The behavior of the electric field in X and O modes in traveling wave tubes filled with magnetized plasma

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Abstract

In this article magnetized plasma filled traveling wave tubes are investigated. The mentioned system, is located inside a cylindrical wall. The space between the helix and the wall is filled with dielectric material. Dispersion relation for the azimuthally symmetric modes is achieved. Using field theory and by applying appropriate boundary conditions, dispersion relation is obtained for the desired configuration. An axial component of the electric field is investigated as an important parameter in the traveling wave tubes numerically. In this paper, the radial electric field changes are analyzed numerically for varying amounts of plasma density, dielectric constant, helix radius and axial magnetic field. [16,17]. The specific geometry under study is illustrated in Fig. 1. A conducting waveguide of radius \( R_w \) encloses a hollow dielectric with an inner radius \( R_s = R_p \). Background plasma with a uniform density distribution of radius \( R_p \) is loaded inside the helix. The purposes are the investigation of the dielectric constant, plasma density, axial guide magnetic field, and helix radius effects on the plasma-loaded helix TWT [18] by solution of the fluid equation and Maxwell’s equations. The boundary condition includes two regions: (1) inside the helix that includes the plasma and (2) between the helix and the wall that is dielectric.

The organization of the paper is as follows. Section ‘Perturbed transfer fields’ is devoted to the derivation of the perturbed transfer fields in region 1. The dispersion relation is determined in section ‘The dispersion relation’ through application of the appropriate boundary conditions upon the solution of Maxwell’s equations. In section ‘Numerical results and conclusions’, we deal with numerical results and conclusion.

Perturbed transfer fields

The plasma-filled tape helix (loaded cylindrical drift tube) is presented in the presence of a uniform axial magnetic field \( B_0 = B_0e_z \). The charge density is described by

\[
\rho_0(r) = n_e H(R_0 - r)
\]

where \( n_e \) and \( R_0 \) are the ambient plasma density and plasma radius, respectively. The \( H \) is the Heaviside function. Fig. 1 shows the...
Fig. 1. Cross-sectional view of the structure. The dielectric fills the region between $R_h$ (helix radius) and $R_c$ (conducting wall radius) and plasma fills the region between 0 and $R_c$.

schematic cross section of the system. The helix is assumed thin enough such that a conducting cylindrical sheet of radius $R_h$, width $\epsilon$, and pitch angle $\phi$ model it. The helix pitch is described by the unit vector $[14]$. 

$$\hat{e}_\phi = \hat{e}_\theta \cos \phi + \hat{e}_z \sin \phi$$ (2)

For the cold plasma the perturbed current density is described with small perturbations about the equilibrium state, $n_p = n_{0p} + \delta n_p$ and $\mathbf{v}_p = \mathbf{v}_{0p} + \delta \mathbf{v}_p$. The nonlinear effects are neglected. The perturbed current density and charge density are obtained by using the linearized continuity, momentum transfer, Maxwell’s equations and Floquent’s theorem, as follows:

$$\delta j_{\perp i} = -\frac{e}{4\pi \lambda_{i\perp}} \delta \mathbf{A}_\perp \left\{ \frac{ck}{o} \left[ \mathbf{\nabla}_i \delta \mathbf{E}_z - \hat{e}_z \times \nabla_i \delta \mathbf{B}_z + i \left( \frac{ck}{o} \hat{e}_z \times \nabla_i \delta \mathbf{E}_z + \nabla_i \delta \mathbf{B}_z \right) \right] \right\},$$ (3)

where

$$R_p(\mathbf{r}, k) = \frac{2\pi A_{0p}}{\lambda_{i\perp}(\mathbf{r}, \alpha)}, \quad A_{i\perp} = 1 - \frac{\omega_\perp^2}{c^2 k^2 (\mathbf{r} + \mathbf{\Omega}_x)};$$

$$\mathbf{A}_\perp = 1 - \frac{\omega_\perp^2}{c^2 k^2 (\mathbf{r} + \mathbf{\Omega}_x)};$$

$$2\mathbf{A}_\perp = \frac{\omega_\perp^2}{c^2 k^2 (\mathbf{r} + \mathbf{\Omega}_x)}; \quad \lambda_{i\perp} = 1 - \frac{\omega_\perp^2}{c^2 k^2 (\mathbf{r} + \mathbf{\Omega}_x)};$$

$$\mathbf{P}_{\perp} = \frac{\omega_\perp^2}{c^2 k^2 (\mathbf{r} + \mathbf{\Omega}_x)}; \quad \mathbf{A}_\perp = 1 - \frac{\omega_\perp^2}{c^2 k^2 (\mathbf{r} + \mathbf{\Omega}_x)}.$$

