Physics Letters B 741 (2015) 290–294

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Inclusive semileptonic B decays from QCD with NLO accuracy for power-suppressed terms

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ARTICLE INFO

Article history:
Received 4 November 2014
Received in revised form 1 December 2014
Accepted 29 December 2014
Available online 7 January 2015
Editor: B. Grinstein

ABSTRACT

We present the results of a calculation of the perturbative QCD corrections for the semileptonic inclusive width of a heavy flavored meson. Within the Heavy Quark Expansion we analytically compute the QCD correction to the coefficient of power-suppressed contribution of chromo-magnetic operator in the limit of vanishing mass of the final state quark. The important phenomenological applications are decays of bottom mesons, and to the lesser extent, charmed mesons.

1. Introduction

With the success of the LHC mission and the Higgs boson discovery the validity of Standard Model (SM) as the theory of particle interactions at the energies below 1 TeV has been convincingly proved. However, it is hard to expect that we shall be able to explore still higher energy regions in the same manner, namely by a direct observation of new physics phenomena. It is conceivable that new phenomena beyond the standard model can only be identified through detecting slight discrepancies between theoretical predictions within the SM and precision measurements at low energy with available machines. For this program to succeed, the availability of accurate theoretical predictions within the standard model is of crucial importance, especially precise numerical values of key parameters of the SM are necessary.

There is a common belief that the flavor physics of quarks is one of the most promising places for search of new physics [1]. The relevant standard model parameters in this sector are the Fermi constant G_F, quark masses, and quark mixing parameters gathered in the CKM matrix. While the quark weak decays are mediated through charged currents at the tree level (which are believed not to have sizable contributions of possible new physics), their study is of paramount importance for precise determination of the numerical values of the CKM matrix elements. In contrast to leptons, obtaining a theoretical prediction for processes with quarks requires the use of genuinely nonperturbative computational methods (like QCD lattice calculations) due to confinement. Nevertheless, for heavy hadrons the theoretical treatment is somewhat simplified because the large mass of the heavy quark opens the possibility for an expansion in powers of Λ/m_Q where m_Q is the heavy quark mass and $\Lambda \sim 500$ MeV is a hadronic scale of QCD. The technique of such an analysis is formalized as the Heavy-Quark Expansion and is formulated in the modern language of effective theory approach as an effective theory of heavy quarks (HQET). The method has been first used to extract the explicit dependence of decay constant of heavy mesons on the heavy quark mass [2] and then developed into a full-scale effective theory framework (see, e.g., [4,3,5–8]). Top quarks do not form long living hadrons due to the short top quark lifetime, charmed mesons are probably not heavy enough, rendering the application of the Heavy-Quark Expansion (HQE) almost marginal but still possible. The case of bottom meson decays is certainly tractable in this way and thus has been intensively studied. The technique is applicable to $b \to u$ and $b \to c$ transition and to both semileptonic and purely hadronic decays. Our general formulas can be used for the case of charm quark decays $c \to s$ of $c \to d$ as well but, for definiteness, we will stick to semileptonic $b \to c$ transitions in our notation below.

Over the last ten years the heavy quark expansion method in inclusive semileptonic $b \to c$ decays has been refined to such an extend that the remaining theoretical uncertainty in the prediction of the total inclusive rate for $B \to X_c \ell \bar{\nu}$ has reached a level of less than two percent. The structure of the HQE in the case at hand is given by the following expression (see, e.g., [9])

$$\Gamma(B \to X_c \ell \bar{\nu}_\ell)/\Gamma^0 = |V_{cb}|^2 \left[a_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2} \right) + a_2 \frac{\mu_G^2}{m_b^2} + a_3 \frac{\bar{\rho}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda^4}{m_b^4} \right) \right] \tag{1}$$

