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Simultaneous solutions of the strong CP and μ problems

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Abstract

The μ parameter of the supersymmetric standard model is replaced by λS, where S is a singlet chiral superfield, introducing a Peccei–Quinn symmetry into the theory. Dynamics at the electroweak scale *naturally* solves both the strong CP and μ problems as long as λ is of order $\sqrt{M_Z/M_{\rm pl}}$ or smaller, and yet this theory has the same number of relevant parameters as the supersymmetric standard model. The theory will be tested at colliders: the μ parameter is predicted and there are long-lived superpartners that decay to gravitinos or axinos at separated vertices. To avoid too much saxion cold dark matter, a large amount of entropy must be produced after the electroweak phase transition. If this is accomplished by decays of a massive particle, the reheat temperature should be no more than a GeV, strongly constraining baryogenesis. Cold dark matter may be composed of both axions, probed by direct detection, and saxions, probed by a soft X-ray background arising from decays to $\gamma\gamma$. There are two known possibilities for avoiding problematic axion domain walls: the introduction of new colored fermions or the assumption that the Peccei–Quinn symmetry was already broken during inflation. In the first case, in our theory the colored particles are expected to be at the weak scale, while in the second case it implies a good chance of discovering isocurvature perturbations in the CMB radiation and a relatively low Hubble parameter during inflation.

1. Introduction

A spontaneously broken global symmetry remains an attractive solution to the strong CP problem [1]. The strong CP parameter $\bar{\theta}$ is canceled by the dynamical relaxation of the resulting pseudo-Goldstone boson, the axion [2]. A global symmetry with a QCD anomaly, $U(1)_{PQ}$, can be implemented in one of the simplest extensions of the standard model—models with two Higgs doublets $h_{1,2}$ [1]. In order that f_a be much larger than the weak scale, $\tilde{m}$, the primary breaking of $U(1)_{PQ}$ must come from an electroweak singlet scalar, s. As in the DFSZ invisible axion models [3], these scalars will have $U(1)_{PQ}$-preserving interactions such as sh_1h_2 or $s^2h_1h_2$, but not $s^*h_1h_2$ or $s^{*2}h_1h_2$. This extension of the standard model fits well with supersymmetry (SUSY), except for an immediate question

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of why the masses of the two Higgs doublets are much smaller than the scale f_a. This corresponds to the μ problem in the supersymmetric standard model.

In this Letter we point out an extremely simple model that simultaneously solves both the strong CP and μ problems without severe fine-tuning. Ours is certainly not the first such theory (i.e., SUSY DFSZ axion models) [4–6], but it is very simple and has important consequences for signals at accelerators and in cosmology. In particular, the gravitino and axino (fermionic SUSY partner of the axion) are much lighter than the weak scale, so that the superpartners produced at hadron colliders end up decaying to either gravitinos or axinos possibly with separated vertices. The saxion, the scalar SUSY partner of the axion, is also lighter than the weak scale. All SUSY particles around the weak scale are unstable, but cold dark matter may be composed of axions,[1] and possibly saxions. The cosmological saxions lead to astrophysical X-ray signals.

The model is presented in Section 2. Limits on the SUSY-breaking scale, and hence on the gravitino mass, are discussed in Section 3. Section 4 is devoted to the thermal history of the model, including late-time entropy production required from saxion evolution. The final section contains a summary of the predictions of our theory, along with remarks on the difference of our theory from those in [4].

2. Model

We consider a supersymmetric theory at the electroweak scale with a superpotential

$$W = \lambda S H_1 H_2 \tag{1}$$

together with Yukawa interactions of the quarks and leptons to the Higgs doublets $H_{1,2}$. This is certainly a very simple theory: it is the minimal supersymmetric standard model (MSSM) with μ replaced by λS, where S is a gauge singlet superfield, and the soft parameter μB replaced by $A\lambda s$. Alternatively it can be viewed as the next-to-MSSM without the κS^3 interaction. There is a $U(1)_{PQ}$ symmetry that is spontaneously broken by $\langle s\rangle$, leading to a Goldstone boson. Therefore, $\langle s\rangle$ must be larger than about 10^{10} GeV to avoid laboratory and astrophysical constraints. One immediate problem is to obtain an appropriate scalar potential for s; $\langle s\rangle$ should be non-zero, but should not be infinite. The superpotential (1) alone neither destabilizes nor stabilizes the flat direction s.

This theory has been studied before as a solution to the strong CP problem [7], and a stable vacuum with finite $\langle s\rangle$ was found in the analysis of the scalar potential involving SUSY breaking. However, the authors of [7] believed that a fine-tune was necessary to obtain a stable vacuum, and that an effective $\mu_{\mathrm{eff}} \equiv \lambda\langle s\rangle$ parameter of order weak scale, $\tilde{m}$, is obtained only as a result of another accidental cancellation between $\langle s\rangle \gg \tilde{m}$ and $\lambda \ll 1$. The μ problem was not solved.

