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The College of Preceptors and the *Educational Times*: Changes for British mathematics education in the mid-nineteenth century

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Abstract

Founded in Britain in 1846 to standardize the teaching profession, the College of Preceptors is little known today. The College was closely linked to the *Educational Times* (hereafter *ET*), a journal of "Education, Science and Literature" launched in 1847. This paper examines in detail a sample of College examinations, articles on mathematics education, and reviews of mathematics textbooks that appeared in the *ET*. Key figures in the mathematical discussion were William Whewell, Augustus De Morgan, and Thomas Tate. The paper shows how the discourse on mathematics education led to the introduction of entrance examinations for Oxford and Cambridge Universities.

Résumé

Fondé en Grande-Bretagne en 1846, pour la standardisation de l'enseignement, le College of Preceptors est peu connu aujourd'hui. Le College était fort lié à l'*Educational Times* (*ET*), un journal « de l'Education, Science et Littérature », lancé en 1847. Cet article examine en détail un échantillon des examens du College, des articles sur l'enseignement de mathématiques, et des critiques de livres des mathématiques parus dans l'*ET*. Trois noms ressortent dans la littérature concernant les mathématiques, ceux de William Whewell, d'Augustus De Morgan et de Thomas Tate. L'article montre aussi comment la discussion sur l'enseignement des mathématiques a mené à l'introduction des examens d'entrée pour Oxford et Cambridge.

MSC: 01A55; 97-03

Keywords: Whewell; De Morgan; Tate; *Educational Times*; College of Preceptors

Introduction

The *Educational Times* (*ET*) is a remarkable source, comprising articles of general as well as educational interest, covering a wide range of disciplines [Grattan-Guinness, 1992, 76–78]. It is a source

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that has been, and still is, difficult to access; not least because of the scarcity of copies of the journal. Where they do exist, these copies are frequently in a very delicate state. Not surprisingly, the journal contents are not generally well known. A study of the journal from its inception reveals that one of its main functions was to report on the activities of the College of Preceptors and to expound the aims and objectives of this institution.

A very clear message transmitted by the *ET* was that the College cared passionately about mathematics education in schools and how it affected subsequent mathematical study at Cambridge. Specifically, the *ET* published articles on arithmetic and algebra that featured the educational ideas of Augustus De Morgan, William Whewell, and Thomas Tate. Each of these pioneers had his own ideology and methods of putting it into practice. These methods were fully contrasted and debated within the articles, shedding new light on how arithmetic and algebra were taught in mid-19th-century Britain and what agents there were for changing the teaching methods for these key subjects.

During the 19th century Britain witnessed many interrelated political, religious, and educational reforms. The three educational pioneers were involved in these reforms to a greater or lesser extent and their religious, social, and political persuasions had a bearing on their work as educators. The College was involved in general educational reform as well as reform in specific areas such as mathematics teaching, and there was interplay between these different levels of reform. Thus it is a complex matter to unravel the contribution of the College to mathematics education, and the College's activities must be considered against a range of backgrounds in order to appreciate their full significance.

We begin with a very brief synopsis indicating what the College has achieved to date as well as its current status. Next a review of 19th-century mainstream education points out the main deficiencies in the system at the time. We examine reforms that were instituted to remedy these deficiencies and consider how and why the College came into being and what it contributed to the reforms. The role the *ET* played as the organ of the College is investigated, particularly with respect to the subject of mathematics examinations. We study in depth a solution guide for a general examination in education written by Joseph Payne, a College council member, who specifically recommended pioneering textbooks by Whewell, De Morgan, and Tate. The teaching of mathematics at Cambridge University is considered in the context of contemporary educational reform. We examine the writings of Whewell, De Morgan, and Tate in some detail. The main focus of the paper is several articles in the *ET* on the teaching of arithmetic and algebra, referring to these pioneers and two prominent College Council members, James Wharton and Richard Wilson. The paper concludes with a discussion of how educational reforms in mathematics led the College to introduce university entrance examinations.

Brief history of the College of Preceptors

The College of Preceptors dates back to 1846 and is the oldest surviving teaching association in the United Kingdom. Its founding principle was to certify teachers, thus assuring a common standard of education. It pioneered a number of important initiatives: introducing public examinations for secondary school pupils (including more recently the C.S.E.[1] examination), opening a training college for secondary teachers, and establishing professorships in education. Many eminent educationalists have been involved

[1] Certificate of Secondary Education. This was combined with the old G.C.E. "O" level (General Certificate of Education, ordinary level) to form the new G.C.S.E. (General Certificate of Secondary Education), in use in Britain today.

in administering the College. An example from the early decades was Dr. B.H. Kennedy (later professor of Greek at Cambridge and Dean of Ely), who was its president from 1859 to 1868 [Chapman, 1985, v, 170].

In recent years some attention has been paid, in monographs, to the College of Preceptors' significant contribution to the development of education in the 19th [Chapman, 1985][2] and 20th [Chapman, 1985; Aldrich, 1995][3] centuries. However, the College features little, if at all, in mainstream texts. For example, the College does not even receive a mention in the relevant two volumes of Simon's oeuvre on the history of education [1974a, 1974b]. Even Aldrich [1982, 58–59, 153] only alludes briefly to it in his general introduction to the history of education. Noting this lack of attention, Chapman remarks, "If there can be such scanty references to a group of men and women who have initiated and influenced such changes in British society, then the picture of the past hundred years or more must be incomplete and perhaps even inaccurate" [1985, xi]. He further observes that "the College... committed the unpardonable sin of the twentieth century: a lack of communication" [1985, xi]. Silver [1983, 29–30], a mainstream historian of education, attributes the historical neglect of the College to the fact that historians of Victorian education have focused almost exclusively on the development of the educational *system*, ignoring, for example, the impact of educational material in the press.

Background—British education

While public schools such as Winchester College and Eton had, from the late Middle Ages onward, offered education for a minority of pupils, school education in Britain was, up until the mid-18th century, more generally provided by means of a system of endowed grammar schools. The situation, by the beginning of the 19th century, was that there were three broad categories of education, provided by (a) the endowed grammar schools, each accommodating a small number of pupils, (b) various national voluntary organizations which gave children of the "laboring classes" some elementary education of a basic nature (the "monitorial" system), and (c) an increasing number of fee-charging, privately owned schools [Birchenough, 1930, 1]. Joseph Lancaster (a Dissenter) and Dr. Andrew Bell were prominent reformers of education for the laboring classes during the early 19th century and independently reintroduced and popularized the monitorial system. Lancaster's system was incorporated with the aid of trustees into the Royal Lancasterian Institution in 1810. The monitorial system was one in which children taught each other and concentrated on the basic subjects—the 3 Rs (*R*eading, w*R*iting, and a*R*ithmetic). Bell published his *Sketch of a National Institution* [1808] for training the children of the poor, and in 1811 "The National Society for Promoting the Education of the Poor in the Principles of the Established Church throughout England and Wales" was founded [Birchenough, 1930, 41–52].

In general, education was determined by social status, with the educational content tailored to suit the eventual occupation and way of life of the young scholar [Aldrich, 1982, 13]. The industrial revolution brought about social changes [Thompson, 1991], resulting in new educational opportunities for the labouring classes, including Sunday schools, the monitorial schools, and mechanics' institutes.[4] The

[2] Chapman looks at the history of the College.

[3] Aldrich's monograph concentrates on Joseph Payne, a central figure in the early life of the College.

[4] The first Mechanics' Institution appeared in Glasgow in the late 18th century and grew out of a series of scientific lectures given by George Birkbeck to working men. He then went on to found the Birkbeck Mechanics' Institution in London in 1824 and

early 19th century was a period of active educational reform [Thompson, 1991; Thomson, 1991, 11–98]. Aldrich [1982, 14] writes,

> [Radicals] including Owenites[5] and Chartists[6] also tried to shape an educational experience which would protect the labouring classes from the worst excesses of political, industrial and religious exploitation. This alternative system was intended to supply 'really useful knowledge.' Its agencies included a free press, itinerant lecturers and discussion centres. Informed parents would thus ensure that the home remained a true educational agency. Schools would not be subservient to the interests of a corrupt church and a tyrannical government. Indeed one of the overall purposes of such educational provision was to secure reform of these two institutions. By the middle of the nineteenth century, however, the failure of these alternatives was apparent and the children of the working classes were shepherded into the official schooling system.

The Anglican Church (Church of England) had many spheres of influence and there was a desire to reform more than its hold on religion. The Church had held a strong grip on education since the Reformation. The 1662 Act of Uniformity required all clergymen, teachers, or tutors in any kind of private or public school (even in the home) to conform to the liturgy of the Anglican Church and subscribe to its Thirty-Nine Articles. The remit of the Act also extended to all tutors, masters, Fellows, and chaplains in any college or university. Teachers who failed to comply were imprisoned and fined £5. This Act remained in force until 1869 and affected those outside the Anglican Church—including Quakers, Methodists, Baptists, and Unitarians—known collectively as Dissenters, and also Catholics and Jews [Smith and Wise, 1989, 3–119; Thompson, 1991, 28–58; Young, 1977, 74]. Many clergy and teachers were removed from their posts because of this Act, but a good number of them went on to establish new schools elsewhere. The Five-Mile Act was passed in 1665 to prevent such new establishments from setting up within 5 miles of a town or city [Howson, 1982, 46]. Dissenters, in particular Unitarians,[7] feature prominently in this article. Both the dissenting and the Anglican clergy wished to control religious teaching in schools and both influenced attempts to regulate education in the 19th century.

Prominent educational reformers of this era included Lord Henry Brougham and Dr. James Kay-Shuttleworth. Brougham's campaign for the education of the poor resulted in the setting up of a Select Committee on the Education of the Lower Orders. English education was found to be severely wanting and Brougham introduced an Education Bill to provide suitable schools, to be run by Anglican clergy but with nonsectarian religious teaching. Neither the Anglicans nor the Dissenters wished to support such an arrangement and the Bill was withdrawn. Brougham continued his campaign undeterred and his pamphlet *Practical Observations on the Education of the People* [1825] inspired the foundation of the Society for the Diffusion of Useful Knowledge (SDUK), which disseminated scientific and other knowledge in cheap, lucid tracts. (The SDUK was seen as a clear challenge to the SPCK—Society for

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this eventually became Birkbeck College. Similar institutions then appeared throughout England providing working men with classes, lectures, and libraries. They were opposed by some members of the Church of England but supported by Dissenters. (Lord Henry Brougham was a strong supporter.) By the 1830s the movement in many places had lost its popularity as public attention shifted to lectures on literary topics [Cardwell, 1972, 40–41].

[5] Followers of Robert Owen (1771–1858), an important pioneer socialist.

[6] The Chartism movement, particularly powerful from 1838–1848, had among its objectives that every man should have a vote.

[7] Unitarians did not accept the doctrine of the Trinity. See Thompson, pp. 29–31, for a discussion of the rise of Unitarianism amongst the dissenting religions. He alluded to "The rational Christianity of the Unitarians, with its preference for 'candour' and its distrust of 'enthusiasm'... it seemed too much associated with the comfortable values of a prosperous class to appeal to the city or village poor."

the Promotion of Christian Knowledge—whose schools provided secular and religious teaching for the poor [Howson, 1982, 85–86].)

Kay-Shuttleworth was appointed secretary to the Committee of Privy Council on Education set up in 1839. Kay-Shuttleworth and his friend E. Carlton-Tufnell used their own resources to set up a training college for teachers at Battersea in 1839–1840. The College was a success and was built upon by the government. "Trained teachers, public inspections, the pupil–teacher system,[8] the combination of religious with secular instruction with liberty of conscience, and the union of local and public contributions were all provided for or foreseen by him" [Stephen, 1885–1903].