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http://dx.doi.org/10.1016/j.physletb.2014.12.058

where $\Gamma^0 = G_F^2 m_b^5/(192\pi^3)$ and m_b is the b-quark mass the precise definition of which is discussed below. Here μ_π^2 is the kinetic energy parameter of a heavy hadron, μ_G^2 is the chromo-magnetic parameter, and $\bar\rho$ is a generalized contribution of higher-dimension operators. These are the nonperturbative power-suppressed contributions whose numerical values are of the order of Λ, the hadronic scale of QCD. The coefficients a_i accumulate the perturbative short-distance physics and are functions of the quark (and, in general, lepton) masses and have a perturbative expansion in the strong coupling constant $\alpha_s(m_Q)$ taken at the high scale of the heavy quark mass m_Q. The leading order term a_0 of the heavy quark expansion is known analytically to $\mathcal{O}(\alpha_s^2)$ precision in the massless limit of the final state quark [10]. At the next-to-next-to-leading order of QCD perturbation theory expansion the corrections due to the nonzero value of the final state quark mass have been analytically accounted for as an expansion in m_c/m_b in Ref. [11] and numerically in [12]. The coefficient of the kinetic energy parameter is linked to the leading order coefficient a_0 by Lorentz invariance (see an explicit analysis in [13]).

The expression (1) is a generalization of the result for the muon decay that is one of most precisely known processes that provide valuable information on the SM parameters. The muon decay is important for the determination of the Fermi constant G_F with high accuracy. To match the precision of the experimental data in this case, the theoretical calculations have to be performed with very high accuracy. In this case this is feasible, since the purely leptonic decays are well described with perturbation theory and the expansion parameter $\alpha \approx 1/137$ is small. The latest theoretical result includes the second order (NNLO) radiative correction in the fine structure constant expansion [14] (as a recent review, see [15])

$$\begin{aligned}
&\Gamma(\mu \to \nu_\mu e \bar\nu_e)/\hat\Gamma^0 \\
&\quad = 1 + \Delta_0^{(0)}(m_e^2/m_\mu^2) + \frac{\alpha}{\pi}\left[\left(\frac{25}{8} - \frac{\pi^2}{2}\right)\right. \\
&\qquad \left. - \frac{m_e^2}{m_\mu^2}\left(34 - 24\ln\left(\frac{m_\mu}{m_e}\right)\right) + \frac{2\alpha}{3\pi}\ln\left(\frac{m_\mu}{m_e}\right)\right] \\
&\qquad + (6.700 \pm 0.002)\left(\frac{\alpha}{\pi}\right)^2
\end{aligned}$$

with $\hat\Gamma^0 = G_F^2 m_\mu^5/(192\pi^3)$, m_μ is the muon mass, m_e is the electron mass. Here the function $\Delta_0^{(0)}(m_e^2/m_\mu^2)$ accounts for the tree-level correction due to the non-vanishing electron mass and has the form

$$\Delta_0^{(0)}(z) = -8z - 12z^2 \ln z + 8z^3 - z^4. \tag{2}$$

This result gives an $\mathcal{O}$ (1 ppm) accuracy of theoretical expression that is competitive for comparison with experimental data. The numerical value of the Fermi constant is now known with unprecedented accuracy $G_F = (1.16637 \pm 0.00001) \times 10^{-5}\ \mathrm{GeV}^{-2}$.

The parametrically largest contribution to the theoretical expression for the semileptonic decay width in Eq. (1) currently unknown is the radiative correction of order α_s to the coefficient a_2 of the chromo-magnetic parameter. This correction has been investigated recently in Ref. [16], where a result for this contribution has been obtained after a numerical integration over the phase space of the final leptons.

In this Letter we report on an analytical calculation of corrections to a_2 in the limit of vanishing charmed quark mass. As it turns out, the precision gained in this approximation is sufficient for phenomenological applications.