On the other hand, it is known [8,9] that the theory with (1) has a natural solution to the μ problem, provided that the soft mass term for s is sufficiently small

$$\left|m_S^2\right| < \lambda^2 \tilde{m}^2 \tag{2}$$

for some dynamical reason (see Section 3). Once the two Higgs doublets $H_{1,2}$ acquire vevs $v_1 = \cos\beta v$ and $v_2 = \sin\beta v$, the potential for s becomes

$$V(s) = -(A\lambda v_1 v_2 s + \text{h.c.}) + \lambda^2\left(v_1^2 + v_2^2\right)s^* s, \tag{3}$$

leading to a stable vacuum with $\langle s\rangle \sim \lambda^{-1}\tilde{m}$ and generating an effective μ parameter

$$\mu_{\mathrm{eff}} \equiv \lambda s = A\cos\beta\sin\beta \tag{4}$$

of order $\tilde{m}$ for *any* value of λ, providing a very elegant solution to the μ problem. The crucial point is that the scale of μ is set by A, not by $\langle s\rangle$. Thus, our observation is that the theory described by (1) provides a simple simultaneous solution to the strong CP and μ problems.

Of course, to satisfy astrophysical limits $f_a \approx \langle s\rangle \gg \tilde{m}$, the coupling λ must be very small. We will demonstrate in Section 5 that some models can yield $\lambda \approx \sqrt{\tilde{m}/M_P}$, so that the axion decay constant f_a lies around 10^{11} GeV, naturally giving rise to axions as cold dark matter.

[1] It is remarkable that the cosmological dynamical relaxation of the axion field during the QCD era can lead to the observed amount of dark matter in cold axions, providing the symmetry breaking scale is $f_a \approx 10^{11}$ GeV, one order of magnitude above the lower bound set by the SN 1987A constraint. This value for f_a required for the dark matter may be obtained as the geometric mean of the supersymmetry (SUSY) breaking scale and the Planck scale [4].

The spectrum of this theory consists of states in the chiral multiplet S, in addition to those of the MSSM [8,9]. The saxion mass, m_s, is of order $\lambda\tilde{m} \sim 100\ \text{eV} \times (\lambda/10^{-9})$, and the axino mass is of order[2,3] $\lambda^2\tilde{m} \sim 10^{-7}\ \text{eV}(\lambda/10^{-9})^2$: these very small masses are a distinctive feature of our theory. There is no stable WIMP dark matter candidate at the TeV scale. The axino is the LSP, but is so light that it will not contribute to cold dark matter—axions and saxions are the candidates for dark matter.

Various astrophysical processes, that led to the constraints on the axion, also provide phenomenological limits on the light saxion field.[4] The saxion is light enough to be emitted from the interior of horizontal branch stars, and energy loss from saxion emission, which would shorten the lifetime of helium-burning stars, sets a limit on the Yukawa coupling of the saxion to the electron. The emission is dominated by the bremsstrahlung-like process $e^- + {}^4\text{He} \to {}^4\text{He} + e^- +$ saxion, and the constraint on the Yukawa coupling is given by [10]

$$\frac{1}{4\pi}\left(\frac{m_e}{f_a}\sin^2\beta\right)^2 \lesssim 1.4 \times 10^{-29}, \tag{5}$$

or equivalently,

$$f_a \gtrsim 4\sin^2\beta \times 10^{10}\ \text{GeV}. \tag{6}$$

Saxions are also emitted from SN 1987A, carrying energy away from the supernova. The energy loss rate through the saxion turns out to be roughly the same as that through the axion [10], so that the astrophysical limit on f_a is a little stronger than in conventional axion models. These more stringent bounds apparently indicate f_a close to 10^{11} GeV so that axions necessarily contribute a significant fraction of the dark matter, but this conclusion requires further scrutiny since our theory requires entropy dilution of saxion field oscillations, as discussed in Section 4.

3. A light gravitino

The stable vacuum of our theory crucially relies on an assumption $|m_S^2| < \lambda^2\tilde{m}^2$. There are models of mediation of SUSY breaking with vanishing m_S^2, but radiative corrections to m_S^2 through the interaction (1) are also of order $\lambda^2\tilde{m}^2$. Thus, one should expect either (i) an accidental cancellation between tree-level and one-loop contributions, or (ii) the SUSY breaking is mediated at a low energy scale, so that the one-loop correction is sufficiently small. Let us briefly see how low the mediation scale should be.

