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Designing optimal routes for cycle-tourists

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Abstract

Bicycles are becoming an increasingly popular mean of transport. Being healthy and affordable, they provide a sustainable alternative way of movement, for both leisure and work commuting. In both cases demand increases when bike devoted tracks are available. Providing bike trails that connect touristic spots is a cheap way of increasing the appeal and promoting the development of those regions featuring beautiful landscapes, strong cultural traditions, and historical monuments within a small area. This is the case of the Trebon region, South Bohemia, whose local administrators face the problem of optimally investing scarce resources to set up a network of cycle-dedicated tracks, exploiting existing trails or by reconstruction works, turning gravel roads or unsurfaced forest tracks into paved bike trails. As a first step, we address the design of a single route, modeled as a path on a directed graph between two given nodes, maximizing a utility function related to the attractiveness of the path. Attractiveness depends on several features, such as a service facility, a restaurant serving typical food, an historical village, or a scenic landscape to be enjoyed along the way. Two kinds of resource constraints bound the solution. A path maximum duration, which depends on how many times each arc is traversed, and a maximum budget for setting up the infrastructure, which depends on which arcs are selected. Since a cyclist may be willing to traverse an edge more than once - think, for example, of a detour from the main way to be travelled back and forth to reach a point of interest – cycles can be part of the route. The attractiveness function is concave and decreases after reaching its maximum at a few traversals. Such features make the problem new and challenging. We present an integer linear programming model and validate it by an experimental campaign on realistic data for the Trebon region.

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Keywords: Cycle tourism; bicycle network design; maximum attractiveness;

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1. Introduction

Most European countries are struggling to overcome the global financial crisis which has affected them for the last few years. In this perspective, administrators long for any measure which can help to boost the economy. In particular, actions devoted to promote the tourism industry are of interest to many countries in Central Eastern Europe, especially when such development projects are environmentally friendly and can provide benefits to the whole local community. A recent survey on the Belgium experience (Cox, 2012), concerning the development of a cycling network in the three regions of the country and its effects on the regional economy, supports the idea that investing in the development of cycle-devoted infrastructures allows for noticeable returns while it requires to spend limited amounts of resources. A wider picture is provided by a study commissioned by the European Parliament's Committee on Transport and Tourism in 2009 and conducted by the Institute of Transport and Tourism at the University of Central Lancashire (UK) and the Center for Sustainable Transport and Tourism, at Breda University, in the Netherlands (ITT, 2009). This study evaluates challenges and opportunities of developing a cycle tourism network across Europe, emphasizing the economic impact of tourist direct spending on local economies, and how this can spur business and increase job opportunities. When setting up such kind of projects, quantitative tools finalized to evaluate the minimal amount of public funding required to support the project implementation and how resources should be invested are as much important as the estimate of the expected potential benefits the project can yield (Shcherbina and Shembeleva, 2014). This paper tries to meet this target: it is part of a feasibility study on the development of a cycle-tourist network in the Trebon region, Czech Boemia, and it is aimed to provide decision makers with a quantitative-based decision support tool able to indicate which links a cycle-network should be made of in order to attract tourists the most. The Trebon region is particular fitted to host such an infrastructure since it is already provided with several features that typically attract cycle tourists. Just to mention a few, a pleasant climate in spring and summer, beautiful landscapes and nature, plain or moderate hill climbing tracks, a diffused offer of cheap lodgings, many historical and cultural points of attraction located close to each other, and a renown traditional cuisine. However, the existing infrastructure, in its present state, is not adequate to attract tourists worldwide. The challenge is to increase the offer of bike trails while optimizing the scarce public resources needed for reconditioning existing tracks and providing access to attraction points. Several actions can be taken to this purpose. Existing trails close to the Austrian border, previously reserved to the army, can be easily reconverted to serve as bike trails at a little cost. New infrastructures such as small wooden bridges connecting existing trails on different sides of small creeks could be built from scratch, extending the network reachability with a minor impact on the environment. Gravel and unsurfaced roads can be cheaply turned into regular bike trails. Since financial resources are limited, a step-wise approach must be taken, starting from minor targets and optimizing the use of resources needed at each step. In this work we focus on the search for the most attractive cycle path to be set up from a given origin to a given destination, given a number of constraints. This is the first step of a broader project that aims at designing the whole cycle tourist network, taking into account several classes of users and connecting several origin - destination pairs. At the same time, the present work builds on a previous study concerning how a consistent utility function can be set up by exploiting information coming from several media, like social networks, in order to anticipate the interest of tourists regarding the different attractions already present in the region, so that the selected path will offer access to the most sought after ones. Regarding the problem structure, even the simplest case here tackled, i.e., the *single commodity, single origin-destination pair, maximum attractiveness path* is a new challenging problem in combinatorial optimization. It can be casted into the class of the *profit maximization problems under distance constraints* (see the devoted chapter by Fischetti, Salazar-Gonzales and Toth. (2007)), one of the several TSP generalizations. However, the fact that the same edge or node can be part of the solution several times, each time with a different reward, is a special feature that rules out all the approaches developed for those problems so far and it calls for tailored solution tools. We shall call our problem the *Most Attractive Cycle-Tourist Path Problem*, MACTPP in the following.