2. Outline of the calculation

The decay rate of semileptonic inclusive decays of a meson containing a heavy quark is a proper observable for QCD treatment in a model independent manner as it can be reduce to a computation of an appropriate correlation function. Indeed, due to unitarity of the S-matrix the rate (1) is obtained from taking the absorptive part of the forward matrix element of the transition operator T responsible for the change of the flavor (e.g., [17])

$$T = i\int d^4x\, T\left[H_{\mathrm{eff}}(x) H_{\mathrm{eff}}(0)\right] \tag{3}$$

where H_{eff} is the effective Hamiltonian for the semileptonic transition with $\Delta B = 1$

$$H_{\mathrm{eff}} = 2\sqrt{2} G_F V_{cb} (\bar b_L \gamma_\mu c_L)(\bar\nu_L \gamma^\mu \ell_L) + h.c. \tag{4}$$

The representation (3) is a very general one and can be used for light mesons like the kaon as well. It is the large mass of the heavy quark that makes it possible to use perturbative QCD for a model independent treatment of the decay process of a heavy hadron. In order to make the dependence of the width on the heavy quark mass m_b explicit and to build up an expansion in Λ/m_b, one matches a time-ordered product of full QCD operators H_{eff} in (4) on an expansion in terms of Heavy Quark Effective Theory

$$(\mathrm{Im}\, T)/R_0 = C_0 \mathcal{O}_0 + C_v \frac{\mathcal{O}_v}{m_b} + C_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + C_G \frac{\mathcal{O}_G}{2m_b^2} \tag{5}$$

where $R_0 = \pi \Gamma_0 |V_{cb}|^2$. The local operators $\mathcal{O}_i$ in the expansion (5) are ordered by their dimensionality $\mathcal{O}_0 = \bar h_v h_v$, $\mathcal{O}_v = \bar h_v v\pi h_v$, $\mathcal{O}_\pi = \bar h_v \pi_\perp^2 h_v$, $\mathcal{O}_G = \bar h_v \frac{1}{2}[\not\pi_\perp, \not\pi_\perp] h_v$. Here v is the velocity of the heavy hadron appearing in the HQET construction, $\pi_\mu = i\partial_\mu + g_s A_\mu$ is the covariant derivative of QCD, $\pi^\mu = v^\mu (v\pi) + \pi_\perp^\mu$, and h_v is the heavy-quark field entering the HQET Lagrangian [18,19]. The expansion (5) is a matching relation from QCD to HQET with proper operators up to dimension five in mass units with the corresponding coefficient functions. Note that the operator $\mathcal{O}_v$ will be eliminated by using the equation of motion for the field h_v once the forward matrix elements with mesonic states are taken. The Lagrangian for the modes h_v is given by

$$\mathcal{L} = \mathcal{O}_v + \frac{1}{2m_b}\left(\mathcal{O}_\pi + C_m(\mu)\mathcal{O}_G\right) + O\left(\frac{\Lambda^2}{m_b^2}\right) \tag{6}$$

with

$$C_m(\mu) = 1 + \frac{\alpha_s(\mu)}{2\pi}\left\{C_F + C_A\left(1 + \ln\frac{\mu}{m_b}\right)\right\} \tag{7}$$

being the coefficient of chromo-magnetic operator $\mathcal{O}_G$ including the $\mathcal{O}(\alpha_s)$ QCD correction [20]. Here $C_F = (N_c^2 - 1)/(2N_c)$ and $C_A = N_c$ are Casimir operators of QCD gauge group $SU(N_c)$ with $N_c = 3$, and μ is a normalization point that appears because of the non-vanishing anomalous dimension of the chromo-magnetic operator $\mathcal{O}_G$. The result in Eq. (7) is given in $\overline{\mathrm{MS}}$ scheme of renormalization. Note that we define the modes h_v such that terms of the order $O(1/m_b^2)$ in the Lagrangian contain no time derivative [19,21].