The tree-level potential of the CP-even scalars shows that the smallest eigenvalue of the mass-squared matrix is positive when [8]

$$\left|\xi \equiv \frac{m_S^2}{\lambda^2(v_1^2+v_2^2)}\right| \lesssim \frac{M_Z^2}{M_A^2}. \tag{7}$$

We roughly[5] take this limit to be $|\xi| \lesssim 0.2$. Since the renormalization-group equation for m_S^2 is given by

$$\frac{\partial m_S^2(\mu)}{\partial \log\mu} = -2\frac{\lambda^2}{8\pi^2}\left(m_1^2 + m_2^2 + A^2 + m_S^2\right), \tag{8}$$

the one-loop contribution to ξ is of order

$$\xi_{1\text{-loop}} \approx -\frac{1}{2\pi^2}\ln\left(\frac{M_S}{\lambda s}\right), \tag{9}$$

where SUSY breaking is assumed to be mediated at some energy scale M_S. Thus, it follows from the vacuum stability condition (7) that to avoid any fine-tuning the "messenger scale" M_S is at most one order of magnitude higher than the electroweak scale. Models with such a low messenger scale are found in [12]. For larger values of the messenger scale, the amount of fine-tuning increases logarithmically. For example, for

[2] The light axino is due to a see-saw mechanism: the fermion component of the S multiplet has a mass of order $(\lambda\langle h\rangle)^2/(\lambda\langle s\rangle) \sim \lambda^2\tilde{m}$. This can also be understood in terms of symmetries. Since S is the only multiplet relevant to the axion, the axino mass must be Majorana. The mass should be proportional to λ^2 because the superpotential (1) has a spurious symmetry under which phases of S and λ are rotated in the opposite directions.

[3] When the Kahler potential has a non-renormalizable term $|S^\dagger S|^2/M^2$, there is another contribution to the axino mass of order $(f_a/M)^2 m_{3/2}$. For M of order the Planck scale, M_{pl}, it does not exceed 10^{-3} eV, due to the upper limits on $m_{3/2}$ and f_a obtained in Sections 3 and 4, respectively. Thus, none of the discussion in this article is changed.

[4] Production of axinos is sufficiently suppressed as R parity requires that they must be produced in pairs.

[5] Detailed analysis using $\tan\beta > 2.5$ and $\mu > 120$ GeV leads to $-0.15 < \xi < 0.12$. For the effects of 1-loop corrections to the scalar potential, see [7,11].

gauge mediated SUSY breaking models with a messenger scale of 10^3 TeV, fine-tuning of order $1/10$ is required between $\xi_{\rm tree}$ and $\xi_{\text{1-loop}}$.

There is no conflict between the requirement of a low messenger scale and the large value for $\langle s\rangle$, which is driven by the soft operators. The soft parameters in the one-loop effective potential are renormalized at λs, because this is the combination that appears in particle masses. Since we are interested in the scalar potential for s with λs of order the weak scale, the soft parameters are evaluated at the weak scale, where they are local no matter how low the messenger scale is. The scalar potential of the s field is essentially given by physics at the electroweak scale, even though the vev $\langle s\rangle \approx f_a$ is much larger than the electroweak scale.

The fine-tuning argument above favors a low mediation scale, but does not directly constrain the fundamental scale of local supersymmetry breaking $\sqrt{F_{\rm SUSY}}$. However, in supergravity theories all scalars fields typically[6] acquire a supersymmetry breaking mass, giving a contribution to m_S^2 of order $[m_{3/2} = F_{\rm SUSY}/\sqrt{3}M_{\rm pl}]^2$, where $M_{\rm pl} \simeq 2.4\times 10^{18}$ GeV is the Planck scale. For this contribution to satisfy (2) without any fine-tuning, the bound on the scale of local supersymmetry breaking is

$$m_{3/2} \lesssim 100\ \text{eV} \times \left(\lambda/10^{-9}\right),$$
$$\sqrt{F_{\rm SUSY}} \lesssim 300\ \text{TeV} \times \left(\lambda/10^{-9}\right)^{1/2}. \tag{10}$$

If this bound is saturated we would normally expect a gravitino problem: the gravitinos are in thermal equilibrium at the weak scale, and although they are somewhat diluted by later annihilations, they still give too much hot dark matter. This would lead to an even stronger bound on $\sqrt{F_{\rm SUSY}}$ than given above. Although the gravitino is not the LSP, it decays to axion-axino with a lifetime longer than the age of the universe, so that the gravitino problem is not alleviated. However, in Section 4 we see that entropy production after the electroweak phase transition is required to dilute saxion oscillations, and this will also dilute the gravitinos.

[6] When the Kahler potential has a certain form, the ordinary gravity-mediated supersymmetry-breaking masses are absent, and m_S^2 acquires only an anomaly-mediated piece, which is of order $(\lambda^2/4\pi)\alpha_L m_{3/2}^2$. In this case, although $m_{3/2}$ of order the weak scale is allowed, the discussion in the following sections is not modified essentially. In Section 4, the gravitino is no longer light, but the entropy production required to dilute the saxion oscillation also dilutes the gravitino number density, and there is no gravitino problem. It is known that the Affleck–Dine mechanism works for baryogenesis [13]. In Section 5, we still expect separated vertices in colliders, although they arise from decays to axinos rather than to gravitinos.

4. Thermal history

4.1. Saxions and late-time entropy production

Supersymmetric axion models always involve a saxion field with a mass at most of order the SUSY breaking scale, $\tilde m$. Thus, there is a flat direction, and its evolution in the early universe must be examined carefully.