What was the state of education following the efforts of these reformers? By the mid-19th century elementary education was centered in day schools. The Newcastle Commission in 1858–1861 carried out a survey of popular education. The Commissioners identified a steady increase in day school provision since the beginning of the century, a development in which the monitorial schools predominated. It was noted that in 1858, half a million children attended private schools, which the Commissioners deemed to provide an inferior and unsuitable education for the poor. At that time a standard curriculum had not yet emerged, but the introduction of the Revised Code of 1862, which provided for payment depending on results achieved by the pupils in the 3 Rs, was instrumental in bringing about reform. The Clarendon Commission (1861–1864) considered the nine public schools of ancient foundation, including Winchester, Eton, and Harrow. The Taunton Commission examined a range of public and elementary schools. The work of the Royal Commissions culminated in the introduction of the Elementary Education Act of 1870, which required children to attend school from the age of 5 and mandated requisite school places of a reasonable quality. By 1891 free elementary education was available for all [Aldrich, 1982, 76–79; Price, 1994, 16–18]. However, the dearth of educational provision for the middle classes remained a problem.

The focus so far has been on the situation of the labouring classes and the role of religion. Other important developments in British educational history concerned the professionalization of education and the place of women. De Bellaigue [2001, 963–1988] argues that attitudes to the professions were influenced by the ways in which late-19th-century men sought to define themselves and that in fact women teachers contributed significantly to the development of their profession [2001, 963]:

> In the nineteenth century, as members of new occupations sought to claim equality of status with the traditional professions, the meaning of the term gradually expanded. By using the term 'profession' to describe their work, they were aspiring to the prestige, the ideals of autonomy and independence, and the intellectual clout attributed to the 'learned professions.' The importance of training, examination, and certification in these occupations complemented the new ideal of employment on the basis of merit, which was gradually replacing patronage-based recruitment, and the professions came to be seen as embodying this ideal. ...The term 'profession' was never simply descriptive. It conferred prestige and suggested moral superiority, intellectual ability, modernity and efficiency [De Bellaigue, 2001, 963–964].

The traditional "learned professions"—medicine, the law, and the clergy—provided those in them with a structured system of promotion and advance and grew in importance in the eighteenth century [De Bellaigue, 2001, 964]. The Apothecaries Act of 1815 allowed the Society of Apothecaries to license practitioners, thus controlling entry into the profession. This improved the standard in medicine and provoked a similar rise in the standard of surgery brought about by the College of Surgeons [Cope, 1959,

[8] The pupil–teacher scheme was set up in 1846. It apprenticed clever pupils to headmasters for up to five years, at the end of which time they could apply for scholarships to a teacher training institution.

135]. Both the Royal College of Physicians [Annals, 1838, 219] and the Royal College of Surgeons [Cope, 1959, 133] introduced written examinations in 1838. The Society of Apothecaries had good regulations and introduced further improvements. As a result general practitioners were held in high esteem [Cope, 1959, 135]. Qualifying examinations were introduced into the legal profession in 1836. The learned professions had thus achieved institutionalization and control, which many other 19th-century occupations desired [De Bellaigue, 2001, 972–973]. Teaching was one such occupation and the College of Preceptors was formed to promote its professionalization.

The inauguration of the College of Preceptors

Teaching was not established as a profession in the first half of the 19th century and was recorded by the census enumerators under "learned occupations" until 1861, when it was categorized under "professions" [De Bellaigue, 2001, 973]. Given the three different educational categories and their attendant various religious influences, there was little conformity among teachers in Britain, a situation which posed a significant problem for the professionalization of teaching. In the case of the monitorial system, Bell and Lancaster advanced rival methods of education, and each had his own system for the training of teachers [Birchenough, 1930, 41–52].

The poor standard of teaching in private schools was drawn to the attention of the public by novels such as Dickens's Oliver Twist [1838] and Nicholas Nickleby [1838–1840]. A group of committed private schoolmasters formed the College of Preceptors in 1846 to regulate teaching and raise its standards [Aldrich, 1995, 96]. Founding members included Richard Stokes, a headmaster in Chipping Ongar, Essex; John Parker, a recently graduated assistant master at the same school; Henry Stein Turrell, a headmaster in Brighton specializing in modern languages, especially French; the Rev. Dr. Richard Wilson, a classicist and headmaster in London; Dr. William Ballantyne Hodgson, a headmaster in Manchester; Stephen C. Freeman, a headmaster in Enfield, Middlesex; Joseph Payne, a headmaster in Camberwell whose area of interest was philology; and James Wharton, a teacher from Kinver in Staffordshire [Chapman, 1985, 11–14].

The most notable of these founders were Wilson and Wharton. Wilson was born in Westmoreland in 1798 [*ET*, November 1879, 313–314; Venn, 1940–1954, 526]. He was mainly educated at Kendal Grammar School and went up to Cambridge with an Exhibition in 1820. At St. John's College he obtained his B.A. in 1825, being 17th wrangler[9] on the Tripos list. Shortly afterwards he became a Fellow of the College and gained his M.A. in 1827. The Bishop of Ely ordained him a deacon of the Anglican Church in 1828 and a priest in 1829. He took private pupils for many years at Cambridge, one such being James Joseph Sylvester. He was awarded the degree of D.D. in 1839. He was then headmaster of St. Peter's Collegiate School, Eaton Square, London. Wilson was one of the earliest Fellows of the College and was a member of the College Council until 1877, holding the office of Dean from 1848 to 1859. He was also an examiner for a range of subjects, including classics and Hebrew, and was an author of mathematical textbooks, including one on trigonometry. He continued to live mainly in London until his death in 1879 [*ET*, November 1879, 313–314].

[9] Wranglers were the top mathematics graduates at Cambridge. See [Barrow-Green, 1999] for a discussion of mathematics prizes at Cambridge, the effect they had on mathematical learning, the balance of pure mathematics and natural philosophy.

Wharton (d. 1862) received his B.A. from St. John's College, Cambridge, in 1834, where he was 14th in the list of Senior Optimes (a lower distinction than wrangler). He was the author of several mathematical textbooks on algebra and arithmetic. He was a member of the College Council for many years and also of the College Board of Examiners [*ET*, April 1862, 21; Venn, 1940–1954, 419].

It was Parker, encouraged by Stokes, who first had the idea of a College of Schoolmasters. He outlined his thoughts at a meeting of the Literary and Scientific Society[10] in Brighton in 1845. Parker asserted that the duties of a school teacher were just as important as those of doctors, lawyers, or the clergy and pointed out that many children of the middle classes were educated at private schools where there was no way of ascertaining the teachers' ability. He made it clear that his objective was to form a body capable of assessing teachers' competency independent of religious issues. His suggestions (which were in line with the popular desire to reform education) were warmly received and he was sufficiently encouraged to try them out on a wider audience. He then promoted his proposals through an advertisement in a national newspaper.[11] With the penny post[12] supported by an effective railway system, Parker's message was rapidly communicated across the country. He received many replies from English schoolmasters in support of his schemes [Chapman, 1985, 10–16].

Following the formation of a provisional committee in February 1846, a general meeting was held in London on 20th June of that year [Aldrich, 1995, 96]. Some 300 schoolmasters attended the meeting, 60 of whom joined the College [Chapman, 1985, 21]. The following four resolutions were passed:

> 1. That, in the opinion of this meeting, it is desirable for the protection of the interests both of the scholastic profession and the public, that some proof of qualification, both as to the amount of knowledge and the art of conveying it to others, should be required, from and after a certain time to be hereafter specified, of all persons, who may be desirous of entering the profession; and that the test, in the first instance, should be applied to the Assistant Masters only.
> 2. That, in the opinion of the meeting, the test of qualification should be referred to a legally authorized or corporate body, or college, consisting of persons engaged in tuition.
> 3. That for the purpose of effecting this object—viz., the formation of a corporate body—the members of the profession who enrol their names at this meeting, do resolve themselves, and are hereby resolved, into the College of Preceptors; and that those persons now enrolled, or who may hereafter be enrolled, shall incur no liability beyond the amount of their respective annual subscriptions.
> 4. That a Council, consisting of the members of the Provisional Committee, with power to add to their number, be now appointed for the purpose of conducting the business of the institution, and that Mr. Turrell be appointed President of the Council [College of Preceptors, 1896, 4].

There was no mention of any church involvement in these resolutions, a fact probably reflecting the religious diversity amongst the Council members, just under half of whom were graduates of British universities. Two were Dissenting ministers and the other twelve Anglican clergymen [Aldrich, 1995, 105]. Implicit in the second resolution was the stipulation that teachers (and not the clergy) were to regulate admission into the teaching profession. This regulation was to be by means of examination.

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10 Societies such as this sprang up towards the end of the 18th century, prompted partly by the wider diffusion of scientific knowledge and partly due to dissatisfaction with the inept Royal Society [Cardwell, 1972, 23]. The membership of such societies varied from city to city—"round the Manchester Literary and Philosophical Society a distinguished and very creative school of scientists had grown up..." [Cardwell, 1972, 65]. However, in the Newcastle Literary and Philosophical Society mathematical teachers and practitioners predominated [Fauvel et al., 1991, 163–178]. A common feature of such societies was the provision of regular educational lectures, which were printed and distributed by the society, and the use of a library.

11 Possibly the *Times* or the *Daily News*, which were used by the College in January 1849 [Council Minute Book, 1848–1857, 15].

12 The postal system introduced by Sir Rowland Hill in 1840.

Payne and Wilson established half-yearly examinations, with Wilson taking the lead in arranging them and reporting back to Council [College of Preceptors, 1848–1857, 6, 27, 39]. There are no surviving minutes of any examination committees for these early years, so it is not possible to ascertain on what basis examiners were chosen (but examiners' fees and expenses were agreed at the Council meeting on January 15, 1849 [College of Preceptors, 1848–1857, 14]). Theory and practice of education was a mandatory examination subject. So too was bible history, despite the secular bias of the College. The first candidates were able to choose from tests at a lower or higher level in mathematics, classics, commerce, and foreign languages. Appropriate qualifications for those passing examinations were consolidated during the first 2 years of the College's life. To qualify for a certificate, a successful candidate had to pass the third class of either the mathematical or classical test; otherwise he or she would receive a diploma. A candidate with a second-class pass became an Associate of the College of Preceptors (ACP), and with a first-class pass, a Licentiate of the College of Preceptors, (LCP). Work of distinction in education, science, literature, or the fine arts was needed to become a Fellow of the College of Preceptors (FCP). These awards were meant to correspond to traditional university awards, ACP to a university degree B.A., LCP to an M.A., and FCP to a doctorate. Each award had its own academic gown [Chapman, 1985, 28–31].

From the start the College welcomed women teachers. This was reflected in the choice of the name "preceptor," which was a more appropriate term than "schoolmaster," given the large number of female teachers [Chapman, 1985, 10]. In reference to the state of female teachers at the time De Bellaigue observes [2001, 963] that the popular image of the 19th-century amateur governess is inaccurate. In fact many female teachers (who would have taught only girls) were highly capable and the midcentury reformers drew on their expertise. They were actively involved in the struggle to professionalize teaching but their contribution was forgotten in the 1870s when a model of boys' education was adopted.

To respond to the needs of the female teacher, the College announced the creation of a Ladies' Department [*ET*, October 1847, 5]. Payne and Freeman were both keen advocates of women's education. At first, a collateral College for women was envisaged, parallel in all aspects to the original College, and managed by a Ladies' Committee [Chapman, 1985, 26]. Wharton and Wilson were part of a separate committee set up to communicate with the Ladies' Committee regarding classes for women (there being separate Gentlemen's classes) [College of Preceptors, 1848–1857, 39]. Wilson was responsible for supervising the "Ladies' Classes" [College of Preceptors, 1848–1857, 27]. Both men were examiners for the female candidates [College of Preceptors, 1848–1857, 53]; indeed Wharton was involved in organizing the examination of Miss Wilson in the theory and practice of education [College of Preceptors, 1848–1857, 66] although she apparently passed music in the second degree [College of Preceptors, 1848–1857, 69]. This was the first time women had been allowed to take such examinations and was a radical step for the profession [Chapman, 1985, 25; De Bellaigue, 2001, 975]. Many women teachers supported the College and by 1862 there were at least 48 women licentiates, associates, and members of the College [De Bellaigue, 2001, 975]. In 1849, however, the separation of the sexes was discontinued and women took their examinations alongside men [Aldrich, 1995, 99], an event which led to a reduced participation of women in the College.