It is convenient to chose the local operator $\bar b \not v b$ defined in full QCD as a leading term of heavy quark expansion [22]. Indeed, the current $\bar b \gamma_\mu b$ is conserved and thus its forward matrix element with hadronic states is absolutely normalized. For implementing this one needs an expansion (matching) of a full QCD local operator $\bar b \not v b$ in HQE through HQET operators. This procedure is similar to the way the expansions have been reorganized in Ref. [23]. The expansion of the local operator reads

$$\bar{b}\slashed{v}b = \mathcal{O}_0 - \tilde{C}_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + \tilde{C}_G \frac{\mathcal{O}_G}{2m_b^2} + O\left(1/m_b^3\right) \tag{8}$$

and is valid including the radiative corrections of order α_s. The leading power operator $\mathcal{O}_0$ has no corrections and the kinetic operator has the same coefficient as the leading one due to Lorentz invariance.

Substituting the expansion (8) into Eq. (5) one obtains after using the equation of motion for the operator $\mathcal{O}_v$ in the forward matrix elements

$$(\mathrm{Im}\, T)/R_0 = C_0 \left\{ \bar{b}\slashed{v}b - \frac{\mathcal{O}_\pi}{2m_b^2} \right\} + \{-C_v C_m + C_G - \tilde{C}_G C_0\} \frac{\mathcal{O}_G}{2m_b^2}. \tag{9}$$

The numerical value for the chromo-magnetic moment parameter μ_G^2 related to the forward matrix element of the operator $\mathcal{O}_G$ is usually taken from the mass splitting between the pseudoscalar and vector ground-state mesons. With $m_{B^*} = 5325$ MeV and $m_B = 5280$ MeV the mass difference of bottom ground-state mesons become $m_{B^*}^2 - m_B^2 = \Delta m_B^2 = 0.48$ GeV^2 and can be written through the forward matrix element of the chromo-magnetic operator in the form

$$\frac{1}{2m_B} C_m(\mu) \langle B(p_B) | \mathcal{O}_G | B(p_B) \rangle = \frac{3}{4} \Delta m_B^2 \tag{10}$$

where we use the usual relativistic normalization of the particle states.

Taking the forward matrix element of (9) one gets

$$\Gamma(B \to X_c \ell \nu_\ell)/\Gamma_0 = |V_{cb}|^2 \left\{ C_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2}\right) + \left(-C_v + \frac{C_G - \tilde{C}_G C_0}{C_m}\right) \frac{3\Delta m_B^2}{8m_b^2} \right\}. \tag{11}$$

The matching procedure of obtaining the coefficients of expansion in Eq. (5) is straightforward and consists in computing matrix elements with partonic states (quarks and gluons on the mass-shell) at both sides of the expansion (5). In this way the coefficient function C_0 of the dimension-three operator $\bar{h}_v h_v$ determines the total semileptonic decay width of the heavy quark and at the same time gives the leading contribution to the width of a bottom hadron in the HQE framework.

Going to order α_s, the calculation of the matching of the transition operator T in Eq. (3) requires to consider three-loop diagrams with external heavy quark lines taken on mass shell.

The leading order result is well known and requires the calculation of the two-loop Feynman integrals of the simplest topology – the sunset type ones (see, e.g., [24]). This can be readily obtained with account of masses of final quarks and even leptons that can be of interest for decays of D-mesons to muons or tau leptons.

The contribution to the coefficient C_0 is proportional to the expression of the general form

$$\int \bar{u}(p)\gamma_\mu(1-\gamma_5)S(x, m_c)\gamma_\nu(1-\gamma_5)u(p) \times \Pi_{\mu\nu}(x)e^{ipx}dx$$

where $u(p)$ is a b-quark spinor, $S(x, m_c)$ is a charmed quark propagator, $p = mv$, and $\Pi_{\mu\nu}(x)$ is the leptonic tensor. The Fourier transform of the leptonic tensor reads

$$\Pi_{\mu\nu}(q) = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi\left(q^2\right)$$

It is transverse in the limit of massless charged leptons. In case of decaying into the final massive leptons (muon or massive neutrino even) one can account for the lepton masses as well. In the massless limit the explicit expression for $\Pi(q^2)$ reads