During inflation the saxion field could be zero or large, for example, of order the Planck scale, depending on its coupling to the inflaton. We begin by supposing that it is at the origin. In this case it stays at the origin until the Peccei–Quinn phase transition is triggered by the Higgs vev at a temperature of order the electroweak scale, as seen from the potential of Eq. (3). Immediately after the phase transition, the saxion field oscillates about the minimum of its potential, with an energy density, V_s, of order $\tilde m^4$. If the saxion were to decay rapidly enough, for instance, with a decay rate of order $\Gamma \sim \tilde m^3/f_a^2$, the field energy would rapidly convert into radiation giving no problem. However, the saxion mass is not of order $\tilde m$, but $\lambda\tilde m$. The lifetime of the saxion is of order $\tau \sim 10^2(10^{-9}/\lambda)^5 \times 10^{10}$ yrs., and is much longer than the present age of the universe. The oscillation of the saxion field, which behaves like matter, overcloses the universe. To avoid this we study the dilution of the saxion field oscillations by large entropy production after the electroweak and Peccei–Quinn phase transitions.

Let us suppose, for simplicity, that the entropy is produced via the decays of a massive particle, X, which could be the inflaton, curvaton or flaton. During and after the electroweak phase transition the universe is dominated by X, and is therefore matter-dominated. While the Hubble parameter is much larger than the decay rate of the X particle, the energy density of X, ρ_X, scales as $\propto 1/a^3$, where a is the scale factor. Some X particles decay at a time much less than the X lifetime, producing entropy. The total energy density of radiation $\rho_\gamma \sim T_\gamma^4$ does not scale as $\propto 1/a^4$, but rather as $\propto 1/a^{3/2}$, because of the continuous entropy supply from the X-particle decays. These X decays to radiation clearly dilute the saxion field oscillation energy density. This dilution continues until the age of the universe becomes comparable to the lifetime of the X particle, when ρ_γ is also comparable to ρ_X (see, e.g., [14]). A long X lifetime, and therefore a low value for the reheating temperature T_R, leads to more dilution. Any initial thermal saxions are diluted to a negligible level, while the current number density of cold saxions in the saxion field oscillation is given by

$$\frac{n_s}{n_\gamma} = c \times 10^{-1} \times \left(\frac{T_R}{1\ \text{GeV}}\right)^5 \times \left(\frac{100\ \text{GeV}}{T_{\text{PQ}}}\right)^4 \left(\frac{100\ \text{eV}}{m_s}\right), \tag{11}$$

where T_{PQ} is the temperature of the Peccei–Quinn phase transition, and the dimensionless coefficient c is given by

$$c \sim \left(1 + \frac{21}{22}\right)\left(\frac{2}{5}\right)^2 \frac{3\zeta(4)}{\zeta(3)} \times \left(\frac{g_S(T_R)}{g_S(T_{\text{PQ}})}\right)\left(\frac{V_s}{\rho_{\text{rad}}}\right)_{\text{PQ}}. \tag{12}$$

Here, the last factor V_s/ρ_γ is evaluated at the epoch of the Peccei–Quinn phase transition, and $g_S(T_{\text{PQ}})$ and $g_S(T_R)$ are the effective statistical degrees of freedom when the temperature is around T_{PQ} and T_R, respectively. The saxion field oscillations contribute to the present energy density an amount

$$\Omega_s h^2 = c' \times 10^{-1} \times \left(\frac{T_R}{1\ \text{GeV}}\right)^5 \left(\frac{100\ \text{GeV}}{T_{\text{PQ}}}\right)^4, \tag{13}$$

where

$$c' \sim \left(1 + \frac{21}{22}\right)\left(\frac{2}{5}\right)^2 \left[\Omega_\gamma h^2 \frac{100\ \text{eV}}{2.73\ \text{K}} \simeq 10.5\right] \times \left(\frac{g_S(T_R)}{g_S(T_{\text{PQ}})}\right)\left(\frac{V_s}{\rho_{\text{rad}}}\right)_{\text{PQ}}. \tag{14}$$

The saxion energy density does not depend on the choice of f_a. $\Omega_{\text{CDM}} h^2 \approx 0.1$ requires $T_R \lesssim 1$ GeV, and when this bound is saturated, the saxion is also a significant component of the CDM.[7]

Such a large entropy production not only dilutes saxions to an acceptable level, but also dilutes other species. The SUSY particles of the MSSM sector annihilate quickly to lighter SUSY particles, which eventually decay to the gravitino or axino. Axino number-changing reactions have already decoupled by the electroweak scale, and gravitino number-changing reactions freeze-out well before T_R, so that both axinos and gravitinos are significantly diluted by the entropy production. Hence, even gravitino masses that saturate the bound of (10) do not lead to an amount of hot dark matter in conflict with observation. Furthermore, gravitinos and axinos provide negligible contributions to the effective number of neutrino generations during BBN and CMB eras.

