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H. Dette and W. J. Studden, *The Theory of Canonical Moments with Applications in Statistics, Probability, and Analysis*, Wiley Series in Probability and Statistics, Wiley, New York, 1997, xvii + 327 pp.

This monograph is concerned with the unfamiliar, and under appreciated, theory and applications of canonical moments of a probability measure  $\mu$  on an interval  $[a, b]$  of the real line and measures on the circle. Roughly speaking, the canonical moments successively define the "relative position"

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of the measure  $\mu$  relative to the interval  $[a, b]$ . In particular, they are more intimately related to the measure  $\mu$  than the ordinary moments. Furthermore, they are invariant under linear transformations of the measure. In this book it is shown that it is often easier to describe measures in terms of canonical moments than ordinary moments and that they have other simple interesting properties. The authors also show how these moments can be very useful in problems of design of experiments, birth and death chains, and approximation theory.

Canonical moments are related to orthogonal polynomials and continued fractions. For a probability measure  $\mu$  on the interval  $[0, 1]$  we have the following relation between the canonical moments  $p_k$  and the Stieltjes transform of  $\mu$

$$\int_0^1 \frac{d\mu(x)}{z-x} = \frac{1}{z} - \frac{p_1}{|1} - \frac{(1-p_1)p_2}{|z} - \frac{(1-p_2)p_3}{|1} - \dots, \quad z \notin [0, 1].$$

The first three chapters of the book provide the theoretical background. Chapter 9, with both theory and applications, is devoted to canonical moments for the circle or trigonometric functions. The other chapters contain various applications. One of the major applications is the determination of optimal designs for polynomial regression, which is presented in Chapters 5 and 6.

A special chapter, Chapter 7, is devoted to applications in approximation theory. It is illustrated that they can be used for deriving the asymptotic zero distribution of classical orthogonal polynomials. Another application

is the derivation of identities for sums of squares of orthogonal polynomials. These generalize the well-known trigonometric identity  $\cos^2 x + \sin^2 x = 1$  and have applications in mathematical physics. As a final application in this chapter, a generalization of the classical Chebyshev approximation problem

$$\min_{a_i} \sup_{x \in [-1, 1]} \left| x^m - \sum_{j=0}^{m-1} a_j x^j \right|,$$

which investigates the best approximation of  $x^m$  (with respect to the sup-norm on the interval  $[-1, 1]$ ) by polynomials of degree  $m-1$ , is considered. A new proof is presented of the fact that the best polynomial  $x^m - \sum_{j=0}^{m-1} a_j x^j$  is given by  $2^{-m+1} T_m(x)$ , where  $T_m(x)$  is the Chebyshev polynomial of the first kind. This proof is based on a game-theoretic argument and the theory of canonical moments.

Chapter 8 deals with the relationship between canonical moments, birth and death chains and orthogonal polynomials. Finally in Chapter 10, two other problems are studied: Bayesian estimation of a binomial probability and the asymptotic distribution of random moment sequences.

This volume clearly illustrates the powerful mathematical role of canonical moments and their beautiful application in statistics, probability, and analysis. The book is well written, well presented, easy to read, and with a lot of clear examples. The bibliography offers a very complete overview of the relevant literature. It can be recommended to anyone who is interested in moment theory and wants to learn, in a limited time, basic things about canonical moments and their applications.

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### *Proceedings*

*General Inequalities 7*, C. Bandle, W. N. Everitt, L. Losonczi, and W. Walter, Eds., International Series of Numerical Mathematics **123**, Birkhäuser Verlag, Basel, 1997, xii + 404 pp.

Since 1976 there have been several meetings at Oberwolfach (Germany) regarding *General Inequalities*. The present volume contains the proceedings of the seventh meeting held in 1995. According to the list of participants there were 51 people attending this meeting. The material in this book is arranged in seven chapters, which are: Inequalities in Analysis; Inequalities for Matrices and Discrete Problems; Inequalities for Eigenvalue Problems; Inequalities for Differential Operators Convexity; Inequalities in