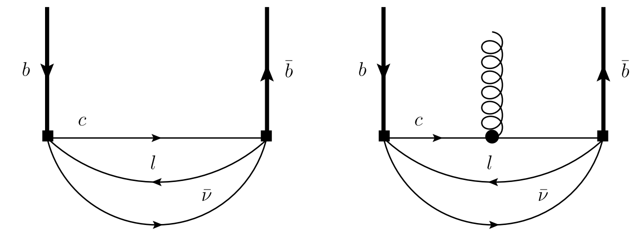

Fig. 1. Perturbation theory diagrams for the matching computation at the leading order (LO), left – width type, right – power correction type (in an external gluon field).

$$\Pi\left(q^2\right) = \frac{C}{(4\pi)^{D/2}} \left(\frac{\mu^2}{-q^2}\right)^{(4-D)/2}$$

in D-dimensional space time, $D = 4 - 2\epsilon$, where

$$C = \left(\frac{2}{3\epsilon} - \frac{2}{9}\right) \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(2-2\epsilon)}.$$

The complete result with the massive charmed quark can be obtained by computing either full integrals with massive lines or through an expansion in the mass ratio m_c/m_b. We checked that the technique of small mass expansion reproduces the known result at the leading order. This technique can be also used at the NLO level. In Fig. 1 we show the leading order diagrams.

When calculating the perturbative coefficients of matching expressions at the NLO level one needs to compute the three-loop on-shell integrals with massive lines. The problem is simplified partly because these are not integral of the most general topology. Further simplification is that one needs only the imaginary parts of emerging integrals.

The computation has been performed within techniques of dimensional regularization used for both ultraviolet and infrared singularities that emerge in separate integrals because of the presence of massless particles and on-shell kinematics. In the actual calculation we used the system of symbolic manipulations REDUCE [25] and the package FeynCalc of Mathematica [26] with original codes written for the present calculation. The general setup of the calculation is by now rather a standard one including first the reduction to a limited set of master integrals that has been done within the integration by parts technique [27]. The original codes have been used for most of the diagrams. The program LiteRed [28] has been used for the check of the results and further application to complicated vertex diagrams. The master integrals have been computed directly. Two of them have a simple topology of sunsets and are almost trivial. The one master integral is nontrivial and has been found with the help of direct integration through Feynman parameters and then checked with the program HypExp [29]. The renormalization is performed on-shell by the multiplication of the bare (direct from diagrams) results by the on-shell renormalization constant Z_2^{OS}

$$Z_2^{\mathrm{OS}} = 1 - C_F \frac{\alpha_s}{4\pi} \left(\frac{3}{\epsilon} + 3\ln\left(\frac{\mu^2}{m_b^2}\right) + 4\right). \tag{12}$$

The whole computation has been done in Feynman (diagonal) gauge for the virtual gluons.

In Fig. 2 we show some typical three loop diagrams of the next-to-leading order. By using the described methods one reproduces the known result for the next-to-leading order expression of the semileptonic decay width of the quark

$$C_0 = 1 + \Delta_0^{(0)}(\rho) + C_F \frac{\alpha_s}{\pi} \left\{ \left(\frac{25}{8} - \frac{\pi^2}{2}\right) + \Delta_0^{(1)}(\rho) \right\} \tag{13}$$

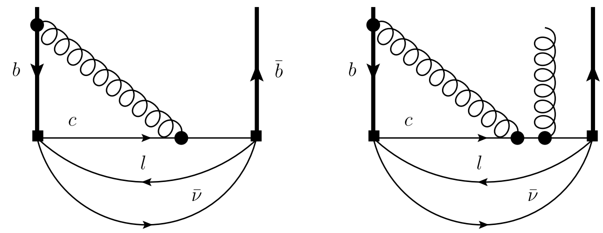

Fig. 2. Perturbation theory diagrams for the matching computation at NLO, left – width type, right – power correction type (in an external gluon field).

