^{M}\sum_{w=1}^{W} X_{imwkh} \geq Y_{ikh} \quad \forall i, k, h. \tag{28}$$

Therefore, the proposed linear mathematical programming model is as follows:

$$\text{Min} = \text{Eq. (1–1)} + \text{Eq. (1–2)} + \sum_{h=1}^{H}\sum_{i=1}^{Q}\left[\left(\sum_{k=1}^{C} F_{ikh}\right) - P_{ih}\right]\theta_i^{\text{inter}} + \text{Eq. (1–4) to Eq. (1–9)}.$$

S.t.:

Table 1
The number of variables in the linearized model.

Variable	Count	Variable	Count	Variable	Count
Y_{ikh}	$Q \times C \times H$	NW_{wkh}	$W \times C \times H$	I_{ih}	$Q \times H$
X_{imwkh}	$Q \times M \times W \times C \times H$	L^{+}_{wkh}	$W \times C \times H$	J_{imwkh}	$Q \times M \times W \times C \times H$
NM_{mkh}	$M \times C \times H$	L^{-}_{wkh}	$W \times C \times H$	F_{ikh}	$Q \times C \times H$
K^{+}_{mkh}	$M \times C \times H$	P_{ih}	$Q \times H$		
K^{-}_{mkh}	$M \times C \times H$	B_{ih}	$Q \times H$		
Sum $= 2(Q \times M \times W \times C \times H) + 3(M \times C \times H) + 3(Q \times H) + 2(Q \times C \times H) + 3(W \times C \times H)$					

Table 2
The number of constraints in the linearized model.

Con.	Count	Con.	Count	Con.	Count
(5)	$Q \times H$	(14)	$C \times H$	(23)	$Q \times M \times W \times C \times H$
(6)	$Q \times M \times W \times H$	(15)	$C \times H$	(24)	$Q \times M \times W \times C \times H$
(7)	$Q \times M \times H$	(16)	$Q \times C \times H$	(25)	$Q \times C \times H$
(8)	$Q \times H$	(17)	$Q \times M \times W \times C \times H$	(26)	$Q \times M \times W \times C \times H$
(9)	$M \times C \times H$	(18)	$3 \times M \times C \times H$	(27)	$Q \times C \times H$
(10)	$M \times H$	(19)	$3 \times W \times C \times H$	(28)	$Q \times C \times H$
(11)	$C \times H$	(20)	$3 \times Q \times H$	(29)	$W \times C \times H$
(12)	$C \times H$	(21)	$Q \times C \times H$	(30)	$M \times H \times C$
(13)	$W \times C \times H$	(22)	$Q \times C \times H$		
Sum $= 4(Q \times M \times W \times C \times H) + (Q \times M \times W \times H) + (Q \times M \times H) + 5(M \times C \times H) + 6(Q \times C \times H) + 5(W \times C \times H) + 5(Q \times H) + (M \times H) + 4(C \times H)$					

Table 3
The machine information for Example 1.

Machine type	Machine information					
	AM_m	α_m	δ^{Ins}_m	η^{Uns}_m	RM_{m1}	RM_{m2}
1	2	400	550	140	30	30
2	2	410	530	130	30	30
3	2	430	560	150	30	40

Constraints (5)–(28) and the new version of constraints (2) and (3) are:

$$\sum_{m=1}^{M}\sum_{i=1}^{Q} J_{imwkh} t_{imw} \le NW_{wkh} RW_{wh} \quad \forall w, h, k; \tag{29}$$

$$\sum_{w=1}^{W}\sum_{i=1}^{Q} J_{imwkh} t_{imw} \le NM_{mkh} RM_{mh} \quad \forall m, h, k. \tag{30}$$

The number of variables and constraints in the linearized model are presented in Tables 1 and 2, respectively.

3. Numerical illustration

To illustrate validity of the proposed model, two examples are solved by branch-and-bound (B&B) method using Lingo 8.0 Software on a PC including two Intel® Core™2 and 2 GB RAM.

Example 1. This example includes two cells, three machines, four parts and four workers. Each part type is assumed to have some operations where each operation can be performed by two alternative workers. Table 3 shows the machine information such as machine availability, maintenance and overhead cost, installing and removing cost and time capacity.

