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A gauge invariant Debye mass and the complex heavy-quark potential

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ABSTRACT

Following the original idea of Debye, we define and extract a gauge-invariant screening mass from the complex static in-medium heavy-quark potential $V_{Q\bar{Q}}$, recently obtained from lattice QCD. To this end we derive a formula using concepts from both effective- and classical field theories that faithfully reproduces both the screened real- and the imaginary part of the lattice potential with a single temperature dependent fit parameter $m_D(T)$. Using values of the real part of $V_{Q\bar{Q}}$ in a gluonic medium, we obtain Debye masses compatible with HTL perturbation theory.

1. Introduction

The concept of a screening mass helps us to intuitively understand the interactions that take place, once a test particle is inserted into a medium of charge carriers. Originally Debye and Hückel [1] investigated the behavior of ions in electrolyte solutions and found that their interactions could be understood by an exponential suppression of the vacuum Coulomb potential due to the presence of classical Boltzmann distributed charges. This in-medium modification amounts to a medium induced dressing of the mediating gauge bosons, bestowing them with an otherwise absent mass.

In perturbative QCD, only the leading order and the logarithm at next to leading order (NLO) of the Debye mass can be computed, constants at NLO already receiving non-perturbative contributions [2]. On the lattice, its definition encounters another difficulty, unlike in QED where the Debye mass can be extracted from the electric field correlator, in QCD the electric field itself is not gauge invariant. Several approaches were proposed to circumvent this problem using e.g. effective theories obtained by dimensional reduction [3], spatial correlation functions of gauge invariant meson correlators [4] or the behavior of the color singlet free energies [5–7].

Here we return to the original idea of Debye and identify a physical observable, the heavy-quark potential $V(r)$ between a static color singlet configuration of a quark and anti-quark, to non-perturbatively define a screening mass for QCD. The fact that such a static $Q\bar{Q}$ may be described by a potential without explicit reference to color degrees of freedom is a manifestation of the simplifying power of the effective field theory pNRQCD [8] underlying the definition of $V(r)$. In the vacuum, the potential exhibits both a perturbative Coulombic and a non-perturbative string-like behavior [9], with corrections due to the running of the coupling and logarithmic contributions being small in the phenomenologically relevant regime between $r = 0.02$–3 fm [10].

In-medium both the Coulombic and string-like parts of $V(r)$ receive modification [11]. An additional complication arises from the fact that the potential is in general a complex quantity [12], due to the presence of scattering of light medium degrees of freedom with the color string spanning in between the heavy quark and anti-quark [13]. A meaningful description of the relevant physics must therefore necessarily capture both the effects of screening of the real part of the potential $\mathrm{Re}V(r)$ and e.g. Landau-damping related to $\mathrm{Im}V(r)$. Our strategy hence is to find a field-theory motivated parametrization of the potential that depends only on a single temperature dependent parameter $m_D(T)$, which we will be able to identify with the Debye screening mass.

In the literature two contributions along this path can be found, which both exploit the applicability of the EFT based singlet potential description. On the one hand, Ref. [7] proposed an analytic function for $\mathrm{Re}[V]$ of a medium-modified Cornell-type potential using the fully classical setup of a Coulombic test charge surrounded by Boltzmann distributed charge carriers, generalized to non-Coulombic potentials in Ref. [14]. While it was shown that the resulting parametrization of $\mathrm{Re}[V]$ can reproduce the lattice

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data quite well, it required the introduction of a second temperature dependent fit parameter and the classical approximation was unable to accommodate an imaginary part of the potential.