with $C_F = 4/3$ and $\rho = m_c^2/m_b^2$. Here $\Delta_0^{(0)}(\rho)$ and $\Delta_0^{(1)}(\rho)$ are corrections due to charmed quark mass at LO and NLO respectively. They are known analytically and normalized such that $\Delta_0^{(0)}(0) = \Delta_0^{(1)}(0) = 0$. At the leading order the whole result is simply as in Eq. (2)

$$\Delta_0^{(0)}(\rho) = -8\rho - 12\rho^2 \ln\rho + 8\rho^3 - \rho^4$$

while the NLO contribution $\Delta_0^{(1)}(\rho)$ given in the original paper is rather lengthy for the general final quark mass dependence [30]. We give only few terms of the small $\rho = m_c^2/m_b^2$ expansion

$$\Delta_0^{(1)}(\rho) = -\rho\left[50 + 12\ln\left(\frac{\mu^2}{m_b^2}\right)\right].$$

Note that in these expressions the definition of the final state quark mass m_c is given in the $\overline{\text{MS}}$ scheme. In principle, for the charmed quark one could use the pole mass as it was in the original paper [30] as well. We have checked the leading order mass correction within our computational techniques.

The coefficient C_v of the dimension-four operator $\mathcal{O}_v$ is singled out by taking the matrix element between quarks on shell and one gluon with vanishing momentum and longitudinal polarization, i.e. the gluon field is chosen to be of the form $A_\mu = v_\mu(vA)$. The coefficient C_v reads

$$C_v = 5 + C_F\frac{\alpha_s}{\pi}\left\{-\frac{25}{24} - \frac{\pi^2}{2}\right\}. \tag{14}$$

It has no μ dependence and no C_A color contribution. It is μ independent because the operator $\mathcal{O}_v$ is RG invariant. The absence of C_A color contribution is due to gauge invariance. Indeed, this matches also the possibility to compute this coefficient using small momentum expansion near the quark mass shell, $p = mv + k$. A powerful check of the result is an explicit cancellation of the contribution proportional to the color structure C_A and the renormalization (cancellation of ϵ-poles) with the same renormalization constant Z_2^{OS} shown in (12).

The final coefficient of the chromo-magnetic operator multiplied by C_m (see Eq. (11)) reads

$$C_{fin} = -C_v + (C_G - \tilde{C}_G C_0)/C_m \tag{15}$$

and

$$C_{fin} = -3 + \Delta_G^{(0)}(\rho) + \frac{\alpha_s}{\pi}\Delta_G^{(1)}(\rho) + \frac{\alpha_s}{\pi}\left\{C_A\left(\frac{31}{18} - \frac{\pi^2}{9}\right) + C_F\left(\frac{43}{144} - \frac{19\pi^2}{36}\right)\right\}. \tag{16}$$

The function $\Delta_G^{(0)}(\rho)$ is known analytically (e.g., [8])

$$\Delta_G^{(0)}(\rho) = 8\rho - 24\rho^2 + 24\rho^3 - 5\rho^4 - 12\rho^2\ln(\rho)$$

The function $\Delta_G^{(1)}(\rho)$ emerges in the analysis of Ref. [16] where the analytical result for the coefficient of the chromo-magnetic operator at the level of hadronic structure functions has been obtained. Both functions are chosen such that they vanish at $m_c = 0$. The final integration over the phase space in Ref. [16] has been done numerically that prevents us from a direct comparison between the two results. Numerically we obtain at the limit $m_c = 0$

$$C_{fin} = -3 + \frac{\alpha_s}{\pi}(0.63C_A - 4.91C_F) = -3 + \frac{\alpha_s}{\pi}(-4.67) = -3\left(1 + 1.56\frac{\alpha_s}{\pi}\right). \tag{17}$$

The μ dependence of the prefactor of $\mathcal{O}_G$ in (9) matches the leading order anomalous dimension of the chromo-magnetic operator [20], such that C_{fin} is μ independent. Furthermore, the mass parameter of the heavy quark m_b is chosen to be the pole mass which is a proper formal parameter for perturbative computations in HQET (see discussion in [9]). After having obtained the results of perturbation theory computation for the coefficients of HQE, one is free to change this parameter to any other [31].