The paper is organized as follows. MACTPP is introduced in Section 2 and related papers are recalled. An Integer Linear Programming model is provided in Section 3. Experimental results are presented in Section 4, where conclusions are drawn and future work is sketched.

2. Problem description

This section describes the practical application which motivated the study of MACTPP, highlighting the decisions to be taken, the constraints to be satisfied, and their impact on the problem structure. In particular, the issues of attractiveness and multiple traversals of the same location are introduced. Then, we recall the research fields to which MACTPP is related, providing references to the most significant papers.

2.1. Problem features

Setting up the Attractiveness Function

The issue of attractiveness was introduced in Cerna and Cerny (2012) where the problem is introduced. Building on Swarbrooke (2002), they propose a procedure for the attractiveness function that we adopt and recall hereafter. The first step concerns the identification of the potential of points of interests (PoIs) for cyclo tourists. To this purpose, a data mining process on the Internet web sites was carried out, and found 182 descriptions of cyclo-tourist trips worldwide, on sites from 29 countries of 6 continents. The mentioned PoIs were classified as natural ones, cultural ones, or related to service facilities. Natural PoIs include: observation decks to a landscape; mountains, volcanoes, valley, canyons; seaside, lakes, and any site good for bathing; forest, animals gathering points. Cultural PoIs include: typical villages, historical buildings, monuments, museums; local markets, vineyards, cellars. Services and infrastructure related PoIs include: high quality surface paths, moderate slopes, reserved lanes, bickers meeting points; resting places, bike rent and assistance spots; eating and lodging facilities; shopping, other sport activities. PoIs have been ranked within each class, according to importance given by cycle tourists on the web. As a second step, a pool of local experts proposed each his/her own list of PoIs in the Trebon region, picking from the previous list. Such people are experienced cycle-tourists with a deep knowledge of the territory. Since the problem's graph models all potential tracks in the area and their intersections, a third step consists of assigning to each node and to each edge of the graph an attractiveness coefficient at the first traversal, based on all the PoIs located on site. As cycle tourists ride for leisure, it may well happen that riding for the second time along a nice track provides some pleasure even though it is not a new experience. Thus, the total reward after the second visit can still be greater than the one after the first visit but, reasonably, less than the double. Indeed, the same ride repeated several times is no longer appealing; then, total reward after a few traversals may decrease with respect to the previous one. This suggests that marginal attractiveness decreases at successive traversals (consider attractiveness after the first traversal equal to the marginal one from time 0, where it is null, to the first time) and might even become negative. One way of quantifying marginal attractiveness is to either poll the expert users, or using a fading function that gives an analytical expression of how marginal attractiveness decreases at successive traversals, according to the kind of PoIs present on site, i.e., a second visit to a museum could be of no interest while a second ride along a panoramic trail could still be enjoyable.

Non-elementary paths

The specific feature of our problem is that multiple traversals of the same location can be part of an optimal solution, which impacts on the structure of MACTPP. First, let us provide a straightforward example of how a cycle can be part of a path which is more attractive than the elementary path obtained by removing the cycle. Let p be an elementary path from origin o_p to destination d_p, fulfilling maximum duration and budget requirements. Let $p^0 = i_1,..,i_n$ be a subpath of p, and let $p^1 = j_1,..,j_m$ be an alternative path from i_1 to i_n, (i_1=j_1 and i_n=j_m) which is also attractive, but less than p^0. If time and budget allow, an alternative itinerary consists of riding along p till node i_n, take a detour along p^1 going back to i_1=j_1 from i_n=j_m, and then travel on p to destination, traversing p^0 for a second time. If the attractiveness of the first traversal of p^1 plus the attractiveness of the second traversal of p^0 (which may be negative) is positive, then the reward associated to the second itinerary is greater than the one of p. and it might be the optimal one.