The dielectric tensor in cylindrical coordinate is obtained by using the Maxwell’s equations and source current as follows:

$$e_{r\perp} = e_{0\perp} = 1 - \frac{\omega_\perp^2}{(\omega^2 - (\mathbf{r} + \mathbf{\Omega}_x)^2)};$$

$$e_{\theta\perp} = -e_{r\perp} = -\frac{\omega_\perp^2}{(\omega^2 - (\mathbf{r} + \mathbf{\Omega}_x)^2)};$$

$$e_{z\perp} = 0, \quad e_{\theta\perp} = 0, \quad e_{0\perp} = 0, \quad e_{2\perp} = 0;$$

$$e_{z\perp} = 1 - \frac{\omega_\perp^2}{c^2};$$

(7)

With the aid of the linearized continuity, momentum transfer, and Maxwell’s equations together with Floquent’s theorem the fluctuating transverse fields are obtained as a function of the axial electric and magnetic fields as follows:

$$\delta \mathbf{E}_\perp = \frac{i}{k} \left[ \frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{E}_\perp}{\partial \mathbf{r}} + \frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{E}_\perp}{\partial \mathbf{r}} \right] \left[ \frac{ck}{o} \delta \mathbf{E}_z + i\delta \mathbf{B}_z \right]$$ (8)

$$\delta \mathbf{B}_\perp = \frac{i}{k} \left[ \frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{B}_\perp}{\partial \mathbf{r}} + i\frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{B}_\perp}{\partial \mathbf{r}} \right] \left[ \frac{ck}{o} \delta \mathbf{E}_z + i\delta \mathbf{B}_z \right]$$ (9)

$$\delta \mathbf{B}_\perp = \frac{i}{k} \left[ \frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{B}_\perp}{\partial \mathbf{r}} + i\frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{B}_\perp}{\partial \mathbf{r}} \right] \left[ \frac{ck}{o} \delta \mathbf{E}_z + i\delta \mathbf{B}_z \right]$$ (10)

$$\delta \mathbf{B}_\perp = \frac{i}{k} \left[ \frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{B}_\perp}{\partial \mathbf{r}} + i\frac{\omega_\perp^2}{c^2} \frac{\partial \mathbf{B}_\perp}{\partial \mathbf{r}} \right] \left[ \frac{ck}{o} \delta \mathbf{E}_z + i\delta \mathbf{B}_z \right]$$ (11)

The dispersion relation

By using Eqs. (8)-(11) and Maxwell’s equations the following fourth-order differential equations are obtained for the electric and magnetic fields as follows:

$$(\nabla^2 + \chi^2_{+1p})\nabla^2 \delta \mathbf{E}_z = 0,$$ (12)

$$(\nabla^2 + \chi^2_{-1p})\nabla^2 \delta \mathbf{B}_z = 0,$$ (13)

where

$$\chi_{\pm 1p} = \delta \left[ \frac{1}{2(1 - \frac{\omega_\perp^2}{c^2 \Omega_x^2})} \left\{ \left( 1 - \frac{\omega_\perp^2}{(\omega^2 - \Omega_x^2)} + \chi_{\pm 1p}(1 - 2\epsilon_p) \right) \right\} \right].$$

Here $\delta \chi_{\pm 1p}(1 - 2\epsilon_p)$. There are two modes in the name of $\chi_{\pm 1p}$. For convenience, the $\chi_{+1p}$ and $\chi_{-1p}$ display the X- and O modes respectively. There are two limiting case (1) in the absence of the magnetic field above equation reduces to $\chi_{\pm 1p} = \chi^2_{\pm 1p}$. In this case there are degenerate modes which correspond to left and right circular waves in uniform plasma (2) in the infinite axial magnetic field we have that $\chi_{+1p} = \chi^2_{\pm 1p}$ and $\chi_{-1p} = \chi^2_{\pm 1p}$. These modes are confined transparent to the plasma and space charge modes, respectively. On the other hand in the infinite magnetic field limit there is the infinite plasma in which we may Fourier transforms in the transverse direction and let $\nabla^2 = -k^2$. In this limit, Eqs. (12) and (13) reduce to the familiar dispersion equation for the Appleton-Hartree magnetoionic modes. If the wave propagates parallel to the axial magnetic field, then $k_\parallel \rightarrow 0$. The following dispersion equation is obtained from Eqs. (12) and (13) as:

$$\chi_{+1p}\chi_{-1p} = \chi^2_{\pm 1p},$$

where $\chi_{\pm 1p}$. is the plasma charge wave dispersion relation and $A_{\pm 1p}$ describes the right and left circularly polarized electromagnetic waves. Dispersion relation using Eqs. (11) and (12) and applying boundary conditions is obtained. For one of the two appropriate modes these equations reduce to the following:

$$(\nabla^2 + \chi^2_{+1p})\delta \mathbf{E}_z = 0,$$ (14)

$$(\nabla^2 + \chi^2_{-1p})\delta \mathbf{B}_z = 0.$$ (15)
Fig. 2. The normalized frequency versus wave number for several values of plasma frequency.

Fig. 3. Normalized axial electric field versus radius for several values of plasma frequency.
The solutions of Eqs. (14) and (15) are as follows:

\[
\delta E_z = \begin{cases} 
J_0(\chi \pm i p r), & 0 \leq r < R_b \\
0, & R_b \leq r \leq R_g
\end{cases}
\]

\[
\delta B_z = \begin{cases} 
J_0(\chi \pm i p r), & 0 \leq r < R_b \\
0, & R_b \leq r \leq R_g
\end{cases}
\]

where

\[
q = \left(c \Omega^2/c^2 - k^2\right)^{1/2}.
\]

By applying the boundary conditions on the waveguide wall, the following equations are obtained:

\[
\delta E_z = \begin{cases} 
J_0(\chi \pm i p r), & 0 \leq r < R_b \\
D W(q R_g, q r), & R_b \leq r \leq R_g
\end{cases}
\]

\[
\delta B_z = \begin{cases} 
J_0(\chi \pm i p r), & 0 \leq r < R_b \\
K' W(q R_g, q r), & R_b \leq r \leq R_g
\end{cases}
\]

where,

\[
W(q R_g, q r) = Y_0(q R_g) J_0(q r) - J_0(q R_g) Y_0(q r).
\]

\[
W'(q R_g, q r) = Y_0'(q R_g) J_0(q r) - J_0'(q R_g) Y_0(q r).
\]

By applying the boundary conditions on the coil, the following equations are obtained:

1-\[\delta \tilde{E}_z(R_b - \varepsilon) = \delta \tilde{E}_z(R_b + \varepsilon)\]

2-\[\delta \tilde{E}_z(R_b - \varepsilon) = \delta \tilde{E}_z(R_b + \varepsilon)\]

Coefficients A and D, are shown by the following equation:

\[
D = A J_0(\chi + i p R_b)/W(q R_g, q R_b)
\]

\[
K = q \chi J_0'[\chi J_0(\chi + i p R_b) + R_d J_0(\chi + i p R_b)]
- i A (ck R_p/\omega) J_0'(\chi + i p R_b))/\chi^2 W'(q R_g, q R_b),
\]

where

\[
W'(q R_g, q r) = Y_0'(q R_g) J_0(q r) - J_0'(q R_g) Y_0(q r),
\]

\[
W(q R_g, q r) = Y_0(q R_g) J_0(q r) - J_0(q R_g) Y_0(q r).
\]

To obtain the dispersion relation, the discontinuity in the axial and azimuthal magnetic field will be used:

\[
2 - \delta B_z(R_b - \varepsilon) - \delta B_z(R_b + \varepsilon) = 4 \pi \frac{i}{c} \delta \tilde{J}_z \Delta R_b \cos \phi,
\]

\[
1 - \delta B_z(R_b + \varepsilon) - \delta B_z(R_b - \varepsilon) = 4 \pi \frac{i}{c} \delta \tilde{J}_z \Delta R_b \sin \phi.
\]

By using Eqs. (22) and (23), one obtains the following equation:

\[
A = 4 \pi \frac{i}{c} \Delta R_b \left[\left(-W'(q R_g, q R_b) \cos \varphi_T - i y^2 \sin \varphi T_1\right) T_1 + T_2 T_3\right],
\]

\[
F = 4 \pi \frac{i}{c} \Delta R_b \left[\left(W'(q R_g, q R_b) \cos \varphi_T - i y^2 \sin \varphi T_2\right) T_1 + T_2 T_3\right],
\]

where \(T_1, T_2, T_3, T_4\) are as follows:

\[
T_1 = J_0(\chi \pm i p R_b) W'(q R_g, q R_b) - \frac{q \chi \pm i p}{\chi^2} W'(q R_g, q R_b) J_0'(\chi \pm i p R_b)
+ R_d J_0(\chi \pm i p R_b)
\]