The College sought patronage to strengthen its political and social standing:

> Its first 'Patron,'[13] the Marquis of Northampton, was not only a peer but also the President of the Royal Society; and amongst the first 'Vice-Patrons' were four other Fellows of the Royal Society, a Queen's Counsel,[14] and twelve Members of Parliament. [Chapman, 1985, vi]

One of the Vice Patrons was Sylvester, who was chosen, perhaps, due to his connection with his former tutor, Wilson. The College further strengthened its constitutional position by the procurement of a Royal Charter on 28 March 1849, which gave it stability and professional standing[15] [Chapman, 1985, 45]. The College made use of the newly founded journal, the *ET*, to advertise its efforts to raise money to gain the Royal Charter [*ET*, April 1848, 137] and later to announce its success in this endeavor [*ET*, December 1850, 49].

All did not welcome the gaining of the charter, however. Aldrich [1995, 102–103] observes:

> The College's determination to seek a charter aroused hostility from many quarters. ...For traditionalists, therefore, the granting of a charter to the College of Preceptors, a body composed in the main of nongraduate lay persons, many of whom were Dissenters, seemed to be an assault upon the principles and persons of the teaching profession, as currently constituted. Equipped with a charter the College might promote the notion that (with respect to teaching) secular was superior to religious, and the College's own qualifications to the degrees of the ancient universities. Opposition, however, came not only from entrenched interests but also from those radicals who feared the establishment of a new set of privileges and interests.

Given the College's involvement in vital teaching reforms, can one conclude that it achieved its fundamental objectives? [Bott et al., 1995, 2] state:

> [G]roups such as school teachers or university teachers, who would like to consider their calling a profession, fall outside it; entry to school teaching depends on recognition by the Department of Education and Science, a government body, rather than by an independent chartered body and there is no independent body which lays down a code of conduct—this is largely a matter for the teachers' employers.

The reasons the College with its Royal Charter is not the body licensing teachers today go right back to its inception and are perhaps best analyzed by one of its founders, Payne, an important critic of the early life of the College. He was involved in the Council from the beginning and frequently argued over its decisions. He condemned the fact that the College did not satisfactorily test for competency in teachers. The College's first resolution only called for *assistant* masters to be examined in the first instance, thus allowing incompetent schoolmasters to join without being tested. In 22 years, only 500 teachers received College certificates. Teachers joining before 1 January 1847 gained the College's highest rank, MCP, and life membership could be purchased for 10 guineas. Not surprisingly the original 60 members had burgeoned to 1000 within the first year. Payne strongly criticized the College for having sought a Royal

[13] The role of a patron was to promote and support the College within his or her sphere of influence.

[14] A barrister (a specialist attorney who speaks in court).

[15] "The decision to grant a Royal Charter to a professional body is taken primarily on the grounds of public interest—is it in the public interest that the activities of the group of practitioners which the body represents should be regulated and, if so, is the professional body a fit, proper and appropriate instrument to do this? This raises subsidiary questions about how representative of practitioners in the field the body is, and the extent to which its members can claim to be a coherent group. A Royal Charter is only granted after extensive investigations; although it confers some privileges, it imposes many responsibilities that the body must be willing and able to accept" [Bott et al., 1995, 2]. For example, the Institution of Civil Engineers, founded in 1818, had as its first president Thomas Telford, who successfully campaigned for the institution to receive the Royal Charter in 1828 [Bott et al., 1995, 5].

Charter and for its dependence on patronage, declaring that an institution with a worthy cause had no need of such influence. He was a firm supporter of female education and did not approve of the closure of the Ladies' Department, a move that resulted in the underrepresentation of women in the life of the College [Aldrich, 1995, 98–99, 104–105].

However, these weaknesses were counterbalanced by one of the College's great strengths: its involvement with the *ET*. It was the unofficial journal of the College until 1861 and as such reflected the early history of the College [Aldrich, 1995, 101].

The launch of the *ET*

The *ET* was launched in 1847 and its mathematics department was created 2 years later. Mathematics was a prominent subject in the *ET*. An examination of British mathematical journals of the period will provide a context for understanding the involvement of the *ET* in the public discussion of mathematics.

In the late 1840s several British mathematical journals had not long ceased publication, for example, the *Mathematical Repository* (1770–1835). Thomas Turner Wilkinson,[16] a deputy headmaster from Burnley, chronicled British mathematical journals in his seminal works [1848, 1849, 1850, 1851, 1852, 1853]. The main journals at this time were the long-standing *Ladies' Diary*, containing varied contributions from mathematicians and philomaths; the *Cambridge (and Dublin) Mathematical Journal*, which had the prominent mathematicians De Morgan, Arthur Cayley, Sylvester, and Gabriel Stokes as early contributors; and last the *Mathematician*, emanating from the Royal Military Academy at Woolwich. Research mathematicians published in the *Proceedings of the Cambridge Philosophical Society*, the *Philosophical Transactions of the Royal Society*, and the *Philosophical Magazine* [Mackay, 1893, 303–308; Glaisher, 1880, 73–75; Perl, 1979, 36–53]. Archibald's survey of British mathematical journals [1929] remains the authority in this area.

General science periodicals were read by many middle-class people, and mechanics magazines by the working classes wanting to improve their social position. The notion of the self-made scientist gave way to that of the professional in the 1860s. The opinions and the aspirations of the editors, who were often respected scientists, had a definite impact on the content of scientific journals. Commercial considerations also shaped science journals, particularly as they were subject to stamp duty, paper tax, and advertisement tax. There was very real pressure on editors to ensure their journals covered at least the cost of printing and publishing. In the 1860s conditions improved for the interchange of journals between Britain and the Continent [Brock, 1980, 105].

The first issue of the *ET* was published on 2 October 1847, with the subtitle "a Monthly, Stamped Journal of Education, Science and Literature" [*ET*, October 1847, 1]. The journal ran from 1847 to 1923, when it became *Education Today*. The editors were not listed and it is not known who exactly in the College was responsible for founding the journal. There was no formal connection stated between the College and the journal until April 1861 (incidentally when James Wharton died), when the *ET* was subtitled "journal of the College of Preceptors." At the Council meeting of 5 January 1849 a resolution was passed to rescind a previous one that stated that the *ET* should pay a yearly rent of £20 [College of Preceptors, 1848–1857, 15]. Two weeks later the Council minutes recorded that the summary of accounts

[16] Wilkinson was a major contributor to the mathematical department of the *ET* and will be the subject of a future monograph.

and balance sheet for the year were to be printed with the Council report in the February issue of the *ET* [College of Preceptors, 1848–1857, 21]. On August 18th the Council agreed that each month the College would buy 200 copies of the *ET* for distribution by the secretary [College of Preceptors, 1848–1857, 42]. The balance sheet in the minutes for Christmas 1850 shows that the cost of the *ET* to the College was £30, and that advertisements in it cost £2 5s 6d [College of Preceptors, 1848–1857, 127]. However, by Christmas 1856, when the College's finances were in difficulties [Chapman, 1985, 47], the *ET* did not appear in the accounts [College of Preceptors, 1848–1857, 344]. These reports point to a working relationship between the College and the *ET*, but do not help to identify the founder(s) or editor(s) of the *ET*.

Aldrich [1995, 101–102] summarizes the contents of the *ET*:

> The *Educational Times* was a serious paper with a serious purpose, although not averse to the inclusion of an occasional humorous piece. It published papers, model answers and pass lists of Preceptors' exams, membership lists and minutes of Preceptors' meetings, accounts of lectures and publications, advertisements of teaching posts and from those seeking employment. It commented on the great educational issues of the day, and printed earnest correspondence from those who would reform the educational establishment and from those who had suffered at its hands.

News from other educational sectors was also reported in the *ET*—there was a section entitled "University Intelligence" which brought information concerning Oxford, Cambridge, Dublin, and the London Universities.

The first number of the *ET* was priced at 6d (making it more expensive than the *Mechanics Magazine*), thus putting it at the cheaper end of the range of contemporary mathematical journals. It was published by Thomas Taylor of 31, Nicholas Lane, London (not to be confused with Taylor and Francis). Each issue contained 16 pages. A notice at the end of the first issue stated: "The *ET* will appear on the first of every month, stamped 6d; unstamped 5d. Parties paying 6s in advance will be supplied with the journal for 12 months" [*ET*, October 1847, 16]. The stamped version carried the red penny post on the front page. The first issue of the *ET* was written with great assurance and purpose, exemplified by a long letter [*ET*, October 1847, 6] from Isaac Reeve commenting on the mission of the College. Reeves wrote as if the *ET* and the College were well-established and not new enterprises.

The contents of the first issue indicated the journal's outlook and aspirations. The first page contained a miscellany of irregularly laid out, boxed-in advertisements for general items (for example, envelopes and beauty products) aimed at a diverse audience of nobility, gentry, managers, merchants, engineers, and the like. Among these general advertisements were those offering school places and also the services of tutors. The very first advertisement promoted a member of the College of Preceptors who had passed the higher classical examination and was seeking employment as a teacher.

A letter from Wilson in his capacity as Dean of the College appeared on page 5. It set out the College's rationale: to train teachers for the middle classes; to separate the function of teacher from that of priest; and to encourage headmasters to produce articled teachers (thus following Kay-Shuttleworth's innovations.) Pages 8–10 presented a short list of the journal's objectives followed by a longer and more detailed prospectus. This editorial highlighted the journal's intent to be a vehicle of communication for issues of current educational interest, and not simply to reflect the views of the College. It emphasized the provision of instruction to teachers and pointed out that the interests of the teacher and the community were connected. Last, the editorial indicated its wish to critically review a wide range of educational works.

In the longer prospectus, six objectives of the journal were stated. The first was to be an organ of official communication for the College of Preceptors both generally and more specifically for the publication of material relating to examinations. The second was to affirm the value of little-known Continental methods in Britain and also to encourage women as teachers and learners. The editors declared the *ET* would include: "papers on female education, and on the duties and provinces of the female teacher; showing their vast importance, especially in a moral point of view, not only upon young women but upon the whole population." This supports De Bellaigue's argument that women had a wide educational influence at this time. The third objective referred to educational politics. Objective five was to produce specific accounts of the influence of educators, educational establishments, and educational literature, and objective six concerned advertisements.

Objective four dealt specifically with scientific material. The *ET* aimed to include:

> Papers for the purpose of conveying exact and scientific information on the subjects of the greatest utility and interest to the enlightened educator. Amongst the subjects in this department will be concise lectures on physics and the natural sciences, intended to give a general view of those branches of knowledge, and to direct the student to the best works upon them. Special attention will be paid to the points most likely to perplex the unaided student; and the best made of presenting them to the youthful pupil, so as to be intelligible to him, will be explained. In this division will be included papers on physical culture, and the intimate connexion between the condition of the body and that of the mind. Lastly, full and faithful reviews of books, especially of those intended to be used in schools, or likely to be serviceable to the instructor, including those which are already well known, but respecting which we may entertain opinions different from those that are prevalent; digests of the transactions of scientific bodies; the proceedings of universities, colleges, chartered and endowed schools, preferments, vacancies etc.

Science was thus singled out for special attention; as it turned out, mathematics and mathematics education were the journal's primary scientific focus. This was reflected in the examinations: there was a strong mathematical component in the general educational examinations, as well as an examination dedicated exclusively to mathematics.