The data set related to the machine–part and machine–worker incidence matrices are shown in Tables 4 and 5, respectively. For example, as seen in Table 4, machine types 1 and 3 are required for part type 3. Also, the quantity of demand, holding and backorder costs of each part in each period are presented in this table. Table 5 indicates capabilities of workers in working with different machines. For example, worker 1 is able to work with machine types 1 and 3. Thus, the term $\sum_w x_{imwkh}$ is equal to the number of alternative workers for processing part type i on machine type m. This table shows the costs of workers in each period.

Table 6 shows the processing time matrix in which each part type is assumed to have some operations that must be processed on machines with the corresponding processing time. For instance, part type 1 must be processed on machine type 1 with processing time 0.04 h by worker 1 or with processing time 0.02 by worker 2. Moreover, the number of cells to be formed is two and the minimum and maximum cell sizes for each cell are 1 and 4, respectively. The minimum size of each cell in terms of the number of workers is assumed to be 1.

Table 4
The input data of machine–part incidence matrix of Example 1.

		Machine type			D_{i1}	D_{i2}	γ_{i1}	γ_{i2}	λ_{i1}	λ_{i2}	θ_i^{inter}
		1	2	3							
Part type	1	1	1	1	0	1550	4	4	14	14	11
	2	1	1	0	900	600	6	6	12	12	9
	3	1	0	1	1700	500	8	8	10	10	8
	4	0	1	1	1700	300	10	10	10	10	10

Table 5
The input data of machine–worker incidence matrix of Example 1.

		Machine			Worker information							
		1	2	3	AW_w	S_{w1}	S_{w2}	HI_{w1}	HI_{w2}	F_{w2}	RW_{w1}	RW_{w2}
Worker	1	1	0	1	2	470	490	270	285	145	30	30
	2	1	0	0	2	460	485	260	290	145	30	30
	3	0	1	1	2	455	475	200	250	155	30	30
	4	0	1	0	2	450	480	265	280	140	30	30

Table 6
The processing time (h) for Example 1.

	Part 1				Part 2				Part 3				Part 4			
	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4
M_1	0.04	0.02			0.04	0.01			0.02	0.03						
M_2			0.02	0.03			0.04	0.03							0.03	0.02
M_3	0.01		0.02						0.01		0.02		0.03		0.04	

Table 7
Optimal production plan for Example 1.

	Period 1				Period 2			
	Part 1	Part 2	Part 3	Part 4	Part 1	Part 2	Part 3	Part 4
Backorder			700	200				
Holding	50							
Production	50	900	1000	1500	1500	600	1200	500
Demand	0	900	1700	1700	1550	600	500	300

Table 8
Objective function value and its components for Example 1.

Total	Backorder	Holding	Inter-cell movement	Maintenance & overhead	Machine relocation	Salary	Hiring	Firing
27 475	9000	200	550	4960	3990	6595	2040	140

Table 9
The results of parts, machines and workers assignment to cells for Example 1.

	Parts assigned to		Machines in		Workers assigned to	
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2
Period 1	1, 3, 4	1, 2	1, 2, 3, 3	1, 2	1, 1, 2, 4	2, 3, 4
Period 2	2, 3, 4	1	1, 2, 3	1, 2, 3	1, 1, 2, 4	2, 3, 3

The results of Example 1 obtained with the proposed model are elaborated in the rest of this section. The objective function values obtained in this paper cannot be compared to the previous studies since different objective costs are involved. The production plan and the objective function value are shown in Tables 7 and 8, respectively. The processing of parts on machines together with the assigned workers for two periods are shown in Fig. 1. Part families, machine groups, worker assignment are also depicted in the cell configurations presented in Table 9.

In period 1, the demand of part type 1 is 0 but 50 units are produced which will be kept for next period. This inventory will be used to satisfy some portion of the demand of period 2 which causes holding cost for part type 1. Moreover, it has to be noted that although demand for part type 3 in period 1 is 1700, the system produces 1000 parts of which 700 parts will be supplied as backorder. This quantity will be produced to satisfy part of the demand in period 2. Therefore, the backorder cost of the system is 7000 for part type 3.

		Part 1				Part 2				Part 3				Part 4			
	Machines	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4
Period1	1		2				2				1						
	2			2					2								1
	3	1[a]								1				1			
	Machines	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4
Period2	1		2				1			1							
	2			2					1								1
	3			2						1				1			

[a] The part movement between cells

Fig. 1. Optimal processing parts on machines which are assigned workers for Example 1.