On the other hand an interesting approach was proposed in Ref. [15]. The authors make the assumption that the in-medium potential arises from the vacuum potential by multiplying it with a field-theory determined complex permittivity in momentum space. Using the hard thermal loop (HTL) permittivity it was possible to reproduce the known real- and imaginary part of the corresponding in-medium potential in HTL. Unfortunately applying the permittivity to the linearly rising potential leads to unphysical results. The real-part does not decay exponentially but retains an $\sim 1/r$ behavior, which does not describe the lattice potential, hinting at the fact that the screening of the vacuum potential is not captured self-consistently. In addition the resulting imaginary part, which must go to a constant at large distance, namely twice the Landau damping of a single quark, diverges logarithmically.

Our study combines the strength of these two approaches bringing together the generalized Gauss law from Ref. [14] with the characterization of in-medium effects through the perturbative HTL permittivity. The use of the Gauss law, a non-local concept, leads to a self-consistent description of both screening and damping effects evading the unphysical behavior of Ref. [15].

1.1. The static in-medium inter-quark potential

The in-medium potential acting between a static quark and anti-quark constitutes the basis for our gauge invariant QCD screening mass. Defining such a potential at finite temperature has been a long standing problem in thermal field theory. The central pillar to its derivation from QCD are the effective field theories NRQCD and pNRQCD [8,16,17], which exploit the separation of scales between the mass of the heavy quark and its typical momenta to set up a non-relativistic description of the $Q\bar{Q}$. In their context it has been shown that if a potential description is valid, the values of $V(r)$ can be obtained from a dynamical QCD observable, the real-time Wilson loop $W(t,r)$ [19]:

$$V(r) = \lim_{t\to\infty} \frac{i\partial_t W(t,r)}{W(t,r)}. \tag{1}$$

This definition was evaluated for the first time in HTL resummed perturbation theory [12] and found to be complex valued:

$$V_{\mathrm{HTL}}(r) = -\tilde{\alpha}_s \left[m_{\mathrm{D}} + \frac{e^{-m_{\mathrm{D}} r}}{r} + iT\phi(m_{\mathrm{D}} r) \right] + \mathcal{O}(g^4), \tag{2}$$

where a factor C_F has been absorbed in the definition of the coupling constant $\tilde{\alpha}_s = \frac{g_s^2 C_F}{4\pi}$ to match the literature on phenomenology and

$$\phi(x) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left(1 - \frac{\sin(xz)}{xz}\right). \tag{3}$$

A non-perturbative, i.e. lattice QCD based determination of the potential remained a conceptual and technical challenge, which has only recently been overcome [11,18–21]. The central hurdle is related to the fact that lattice simulations are performed in Euclidean time and have no direct access to $W(r,t)$.

A possible way around this limitation was proposed in Refs. [22, 18]. The underlying idea is to use a spectral decomposition of the Euclidean Wilson loop $W(\tau,r)$ to relate the Euclidean and Minkowski time domain

$$W(\tau,r) = \int d\omega e^{-\omega\tau}\rho(\omega,r) \leftrightarrow \int d\omega e^{-i\omega t}\rho(\omega,r) = W(t,r).$$

This equation can be combined with Eq. (1) to define the potential in terms of the Wilson loop spectral function $\rho(\omega,r)$:

$$V(r) = \lim_{t\to\infty} \int d\omega\, \omega e^{-i\omega t}\rho(\omega,r) \Big/ \int d\omega\, e^{-i\omega t}\rho(\omega,r). \tag{4}$$

This definition requires precise knowledge of the spectrum $\rho(\omega,r)$, which can in principle be obtained from an inverse Laplace transform of datapoints $W(\tau_n,r)$, $n = 1..N_\tau$ simulated in lattice QCD. In practice however carrying out this ill-defined deconvolution poses a formidable challenge to standard methods, such as the Maximum Entropy Method [23] or extended MEM [24]. In fact, it required the development of a novel Bayesian inference method [21].

A second difficulty lies in taking the infinite time limit in Eq. (1). Using the symmetries of the real-time Wilson loop, it was shown in Ref. [19] that the physics of the potential manifests itself in the position and width of the lowest lying peak in the spectrum, which has the shape of a skewed Lorentzian. Fitting this peak gives access to the real and imaginary part of the potential [25]. In turn, if a well defined Lorentzian peak is found in the spectrum of the Wilson-loop a meaning full static potential can be defined non-perturbatively.