College examinations

The College mathematics examiners for June 1847, printed on the front page of the first *ET* [1847, 1], were John Hind,[17] James Wharton, and George Boole. By June 1849, the examiners were Hind, Boole, and Sylvester [*ET*, June 1849, 243]. In December 1850, the geometer Thomas Stephen Davies[18] was added to this trio [*ET*, December 1850, 49].

Boole, an established mathematician appointed in 1849 to Queen's College in Ireland, was also a passionate teacher who was interested in social issues [MacHale, 1985]. Sylvester was a London actuary with a very substantial mathematical reputation based on his work with Arthur Cayley on the theory of invariants. The mathematical ability of Boole and Sylvester was exceptionally high and contrasted

[17] John Hind (1796–1866) graduated from St. John's College, Cambridge, in 1818, a second wrangler and second Smith's prizeman. After a period of teaching in Sydney, he was ordained Deacon of the Anglican Church in Cambridge [Stephen, 1885–1903; Venn, 1940–1954, 419]. An author of mathematical works on algebra [1830] and the comets, he was also a member of the Royal Astronomical Society.

[18] Davies (1795–1851) was a frequent scientific contributor to the *Gentleman's and Lady's Diary, Philosophical Magazine* and *Mechanics' Magazine*. He was appointed mathematical master at Woolwich in 1834 [Stephen, 1885–1903].

sharply with the abilities of examiners in other subjects, a fact indicative of the important position of mathematics within the College. In a letter to William Thomson in September 1846 Boole stated:

> The Royal College of Preceptors (...) have elected me as an examiner. I need scarcely say that the appointment was quite unsolicited on my part. I have taken no share in their proceedings beyond sending them a guinea[19] which I did as approving of their object though never intending to avail myself of it.

He wrote to Thomson again in November 1846,

> I believe after all I am not appointed as an examiner to the College of Preceptors. The post was certainly offered me by one of the council, but I suppose that he had mistaken his directions in some way. [MacHale, 1985, 284]

The misunderstanding was presumably cleared up as Boole's name appeared as examiner for June 1847. It seems Boole valued the examining position and the College. Sylvester also welcomed his involvement with the College, and may well have seen the examining post as a way of leaving the legal profession and getting back into academia.

In the early Council meetings Wharton was involved in drawing up bylaws for the election of Fellows to the College [College of Preceptors, 1848–1857, 57, 62–63]. The minutes for 4 January 1851 recorded that Mr. Wharton gave the chairman a list of nominees for Fellowships. This list included the Marquis of Northampton, Lubbock, Hind, Whewell, and Boole and was signed "R Wilson and Wharton" [College of Preceptors, 1848–1857, 124]. But written across the list were the words "Cancelled by order of Council. Feb, 15th 1851." The whole Council did apparently not share Wharton and Wilson's desire for the College to offer fellowships to some of the country's finest mathematicians.

The subjects tested on the mathematical examinations were reviewed in the first issue of [*ET*, October 1847, 5]. The higher test covered algebra, conic sections, the differential and integral calculus, statics, and dynamics, together with applications of these subjects; while the lower test covered arithmetic, the first four books of Euclid with deductions and problems, the elements of algebra, and the elements of plane trigonometry. These subjects were also taught in De Morgan's lower junior class at the University of London.

The higher test included material studied by the more able students at Cambridge under Whewell's reforms of 1848. The lower test was similar to the elementary mathematics introduced at Cambridge by Whewell in 1848, except that it lacked applied mathematics. Similarly, it resembled the course studied at Woolwich, but the Woolwich course also contained applied mathematics. (Details of the examinations carried out at Woolwich were published in [*ET*, November 1859, 257]. Pure mathematics was worth 2,000 marks and "mixed" mathematics 1,500.) It is possible that the College mathematical examiners wanted to promote pure mathematics, in reaction to Whewell's emphasis on applied mathematics.

Excellent examiners, however, did not entice excellent candidates, and there were none for the first mathematical higher test. Six men did pass the lower test in mathematics in June 1847, as did two men in June 1848. In [*ET*, April 1849, 150], Wilson referred to the introduction of mathematical prizes. The list of those passing the mathematical examinations in August 1849 showed that one R. Jones, MCP, was the Sir John Lubbock's Mathematical Prizeman, and A. Smith, MCP, the Mr. Wire's Mathematical Prizeman. The Lubbock prize was presumably a high honour, since its namesake was the Vice President

19 This refers to the guinea subscription paid to help the College acquire its Royal Charter.

of the Royal Society and Vice Patron of the College. David William Wire, of Lewisham, was also a Vice Patron of the College. The introduction of mathematical prizes may have been an attempt to emulate the Smith's prize awarded for mathematical prowess at Cambridge.

To promote mathematical understanding, the *ET* published model answers to the mathematics exam questions. The arithmetic test and the solutions to the questions on algebra from January's exam, 1847, were published in [*ET*, November 1847, 21–22]. There was an examination paper in "Arithmetic" and one in "Arithmetic and Algebra" in [March 1849, 138]. The questions asked for definitions of arithmetical and algebraic terms with supporting examples. On the arithmetic paper, which contained 21 questions, question one asked for a definition of "least common multiple" and "greatest common measure," with examples. Question four read "Shew, by arithmetical example, that multiplying a given quantity by a proper fraction decreases its value, and dividing it increases its value. Why is this?" There were then several questions involving a practical application of arithmetic. One of these was a practical problem requiring ratios, but at the end of the problem the candidate was asked to state the reason for the method of solution. The arithmetic exam included four separate questions on "Artificers and Work" related to building problems. All of the questions demanded a thorough understanding of the underlying principles.

The arithmetic and algebra examination (17 questions) was difficult and required definitions, explanation, and investigation of methods, not just arithmetical or algebraic manipulation. Questions were again set in a practical context—the number of bricks in a wall, the cost of cloth, and the weight of gold being examples. No questions appeared contrived. Question 16 requested an explanation of the signs and symbols used in arithmetical algebra.

The article "Euclid with Deductions" [*ET*, April 1850, 164] contained a selection of questions from examination papers. Definitions and explanations were required. None of Euclid's propositions was referred to by number and there was no demand for rote recital from the *Elements*. The first question asked for information on the axioms, including the fifth postulate, and asked for an alternate definition of the latter. The majority of the questions involved straight lines, triangles, circles, and squares, an example being question four. "To draw a straight line through a given point parallel to a straight line. Describe a circle which shall touch two given parallel lines, and pass through a given point between them." A basic knowledge of the first books of Euclid would have been sufficient to answer these two easy problems.

The general examination on education also contained significant mathematical material. The third page of the first issue contained a selection of the College examination papers for June 1847. The "Theory and Practice of Education" paper was set by Payne [Aldrich, 1995, 108] and demanded a wide knowledge of several subjects, including Latin and French, and an understanding of the history of education as well as current Continental educational practice. Several of the 21 questions specifically concerned mathematics:

> 10. Explain and contrast the analytical and synthetical (or constructive) methods of teaching. Illustrate the application of each to teaching Arithmetic or Latin, and trace their respective moral effects on the mind of the pupil.
> 14. What is the nature and range of the mental discipline effected by the study of Mathematics?
> 15. What are the main deficiencies in our common methods of teaching Mathematics? How would you propose to remedy them? As an illustration explain how you would teach Arithmetic to an elementary class.
> 17. Which do you consider, from personal experience, the best books for teaching the following subjects: Greek, Latin, German, Algebra, Trigonometry, Arithmetic, Geography, General History, and English Grammar?

The Council deemed Payne's examination far too taxing and requested model answers, as had been supplied for the algebra examination. He declined, saying such answers could be learned by heart and regurgitated in future examinations. Turrell had set the previous paper in the Theory and Practice of

Education. His questions were of a completely different sort—he examined teachers on their practical experience of the schoolroom (even down to heating it) [Aldrich, 1995, 107–112]. Payne later relented and provided outline answers.

Payne's solutions appeared in [*ET*, December 1847, 40–41]. For question 10 Payne provided a full discussion of analytical and synthetical methods, noting Kay-Shuttleworth's preference for the latter. For an illustration of a synthetical method in arithmetic,[20] Payne recommended Thomas Tate's *Exercises in Arithmetic*, published under the sanction of the Committee of Council. For question 14 he suggested reading Whewell's *Study of Mathematics as Part of a Liberal Education*, De Morgan's *The Mathematics, Their Value in Education*, and Pott's edition of *Euclid*. For question 15 he proposed the students read four articles by De Morgan from the *Quarterly Review of Education*, "On Mathematical Instruction" [1831], "On Teaching Arithmetic," "On Teaching Fractional Arithmetic" [1833], and "On teaching Geometry."

For question 17, Payne specified, for algebra, Tate's *Algebra Made Easy* [1847a], Colenso's *Algebra* [1843], especially, the fifth edition, and Wood's *Algebra*, edited by Lund [1840]; for geometry, Pease's *Practical Geometry* [1844], Potts's *Euclid* [1846], and Lardner's *Euclid* [1828]; for trigonometry, Hall's, Snowball's, and Hymer's trigonometry, respectively; and for arithmetic, Tate's *First Principles of Arithmetic* [1847b], Macleod's *Mental Arithmetic* in two parts, Hopkin's *Manual of Mental Arithmetic*,[21] Crossley's *Arithmetic* [1833], and then De Morgan's, Colenso's, Hind's, and Wharton's arithmetic, respectively.

Hind's book on arithmetic was of a very high standard. His *Elements of Algebra* [Hind, 1830] required a working knowledge of arithmetic and progressed from elementary rules and theorems in algebra, grounded in arithmetic, to symbolic algebra, the basis for analytical investigations [Hind, 1830, preface]. It contained appendices with examples. Bishop John William Colenso (1814–1883) was mathematics master at Harrow from 1838–1842. His textbooks were widely used in Victorian schools, particularly his *Arithmetic* [Howson, 1982, 259]. Wood's *Algebra* contained a chapter on analytic geometry and was used at Cambridge [Becher, 1980, 7].

Payne also advocated De Morgan's works for questions 10 and 12 and Whewell's for question 12. To appreciate the value Payne placed on the writings of De Morgan and Tate it is necessary to examine some developments in mathematics education generally, and arithmetic and algebra teaching specifically, prior to the period in question.

Background to mathematics education

Mathematics gained a secure place in the curriculum at secondary level and in the universities due to the efforts of the 18th-century Dissenter Philip Doddridge, who was not a mathematician as such [Howson, 1982, 45].[22] He succeeded in this despite opposition to the subject by some Dissenters. Often,

[20] Tate and other 19th-century educators used the term "synthetical" in a conventional Kantian sense, to refer to objects that are synthesized from experience, or given synthetically. Such a usage was in general accordance with Heinrich Pestalozzi's views on how arithmetic should be taught, according to which the abstract truths of arithmetic should be presented to the students only after they have had some sensory experience with numbers and operations in a concrete physical setting. See also the passage from Kay-Shuttleworth [1841, 38–39] quoted on page 159.

[21] It has proved impossible to locate further bibliographical details regarding the textbooks of Macleod and Hopkin.

[22] Howson's *A History of Mathematics Education in England* [1982] presents a series of accounts of mathematical educators, several of which are key to this discussion and are used here. In particular, Howson discusses the relationship between

the utilitarian aspect of mathematics was emphasized to justify its teaching [Howson, 1982, 59]. However, Howson [1982, 124] declares:

> Charting the development of mathematics in the first half of the nineteenth century is not easy. All that can be done is to consider the histories of individual schools and, again, the position is confused because often mathematics was taught only temporarily according to the whim of the master. . . . [I]n the eighteenth century most mathematics teaching took place in academies, although a number of grammar schools taught the subject. Moreover, the Eldon ruling[23] served to buttress the classical curriculum and to keep mathematics as an optional subject.