Entropy production from X decays also dilutes the baryon asymmetry, by a factor $(T_R/T_B)^5$, where T_B is the temperature at which the baryon asymmetry is generated. This severe dilution implies that the baryon asymmetry cannot be created above the weak scale. One possibility is that a baryon asymmetry of order unity is created at the weak scale. Another is that the observed small asymmetry is created in the out-of-equilibrium decays of the X particles; see, [15] as an example, where R-parity-violating interactions $W = UDD$ are required, and T_R has to be less than 1 GeV.

4.2. Relic axion energy density

Let us now turn our attention to the energy density carried by the axion field. There are two significant components: one from axions emitted by axionic strings, and the other from the misalignment of the initial value of the axion field from that of the true potential minimum.

[7] To be more precise, the vev's of the Higgs fields are determined by a thermal potential, and they change as the temperature falls. The minimum of the s field is also changing accordingly. Thus, even if the entropy production dilutes the energy of the saxion oscillation when most of the X particles have decayed, one has to further make sure that such readjustments after the entropy production do not release too much oscillation energy for the CDM. It turns out that the Higgs field values are close enough to the vacuum values when the temperature is around 1 GeV or lower, and the saxion oscillation due to this late-time readjustments is not cosmologically important.

The energy density from the axion phase relaxation is known to be [18]

$$\Omega_{\text{mis.}} h^2 = 0.10 \times 10^{\pm 0.4} \left(\frac{\Lambda_{\text{QCD}}}{200\ \text{MeV}} \right)^{-0.7} \times \left(\frac{f_a}{10^{11}\ \text{GeV}} \right)^{1.18}, \tag{15}$$

in the absence of the entropy production from X decays. This calculation assumes a radiation-dominated background at the epoch of relaxation, when the effective mass of the axion $m_{a,\text{eff}} \sim 0.1 m_a \times (\Lambda_{\text{QCD}}/T_\gamma)^{3.7}$ becomes comparable to the Hubble parameter. If T_R is around its upper bound ~ 1 GeV, then this assumption is justified. Given the lower bound on f_a at the end of Section 2, which is more stringent than in conventional models, in our theory the axion energy density accounts for a significant fraction of CDM, and axions are clearly a natural candidate for CDM. When the decay temperature is well below 1 GeV, massive X particles affect the axion energy density in two ways [19]. The extra energy density from X particles delays the epoch of the relaxation of the axion field, and the entropy from X decays dilutes the axion energy density. Combining both effects, the axion energy density is given by

$$\Omega_{\text{mis.}} h^2 \approx 0.1 \times \left(\frac{f_a}{10^{11}\ \text{GeV}} \right)^{1.5} \left(\frac{T_R}{1\ \text{GeV}} \right)^{2.0}, \tag{16}$$

and in order to have axion CDM we require that f_a be raised to

$$f_a \sim 10^{11}\ \text{GeV} \left(\frac{1\ \text{GeV}}{T_R} \right)^{1.3}. \tag{17}$$

Imposing $T_R \gtrsim 1$ MeV from big bang nucleosynthesis, the upper bound on f_a is $f_a \lesssim 10^{15}$ GeV [19].

If s is large during inflation then the axionic strings from the PQ phase transition are inflated away, and the axion energy density is from the misalignment of the initial phase. However, if s vanishes during inflation, axionic strings are formed after the PQ phase transition is triggered at the electroweak scale. After further cooling during the X dominated era, the energy density in strings reaches a fraction $(f_a/M_{\text{pl}})^2 \ln(f_a/H)$ of the total energy density. When the temperature falls so that $m_{a,\text{eff}}$ becomes comparable to the Hubble parameter, axionic domain walls emerge and the string network turns into the boundary of the domain walls. This string/domain wall system rapidly disappears by radiating axions,[8] and the number density of axions is fixed:

$$n_{a,\text{string}} \approx H f_a^2 \Delta, \tag{18}$$

where Δ ranges from $\ln(f_a/H)$ to unity. This large uncertainty in Δ corresponds to the disagreement between [16] and [17] about the typical energy of axions emitted from the string network. Since the axion number density from misalignment is also $H f_a^2$ at that epoch, the resulting relic density from strings is Δ times that of the misalignment axions, just as in the case of radiation dominance. Note also that another uncertainty in the relic density arises from assuming that the energy density of the string network is converted into axion particles when $m_{a,\text{eff}}$ is equal to the Hubble parameter.

In the case that s vanishes after inflation, we have assumed that there is only a single vacuum in the direction of the axion field. However, the model presented in Section 2 has three vacua, due to three families contributing to the $\text{U}(1)_{\text{PQ}}[\text{SU}(3)_C]^2$ anomaly. The number of vacua can be reduced to one by introducing extra colored particles. For instance, introducing two vector-like pairs of chiral multiplets $\Phi_i(\mathbf{3}) + \Phi_i^c(\mathbf{3}^*)$ $(i = 1, 2)$, with a coupling

$$\Delta W = \lambda_i S \Phi_i \Phi_i^c, \tag{19}$$

gives an anomaly coefficient of 1, so that there is a unique vacuum. If the couplings λ_i $(i = 1, 2)$ are much smaller than λ, then these extra particles have already been excluded by data. On the other hand, if λ_i are larger than λ, the SUSY-breaking masses[9] of the extra particles contribute to $\xi_{1\text{-loop}}$ at order λ_i^2/λ^2, requiring excessive fine-tuning. Thus, $\lambda_i \approx \lambda$, and, as a consequence, these extra colored chiral multiplets are expected at the electroweak scale. If these vector-like particles have the same electric and PQ charges as the up or down-type quarks, then they can decay by mixing with the known quarks.