This strategy has been successfully applied to extract the values of the in-medium potential in quenched and full QCD, as reported on in Ref. [11]. For the present study we have generated an additional set of $N_{\mathrm{conf}} = 900$ low temperature quenched configurations at $N_\tau = 192$, i.e. $T = 105$ MeV, which play an essential role for calibration and the determination of the Debye mass.

2. An analytic parametrization of the heavy-quark potential

The starting point of our derivation is the generalized Gauss law introduced in Ref. [14]

$$\vec{\nabla}\left(\frac{\vec{E}}{r^{a+1}}\right) = 4\pi q\delta(\vec{r}). \tag{5}$$

It is formulated in terms of an auxiliary vector field of the form $\vec{E} = qr^{a-1}\hat{r}$, which is derived from the color-singlet heavy-quark potential $-\vec{\nabla}V(\mathbf{r}) = \vec{E}(\mathbf{r})$. Eq. (5) reduces to the well known expression for the Coulomb potential for $a = -1$, $q = \tilde{\alpha}_s$, $[\tilde{\alpha}_s] = 1$, while a linearly rising potential corresponds to $a = 1$, $q = \sigma$, $[\sigma] = \mathrm{GeV}^2$.

Let us first look at the original argument by Debye and Hückel. The presence of a background charge density $\langle\rho(\vec{r})\rangle$ modifies the above equation such that

$$\vec{\nabla}\left(\frac{\vec{E}}{r^{a+1}}\right) = 4\pi q\big(\delta(\vec{r}) + \langle\rho(\vec{r})\rangle\big). \tag{6}$$

In the purely classical case of Boltzmann distributed charges at $T = 1/\beta$

$$\langle\rho(\vec{r})\rangle = \left(n_0 e^{-\beta V(\vec{r})} - n_0\right) - \left(n_0 e^{\beta V(\vec{r})} - n_0\right), \tag{7}$$

where in the present context the first term stands for particles and the second for antiparticles. If the resulting in-medium potential is weak, we can expand the exponential in Eq. (7) as $\langle\rho(\vec{r})\rangle = -2q\beta n_0 V(\vec{r})$. In this linear regime we can interpret n_0 as the charge density in the absence of the test charge, which hence is independent of the vacuum potential being Coulombic or string-like.

When plugged into Eq. (6), we obtain [14]:

$$-\frac{1}{r^{a+1}}\nabla^2 V(r) + \frac{1+a}{r^{a+2}}\nabla V(r) + AV(r) = 4\pi q\delta(\vec{r}), \tag{8}$$

where $A = 8\pi q n_0 \beta$. The appearance of the term with prefactor A is a manifestation of the linear-response character of this approximation. For the Coulombic part of the potential ($a = -1$, $q = \tilde{\alpha}_s$) we have

$$-\nabla^2 V_C(r) + A_C V_C = 4\pi\tilde{\alpha}_s \delta(\vec{r}), \quad A_C = 8\pi\tilde{\alpha}_s n_0 \beta. \tag{9}$$

On the other hand for the string case ($a = 1$, $q = \sigma$) one finds

$$-\frac{1}{r^2}\frac{d^2 V_s(r)}{dr^2} + A_s V_s(r) = 4\pi\sigma\delta(\vec{r}), \quad A_s = 8\pi\sigma n_0 \beta. \tag{10}$$

Note that in both eqs. (9) and (10) the same n_0 appears, representing the medium charge density in the absence of the test charge.