\[
T_2 = J_0(\chi \pm i p R_b) W'(q R_g, q R_b) + \frac{q \chi \pm i p}{\chi^2} W'(q R_g, q R_b) J_0'(\chi \pm i p R_b)
- \frac{ck R_p}{\omega} J_0'(\chi \pm i p R_b)
\]

\[
T_3 = -k R_p J_0(\chi \pm i p R_b)
\]

\[
T_4 = \frac{\chi^2 \xi \omega}{c q W'(q R_g, q R_b)} Y_0(q R_g) J_0'(q R_b) - J_0(q R_g) Y_0(q R_b) J_0'(\chi \pm i p R_b)
- \frac{\chi \pm i p}{\chi^2} W'(q R_g, q R_b) J_0'(q R_b)
+ k R_p \frac{ck}{\omega} J_0'(\chi \pm i p R_b)
\]

\[
\delta E_z \sin \varphi + \delta \tilde{E}_z \cos \varphi = 0,
\]

Dispersion relation is obtained by using Eqs. (24)–(26) as follows:

\[
\psi^{(1)} + \psi^{(2)} = 0
\]

where

\[
\psi^{(1)} = 4 \pi \frac{i}{c} \delta \tilde{J}_z \left[\left(-W'(q R_g, q R_b) \cos \varphi_T - i y^2 \sin \varphi T_1\right) T_1 + T_2 T_3\right]
J_0(\chi \pm i p R_b) \sin \varphi - \frac{ck R_p}{\omega} R_0(\omega, k) \cos \varphi J_0'(\chi \pm i p R_b),
\]

\[
\psi^{(2)} = 4 \pi \frac{i}{c} \delta \tilde{J}_z \left[\left(W'(q R_g, q R_b) \cos \varphi_T - i y^2 \sin \varphi T_2\right) T_1 + T_2 T_3\right]
J_0(\chi \pm i p R_b) \sin \varphi - \frac{ck R_p}{\omega} R_0(\omega, k) \cos \varphi J_0'(\chi \pm i p R_b),
\]
**Fig. 6.** Comparison of axial electric field as a function of radius between x-mode and O-mode.

**Fig. 7.** Normalized axial electric field versus radius with and without the electron beam.
And
\[
q^{(2)} = e \frac{\text{Re} \left\{ \epsilon \right\}}{4} \left[ W'\left( q R_h, q R_h, \cos \theta, \frac{r}{R_h} \right) + \frac{j}{\sin \theta} \sin \theta \frac{1}{1 + j \frac{r}{R_h}} \right]
\]

where
\[
q = \left( \epsilon \sigma^2 / c^2 - k^2 \right)^{1/2}, \quad W'\left( q R_h, q R_h \right) = Y_0(q R_h) I_0(q R_h) - \frac{j \sigma}{q R_h} Y_0(q R_h).
\]

\( I_{\sigma} \) is the surface current density parallel to the helix [2].

**Numerical results and conclusions**

The nominal parameters of this system correspond to a plasma column with radius \( R_p = 0.05 \) cm that completely filled the helix with a period \( R_h = 0.080137 \) cm, a width \( R_c = 0.035 \) cm and a radius of \( R_0 = 0.12446 \) cm. The entire system is enclosed within a wall of radius \( R_w = 0.2794 \) cm. By numerical computation for Eq. (27), we can obtain the dispersion relation of the slow wave structure filled with plasma. The relative amplitude of the axial electric field can be obtained from Eqs. (16)-(19) when the frequency is fixed (\( f = 10 \) GHz).

We first consider the X-mode that corresponding to \( \omega_c > \omega \). Fig. 2 shows the effect of plasma on the dispersion characteristics in the absence of an electron beam. The chosen parameters are \( \Omega_x = 0.1 \) and \( \epsilon = 1.0 \). In the absence of the plasma there is only the helix mode. In the presence of the plasma, two curves corresponding to the lowest frequency solutions for the plasma mode and the highest frequency for the helix mode. The plasma modes between \( \omega = 0 \) and \( \omega = \omega_p \) depend on the presence of the plasma, and it represents the plasma oscillation in a finite plasma. The finite boundary has caused the Langmuir oscillation now propagates. As seen in this figure there is a coupling of an electromagnetic wave supported by the helix and an electrostatic wave corresponding to plasma.