The midcentury Royal Commissions had an effect on mathematics education. As a result of the Clarendon Commission, mathematics teaching in the public schools gained a firmer footing with the appointment of specific mathematics masters. However, most boys only studied four books of Euclid, some algebra, and some arithmetic. The Taunton Commission also reported the same narrow range of topics being taught, arithmetic being dominant. Pupils were observed to perform more poorly in geometry than in arithmetic. Although schoolteachers admitted that mathematics was useful and valuable, there was no enthusiasm to extend mathematics teaching [Price, 1994, 16–18]. The Educational Census, 1851, showed that 61% of elementary schools taught arithmetic, compared with 68% for writing and 98% for reading [Aldrich, 1995, 78]. Mathematics teaching was not uniform across the various schools in England. A lack of uniformity was also a feature of mathematics education in the universities.

Although the universities of Oxford and Cambridge experienced a general decline in the 18th century, mathematical studies increased in importance at Cambridge. Examinations were instituted there, the first "Mathematical Tripos List" appearing in 1747. Mathematics was prized as part of "a liberal education" and was felt to be useful for preparing men for the legal profession. However, there was no sense of pursuing mathematics for its own sake. Newton's notation for the calculus was taught, without heed to the new analysis developed in France [Cardwell, 1972, 18–20]. Mathematics at Oxford was neglected, and the level of this subject was much lower than at Cambridge [Howson, 1982, 126].

Mathematics teaching underwent significant reform in the first half of the 19th century [Howson, 1982, 75]. Charles Babbage, George Peacock, and John Herschel started the short-lived Analytical Society in 1812. This society advocated the introduction of Joseph Louis Lagrange's algebraic methods of calculus into the Cambridge syllabus, in place of Newton's methods, which had predominated in Britain during the 18th century [Rice, 1999 June–July, 535]. Beginning in the 1820s, the innovations inspired by the Analytical Society had an effect on several generations of undergraduates, particularly through Peacock's teaching. De Morgan and Whewell were among the first students to be influenced while Arthur Cayley, Sylvester, William Thomson, and George Gabriel Stokes were student beneficiaries of the new analysis in the 1830s and 1840s. The Tripos course was affected by these reforms and subsequently involved intense mathematical training. The attendant Tripos examination became famous, the top student earning the title of "senior wrangler" [Cardwell, 1972, 53–54].

mathematics and religion. The development of mathematics education differed in schools and universities and the two subjects are thus considered separately.

[23] In 1805 Lord Eldon ruled that the chief purpose of grammar schools was to teach the classics, with the result that modern languages and mathematics were optional subjects that had to be paid for. An Act of 1840 gave schools the freedom to revise their curricula [Howson, 1982, 252].

Whewell, De Morgan, and Tate

As a result of the reforms, it became increasingly difficult for the curriculum to cover both the new analysis and the old 18th-century geometry. The outcome was a destabilizing effect on the mathematical Tripos. Whewell was the prime mover in restoring mathematical equilibrium to the situation. Whewell studied at Trinity College, was second wrangler in 1816, and became a Fellow of Trinity in 1817. He was a college tutor from 1818 to 1839, professor of Moral Philosophy from 1837 to 1855, and Master of Trinity from 1841 to 1866. Having introduced Continental mechanics at Cambridge, he pressed for the continued expansion of applied mathematics in the curriculum in the 1830s, but felt that a similar expansion of analysis, especially pure analysis, would destroy the foundations of a liberal education [Becher, 1980, 3].

Whewell believed in a liberal education for the upper classes, a weakened version of it for the middle classes, and elementary education for the working classes. Mathematics was to be the foundation for all types of education. Geometry should be taught as in Newton's *Principia* [1687], so that the underlying physical process would not be obscured by any analytical symbolism. Mathematics was valued for building the solid and certain reasoning that was at the basis of a good mental discipline [Cardwell, 1972, 54–55]. In defending the liberal education, Whewell became a fierce opponent of the analysis that had come to dominate the curriculum. To address this problem, he reformed the Tripos in 1848, putting a renewed emphasis on the geometrical mathematics of the 18th century [Becher, 1980, 3]. Geometry was emphasized over algebra, intuition over abstract rigor, and problem-solving over formal algorithms [Becher, 1980, 42]. Whewell also preferred the physical sciences with their practical utility to pure mathematics [Fisch, 1991, 39].

Whewell was a very powerful figure at Cambridge in the 1840s as Master of Trinity College. He disseminated his ideas in eight textbooks published between 1819 and 1837 [Fisch, 1991, 15]. During this period textbooks played an extremely important role in Cambridge mathematics instruction [Rouse Ball, 1889, 128]. New textbooks outlined contemporary analytical developments, while Whewell's own books on mechanics applied new philosophical thinking and mathematical techniques to Newtonian mechanics.

The work of Whewell's that Payne frequently referred to in his model answers to his examination questions was *Of a Liberal Education in General and with Particular Reference to the Leading Studies of the University of Cambridge* [1850]. Whewell described "permanent" and "progressive" studies, saying that only permanent studies held a place in his liberal education. Geometry was the core element of his permanent studies but he consigned contemporary developments in algebra to "progressive," studies where they remained as

> novelties which cannot take the place of the Permanent portions, in our Higher Education, without destroying the value of our system. ... For Geometry really consists entirely of manifest examples of perfect reasoning ...But in Algebra, on the contrary, and in all the branches of Mathematics which have been derived from Algebra, we have, not so much examples of Reasoning, as of Applications of Rules; for the rules being at first proved by reasoning, once for all, the application of them no longer comes before us an example of reasoning. (29)

He continued by praising algebraic methods as "ingenious and beautiful" but saying that a "professional mathematical education" was needed for this subject, rather than a liberal education (p. 30). His thoughts on arithmetic were more prosaic:

> Arithmetic has usually been a portion of education on somewhat different grounds; namely, not so much on account of its being an example of reasoning, as on account of its practical use in the business of life. To know and be able familiarly to apply the rules of Arithmetic, is requisite on innumerable occasions of private and public business; and since this ability can never be so easily or completely acquired as in early youth, it ought to be part of the business of the boy at school ...There is another reason for making Arithmetic a part of the school-learning of all who are to have a Liberal Education: namely, that without a very complete familiarity with actual arithmetical processes, none of the branches of Algebra can be at all understood. Algebra was, at first, a generalization and abstraction of Arithmetic; and whatever other shape it may take by successive steps in the minds of mathematicians, it will never be really understood by those students who do not go through this step. (pp. 31–32)

De Morgan, a leading student of Whewell, was one such mathematician whose mind was actively at work on the place of algebra in a liberal education [De Morgan, 1835, 91–110; 293–311]. De Morgan had graduated fourth wrangler at Trinity College Cambridge in 1827. While at Cambridge, he turned away from the Anglican Church and refused to subscribe to the Thirty-Nine Articles. Although a career at Cambridge was closed to him, he was able to obtain a position at London University, a newly founded nonconformist institution [Howson, 1982, 79].

It was a correspondent of De Morgan's, Lord Brougham, who was instrumental in setting up the University of London in 1826. This institution was for the financially disadvantaged middle classes (being unable to support their children at university) and also for those excluded on other grounds from the universities. Men and women of any religious persuasion, including Anglicans, Dissenters, Catholics, and Jews, were able to study there. This new university was vociferously resisted by Anglicans, who founded King's College London in opposition to it. In 1836 the University of London was renamed University College London [Cardwell, 1972, 45; Rice, 1996, 377]. De Morgan became professor of mathematics at the University of London in February 1828, but resigned on principle in 1831 over the dismissal of a colleague. Five years later De Morgan's replacement died and he returned to his professorship, which he held for 30 years. He then resigned again on principle when a colleague was dismissed on religious grounds. De Morgan had converted to Unitarianism and he protested against the treatment of his colleague, who was a controversial Unitarian minister [Rice, 1999 June–July, 535; Howson, 1982, 79].[24]

De Morgan was involved with popular education for the working classes through the Society for the Dissemination of Useful Knowledge (SDUK), an organization that published many of his books. He also contributed extensively to the SDUK's *Quarterly Journal of Education* and *Penny Cyclopaedia* and recommended parts of the latter to Brougham in his letter of [19th May 1842, Brougham Collection].

Influenced by the algebraist George Peacock, De Morgan strove to establish algebra as a basic component of liberal education. He believed that Whewell's emphasis on geometry was detrimental to the development of pure mathematics generally and specifically to algebra and analysis. De Morgan's work in algebra was encapsulated in his *Trigonometry and Double Algebra* [1849], in which he laid down 14 laws of symbolic algebra. "Single" algebra concerned algebra without the use of angles and "double" algebra with them, that is, when complex numbers were involved. In contrast to the formalist Peacock, he was more concerned with how symbolical forms were interpreted and less with how they were manipulated [Panteki, 1991, 191].

In addition to being an eminent mathematician, De Morgan was also an outstanding teacher. Rice [1999 June–July, 534–552] calls attention to De Morgan's large collection of notebooks that

[24] De Morgan's Unitarian inclinations are evident in an 1854 letter to Brougham [11 July 1854, Brougham Collection], "I brought together all the evidence for I.N. being a Unitarian—of a deeper cast than Arianism—in my life of Newton in Knight's British Worthies—It has been stated in *Unitarian books*—but rarely in orthodox ones."

supplemented his course textbooks. These materials showed how thoroughly he covered a broad curriculum. De Morgan was a natural communicator and could present complex ideas from any area of mathematics in a manner understandable to his students [Rice, 1999 June–July, 549]. His interest in teaching derived not from an interest in the utilitarian value of mathematics, but rather from its value in exercising the reasoning powers [Howson, 1982, 88–89]. His point of view was similar to Whewell's and contrasted with that of Brougham, who emphasized not just the intrinsic value of mathematics but also the importance of its applications [Howson, 1982, 86].

De Morgan had very firm opinions on examinations. While accepting the need for them, he openly detested the Cambridge system where students competed against each other. The students had to "cram" for examinations, learning the right techniques in order to solve complex mathematical problems in a small amount of time. This practice prevented them from reading widely around the subject. De Morgan preferred instead the Oxford approach where the students were examined "against the mathematics." He made it clear that "cramming" would be of no use in his examinations [Rice, 1996, 381–382].

A natural consequence of De Morgan's educational principles was an aversion to the rote learning of mathematics. According to [Howson, 1982, 89], in many schools:

> [a]rithmetic had been broken down into a number of rules, many of them so unintelligible that by themselves 'they could be Hebrew.' Thus inverse proportion was described as when 'less requires more, and more requires less.' The pupil was not expected to understand the reasons for the rules, but merely to be able to apply them. If he could not he was flogged.... Teachers feared to teach principles, for to do so required knowledge and understanding. It was much easier to teach rules and with the help of various books of worked solutions avoid any troublesome questions.

De Morgan favoured teaching arithmetic by principles, based on data or assumptions known to the student from the start. He also found similar difficulties with algebra teaching and ridiculed so-called "practical" problems in algebra, which were in reality contrived. De Morgan's remedy for these deficiencies lay in using clear language and solid logic. Instruction in arithmetic should commence with a lucid exposition of the methods of numeration, for example addition, subtraction, multiplication, division, fractions, and decimals, as found in his *Elements of Arithmetic* [1830]. The goal was to instill in the student an understanding of arithmetical processes and principles. Algebra was to be taught in such a way that the symbolism was fully comprehended [Howson, 1982, 89–92].