Now let us return to the generalized Gauss law of Eq. (5) for a Coulomb charge ($a = -1$) in momentum space, $p^2 V_C(\vec{p}) = 4\pi\tilde{\alpha}_s$. In our approach the medium effects are incorporated by an in-medium permittivity $\epsilon(\vec{p}, m_D)$ as is commonplace in e.g. electrodynamics [26]

$$p^2 V_C(\vec{p}) = 4\pi \frac{\tilde{\alpha}_s}{\varepsilon(\vec{p}, m_D)}. \tag{11}$$

Similar to Ref. [15], we use the following perturbative HTL expression, originally derived for the case where temperature is the largest soft scale

$$\varepsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2 + m_D^2} - i\pi T \frac{p\, m_D^2}{(p^2 + m_D^2)^2}. \tag{12}$$

I.e. we assume a medium of weakly coupled quarks and gluons, in which our non-Abelian test charge is immersed. Inserting this formula into equation (11) and multiplying by $\frac{p^2+m_D^2}{p^2}$, we obtain:

$$p^2 V_C(\vec{p}) + m_D^2 V_C(\vec{p}) = 4\pi\tilde{\alpha}_s\Big(1 - i\pi T \frac{m_D^2}{p(p^2 + m_D^2)}\Big). \tag{13}$$

The inverse Fourier transform of the real part of Eq. (13) exactly reproduces the linear-response expression of (9), allowing us to identify $A_C = m_D^2$ and in turn gives an expression for the charge density

$$n_0 = \frac{m_D^2 T}{8\pi\tilde{\alpha}_s}. \tag{14}$$

The imaginary part arising on the RHS of Eq. (13) can then also be interpreted as a modification of the charge density. When Fourier transformed to coordinate space, it completes our generalized formula for the in-medium modification of a Coulombic test charge

$$-\nabla^2 V_C(r) + m_D^2 V_C(r) = \tilde{\alpha}_s\Big(4\pi\delta(\vec{r}) - iT m_D^2 g(m_D r)\Big), \tag{15}$$

with

$$g(x) = 2\int_0^\infty dp \frac{\sin(px)}{px}\frac{p}{p^2+1}. \tag{16}$$

The solution to Eq. (15) with the physical boundary condition $\mathrm{Re}V_C(r)|_{r=\infty} = 0$, $\mathrm{Im}V_C(r)|_{r=0} = 0$ and $\partial_r \mathrm{Im}V_C(r)|_{r=\infty} = 0$ coincides with the HTL potential obtained by Laine et al. [12], given in Eq. (2).[1]

Now we focus on the string-like part of the potential by returning to Eq. (10). With the medium charge density n_0 being independent of the $T = 0$ potential being Coulombic or string-like, one obtains from Eq. (10) and Eq. (14):

$$A_s = \mu^4 = m_D^2 \frac{\sigma}{\tilde{\alpha}_s}. \tag{17}$$

On the other hand the Gauss law operator for the string-potential does not permit a similar straightforward Fourier transform as in the Coulomb case. Motivated by the relation between eqs. (9) and (15) we instead assume the validity of the linear response approximation with a similar change in the in-medium charge density on the RHS. This leads us from eq. (10) to the defining equation for the in-medium string-potential

$$-\frac{1}{r^2}\frac{d^2 V_s(r)}{dr^2} + \mu^4 V_s(r) = \sigma\Big(4\pi\delta(\vec{r}) - iT m_D^2 g(m_D r)\Big). \tag{18}$$

Note that these expressions differ from the ones used in [7], where A_s was chosen on purely dimensional grounds to be m_D^4.