Our results (16), (17) still depend on μ through the strong coupling α_s defined in the $\overline{\text{MS}}$-scheme; however, this remaining scale dependence can only be resolved at the next order in α_s.

3. Discussion of the results

The radiative corrections are of reasonable magnitude and are well under control for the numerical values of the coupling constant for $\mu \sim 2$–4 GeV. This provides a clean application of the results to the decays into final light quarks u for bottom mesons and s or d for charmed mesons.

For the applications to the $b \to c$ transitions an important question is the magnitude of corrections due to non-vanishing charmed quark mass. It seems that mass corrections are important numerically. The small ρ expansion reads $\Delta_G^{(0)}(\rho) = 8\rho + \ldots$, and $\Delta_G^{(1)}(\rho) = A\rho + \ldots$ where the factor A is not known analytically. Assuming $|A| \leq 50$ one sees that the massless approximation dominates the radiative correction for typical values of ρ in the range $\rho = 0.06 \pm 0.02$ [32],

$$C_{fin} = -3 + \frac{\alpha_s}{\pi}(-4.67 + \rho A). \tag{18}$$

While a literal comparison with the results of [16] is difficult, the phase space reduction factor of 1/3 suffices to reproduce the result of massive computation. Note that within our approach the analytical computation of mass corrections is possible in the form of a series expansion in ρ.

At present the value of $|V_{ub}|$ from inclusive decays is $|V_{ub}| = (4.41 \pm 0.15 \pm 0.16) \times 10^{-3}$ [33] while the extraction from exclusive $B \to \pi\ell\bar{\nu}$ yields $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$. However, the exclusive determination does not rely on the local OPE considered here, so from our results we cannot really draw a definite conclusion. Nevertheless, if our result indicates the size of the expected corrections, it cannot resolve the tension between the inclusive and the exclusive value.

More important are the implications for inclusive semileptonic B meson decays to charm, since here the precision is high enough to worry about the correction computed above. Indeed, the inclusive determination has a precision at the level of roughly 2%, the value being $|V_{cb}| = (42.4 \pm 0.9) \times 10^{-3}$ [34,33]. Since we only have the analytical result in the limit $m_c \to 0$ at hand, we estimate the impact of our correction in a simplified manner. Because it is a small correction, we only account for charmed quark mass at tree approximation, taking into account the kinematic function

$\Delta_0^{(0)}(\rho) = -8\rho - 12\rho^2 \ln\rho + 8\rho^3 - \rho^4$. The determination of $|V_{cb}|$ uses the total rate only, so we get for the shift in $|V_{cb}|$ through the α_s correction in the coefficient of chromo-magnetic operator

$$\frac{\Delta|V_{cb}|}{|V_{cb}|} = 4.67\frac{\alpha_s}{\pi}\frac{3\Delta m_B^2}{8m_b^2}\frac{1}{2(1+\Delta_0^{(0)}(\rho))} \tag{19}$$

which yields for $\rho = 0.07$ and $\alpha_s/\pi = 0.1$ a relative shift of $+0.3\%$ in the value of $|V_{cb}|$.

The shift in $|V_{cb}|$ has to be compared to the corrections of order $(\Lambda/m_b)^n$, $n = 3, 4$, at tree level. The $(\Lambda/m_b)^3$ contributions induce a relative shift in $|V_{cb}|$ of about -1.5% which is included in the current analysis. The terms of order $(\Lambda/m_b)^4$ are not yet included and shift the value of $|V_{cb}|$ by about 0.3% [35], which is roughly of the same order as the corrections considered here.

Acknowledgements

We acknowledge interesting discussions with A.G. Grozin, B.O. Lange, J. Heinonen, and T. Huber. We thank P. Gambino for a careful reading of the manuscript, valuable comments, and discussion of the results of the small charm quark mass extrapolation. This work is supported by DFG Research Unit FOR 1873 "Quark Flavour Physics and Effective Field Theories".

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