A bound on the total duration of the itinerary ensures that the problem is well posed and it has a finite solution even if the marginal attractiveness of some location would never become negative. This enlarges the feasible set, as any solution can be seen as a connected subgraph, made of an elementary path from origin to destination plus a set of cycles, each potentially repeated a few times, each time yielding a different reward. Any mathematical model and solution approach to MACTPP must consider this feature and handle non elementary paths and concave objective functions required to model attractiveness functions.

2.2. Related works

MACTPP is original and involves several areas of research: i) as a combinatorial optimization problem, it is related to resource constrained optimal paths and to the orienteering problem ii) the bicycles network design problem has been studied in the framework of soft mobility iii) the objective function, describing the attractiveness of the solution and how reward decreases at the next visit, involves the issue of how to model utility functions.

Network problems

MACTPP is a special profit-maximization, resource-constrained problem. Its closest problem is the *Orienteering Problem* (OP) Vansteenwegen et al. (2011), which MACTPP generalizes. The OP searches for a cycle that maximizes the profits of the visited nodes, subject to a bounded duration. In particular, we refer to the variant of the OP where the depot must belong to the cycle. In our case, the origin and the destination nodes of the path would collapse into the depot node. MACTPP generalizes OP in several aspects. It allows structures other that simple cycles. Reward is on both nodes and edges, and each further passage modifies the profit, according to a concave function. Resource constraints are imposed not only on the design variables (which edges are used) as in OP, but also on the flow variables (how many times the route goes through an arc). Therefore, the OP mathematical formulation provided in Fischetti et al. (2007), although inspiring, must be modified to capture the MACTP features.

MACTPP is also related to the *Elementary Resource Constrained Shortest Path Problem* (ERCSPP) where a minimum cost acyclic path is sought such that the accumulated quantity of each resource along the path is below a given threshold. ERCSPP is NP-Hard in the strong sense, and often arises in the sub-problem of column generation approaches. In MACTP, travel time and design cost are constrained, and we maximize path attractiveness, made of the reward at the first visit plus the marginal rewards at the following ones. Therefore, MACTP can be reformulated as a particular ERCSPP over an expanded network, where nodes and arcs are replicated each time they are time traversed. Although the two problems are so close, the size of the expanded network would considerably affect running time of the efficient label setting algorithms developed for ERCSPP, as in Boland et al. (2006).

Bicycle network design

The few studies on bicycle network design we are aware of, take a different perspective other than MACTP since bicycles are used for utilitarian travel and not for leisure. The bicycle is seen as an additional mean of transport for multi-modal commuters who reach by bike a stop of the public transport network or use bike sharing services to integrate it, or use the bike as an alternative tout court to vehicular trips on short distances, where bikes can compete with cars regarding travel times. Several problems arise in this framework. A line of research focuses on how to improve the cyclist network infrastructure in the cities by providing reserved lanes. It is well known that there is a positive correlation between the length of bikeways and the number of bicycle commuters, as in Nelson and Allen (1997). However, the mathematical modeling of the commuting cyclist preference structure is still debatable: travel time is not the only criterion used to rank different itineraries but safety related features are also relevant. A recent paper by Smith and Haghani (2012) presents a M model to optimally select which road segments should be improved to increase their “bicycle level of service” in order to shift to the bike mode the maximum number of vehicular trips. Other studies such as Shu et al. (2013) provide mathematical based methodologies to tackle the optimal design of a bike sharing system, again with the objective of capturing commuter travel demand on short distances and increase the number of trips that shift from motorized vehicles to bicycles. Finally, and closer to our setting, other works address the issue of how to extend existing long-distance bike networks to be used for leisure, focusing on the economic impact of these investments, and on the integration of the cycle network with the public transport network. Unfortunately, no quantitative methods nor optimization tools are used or even suggested for optimal planning. Among those studies we just mentioned Cox (2012) and the European Commission report ITT (2009).

Attractiveness and utility functions

Regarding the utility function, this study exploits the methodology developed in Cerna and Cerny (2012) which builds on the assessments of Swarbrooke (2002), where visitors attractions are enlisted and the decision making process according to which visitors select their destination is analyzed. Other hints could be taken from the field of semantic web technologies, where information on consumer behaviors on the web are used to infer user preferences, as in Williams and Al-Sharawneh (2009). Finally, in a recent paper, Shcherbina and Shembeleva (2014) provide a review of optimization techniques applied to recreational systems.