Solving for the real part of Eq. (18) with the same physical boundary conditions as in the Coulomb case we find

$$\mathrm{Re}V_s(r) = -\frac{\Gamma[\frac{1}{4}]}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu} D_{-\frac{1}{2}}\big(\sqrt{2}\mu r\big) + \frac{\Gamma[\frac{1}{4}]}{2\Gamma[\frac{3}{4}]}\frac{\sigma}{\mu}, \tag{19}$$

which furthermore differs in a factor $\sqrt{2}$ in the argument of the parabolic cylinder function $D_\nu(x)$ compared to Ref. [7]. The imaginary part of the in-medium string potential can be written in a closed form as a Wronskian solution:

$$\mathrm{Im}V_s(r) = -i\frac{\sigma m_D^2 T}{\mu^4}\psi(\mu r) = -i\tilde{\alpha}_s T\psi(\mu r), \tag{20}$$

with

$$\begin{aligned}\psi(x) = {} & D_{-1/2}(\sqrt{2}x)\int_0^x dy\, \mathrm{Re}D_{-1/2}(i\sqrt{2}y)y^2 g(y m_D/\mu) \\ & + \mathrm{Re}D_{-1/2}(i\sqrt{2}x)\int_x^\infty dy\, D_{-1/2}(\sqrt{2}y)y^2 g(y m_D/\mu) \\ & - D_{-1/2}(0)\int_0^\infty dy\, D_{-1/2}(\sqrt{2}y)y^2 g(y m_D/\mu).\end{aligned} \tag{21}$$

Let us have a look at the behavior of the solution just obtained. In the limit of zero temperature, i.e. vanishing m_D, we recover the Cornell potential in the real part. As expected, at small distances the Coulombic real part behaves as $1/r$ whereas the string shows a linear rise with r. The imaginary part on the other hand rises according to r^2 for the Coulombic part and with r^3 for the string. I.e. the Coulombic HTL part (2) dominates at small r. At large distances we again find that the Coulombic part dominates the real part and behaves just like the naive Debye screened potential $\exp(-m_D r)/r$. The fact that the string part dies off much more rapidly as $\exp(-m_D^2 r^2/2)$ is the reason why we can identify the parameter m_D with the Debye mass, when fitted to the functional form of the lattice potential. At asymptotically large distances both the Coulombic and string imaginary parts saturate to a constant as required.

3. A Debye mass from the lattice in-medium potential

Our goal is to use the derived expression for the in-medium potential to extract the Debye mass from the static inter-quark potential recently measured in lattice QCD. In this work we focus on the case of a purely gluonic medium, for which both $\mathrm{Re}V$, as well

[1] A running of α_s can in principle be introduced by replacing r^{a+1} in Eq. (5) by a more complicated function $\beta(r)$, here e.g. $\beta(r) \sim 1 - r\alpha_s'(r)/\alpha_s(r) \sim 1 + 1/\log(r)$.

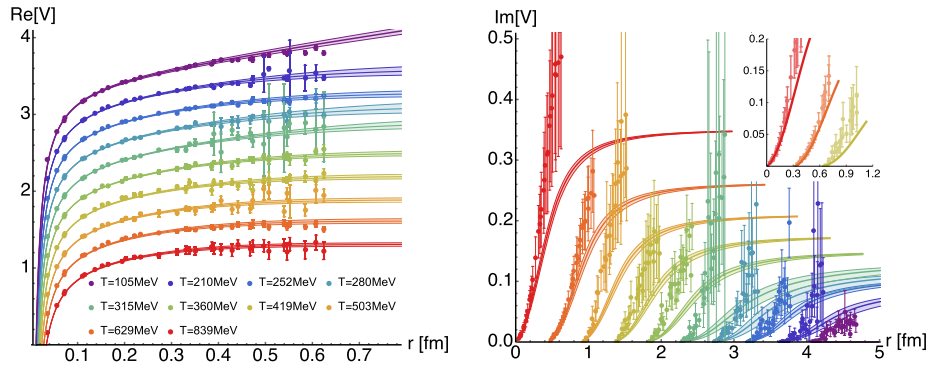

Fig. 1. (left) One parameter fit (solid lines) to the $T > 0$ real part of the potential (data points) obtained in quenched QCD. (right) Im[V] in quenched QCD (data points) and the values obtained from our analytic expression using the Debye mass fitted in ReV.