Tate was another influential educator who was concerned with the teaching of arithmetic. He was one of the country's first teacher trainers, working at Kay-Shuttleworth's Battersea Training School. A gifted teacher who used practical problems in his teaching, he also advocated arithmetic for the development of reasoning power and to produce habits of order and exactness. Mental arithmetic was valued as an aid to memory, reasoning, and the powers of conception [Howson, 1982, 114]. The syllabus at Battersea included algebra, arithmetic, mechanics, and mental arithmetic. Payne's solution to question 10 of the June 1847 general examination on education, published in [*ET*, December 1847, 40–41], provides insight into Tate's views on teaching:

> As a familiar illustration of both these processes, we may refer to the case of a pupil writing a Latin exercise, in accordance with directions supplied to him, and afterwards parsing the exercise when completed—first employing synthesis and then analysis. In the ordinary routine of instruction both methods are used, according to circumstances; but different teachers—as for instance, Pestalozzi and Jacotot—have so strongly insisted upon a preference for one or the other principle, that the question between them has become of considerable importance. More recently, too, the selection of the synthetic mode by Dr Kay Shuttleworth, the educational engineer of the operations of the Committee of the Council on Education, as the foundation of all the works published under the sanction of the Govt, has brought the subject prominently forward. Dr Shuttleworth's Report on this and kindred topics, in the Minutes of the Council, and in the volume on the training of pauper children, are well worthy of attention. The Pestalozzian method of teaching

> Arithmetic, as exemplified in Mr. Tate's Exercises in Arithmetic, published under the sanction of the Committee of Council, is purely synthetic; and parallel specimens applied to the Greek and Latin languages may be seen in the late Dr Allen's Constructive Greek Exercises....

Tate succeeded in making arithmetic widely accessible to children by the way he presented the rules of arithmetic, and his textbooks were widely adopted. In his Report of 1841 on the Battersea Training School, Kay-Shuttleworth discussed the syllabus used by Tate:

> In ... *arithmetic* it has been deemed desirable to put [the students] in possession of the pre-eminently synthetical method of Pestalozzi ... The use of such a method dispels the gloom which might attend the most expert use of the common rules of arithmetic, and which commonly afford the pupil little light to guide his steps off the beaten path illuminated by the rule...
>
> The opposite practice of dogmatic teaching is so ruinous ... to the intellectual habits, and so imperfect a means of developing the intelligence, that it ought, we think, at all expense of time, to be avoided. With this conviction, the method of Pestalozzi has been diligently pursued.
>
> Whilst these lessons have been in progress, the common rules of arithmetic have been examined by the light of this method. Their theory has been explained, and by constant practice the pupils have been led to acquire expertness in them, as well as to pursue the common principles on which they rest, and to ascertain the practical range within which each rule ought to be employed. The ordinary lessons on mental arithmetic have taken their place in the course of instruction separately from the peculiar rules which belong to Pestalozzi's series.
>
> These lessons also prepared the pupil's for proceeding at an early period in a similar manner with the elements of algebra, and with practical lessons in mensuration and land surveying. [Kay-Shuttleworth, 1841, in Howson, 1982, 107, 110, 119]

In recommending Whewell, De Morgan, and Tate, Payne was advocating very different methods and, in some instances, diametrically opposed points of view regarding mathematics education. However, Payne was not the only College council member to discuss the views of these three mathematics educators—Wilson and Wharton were also involved. Whewell's advocacy of a liberal education and its repercussions for school mathematics were debated at length in the *ET*.

Mathematics education in the *ET*

Mathematics education was prominent in the *ET* right from the start. On the first page of the first issue there was an advertisement for the winter lectures (1847–1848) of the "South Islington Commercial and Mathematical School." A long item on education by "A Collegian, London" appeared in [*ET*, April 1848, 149–151]. It contained a large section on the importance of mathematics in both schools and commercial establishments. The College was encouraged to take the lead in demonstrating *how* to teach mathematics in a lively and appropriate manner. The author summarized the College's mathematical aims:

> The College has already by its Dean (Rev. Dr. Richard Wilson), declared that it considers mathematical studies amongst the most important, yet at present most neglected, branches of education; and an illustrious mathematical scholar has shown his opinion on the subject by offering a prize for the greatest proficients in mathematics at the College examinations. At Cambridge, Dr Whewell and others hail the proceeding of the College in this direction as an earnest of improvements in primary instruction. Already many of our commercial schools are devoting their attention to a sound and diligent course of mathematics; and the supplementary educational establishments which have arisen under the appellations of athenaeums and mechanics' institutions, at which thousands of youths are labouring to compensate the defects of instruction by a diligent application to mathematical as well as classical and other studies, are sufficient proofs of the necessity of some change in our methods of school education. Let it not be supposed that we would wish mathematics only to be studied, but we are aware of opinion that they ought to form the staple in commercial schools, and that in classical schools they ought to engross at least one-half of the hours of study.

The article then continued with a discussion on the desirability of concentrating on a few subject areas—mathematics, classics, and chemistry—and of providing lectures that could be published in the form of model lessons [*ET*, April 1848, 150]. Wilson's report in his capacity as Dean [*ET*, July 1854, 224] recommended "that schoolmasters will now universally instruct their pupils through principles rather than rules." Wilson had thus joined Tate, Brougham, and De Morgan in the general campaign to reform mathematics teaching, specifically echoing De Morgan's and Tate's preference for principles. He was also advocating a significant place for mathematics in the educational curriculum.

In the early issues, there were many articles on the teaching of arithmetic. A lengthy article entitled "A Model Lesson in Arithmetic," attributed to one "B.A," appeared in [*ET*, October 1848, 5], possibly in response to Wilson's call. It was in the format of a teacher–pupil dialogue, where the teacher asked a question of a class of about 30 boys and pupils were selected at random to answer. The answers given were idealized and showed a full understanding of why the rules of arithmetic were effective. The subject of the lesson was decimal fractions, with considerable detail concerning all the different situations in which they occurred. The model students explained the terms involved, expanded on them, and then gave examples. Although the answers were too good to have been the responses of real-life students, they were very informative and contrasted favourably with the presentation of incomprehensible rules prevalent at the time.

"Notes on Teaching Arithmetic" by J.B. appeared in [*ET*, May 1857, 99] and contained hints for the successful organization of an arithmetic class. It recommended mental arithmetic, as well as "sound and thorough Arithmetic," but was chiefly concerned with the mechanics of the lesson itself. It stated that arithmetic lessons used the power of the mind rather than the memory and could therefore be carried out in the afternoon when the memory was flagging. A variety of activities were suggested for the lesson, which was preferably to be held in a separate room with a blackboard. The old practice of leaving the student to work on problems unassisted day after day was deplored.[25] But the author also warned against the antithesis of this practice, of explaining everything to the pupil. Instead the teacher was encouraged to observe the progress of the pupils and be ready to guide them in their own learning so that the pupils themselves would express the ideas taught them by their master. This point of view recalled Tate's methods.

Another article by J.B., "Hints on teaching Algebra," appeared in [*ET*, June 1857, 123]. The author noted the importance of establishing a firm foundation in arithmetic before attempting algebra. A justification for teaching algebra was given:

> The reasons why Algebra should form a branch of study are numerous. It is now taking a place in not only a liberal, but a moderate education; its aid in Arithmetic is invaluable, being the royal road by which many complicated and high-sounding rules are resolved; it stands on the threshold of all mathematical studies; it quickens the intellect and develops latent genius. And, again, the pursuit of this branch is recognized as a feature of the educational movement of the present age. The Committee of Council fosters its cultivation, and the Royal College of Preceptors must deem its importance great, since in their scholastic examinations, it is required not only for First and Second Classes, but even for the Third.

[25] A description of the traditional organization of the school day is contained in a letter De Morgan wrote to Lord Brougham [18 October 1862, Brougham Collection]: "As to the preparatory schools, the bare fact, which is easily proved, which shew that mischief is going on. As follows—when I was a boy, the school day was mostly spent in *preparing*, and the far smaller part of the school time was passed *coram recapitro*. In our time, nearly or quite the whole day is before the master, and the preparation is all thrown into the evening. The consequence is that a boy's work keeps him 'til late at night. Now after six hours' homework, a boy of 13 or 14—not to speak of the younger ones—is not fit for *three* or *four* hours' work in the evening. In our University school there is less of this than anywhere I know: but I am credibly informed that it is not uncommon elsewhere for lads to be up 'til 11 O clock and 'til midnight. This will tell very readily by and bye."

These sentiments were very much in line with De Morgan's desire to see algebra as a core subject in a liberal education.

J.B. recommended teaching algebra at an early age and exhorted the teacher to keep his pupils' interest. The simple equation was stipulated no longer to be a boundary, nor was the quadratic regarded as invincible. Formulae and progressions were said to be more appealing than horseshoe problems, an approach very different from the practical one taken in the arithmetic and algebra examinations 10 years earlier. Less time spent on practical problems would result in more of this "almost boundless science" being taught in schools. Three hours a week was given as the right time to spend on algebra and it was strongly recommended that pupils study independently, so their abilities would be "uncrammed." J.B. expressed in conclusion the hope that a wide range of pupils would start to study algebra. His allusion to "the royal road" recalled Euclid's geometry and its prominent position in British mathematical education and seemed to suggest that algebra should be accorded similar status. The whole tenor of the article recalled De Morgan's emphasis on the value of teaching algebra.

J.B. published a third article in [*ET*, July 1857, 146] on the "Use of Geometry Theoretical and Practical." The author discussed the best time of day for a geometry lesson and the best age to start teaching geometry and then progressed to the subject's practical applications.

J.B.'s three articles expressed clear support for the changes in mathematics education advocated by De Morgan and Tate. Their publication indicated the importance the editors of *ET* attached to the teaching of mathematics. Mental arithmetic was promoted in opposition to the prevailing sentiment concerning the irrelevance and difficulty of mental training [Aldrich, 1982, 104]. The editors were also keen to publish articles on practical ways of teaching arithmetic, algebra, and geometry and recommended the best textbooks for these subjects.

Mathematical textbooks reviewed in the *ET*

A letter in [*ET*, December 1847, 55] asked the editor which books he would recommend to prepare for the mathematics exam. In his reply Wilson suggested Hind's *Algebra* [1830] or Lund's edition of Wood's *Algebra* [1840] and referred to a list given earlier in the same volume. He also observed that Euclid should be studied in *viva voce* and students should be asked to write out propositions. Below the list of books recommended by Payne in [*ET*, December 1847, 41] there was a note announcing the publication of Wharton's *The Principles and Practice of Arithmetic and Mensuration with the Use of Logarithms*, a book which included a written endorsement by the College.

Good mathematical textbooks, both new and old, were reviewed in detail in the *ET* and those considered beneficial and instructive for personal study were recommended. A review in February 1848 of *The Young Ladies' Arithmetic* [date not known, probably 1847 or 1848] by Samuel Goodwin expressed the following sentiments:

> ...[this] work adds another to the list of treatises which proceed upon the now happily disappearing system of teaching arithmetic by mere rule.... We can see no reason why girls as well as boys should not be taught the principles of arithmetic; we believe their minds to be equally capable of development and that it is quite as important that their powers be educed [*ET*, February 1848, 107].

De Morgan taught women arithmetic and algebra for a short while at the Ladies' College, Bedford Square, London, an institution started in 1849 [Rice, 1996, 395]. The College founder William Hodgson had enjoyed success earlier teaching a class of women in Liverpool. He found them attentive, hardworking, and quick to understand and reported that they made good progress [Chapman, 1985, 26]. The reviewer criticized Goodwin's work for neglecting to explain the principles behind mechanical methods of arithmetic.

Books on both arithmetic and mental arithmetic appeared prominently in Payne's list of recommended books for question 17 of the June 1847 examination. The Rev. I. Steen of the Belfast Royal Academical Institution wrote a *Treatise on Mental Arithmetic* [1846], which was reviewed positively in [*ET*, February 1848, 108], with the proviso that little time should be spent in teaching mental arithmetic in schools, as this would detract from teaching the principles of arithmetic. In his response [*ET*, March 1848, 125] Steen asserted that mental arithmetic, if taught correctly, could help develop the powers of the mind. He expressed concern that teachers would be turned against this new discipline, and claimed that his teaching methods were based on clear principles, sentiments that recalled Tate's emphasis on teaching both mental arithmetic and arithmetic. The response of the reviewer, published below this letter, stated that it was more important to put arithmetic on a sound footing and that concentrating on mental arithmetic, even if taught well as Steen advocated, would detract from this endeavour. The reviewer reiterated the College's determination to free the teaching of arithmetic from all defects.