[8] There must be only one vacuum in the phase direction of s. Otherwise, such string/domain wall system cannot disappear.

[9] This argument does not apply if λ_i are so large that their masses due to (19) are larger than the "messenger scale" M_S.

4.3. Early breaking of PQ symmetry

It has been assumed so far in this section that the s field vanishes after inflation until later times. If s takes a very large value s_0 during and after inflation, the radial direction of the s field, i.e., the saxion field, starts to oscillate when the Hubble parameter is comparable to the curvature of its scalar potential. The energy of the saxion oscillation has to be diluted in this case, as well, by an amount that depends on s_0. If s_0 is of order of the Planck scale, even the entropy production from the X decays is not enough. Indeed, $\rho_s/\rho_X \approx (s_0/M_{\rm pl})^2$ when the s field starts to oscillate,[10] and for X decays to provide sufficient dilution for saxions, we find that the following condition should be satisfied:

$$s_0 \lesssim s_{0,\max} \equiv 10^{-3} M_{\rm pl} \times \sqrt{\frac{\rm MeV}{T_R}}. \tag{20}$$

The initial phase of s_0 is well-defined and almost homogeneous inside the present horizon provided the Hubble parameter during the inflation H_I is smaller than s_0; the phase fluctuation that might be generated during inflation is of order $\delta\theta \sim H_I/(2\pi s_0) < 1$. The initial phase is preserved in classical evolution of the s field until the QCD phase transition, when the potential in the phase direction emerges. Since the whole universe inside the horizon falls into the single vacuum, there is no problem of domain walls even in the absence of the extra colored particles. The estimate (15) or (16) for $\Omega_{\rm mis.}$ applies to this case, except that (i) the initial phase of s_0 is used instead of the average of random phases $\pi/\sqrt{3}$, and (ii) the normalization of f_a can be different because of the different number of vacua in the phase direction [18]. Note that axionic string network is not formed after inflation in this case, and the misalignment of the initial phase is the only source of the cosmological axions.

The phase fluctuation $\delta\theta$ leads to isocurvature density perturbations in the axions and radiation [20], giving

$$\left(\frac{\delta T}{T}\right)_{\rm isocurv} \approx 10^3 \frac{H_I}{2\pi M_{\rm pl}} \sqrt{\frac{T_R}{\rm MeV}} \frac{s_{0,\max}}{s_0}. \tag{21}$$

Since the current CMB data is consistent with purely adiabatic density perturbation, this implies $H_I \lesssim 10^{11}\,{\rm GeV} \sqrt{\frac{\rm MeV}{T_R}} \frac{s_0}{s_{0,\max}}$, which is quite a non-trivial constraint on many models of inflation. If this bound is saturated, the isocurvature perturbation may be observed in future CMB data.

5. Predictions and conclusions

In this Letter we have made the observation that by promoting the μ parameter of the minimal supersymmetric standard model to a field, both the strong CP and μ problems are solved. How will we know whether this theory is correct?

The first, and most important, test that the theory must pass is that the three parameters μ, A and $\tan\beta$, which are independent in the MSSM, must satisfy the relation of Eq. (4) [7]. This signals that the electroweak symmetry breaking sector of the theory is governed by the superpotential interaction $\lambda S H_1 H_2$, without the S^3 interaction of the next-to-MSSM, and that the vacuum is the one with $\langle s\rangle = \tilde{m}/\lambda$ that occurs when $|m_S^2| < \lambda^2 v^2$ [8,9]. While verification of this relation will show how the μ problem is solved, it is clearly insufficient to demonstrate that there is a PQ solution to the strong CP problem. For example, it could be that $\lambda \approx 10^{-3}$ and that small explicit symmetry breaking terms give the would-be axion a mass of order a GeV [9], so that the strong CP problem is not solved. On the other hand, observing a very small value for λ would provide a strong indication that $\langle s\rangle = \tilde{m}/\lambda$ is large and is consistent with astrophysical constraints on the PQ solution. Such evidence for small λ could be found in the cascade decays of the superpartners produced at hadron colliders, as we now discuss.