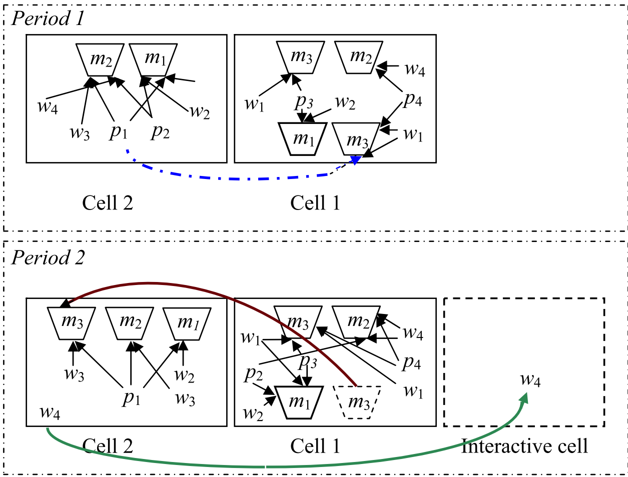

Fig. 2. Cell reconfiguration schema for Example 1.

Part type 3 is assigned to cell 1 in period 1. This part type should be processed on machine types 1 and 3. In the first period, the processing of part type 3 on machine type 1 is done by worker type 2 in cell 1. This operation in the second period, however, is processed on machine type 1 by worker type 1 in cell 1. Moreover, part type 2 is assigned to cell 2 in period 1 which is processed on machine type 1 by worker 2. But, in period 2, part type 2 is assigned to cell 1 for processing on machine type 1 by worker 2.

As seen in Table 9, in the first period, one machine of types 1, 2 and two machines of type 3 are assigned to cell 1. Also, one machine of types 1 and 2 are assigned to cell 2. As mentioned before, variation in part demand results in system reconfiguration. In the second period, one machine of type 3 is removed from cell 1 and one machine of type 3 is added to cell 2. On the other hand, the configuration of workers in periods 1 and 2 is the same to cell 1. In period 1, one worker of types 2, 3 and 4 are hired to cell 2. Moreover, worker type 4 is fired from cell 2 and one worker of type 3 is hired to cell 2 in the second period.

Fig. 1 and Table 9 show that part type 1 is processed in two cells in period 1. This part type has three operations which are processed on machine types 1 and 2 within cell 2 and machine type 3 within cell 1. The Eqs. (1)–(3) in the objective function calculates the inter-cell material handling cost in which the value of decision variables Y_{111} and Y_{121} is 1. The inter-cell material handling cost for part type 1 in period 1 is calculated:

$$\left[\left(\sum_{k=1}^{C} Y_{1k1}\right) - 1\right] \theta_1^{\text{inter}} P_{11} = [(Y_{111} + Y_{121}) - 1]\, 11 \times 50 = (2-1) \times 550 = 550.$$

Fig. 2 presents the configuration of this example and its properties.

We implemented the sensitivity analysis of model features in three parts: (1) Machine relocation, (2) worker reconfiguration, and (3) both machine relocation and worker reconfiguration. To demonstrate the effect of these features on the performance of model, we investigated cost savings which may be originated from those. To investigate the cost saving as a result of machine relocation, we solved the model by eliminating these features one at a time. If we add constraint (31) to the basic model, all necessary machines are assigned at the beginning of period 1 and no machine relocation afterwards. If constraint (32) is added to the proposed model, all necessary workers will be hired at the first period with no worker

Table 10
Features eliminating from the proposed model and related cost.

Features eliminated from the basic model	Objective function value	Cost saving
None	27 475	None
Machine relocation	39 228	43%
Worker reconfiguration	28 220	3%
Both of them	44 010	60%

Table 11
The machine information for Example 2.

Machine type	Machine information						
	AM_m	α_m	δ_m^{Ins}	η_m^{Uns}	RM_{m1}	RM_{m2}	RM_{m3}
1	2	520	600	100	40	40	40
2	2	510	650	150	40	40	40
3	2	550	660	200	40	40	40

Table 12
The input data of machine–part incidence matrix for Example 2.

		Machine type			D_{i1}	D_{i2}	D_{i3}	γ_{i1}	γ_{i2}	λ_{i1}	λ_{i2}	θ_i^{inter}
		1	2	3								
Part type	1	1	1	1	0	1550	500	1	1	14	14	5
	2	1	1	0	600	800	1000	2	2	12	12	6
	3	1	0	1	1200	1000	500	3	3	10	10	4
	4	0	1	1	1200	900	900	4	4	8	8	3
	5	0	0	1	1400	900	600	5	5	7	7	4
	6	0	1	0	1500	600	1000	6	6	6	6	5

Table 13
The input data of machine–worker incidence matrix for Example 2.