Other important members of the College were involved in mathematics education. John Parker was the author of three mathematics books that were advertised in [*ET*, November 1847, 31]. It was Wharton, however, who was the most active College Council member in the field. A paper read by him at the Conversazione held by the College of Preceptors, 26 June 1848 was published in [*ET*, August 1848, 242–243]. Wharton pointed out that mathematics was necessary to engender power of sustained thought and accuracy of reasoning. He stated "Cambridge has of late expressed its determination to encourage only accurate mathematical learning, and the College will do its utmost with that University for the attainment of so desirable an object." Wharton was also classics examiner in June 1850 and was a prolific textbook writer.

Wharton, De Morgan, and rules in arithmetic teaching

An outline of a lecture by Wharton on the study of mathematics that was given at a College meeting was reported in [*ET*, January 1849, 82; Wharton, 1849a]. Wharton started with a short review of the history of mathematics and then discussed the lack of mathematical study at the public schools, at Oxford, and in the education of members of the armed forces and future parliamentarians. He criticized teaching that involved "a never-ending succession of rules" and that lacked a logical method. Cambridge wranglers who had "crammed" for exams were pronounced unfit to teach. The Cambridge system was denounced for its "*grinders*" who prepared pupils for the examination by making them copy out entire mathematics books without the examples.

Wharton advocated a strict matriculation examination to improve standards of students before they entered Cambridge. He called for the provision of good textbooks beyond the elementary level, instead of the mere translation of French textbooks. He claimed that Cambridge was held in poor esteem compared with the French "schools." (He was referring here to the French "*grands écoles*," e.g., *L'Ecole Polytechnique* and other specialist engineering schools [Grattan-Guinness, 1985, 84–111].) Cambridge tutors required knowledge only of the first three books of Euclid and simple equations; a clear statement of the mathematical level appropriate for an entering student was needed. Putting principles in the form of rules was censured and teachers were encouraged to cultivate their pupils' reasoning, even if this produced a profusion of questions. Wharton suggested that arithmetic would fill the void in female education and teach girls to think accurately and consecutively. The report cited an eminent Cambridge man who stated that it was useless trying to teach higher mathematics at Cambridge when the basics had not been grasped. Wharton called for the compilation of a large collection of problems, such as his own textbook *Examples in Algebra* [1848].

Wharton published around a dozen books, some of which ran to many editions and several of which contained examples from examination papers, for example for the Civil Service. His *Elements of Plane Trigonometry* [1849b], though aimed at school children, also contained an introduction to analytic geometry, a subject he felt had been neglected. This book covered the standard definitions and formulae in plane trigonometry and included examination papers from Emanuel College, Oxford, the College of Preceptors, and St. John's College, Cambridge. Wharton recommended Wilson's, Hind's, Snowball's, and Hymer's trigonometry books for further reference.

The purpose of Wharton's *Logical Arithmetic,* fifth edition, 1859 was "to render youth familiar with the use of numbers, to make them independent of rules in their calculations, and to give them method in writing out at examinations" (preface). The book commenced with definitions, then considered fractions, decimals, logarithms, proportions, and compound interest. It contained some Civil Service, College of Preceptor, and Oxford middle-class examination papers. The definitions were elementary; one, for example, declared there to be three kinds of number: odd, even, and fractional. The 1861 edition started with a history of number systems and had additional material on mental arithmetic. The book was slim and the explanations and definitions were of a low standard, below the caliber of Wilson's work on trigonometry.

Logical Arithmetic [1847] was unfavorably reviewed in the *ET* [November 1847]. Wharton answered his reviewers in [*ET*, February 1848, 99–100]. His reply, entitled "On Arithmetical Instruction," was somewhat arrogant and discourteous, immediately claiming that the reviewer had not read his book carefully and had also made errors. He denounced the reviewer as being afraid of new methods and wishing to retain the old rules, someone who "wants to have a large mess of principles, something like Professor De Morgan's book, which, at his recommendation I have inspected; and my opinion of which is that, with all its excellence, there is too much explanation." Given his emphasis on explanation in the examinations of 1847, this seems a strange line for Wharton to have taken. It was also at odds with Wilson's later emphasis on principles in 1857. Wharton included a section of a Cambridge book that proved that

$$am/bm = a/b \quad (a, b, m \text{ integers})$$

and criticized the demonstration as being unnecessary, as such an identity should be regarded as an axiom, or at least as data. He asserted that a statement such as $1/2 = 2/4$ needed no further explanation and that arithmetic could be taught satisfactorily from such a basis. He suggested that "arithmetic be brought as nearly as possible to common sense, and let it exercise common reason, and be made the ladder to higher subjects." He emphasized that boys should be made to reason, not to understand. He felt the College examinations had already "done very great good" but that lengthy textbooks were detrimental, even if they did attempt to propound the principles of arithmetic. He claimed that boys he had taught for 3 months were capable of tackling difficult problems from other books, such boys being potential counterexamples to Dr. Whewell's observation that "young men come to Cambridge entirely unprepared to proceed in the University course, because they are often entirely devoid of any previous mental cultivation." Note here Wharton's emphasis on the students' ability to handle worked examples. It was consistent with the

Cambridge ethos of studying examples to cram for exams, and was in line with Whewell's conception of mathematics as something that is primarily a means to an end.

Wharton's reliance on "axioms" compared poorly with De Morgan's desire to set out a firm foundation for his arithmetic. Rice [1997, 112] describes De Morgan's concern over teaching fractions:

> He [the pupil] has been accustomed to the consideration of several things of the same kind, but rarely to that of the division of one of these objects into *equal* parts. His *half* has, most probably, been merely a division into any two parts whatsoever, and can accordingly, with perfect consistency, talk of the larger and the smaller half" [De Morgan, 1833, 210]. This point was of no small consequence since he had observed from experience that "the want of a familiar acquaintance with common and decimal fractions is the source of nine out of ten of the difficulties which are commonly found in the study of algebra" [De Morgan, 1833, 221–222]. The first step, therefore, in training sound mathematicians was to ensure that their notion of all the concepts they were required to employ was rigorous and exact.

De Morgan and Wharton both used the terms "data" and "axiom" in the same general way, but disagreed as to what they meant in the particular case of arithmetic. De Morgan clearly stated his rules for manipulating fractions in his *Elements of Arithmetic* [1830, 66].

De Morgan set out his view of the roles of rules, reason, principles, and arguments in the preface of the same book (iii, iv):

> This little work is an attempt to give the young student the common rules of Arithmetic, accompanied by the reasoning to which he must habituate his mind before he can make progress in any science ...But as almost all agree in opinion, that this science [arithmetic] ought not to be, as it is in this country, degraded into a mass of rules, one half of which are of no use but in commercial business, and rarely even there ...There are some who defend the present system of teaching this science upon the ground that children retain and apply a rule with more facility than a principle or an argument.

Wharton's defence of his article was followed by a reply from the reviewer, who was at a loss to discover his so-called errors or understand why he was deemed to approve of arithmetic taught by rule, which he said he deplored. He criticized Wharton's practice of using axioms for arithmetical properties that should be proved, declaring Wharton's reasoning to be somewhat loose. He also criticized Wharton's error of going from the particular to the general, but did agree with him about the poor state of mathematical instruction in most of the private and public schools. The reviewer's response was lucidly and cogently argued, exposing Wharton's inferior grasp of how to teach arithmetic. The board's decision to publish both the review and the subsequent exchange showed a certain impartiality with respect to the question of reform in the teaching of mathematics.

Wharton's *Examples in Algebra: Being a Collection of More than 2000 Examples* [1848] came mostly from Cambridge examination papers and was intended to allow progression from elementary to more complex problems. It covered fractions, surds, permutations and combinations, the binomial theorem, series, simple equations, quadratic equations, simultaneous equations, and indeterminate equations. Wharton's *Complete Solutions of Every Class of Examples in Algebra*.... was published posthumously in 1863 and contained an anonymous preface marked "Cambridge":

> This Publication, entitled "Solutions of Examples in Algebra," is the first part of a work designed as a help to Students of that science. The first part, complete in itself, was nearly finished when the Author was, after a painful illness, removed by death. The book will be a useful companion, more especially to those Students who have not access to a living Instructor to explain their difficulties.

> The late James Wharton, B.A., M.C.P., was a Schoolmaster, and the author of several Elementary Books.... He was one of the originators of the Royal College of Preceptors and a most active and zealous promoter of its success. He was also, for sometime, one of its Mathematical Examiners, and he continued to labour for the interests of the College as long as his health enabled him to do so.

De Morgan was strongly critical of the value of examples in mathematics textbooks, a point on which he differed greatly with Wharton. In the preface to his *Elements of Arithmetic* (iv) De Morgan referred to the "seven or eight years during which they [the pupils] are employed in toiling from rule to rule through countless examples, and dragging at each remove a lengthened chain."

De Morgan was the subject of other articles in the *ET.* His "Decimal Coinage" in the *Companion to the Almanac* [1848] formed the basis for discussion in [*ET*, February 1848, 102]. (He discussed this issue at length in his letter to Brougham [15 July 1855, Brougham Collection], where he alluded to Brougham's political support.) His lecture in October 1848 at the commencement of the Session of the Faculty of Arts of University College London, "On the Effects of Competitory Examinations, Employed as Instruments in Education," was reported in full in [*ET*, December 1848, 56–59]. This article emphasized the need for depth of study in a particular area, not breadth of study as in the liberal education at Cambridge. De Morgan preferred private study to lectures, the latter being valued mainly as a source of useful hints.

There was an anonymous article on "Double Algebra" (the algebra of complex numbers) in [*ET*, September 1854, 273–274] which discussed De Morgan's work of that title. The author denounced learning arithmetic by rule and stated that De Morgan was "the only Englishman who has attempted to raise the standard of Elementary Algebra." He especially recommended De Morgan's *Trigonometry and Double Algebra* and works by the German mathematician Martin Ohm, pronouncing these to be admirable schoolbooks. However, he claimed that De Morgan fell short of establishing algebra as a significant and symbolic calculus. He praised Colenso's book as being the best schoolbook on the subject in England, but claimed that it failed to fulfill the logical requirements of the science. He then called upon the College to go beyond Cambridge and "demand for its First Class, a thoroughly philosophic knowledge of Algebra as a science." This was to include (a) definitions of symbols, (b) the use of infinite series, (c) the notion of limits, (d) the notation of functions, (e) different scales of origin, and (f) equations of the first and second degree. He praised De Morgan for producing problems for such equations on topics such as levers and specific gravities—mechanical problems, instead of examples involving "old ladies dealing in eggs." As De Morgan had very definite views on the place of examples in algebra teaching, it is worth looking in detail at what they were.

In his letter to Mr. Coates [7 December 1836, SDUK Collection], De Morgan declared "it is almost impossible to manufacture useful examples in Algebra, except in the details of some other science, the application of Algebra to Geometry is a good one, the Differential calculus still better." He then went on to explain that books of examples normally had a chapter on each different type of problem, but that in reality several different problems may occur at the same time, leaving the student confounded as to how to proceed. To cope with this situation, he recommended the student be "regularly pelted with problems." A problem required the student to discover a solution and to work things out for himself, whereas an example simply displayed the answer in final form. He pointed out that the French did not put a single example in their books, but produced far superior mathematicians. He remarked "But then they [the French] study well what they have: whereas our people want to be guided from example to example, and they think they get on because they *do* something." He advocated that students read the "great books of science" and that they then be supplied with just sufficient examples to make the principles expounded in these works understood.