Table 1
Quenched lattice parameters ($\beta = 7$, $\xi_b = 3.5$, $N_s = 32$) and Debye masses.

SU(3): N_τ	24	32	40	48	56	64	72	80	96	192
T [MeV]	839	629	503	419	360	315	280	252	210	105
N_{meas}	3270	2030	1940	1110	1410	1520	860	1190	1800	900
m_D [MeV]	852 ± 60	709 ± 70	654 ± 62	537 ± 59	444 ± 52	264 ± 62	309 ± 86	401 ± 62	328 ± 78	0

as ImV have been determined at various temperatures around the deconfinement transition. With only a single parameter m_D, we will carry out fits solely to the real-part of the potential, so that the agreement or disagreement in the imaginary part serves as a crosscheck of our approach.

We assume the values of the strong coupling $\tilde{\alpha}_s$ and string tension σ not to vary with temperature T, as they characterize the properties of the test charge to be inserted in the medium. Their values hence have to be determined in vacuum. In the absence of a true $T = 0$ lattice measurement, we use the newly generated lattice ensemble at $T = 105$ MeV instead and fit the small to intermediate r region of ReV, where the remaining thermal effects are negligible. The particular nature of the lattice normalization of the potential introduces a constant shift c, which we also determine

$$\tilde{\alpha}_s = 0.206 \pm 0.011, \quad \sigma = 0.174 \pm 0.011\ \text{GeV}^2,$$
$$c = 2.60 \pm 0.023\ \text{GeV}. \tag{22}$$

Varying the fitting range up to a maximum of six steps at the upper and lower end of the fitting interval yields the error estimates shown. The only remaining free parameter at finite temperature then is the Debye mass m_D. As can be seen by the agreement of the solid lines and data points in the left panel of Fig. 1, its tuning alone allows us to achieve an excellent fit of the real-part of the potential at all temperatures. To account for the propagation of the error on the low temperature parameters, besides changing the fitting range on the finite T potential, we also use in each range different combinations of the values for $\tilde{\alpha}_s$, σ and c according to the uncertainties from the fits at $T \simeq 0$. The values we obtain for the Debye mass and their error estimates are given in Table 1.

Note that while we have determined m_D solely from an inspection of the real-part, the resulting values for ImV also agree reasonably well with the lattice data (Fig. 1 right). At small distances, where the lattice reconstruction of the potential is most reliable, we find quantitative agreement with the analytical form within statistical errors. At larger distances we expect that the lattice data-points are indeed larger than the actual values of ImV, as the underlying extraction from spectral widths leads to unphysically large values due to a diminishing signal to noise ratio.

Close to T_C at $T = 315$ MeV our analytic postdiction of ImV appears to lie rather far away from the lattice data. The reason for this, is twofold. On the one hand the extracted values for ImV at this temperature still contain large statistical errors. On the other hand the determination of m_D from ReV seems to lack accuracy, as the slope at large r is steeper than at both neighboring temperatures. We expect that higher statistics will lead to closer agreement.

The temperature dependence of the extracted Debye masses can be compared to HTL perturbation theory. According to Ref. [2] the Debye mass at leading log order can be written as:

$$m_D = Tg(\mu T)\sqrt{\frac{N_c}{3} + \frac{N_f}{6}} + \frac{N_c T g(\mu T)^2}{4\pi} \log\left(\frac{\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}}{g(\mu T)}\right)$$
$$+ \kappa_1\, Tg(\mu T)^2 + \kappa_2\, Tg(\mu T)^3, \tag{23}$$

where μ denotes the renormalization scale, g the running QCD coupling, and κ_1 and κ_2 are constants that represent non-perturbative contributions which need to be fitted to the data. As there exists a clear degeneracy between the variation of the renormalization scale and the variation of the higher order contributions, parametrized by κ_1 and κ_2, we choose to fix μ according to the usual convention $\mu = \pi T$ and fit c, d from the obtained Debye masses in the previous section.