Fig. 3 illustrates the radial profiles of the axial electric field for several values of plasma density for X-mode. As seen in this figure, there are two regions. The region 1 \((0 < r < R_0)\) that is completely filled with plasma and the region 2 \((R_0 < r < R_h)\) that is dielectric. Inside the region 1 the electric field increases by increasing of the \( r \) until it reaches its maximum value on the helix. This behavior of the electric field in the region 2 is reverse. It is apparent from the figure that in the fixed radius as the plasma electron density increases, the value of the axial electric field also increases. This is because of the fact that the density of the plasma electron has direct relation with the electric field. The transverse structure of such wave electric field in cylindrical coordinates is described by modified Bessel functions. As the plasma frequency increases the \( k_{\text{pl}}^{(2)} \) also increases and, by considering mathematical properties of modified Bessel function, the electric field axis inside the plasma increases. In the absence of plasma, the axial field of a slow wave is strongest near the dielectric and a beam would have to be placed close to the dielectric to experience a strong interaction with the wave. With an over dense plasma present the field within the plasma can be enhanced. The coupling impedance on axis (in ohms) is defined as
\[
Z = 377 \left| E_0(0) \right|^2 \left[ k^2 \text{Re} \left\{ \int d \tau \cdot \left( \vec{E} \times \vec{B} \right) \right\} \right]
\]

where \( \vec{B} \) and \( \vec{E} \) are the magnetic and electric fields of the mode. The strength of the component of the field, which interacts with the beam, is normalized to the power being carried by the wave measured by the coupling impedance. As shown in Fig. 3, the presence of plasma increases the axial electric field near the axis. According to this relation, the presence of the plasma increases the coupling impedance. On the other hand, Fig. 3 shows that the effect of plasma is to flat the profile of the electric field. This is of interest since in this case all beam particles immersed into the plasma can experience the same deceleration field. Numerical results show that in this case there is no difference between the X- and O-modes.

Now we want to response the question of axial magnetic field on the radial variation of the axial electric field. It is apparent that at sufficiently strong axial magnetic fields the electrons motion will be drastically one-dimensional since the transverse motion of the electron in the presence of the axial magnetic field and electromagnetic field of the helix will be suppressed. As a result, the specifications of the interaction can be expected to change with increasing values of the axial magnetic field until the field becomes strong enough to suppress the transverse electron motion and to remain relatively constant thereafter. In Fig. 4, \( \omega_p = 0.01 \) and \( \epsilon = 1.75 \). This figure shows the radial profile of axial electric field for different values of cyclotron frequency for X-mode. As seen in this figure in region 1 the axial electric field increases by increasing the axial guide field until it reaches its maximum at \( \Omega_x = 0.03 \). It is shown that the axial electric field remains constant in strong magnetic field limits. It is apparent from the Fig. 4 that the radial profile of the axial electric field in region 2 is approximately constant with the variation of the cyclotron frequency.

In Fig. 5, \( \omega_p = 0.01 \) and \( \epsilon = 1.75 \). This figure shows the radial profile of axial electric field in regions 1 and 2 for several values of cyclotron frequency for O-mode. As seen in this figure in region 1 the axial electric field decrease by increasing the axial guide field until reaches its minimum at \( \Omega_x = 0.02 \) and after this increase by increasing the magnetic field. This figure shows that the electric field in the strong magnetic field limit remains constant. The radial profile of the axial electric field in region 2 is not very sensitive to the variation of the cyclotron frequency. As seen from Figs. 4 and 5 the radial profile of the electric field for several values of axial electric field for X- and O-modes have different behavior.

Fig. 6 shows the comparison of radial profile of electric field for X- and O-modes. It is clear in regions 1 and 2 that the X-mode axial electric field is greater than the O-mode.

Fig. 7 shows the radial profile of the axial electric fields with and without the electron beam for X mode in region 1. The chosen parameters are \( \gamma = 1.005558, \omega_p = 0.005 \) and \( \Omega_x = 0.1 \). As seen in this figure the presence of the electron beam decrease the electric field. This is because of the fact that the presence of the electron beam removes the plasma electrons and finally decreases the electric field inside the plasma.

**Summary**

The radial profile of the axial electric field plays the important role in the interaction of electron beam and electromagnetic wave in the traveling wave tube. In the absence of plasma, the radial profile of the electric field is strongest near the helix wall [19] and causes some problem in the operation of the traveling wave tube. In the present work, we investigate the effect of plasma and system parameters on the radial profile of axial electric field in the absence of the electron beam. The scope is to increase the amplitude of the axial electric field near the center of the helix axis. The results show that the presence of plasma and dielectric considerably increases the amplitude of the axial electric field near the axis of the helix. The results show that the behavior of the X-mode and O-mode are different.

**References**