		Machine			Worker information											
		1	2	3	AW_w	S_{w1}	S_{w2}	S_{w3}	HI_{w1}	HI_{w2}	HI_{w3}	F_{w2}	F_{w3}	RW_{w1}	RW_{w2}	RW_{w3}
Worker	1	1	0	1	2	400	450	450	230	285	285	110	145	40	40	40
	2	1	0	0	2	420	465	465	220	290	290	120	145	40	40	40
	3	0	1	1	2	415	475	475	200	250	250	115	155	40	40	40
	4	0	1	0	2	430	480	480	245	280	280	120	140	40	40	40

Table 14
The processing time (h) for Example 2.

	Part 1				Part 2				Part 3				Part 4				Part 5				Part 6			
	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4	W_1	W_2	W_3	W_4
M_1	0.04	0.02			0.04	0.01			0.02	0.03														
M_2			0.02	0.03			0.04	0.03							0.03	0.02							0.01	0.03
M_3	0.01		0.02						0.01		0.02		0.03		0.04		0.03		0.04					

relocation, hiring or firing in other periods.

$$NM_{mkh} = NM_{mk,h+1} \quad \forall m, k, h = 1, \dots, H-1 \tag{31}$$

$$NW_{wkh} = NW_{wk,h+1} \quad \forall w, k, h = 1, \dots, H-1. \tag{32}$$

By eliminating the features mentioned above one at a time from the basic model using the corresponding constraints, we recalculated the first example to observe their impacts on the solution of the model. The results are summarized in Table 10 and cost saving is significant for the first example if machine relocation and worker reconfiguration is allowed.

Example 2. The second example includes three machine types, six part types, four workers types and three periods and the related information is given in Tables 11–14 which consist the machine, part and worker information and processing time. Moreover, the number of cells to be formed is two and the minimum and maximum cell sizes for each cell are 1 and 5, respectively. The minimum size of each cell in terms of the number of workers is assumed to be 1.

Table 15 shows how demand is satisfied for part type 5 through production and backorder in period 1. Moreover, part types 2 and 4 are produced as surplus to be carried to period 2 to satisfy demands of these parts in period 2. Finally, the quantity of production of all part types equal demands in the third period.

Table 15
Optimal production plan for Example 2.

	Period 1						Period 2						Period 3					
	P_1	P_2	P_3	P_4	P_5	P_6	P_1	P_2	P_3	P_4	P_5	P_6	P_1	P_2	P_3	P_4	P_5	P_6
Backorder					67													
Holding		25		63														
Production	0	625	1200	1263	1333	1500	1550	775	1000	837	967	600	500	1000	500	900	600	1000
Demand	0	600	1200	1200	1400	1500	1550	800	1000	900	900	600	500	1000	500	900	600	1000

Table 16
Objective function value and its components for Example 2.

Total	Backorder	Holding	Inter-cell movement	Maintenance & overhead	Machine relocation	Salary	Hiring	Firing
22 251	469	302	0	8440	3920	7205	1660	255

Table 17
The results of parts, machines and workers assignment to cells for Example 2.

	Parts assigned to		Machines in		Workers assigned to	
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2
Period 1	5	1, 2, 3, 4, 6	3	1, 2, 2, 3	1	1, 2, 3, 4
Period 2	3, 5	1, 2, 4, 6	1, 3	1, 2, 2, 3	1, 1	2, 3, 3, 4
Period 3	5	1, 2, 3, 4, 6	3	1, 2, 2, 3	1	2, 3, 3, 4

		Part 1				Part 2				Part 3				Part 4				Part 5				Part 6			
	M	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4	w_1	w_2	w_3	w_4
Period1	**1**	2					2			2															
	2				2			2									2							2	
	3	2								2					2			1							
Period2	**1**		2				2			1															
	2			2					2								2							2	
	3			2						1					2			1							
Period3	**1**		2				2				2														
	2			2					2							2								2	
	3			2								2			2			1							

Fig. 3. Optimal processing parts on machines which are assigned workers for Example 2.