3. A Mixed-Integer Linear Programming model for the MACTP problem

Let us introduce the mathematical notation required to formulate MACTPP as an Integer Linear Programming (ILP) problem. The set of potential bike trails and their intersections is modeled as a mixed graph, with both undirected edges and directed arcs. Each track between two intersections is associated to an edge $[i,j]\in E$, $i<j$, where E is the set of edges. For each edge $[i,j]\in E$ a pair of directed arcs (i,j) and (j,i) are given, modeling the two opposite directions of travelling on the track, and yielding the arc set A, with $|A|=2|E|$. Junctions are modeled as the nodes N of the graph and nodes s, $t\in N$ denote the origin and destination of the path. The resulting mixed graph is denoted as $G = (N, A\cup E)$. For each arc $(i,j)\in A$ the travel time t_{ij} is known, and let T denote the maximum duration of a path. Note that travel time may depend on the orientation of the arc ($t_{ij}\neq t_{ji}$) when the altitude changes along the track associated to the arc.

Since attractiveness does not depend on the direction a track is traversed, it is an edge attribute. We have already introduced nodes and edges attractiveness functions. Formally, denote by $\varphi^i(1)$ and by $\phi^{ij}(1)$ the attractiveness at first traversal of node $i\in N$ and edge $[i,j]\in E$, respectively. Given $\varphi^i(0)=\phi^{ij}(0)=0$, attractiveness at further traversals is such that marginal attractiveness decreases, i.e., $\varphi^i(k)$- $\varphi^i(k-1) >$ $\varphi^i(k+1)$- $\varphi^i(k)$ $\forall i\in N$ and $\phi^{ij}(k)$-$\phi^{ij}(k-1) > \phi^{ij}(k+1)$-$\phi^{ij}(k)$ $\forall[i,j] \in E$ with $k \in\{1,..,k_{max}-1\}$, where $k_{\max}$ is the maximum number of traversals (typically $\phi^{ij}(k_{max})=0$ and $\varphi^i(k_{max})=0$). For sake of clarity, we use a specific notation for edge and node marginal attractiveness: $a_{ij}^k=\phi^{ij}(k)-\phi^{ij}(k-1)$ and $d_i^k=\varphi^i(k)-\varphi^i(k-1)$, that we will exploit to model the objective function. Note that the piecewise linear function obtained for each node i by connecting $\varphi^i(k)$ to $\varphi^i(k+1)$ by a segment $\forall$ k, is concave, and the same holds for edges.

Then we introduce binary variables associated to each one of the k^{th} traversals of each edge and node. Let $\chi_{ij}^k\in\{0,1\}$ be the binary variable associated to the k^{th} traversal of edge $[i,j]$ and let γ^k_i be the one associated to the k^{th} traversal of node i. In particular, $\chi_{ij}^k=1$ if the path goes through edge $[i,j]$ at least k times; likewise, $\gamma^k_i=1$ if the flow through i is at least k. This meaning is enforced by stating that $\chi_{ij}^{h-1}\geq\chi_{ij}^h$ and $\gamma_i^{h-1}\geq\gamma_i^h$, $\forall$ $h\in\{2..k_{max}\}$. These constraints ensure that an edge cannot be traversed k times unless it has been traversed at least k-1 times (the same holds for nodes). Clearly, it follows that $\phi^{ij}(k) = \sum_{h=1..k} a_{ij}^h = \sum_{h=1..kmax} a_{ij}^h\chi_{ij}^h$, provided that $k=\sum_{h=1..kmax}\chi_{ij}^h$. Likewise, it holds that $\varphi^i(k)=\sum_{h=1..kmax} d_{ij}^h\gamma^h_i$ with $k=\sum_{h=1..kmax}\gamma_i^h$. Note that variables χ_{ij}^1 (γ_i^1) also deliver the information about the fact "edge $[i,j]$ (node i) belongs or does not belong to the path" and will be used to model design related issues. Integer flow variables x_{ij} denote the number of times each arc (i,j) is traversed from i to j along the path. Then, the sum over k of variables χ_{ij}^k is equal to the sum of x_{ij} and x_{ji} (see Eq. (4)), while the sum over k of γ^k_i is equal to the flow outgoing from node i (see Eq. (7) in the model). For each edge $[i,j]\in E$, let $c_{ij}\geq 0$ be the design cost required to recondition the track in between i and j, and let B be the available budget. Finally, let N^{st} denote any node subset containing both s and t.