Recall that all the superpartners have masses of order $\tilde{m}$, except for the axino and the gravitino which are much lighter. An important question is the decay mode and decay rate of the lightest superpartner amongst those that have masses of order $\tilde{m}$, the LSP′. For large λ the dominant decay of the LSP′ will be to the axino, $\tilde{a}$. For example, if the LSP′ is a neutralino, it would decay with a rate $\lambda^2\tilde{m}$ to either $h\tilde{a}$ or $Z\tilde{a}$, leading to the spectacular events discussed in [9]. The last decay process of the cascade chain takes place well inside

[10] Here, the gravity-mediated quadratic and/or gauge-mediated logarithmic SUSY-breaking potential for s is assumed.

the beam pipe. However, in axionic theories we have $\lambda \lesssim 10^{-9}$ and a low value for $\sqrt{F_{\rm SUSY}}$ so that LSP′ decays to gravitinos, $\tilde{G}$, have a branching ratio comparable to or higher than that of decays to axinos. The decay rate to the gravitino is $\tilde{m}^5/F_{\rm SUSY}^2$, and hence

$$\frac{\Gamma(\mathrm{LSP}' \to \tilde{a})}{\Gamma(\mathrm{LSP}' \to \tilde{G})} \simeq \left(\frac{\lambda}{10^{-9}}\right)^2 \left(\frac{\sqrt{F_{\rm SUSY}}}{300\ \mathrm{TeV}}\right)^4 \lesssim 1. \quad (22)$$

When $\sqrt{F_{\rm SUSY}} \ll 300$ TeV, the LSP′ decays are mainly to gravitinos. The decay vertices are still within the beam pipe, and the event configuration of decays to gravitinos is quite similar to that of decays to axinos. Thus, it may be difficult to distinguish this axionic theory with $\sqrt{F_{\rm SUSY}} \ll 300$ TeV from the theory with $\lambda \sim 10^{-3}$ in [8,9]. In cases with photino LSP′, however, $\mathrm{Br}(\mathrm{LSP}' \to \tilde{a} + \gamma)/\mathrm{Br}(\mathrm{LSP}' \to \tilde{a} + Z)$ is different from $\mathrm{Br}(\mathrm{LSP}' \to \tilde{G} + \gamma)/\mathrm{Br}(\mathrm{LSP}' \to \tilde{G} + Z)$. Determination of the mixing in the neutralino sector might be able to exclude one of the two possibilities above. When $\sqrt{F_{\rm SUSY}}$ is close to its upper bound of 300 TeV, the LSP′ decay to a gravitino occurs at vertices separated from the primary vertex.[11] While this is also a typical signal of gauge-mediated SUSY breaking with such a value of $F_{\rm SUSY}$, decays at separated vertices, combined with a positive test of the relation (4) for μ, would give a strong indication that λ is small, and that our proposal for simultaneous solutions of strong CP and μ problems is correct.

It may also happen that the QCD-charged extra particles introduced in Section 4 are within the reach of hadron colliders.

If our theory is correct, hadron colliders will tell us that the LSP is not in the MSSM sector. They will also tell us, from the LSP′ decay rate, that the gravitino mass is too small for gravitino cold dark matter. The axion will be the natural remaining candidate for cold dark matter.

When f_a saturates the lower bound 10^{11} GeV, and $T_R \sim 1$ GeV, the cold dark matter consists of both saxions and axions. The standard axion dark matter search [21] can detect the axion of this theory. The saxion can be detected indirectly from the soft X-rays produced by its subdominant decay mode $s \to \gamma\gamma$. There are two sources of this X-ray flux. One is from the saxions distributed uniformly throughout the universe, and the other arises from saxions which have fallen into the gravitational potential of clusters of galaxies (cf. [22]). The former is observed as an isotropic flux with a continuous spectrum, because the X-rays emitted long ago and far away appear red-shifted. A rough estimate for the photon flux is given by

$$\begin{aligned}\frac{d\Phi_{\rm isotropic}}{d\Omega\, dE_\gamma} &\approx \frac{3}{8\pi} \frac{n_s \Gamma(s\to\gamma\gamma) c t_0}{m_s c^2/2} \sqrt{\frac{E_\gamma}{m_s c^2/2}} \\ &\approx 10^3 \sqrt{\frac{E_\gamma}{m_s c^2/2}}\ \mathrm{cm^{-2}\,s^{-1}\,str^{-1}\,keV^{-1}} \\ &\quad \times \left(\frac{100\ \mathrm{GeV}}{T_{\rm PQ}}\right)^4 \left(\frac{T_R}{1\ \mathrm{GeV}}\right)^5 \\ &\quad \times \left(\frac{m_s c^2}{100\ \mathrm{eV}}\right) \left(\frac{10^{11}\ \mathrm{GeV}}{f_a}\right)^2 \end{aligned} \quad (23)$$

for $E_\gamma < m_s/2$, where matter dominance is assumed in the first line, and the number density of the saxions in (11) is used in the second line. This is the flux predicted in extragalactic space—the flux on Earth is reduced by absorption in the Galaxy, and will have a modified energy spectrum. Using ROSAT data, the observed extragalactic soft X-ray background is found to be 30–65 $\mathrm{keV\,cm^{-2}\,s^{-1}\,str^{-1}\,keV^{-1}}$ for the 1/4 keV energy region [23]. Given the uncertainties in the predicted flux of (23), the X-ray flux from saxion decays is consistent with observation. We note here that the saxion mass m_s could be as high as 1 keV in a parameter region, although we have typically considered $m_s \approx 100$ eV.