For the running of the coupling $g(\mu)$ we utilize the four loop result of Ref. [27] setting $\Lambda_{QCD} = 0.216$ GeV, appropriate for quenched QCD. The fit yields $\kappa_1 = -0.40 \pm 0.06$ and $\kappa_2 = 0.21 \pm 0.06$, which is shown in Fig. 2. Note that the values obtained for κ_1 and κ_2 are small, which implies quite good agreement between hard thermal loop perturbation theory and the lattice extraction, even at the low temperatures probed here.

In previous lattice studies Debye masses were e.g. obtained by fitting the color-singlet free-energies [6] with a simple Coulombic Debye-screened form, even though these quantities have been shown to differ from the real-part of the proper heavy-quark potential already in perturbation theory [28,29]. Comparing, we find that our values lie consistently lower than these previous estimates. Note also that while the quenched lattices used to determine the heavy-quark potential deployed here are finely spaced ($a_s = 0.039$ fm), no continuum extrapolation has been carried out. As a possible remedy we follow Ref. [7] in providing the ratio of

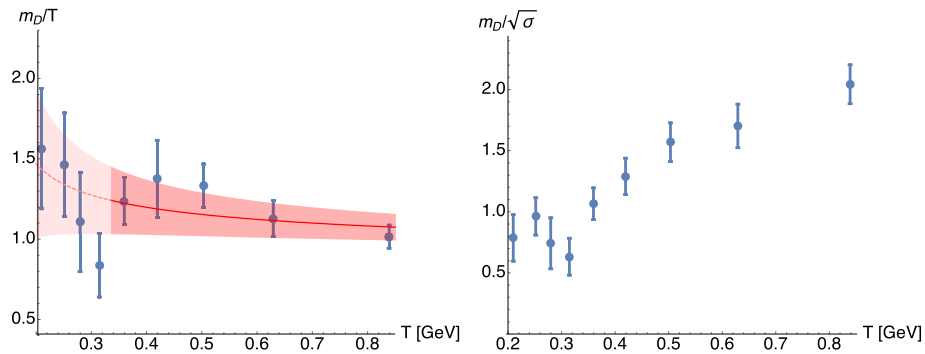

Fig. 2. (left) Debye masses (data points) obtained on quenched lattices and HTL fit (solid line) of their temperature dependence which is meaningful only well above the 1st order transition. Error bars include statistical errors, as well as those from changing the fitting range and propagation of the error of $\tilde{\alpha}_s$ and σ determined at $T \simeq 0$. (right) The dimensionless ratio $m_D/\sqrt{\sigma}$.

$m_D/\sqrt{\sigma}$ on the right of Fig. 2, in which some of the systematic uncertainties arising from a finite lattice spacing might be expected to cancel.

4. Conclusion

Based on a combination of the generalized Gauss law, introduced in [14] and the in-medium modification of its point charge distribution by a weakly interacting medium of light quarks and gluons, described by the HTL permittivity, we derived an analytic expression for the real- and imaginary parts of the static inter-quark potential at finite temperature. After fixing the strong coupling and string tension at low temperature, we are able to qualitatively and quantitatively reproduce the real-part of the potential measured in lattice QCD by fitting a single temperature dependent parameter $m_D(T)$, which is proposed as gauge invariant screening mass in QCD. The temperature dependence of m_D we obtained in a purely gluonic medium agrees well with HTL perturbation theory. Using the fitted values for m_D we furthermore find that a quite successful postdiction of ImV at high temperatures is possible. Agreement with ImV at smaller temperatures, currently hampered by uncertainties in the fit of m_D to the real-part, should improve once higher statistics become available.

We hope that phenomenological modeling will benefit from the derivation of a well motivated and lattice data validated parametrization of both ReV and ImV. In addition our study opens up the possibility to extract the imaginary part of the potential in full QCD simulations, in which up to date only the real part has been determined in a reliable fashion.

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