As can be seen in Table 16, the inter-cell material handling cost is zero. This means that all part types are processed in only one cell and the value of decision variable Y_{ikh} for each part type and each period is 1 only once.

The processing of parts on machines along with the assigned workers for three periods are shown in Fig. 3. For instance, part types 5 and 6 are produced in the same position and workers in any periods. Part families, machine groups, worker assignment are also depicted in the cell configurations presented in Table 17. Based on this Table, relocation is occurred for machine type 1 where it is added to cell 1 in period 2. One worker of type 1 is hired to cell 2 in period 1 and fired from the system in period 3, and worker type 1 is hired to cell 2 in period 2. Moreover, in cell 2, one worker of type 1 is hired and one worker of type 3 is hired in the second period.

The linearized proposed model consists of 524 variables and 1186 constraints in the first example and its CPU time is 37 min. The second example consists of 1062 variables and 2475 constraints and its computation time is 980 min. The proposed model is computationally complex as it integrates the dynamic cell formation problem along with other manufacturing features including the cell reconfiguration and the part routing problem with alternate workers. The cell formation problem has been reported as a NP-hard problem [25,26]. Logendran et al. [27] have shown that the problem of the determination of the process routing from alternate routings is also NP-hard. Chen [10] described that solving the model considering system reconfiguration in terms of machine relocation is NP-hard as well. Therefore, the proposed model in this paper is NP-hard since it integrates all these NP-hard problems.

4. Conclusions

This paper presents a novel integer nonlinear programming model for dynamic cellular manufacturing systems in the presence of worker assignment. The proposed model incorporates several design features including operation time, alternative workers, duplicate machines, removing idle machines from system or returning them to system, machine capacity, hiring and firing of workers, production volume of parts, part movements between cells, cell reconfiguration and production planning. A review of the literature reveals few attempts integrating these important design features during cell formation, simultaneously. The objective is to minimize the total costs of inter-cellular material handling, holding and backorder and manages machines and workers over a certain planning horizon.

This model is capable to determine the optimal cell configurations, worker assignment and process plan for each part type at each period over the planning horizon. The nonlinear formulation of the proposed model was linearized using some auxiliary variables. The performance of the model is illustrated by two numerical examples. CPU time required to reach the optimal solution of the attempted examples show that obtaining an optimal solution in a reasonable time is computationally intractable. Therefore, it is necessary to develop a heuristic or metaheuristic approach to solve the proposed model for large-sized problems. Moreover, the originality of paper is as follows:

- Considering the cubic space of machine–part–worker in CFP.
- Designing a comprehensive model for CFP and production planning.
- Balancing machines and workers with respect to relocation and reconfiguration in multi-period production planning.

The proposed model is still open for incorporating other features in future researches. Some guidelines for future researches can be outlined as follows.

- Application of metaheuristic approach (Simulated Annealing, Genetic Algorithm, etc.) to solve the proposed model for real-sized problems.
- Incorporation of sequence data (sequence of operations) for CFP which provides additional information to the cell designer.
- Incorporating intra-cell layout of machines to exactly calculate inter-cell material handling cost.

Acknowledgements

The authors would like to acknowledge the Iran National Science Foundation (INSF) for the financial support of this work. Moreover, the authors are cordially thankful for the constructive comments of the anonymous reviewers which have substantially improved the quality of paper over its earlier versions.