De Morgan's publications on algebra seem to have been better received in the *ET* than his writings on arithmetic. De Morgan was very clear that rules could be used in arithmetic if the accompanying reasons and principles were lucidly explained at the same time. Wharton purported to have the same aims, but in practice he required the students to tackle masses of examples.

De Morgan, Wharton, and Tate were all opposed to the use of mechanical, unexplained rules in the teaching of arithmetic. De Morgan [1830, iv] mentioned that *almost* everyone agreed that arithmetic teaching should not be "degraded into a mass of rules learnt by rote." Which influential mathematical educator might have disagreed? Whewell allowed that:

> One man speaks with ridicule of the Rules of Arithmetic as commonly given; for instance, the Rule of Three, Inverse, the Rule of Five, Alligation, Barter, and the like ... it usually happens that boys who are made to learn and apply rules, begin to see the meaning of the rules, when their habits of thought are further unfolded: and though this may lead their friends or themselves to suppose that the rules at first were of no value to them, this supposition would be a great mistake. Boys can easily learn to apply rules, before they can easily learn to understand them; and are likely to understand them the better, from being already familiar with the mode in which they are applied. The memory may be brought into extensive action, before the understanding can; and may be made to assist powerfully in unfolding the understanding, by supplying it with materials to operate upon. If no boy was allowed to learn anything of which he did not, at the time, understand the reason, no general system of teaching could be applied; the progress of learners must be slow and irregular; and after all, there is no ground to believe that boys so taught would understand the rules better than those who begin by applying, and end by understanding the reasons of them. For it can admit of no doubt that to understand the rules and their reason *at a subsequent period* is a necessary portion of the system of education to which they belong. To make the student understand fully both the rules of arithmetic and grammar, and their reasons, is an important step in that higher education which succeeds the education of the school boy. But on this ground, no valid argument against any particular form of such rules can be drawn from the ridicule to which they are subject, as being unintelligible to the boys who use them. [1850, 102–103]

Whewell's views on rules were thus diametrically opposed to those of De Morgan, Wharton, and Tate, and it may well have been that De Morgan was referring to Whewell when he wrote of someone who did not agree that a mass of rules was a bad thing.

Whewell [1850, 158–162] also discussed arithmetic teaching in general in his book on liberal education. First, he drew a somewhat disparaging distinction between the breadth of study at Oxford and Cambridge, and the emphasis on just obtaining a degree at the Universities of London. Whewell affirmed that boys should have a good knowledge of, and be able to perform the common operations of, arithmetic. They needed to be able to apply "the rules in a correct and intelligent manner." He lamented that such was not the case with many boys, thus prohibiting their mathematical learning later on. He observed that arithmetic was a matter of habit and should be learned by copious practice. By contrast, algebra and geometry could be picked up easily by a pupil already familiar with arithmetic. A pupil would learn to perform arithmetical calculations at school, but would only understand why they worked when at university, with the development of his reasoning powers. Whewell believed that such a system aided the mental processes and suggested that boys at school should also study the practical rules of mensuration. He felt this would make rules more intelligible to the pupils and be of use in the business of life.

De Morgan, Tate, and Wharton all wanted to improve arithmetic instruction in schools by teaching rules together with the reasons why they held. By contrast, Whewell was content to give the schoolboy only the "ridiculous" rules, leaving the reasons to be revealed later at university. Pressure was mounting on Whewell to respond appropriately to the movement to reform mathematics education.

Oxford, Cambridge, and the school matriculation examinations

Whewell was reported as approving of the improvements in primary instruction brought about by the College [*ET*, April 1848, 150]. According to Wharton [*ET*, February 1848, 99–100], Whewell also desired that boys should arrive at Cambridge with a satisfactory mathematical grounding.

Wharton [1850] wrote to Whewell on 3 May 1850 (Wharton–Whewell correspondence):

> My name may perhaps be unknown to you, or rather forgotten amidst the multitude of your important occupations. I now however write to you that having been for years the constant corrector both by Pamphlets and by means of the *Educational Times*, of an improved method of study and teaching both in the University and in the Public Schools, and also having introduced considerable improvement in Private Schools, I should be particularly glad of an opportunity of taking a place as Mathematical Assistant in a Public School, and then to have an opportunity of trying effectually, what can be effective in that way by diligence and authority in our Public Schools.

The outcome of this letter is not known, but it shows Wharton's eagerness to put his educational reforms into practice where he felt they were needed.

John Parker reviewed Whewell's *A Liberal Education in General, and with Especial Reference to the University of Cambridge, Part II* in [*ET*, April 1850, 158–159]. He emphasized Whewell's important position at Cambridge and took pleasure in the fact that Whewell advocated the same aims as those of the *ET*, quoting Whewell's assertion that the faculty of reason is cultivated by the study of mathematics. Parker argued that the Classical Tripos should concentrate on logical reasoning and systematic arrangement rather than mere translation, putting it on the same footing as the Mathematical Tripos.

Parker discussed Whewell's opinion of the need for improvement of the Cambridge system, and agreed with him that Cambridge should produce sound textbooks on every aspect of elementary mathematics. Parker applauded Whewell's boldness in criticizing the teaching in the public schools and reported his observation that improvement in education would only occur if there were concomitant improvement in the schools and the universities. He reproduced a passage from the book in which Whewell called attention to the lack of sound mathematical teaching at several of the great schools and emphasized the utility of arithmetic for practical purposes. According to Whewell, "Arithmetic is in itself a good discipline of attention and application of mind." Parker discussed Whewell's insistence that school pupils should be sufficiently mathematically prepared before going to Cambridge and drew attention to the need for a matriculation examination. He concluded the review by recommending Whewell's work to anyone interested in education, particularly of the middle classes.

In [*ET*, July 1850, 223–234] a certain F.C., a Cambridge alumnus, responded to Parker's review, praising Whewell for balancing the claims of both the classics and mathematics for a place in a liberal education. F.C. contended that boys should be classified at school according to their mathematical ability, not their classical ability, and taught at an appropriate level. He agreed with Whewell that the grammar schools should provide a satisfactory grounding for the universities.

In the review Parker had quoted the following remarks from Whewell's book:

> It may be said that the University and the Colleges ought to compel the schools to teach the Elements of Mathematics, by requiring a certain quantity of Mathematics of all their students. This may be said, and it is true, and I hope will be acted upon. I hope the Colleges or the University will require of all the students a real and practical acquaintance with Arithmetic and Geometry. [*ET*, April 1850, 159]

Parker reinforced these statements and added a warning:

> The difficult point respecting a matriculation examination is now fully shown; and it is hoped that the University will be induced to make themselves equal to the times in which they live; and if not, either some violent change will be imposed, or the University will gradually become disused and neglected, except by those who fatten on its income.

The theme of school matriculation had already been taken up by Wharton in [*ET*, January 1849, 82] and was developed in a lecture reported in [*ET*, February 1850, 102]. Wharton emphasized sound arithmetical instruction, observing that arithmetic was the only mathematical subject many members of the College taught. The report concluded with a call to Cambridge:

> Let Cambridge fit and enact a certain standard by means of a matriculation examination, and their requirements are sure to be met. It is their neglect and indifference which keeps down the standard—they demand accurate Arithmetic and a thorough knowledge of equations and Algebra, why then do they not exact that or its equivalent from everyone.

According to Chapman [1985, 38–42], it was a founding member of the College, Mr. Hall, who had originally written to the College Council advocating school matriculation examinations. The College had thus started off examining teachers and was now moving into examining secondary students in mathematics in preparation for study at Oxford or Cambridge. Hall's ideas clearly coincided with Wharton's and Parker's and became reality in August 1850, when a committee set up to look into the matter produced a full scheme for their adoption. The plans for school examinations were very similar to those already in place. The school principal or his nominated assistant would conduct the examination while an examiner appointed by the College would supervise the entire proceedings. Additionally, the supervisor would add his own exam questions to ensure no "cramming" of pupils took place. In November 1850, a circular was sent to all members of the College who were headmasters, asking them if they wished to participate. Only William Goodacre, a Dissenter of the Standard Hill Academy, Nottingham, replied [College of Preceptors, 1848–1857, 118]. The Rev. Dr. Wilson and Parker paid their first visit to this school in December 1850, examining the boys on 23 and 24 December. This visit attracted publicity and by 1853, 20 more schools had been added to the one in Nottingham. The testing of students grew, and in 1856 the Society of Arts also began to examine secondary school pupils. Soon the administration of formal entrance examination was taken over by Oxford and Cambridge, an event that aggrieved members of the College, who saw the examination of school pupils as their territory. The front page of [*ET*, June 1857, 121] carried an article on "Oxford Examination of Middle Class Schools" claiming that the College and its school examinations had been ignored by Oxford, ostensibly because the College ran on nonsectarian principles. It was pointed out most forcibly that University tutors were not equipped to examine school pupils, having never taught in secondary institutions. Such a clash with Oxford and Cambridge threw the College into a crisis, and both Wharton and Wilson suffered as a result [Chapman, 1985, 49, 53], Wilson resigning as Dean of the College. Both Wilson and Wharton seemed to lose heart. Wharton died in 1861, apparently after a long and painful illness.

Conclusions

There were several layers of reform at work in mid-19th-century mathematics education. Whewell was an establishment figure who brought about significant reforms of the Cambridge curriculum, but always on his own terms. He was keen to uphold the values of his "liberal education" and openly despised the London universities which he held as being there only to give out degrees, and not to provide a rounded education. He had a condescending attitude toward the teaching of arithmetic in schools and promoted the use of rules without understanding. He actively preferred geometry to algebra for inducing the reasoning powers and did not encourage graduates to become professional mathematicians.

De Morgan, on the other hand, was an active and prominent research mathematician. He was not afraid to stand out against the establishment and showed great conviction regarding moral and religious principles. He was a supporter of the radical Lord Brougham, and their correspondence contains exchanges on mathematical, social, religious, and political subjects. De Morgan was a celebrated historian of mathematics and thought deeply about how mathematics should be taught. He admired the French higher education system where students were made to reflect on the subject, instead of merely copying out exercise after exercise, as was the case at Cambridge. The French system produced research mathematicians, and Cambridge by and large did not.

Tate was supported by the reformer Kay-Shuttleworth, and he was also influenced by Pestalozzi's synthetical method of teaching. His teaching of arithmetic, aimed at poor children, used clear rules with guiding explanations, and his textbooks based on these principles became very popular. Tate firmly advocated mental arithmetic, which was unusual for the time.

The College drew attention to these reformers in the *ET* and thus participated in the debate amongst educators on the teaching of arithmetic and algebra. Payne, who was to become the country's first professor of education, somewhat surprisingly recommended all three pioneers in the field of mathematics education. Wilson and Wharton were much more focused—they both deplored rules without principles, and thus supported De Morgan and Tate, but Wharton found De Morgan's textbooks had too much discussion of underlying principles. Wharton differed from De Morgan in another fundamental way—Wharton actively supported teaching with examples, whereas De Morgan preferred the approach of French textbooks with no examples in at all. Despite these differences, there was a consensus among the participants that teaching arithmetic using unexplained rules was quite unacceptable and needed to be stopped.

In rejecting the rote teaching of arithmetic, Wharton, Wilson, and Parker found themselves in opposition to Whewell's system. Their convictions would eventually lead to entrance examinations to Oxford and Cambridge intended to ensure an even standard of mathematics education in the schools. These three College council members showed considerable courage in bringing about this reform. When they first introduced the examinations there was only one Dissenting school prepared to avail itself of them. Wilson and Wharton appeared to have paid for their beliefs, both being very disappointed when Oxford and Cambridge took over their innovations. The establishment that they had goaded into change had responded by annexing their efforts.

Wharton and Wilson also supported arithmetical instruction for girls and organized examinations for them, both highly unusual activities for the time. The College and its council members certainly deserve to be remembered for their reforms in both education generally and mathematics education in particular.

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