Observations with high angular resolution suggest that 60% (and perhaps more) of the extragalactic X-rays can be attributed to the flux from discrete sources, such as AGN's, in the 1–2 keV energy region [24]. But it is not clear whether all of the soft X-ray flux is accounted for by such sources in the sub-keV energy region [23–26], and there is still room for extra fluxes with particle-physics origins. Further observation with high angular resolution and long exposure time will certainly help determine the purely isotropic extragalactic component while removing foreground contamination and contributions

[11] In the previous section we pointed out that one way to accomplish baryogenesis at low temperatures is by introducing large R parity violating operators of the form UDD. In this case the LSP′ is expected to decay dominantly via these interactions, so that the separated vertex signal is replaced by the signals of R parity violation.

from discrete sources. Furthermore, our X-ray signal from CDM saxions has a different spectrum from those of the discrete sources identified in [24]. Those fluxes decrease with increasing energy and have tails that extend above 1 keV [24], while our signal's flux increases with energy until it is sharply cut off at an energy of half the saxion mass. Thus, observations with high energy resolution [27] will help identify the X-ray signal from CDM saxions, when combined with a better understanding of the foreground absorption and emission.

The X-rays from saxions bound to a galactic cluster produce a "line spectrum", with a width $\Delta E_\gamma/E_\gamma \sim \mathcal{O}(v/c) \sim 10^{-2}$, and an energy $m_s/(2(1+z))$, with z the redshift of the cluster. The photon flux at the peak energy is roughly given by

$$\frac{d\Phi_{\rm cluster}}{d\Omega\, dE_\gamma} \approx \frac{1}{4\pi}\frac{1}{10^{-2}E_\gamma}\int n_s|_{\rm cluster}\Gamma(s\to\gamma\gamma), \\ \approx 3\times 10^4\ {\rm cm}^{-2}\,{\rm s}^{-1}\,{\rm str}^{-1}\,{\rm keV}^{-1} \\ \times\left(\frac{\Omega_s h^2}{5\Omega_B h^2}\right)\left(\frac{\text{cluster size}}{1\ \text{Mpc}}\right) \\ \times\left(\frac{n_B|_{\rm cluster}}{10^{-3}/{\rm cm}^3}\right)\left(\frac{m_s c^2}{100\ {\rm eV}}\right) \\ \times\left(\frac{10^{11}\ {\rm GeV}}{f_a}\right)^2, \qquad (24)$$

where the integration in the first line is along the line of sight. In order to identify these line-spectrum X-rays, observations with both high angular resolution and high energy resolution are required.

As T_R is lowered below 1 GeV, f_a increases above 10^{11} GeV and these axion and saxion signals become too difficult to see: the axion detection rate is proportional to $\lambda \approx 1/(m_a f_a^2)$ [21], and the extragalactic photon flux from saxion decay $\propto \lambda^{6.8}$, where Eq. (17) and $E_\gamma \propto m_s \approx \lambda v$ are used. However, the lowest value of f_a (and hence the largest of $\lambda \sim 10^{-9}$) is the most theoretically well-motivated. So far in this Letter, the extremely small value for λ has been put by hand, but it can be obtained naturally as $\sqrt{\tilde{m}/M_{\rm pl}} \sim 10^{-9}$, where $M_{\rm pl}$ is the Planck scale. Indeed, one can think of a theory with a superpotential

$$W = \frac{1}{M}S'SH_1H_2 + \frac{1}{M'}S'^4, \qquad (25)$$

where $M \sim M' \sim M_{\rm pl}$. With a negative SUSY-breaking mass-squared of order $-\tilde{m}^2$ for S', one can easily see that $\langle S'\rangle/M \sim \sqrt{\tilde{m}/M_{\rm pl}}$. The mixing between S and S' is so small in this theory that the phenomenological analysis given in this article with an effective coupling $\lambda \equiv \langle S'\rangle/M$ is completely valid.

The theory with the superpotential (1) is quite similar to those [4] with

$$W = \frac{1}{M}S'SH_1H_2 + \frac{1}{M'}S^nS'^{(4-n)} \quad (n \neq 0,4) \qquad (26)$$

at first sight, but these two classes of theories are quite different. In our theory ($n = 0$), S' is neutral under the Peccei–Quinn symmetry, and the mixing between S' and S is quite small. Thus, the chiral multiplet S is virtually the only one responsible for the spontaneous Peccei–Quinn symmetry breaking. This is one of the most important reasons why the axino, which is the LSP, is extremely light in our theory (cf. [28]). Another important aspect of our theory is that the stabilization of s results only after the electroweak phase transition, so that the μ parameter is predicted in terms of $\tan\beta$ and the A parameter. Therefore, our theory is not merely a particular case of the theories in [4], but is essentially different. Indeed, the theory of the invisible axion presented in this article has several predictions that can be tested in the near future.

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