References

[1] U. Wemmerlov, N.L. Hyer, Procedures for the part-family/machine group identification problem in cellular manufacturing, Journal of Operations Management 6 (2) (1986) 125–147.
[2] A. Kusiak, W.S. Chow, Efficient solving of the group technology problem, Journal of Manufacturing Systems 6 (2) (1987) 117–124.
[3] A. Reisman, A. Kumar, J. Motwani, C.H. Cheng, Cellular manufacturing: a statistical review of the literature (1965–1995), Operations Research 45 (4) (1997) 508–520.
[4] H.M. Selim, R.G. Askin, A.J. Vakharia, Cell formation in group technology: review, evaluation and directions for future research, Computers & Industrial Engineering 34 (1998) 3–20.
[5] N. Singh, Design of cellular manufacturing systems: an invited review, European Journal of Operational Research 69 (3) (1993) 281–511.
[6] S.A. Mansouri, S.M. Moattar Husseini, S.T. Newman, A review of the modern approach to multi-criteria cell design, International Journal of Production Research 38 (2000) 1201–1218.
[7] Y. Yin, K. Yasuda, Similarity coefficient methods applied to the cell formation problem: a taxonomy and review, International Journal of Production Economics 101 (2006) 329–352.
[8] J. Balakrishnan, C.H. Cheng, Multi-period planning and uncertainty issues in cellular manufacturing: a review and future directions, European Journal of Operational Research 177 (2007) 281–309.
[9] M. Rheault, J. Drolet, G. Abdulnour, Physically reconfigurable virtual cells: a dynamic model for a highly dynamic environment, Computers & Industrial Engineering 29 (1–4) (1995) 221–225.
[10] M. Chen, A mathematical programming model for systems reconfiguration in a dynamic cell formation condition, Annals of Operations Research 77 (1) (1998) 109–128.
[11] E.M. Wicks, R.J. Reasor, Designing cellular manufacturing systems with dynamic part populations, IIE Transactions 31 (1999) 11–20.
[12] A. Mungwattana, Design of cellular manufacturing systems for dynamic and uncertain production requirements with presence of routing flexibility, Ph.D. Thesis, Virginia Polytechnic Institute and State University, Blackburg, VA, 2000.
[13] J. Balakrishnan, C.H. Cheng, Dynamic cellular manufacturing under multi-period planning horizons, Journal of Manufacturing Technology Management 16 (5) (2005) 516–530.
[14] F. Defersha, M. Chen, A comprehensive mathematical model for the design of cellular manufacturing systems, International Journal of Production Economics 103 (2006) 767–783.
[15] N. Safaei, M. Saidi-Mehrabad, M.S. Jabal-Ameli, A hybrid simulated annealing for solving an extended model of dynamic cellular manufacturing system, European Journal of Operational Research 185 (2008) 563–592.
[16] F.M. Defersha, M. Chen, A linear programming embedded genetic algorithm for an integrated cell formation and lot sizing considering product quality, European Journal of Operational Research 187 (1) (2008) 46–69.
[17] S. Ahkioon, A.A. Bulgak, T. Bektas, Integrated cellular manufacturing systems design with production planning and dynamic system reconfiguration, European Journal of Operational Research 192 (2009) 414–428.
[18] N. Safaei, R. Tavakkoli-Moghaddam, Integrated multi-period cell formation and subcontracting production planning in dynamic cellular manufacturing, International Journal of Production Economics 120 (2) (2009) 301–314.

[19] D.A. Nembhard, Heuristic approach for assigning workers to task based on individual learning rates, International Journal of Production Research 39 (8) (2001) 1968–1995.
[20] B.A. Norman, W. Tharmmaphornphilas, K.L. Needy, B. Bidanda, R.C. Warner, Worker assignment in cellular manufacturing considering technical and human skills, International Journal of Production Research 40 (6) (2002) 1479–1492.
[21] B. Bidanda, P. Ariyawongrat, K.M. Needy, B.A. Norman, W. Tharmmaphornphilas, Human-related issues in manufacturing cell design, implementation, and operation: a review and survey, Computer & Industrial Engineering 48 (2005) 507–523.
[22] P. Wirojanagud, E.S. Gel, J.W. Fowler, R.L. Cardy, Modeling inherent worker difference for workforce planning, International Journal of Production Research 45 (3) (2007) 525–553.
[23] M.B. Aryanezhad, V. Deljoo, S.M.J. Mirzapour Al-e-hashem, Dynamic cell formation and the worker assignment problem: a new model, International Journal of Advanced Manufacturing Technology 38 (3–4) (2008) doi:10.1007/s00170-008-1479-4.
[24] M. Solimanpur, I. Mahdavi, A. Aalaei, M.M. Paydar, Multi-objective cell formation and production planning in dynamic virtual cellular manufacturing systems, in: International Conference on Business and Information, BAI 2009, Kuala Lumpur, Malaysia, July 6–8, 2009.
[25] A. Ballakur, An investigation of part family/machine group formation in designing a cellular manufacturing system, Ph.D. Thesis, University of Wisconsin, Madison, WI, 1985.
[26] J. King, V. Nakornchai, Machine-component group formation in group technology: review and extension, International Journal of Production Research 20 (2) (1982) 117–133.
[27] R. Logendran, P. Ramakrishna, C. Sriskandarajah, Tabu search-based heuristics for cellular manufacturing systems in the presence of alternative process plans, International Journal of Production Research 32 (1994) 273–297.