Now we can present an ILP model for MACTPP. The objective function (1) exploits marginal attractiveness. The constraints part of the model (2-13) can be seen as made of three parts. The first one is a regular network flow model used to represent a path as a unit of flow traversing the graph from origin to destination, potentially with cycles. This part of the model uses integer flow variables x_{ij}. The path duration constraint is also expressed as a function of such variables. The second part of the model introduces binary variables χ_{ij}^h and γ_i^h which are necessary to model the objective function and, for h=1, provide the network design information to which budget depends on. Finally, a third part of the model ensures connectivity, which must be explicitly enforced in this problem. As marginal attractiveness – at least at the first traversal – is usually positive, additional cycles provide additional benefit and tend to be part of good quality solutions. However, cycles must be connected to the origin-destination path to represent a feasible itinerary. A well-known way of modeling connectivity in ILP models is to ensure that, for any cut (S, N/S) such that S contains at least one node that belongs to the solution (i.e., γ_i^1.=1) and both the origin and the destination nodes belong to the other set, at least one edge of the cut belongs to the route. The design variables χ_{ij}^1 and γ_i^1 are used to formalize these constraints. Finally, note the constraints $\chi_{ij}^{h-1}\geq\chi_{ij}^h$ and $\gamma_i^{h-1}\geq\gamma_i^h$ $\forall$ $h\in\{2..k_{max}\}$ are redundant, since, due to decreasing marginal attractiveness, the optimal solution will always satisfy them, and thus will be omitted.

$$Max \quad \sum_{k=1..k_{max}} \sum_{[i,j]\in E} a_{ij}^k \chi_{ij}^k + \sum_{k=1..k_{max}} \sum_{i\in N} d_i^k \gamma_i^k \tag{1}$$

$$\sum_{(ji)\in BS(i)} x_{ji} - \sum_{(ij)\in FS(i)} x_{ij} = b_i \qquad \forall i \in N \tag{2}$$

$$\sum_{(ij)\in A} t_{ij} x_{ij} \le T \tag{3}$$

$$x_{ij} + x_{ji} = \sum_{k=1..k_{max}} \chi_{ij}^k \qquad \forall [i,j] \in E \tag{4}$$

$$\sum_{[ij]\in E} \chi_{ij}^1 c_{ij} \le B \tag{5}$$

$$\sum_{(ij)\in FS(i)} x_{ij} = \sum_{k=1..k_{max}} \gamma_i^k \qquad \forall i \in N, \quad i \ne t \tag{6}$$

$$\sum_{(it)\in BS(t)} x_{it} = \sum_{k=1..k_{max}} \gamma_t^k \tag{7}$$

$$\gamma_v^1 \le \sum_{[ij]\in E: i\notin N^{st} j\in N^{st}} \chi_{ij}^1 \qquad \forall v \notin N^{st}, \quad \forall N^{st} \subseteq N \tag{8}$$

$$\chi_{ij}^k \in \{0,1\} \qquad \forall [i,j] \in E, \quad \forall k \in \{1..k_{max}\} \tag{9}$$

$$\gamma_i^k \in \{0,1\} \qquad \forall i \in N, \quad \forall k \in \{1..k_{max}\} \tag{10}$$

$$x_{ij} \in Z^+ \qquad \forall (i,j) \in A \tag{11}$$

Eq.s (2) are flow balance constraints, where *BS(i)* and *FS(i)* denote the backward and the forward star of node *i*, and b_i=-1 for i=s, b_i=+1 for i=t and b_i=0 elsewhere. Eq. (3) imposes a maximum riding time. Eq.s (4) introduce variables χ_{ij}^k. Eq. (5) formalizes the budget constraint. Eq.s (6-7) define variables γ_i^k. Eq.s (8) impose that the resulting subgraph is connected (connectivity cuts) and their size is exponential. As mentioned, for each bipartition of the node set *N* such that *s* and *t* belong to same subset N^{st}, if at least one node is selected in the other node subset, at least one edge in the cut must be part of the solution. Note that the number of these constraints is exponential.

4. Computational results

4.1. Problem data

The chosen area is located southwest of Trebon, in South Bohemia, Czech Republic. The individual tracks, corresponding to the graph edges, are either paved roads with low vehicular traffic, unpaved roads, or natural trails that are already being used for cycling or hiking. Their surface may be either asphalt, gravel, or they can be field/forest paths of bad quality, single-track (i.e. narrow, one-way) cycling paths which must be turned two-lane wide, or concrete panel path. The design cost depends on present condition and path length. Nodes are interesting points for tourists or cross-roads. E.g. vertex n. 1 is the historical town of Trebon and n. 18 is the typical village of Majdalena. The set of edges is such that all different types of existing tracks are considered as potential arcs of the path and all main natural and cultural points of interest are reachable. For example, edge 68-71 Borovany - Ostrolovskjezd, partly runs by a watercourse and a museum is located along it. The resulting network is depicted in

Figure 1. The network is made of about 80 vertices and 150 edges. For each of them, attractiveness was computed as described before; edge/arc length is computed by a web map system; arcs traveling time is obtained from distance using an average speed of 18 km/h and adapted according to altitude change along the way on each direction.

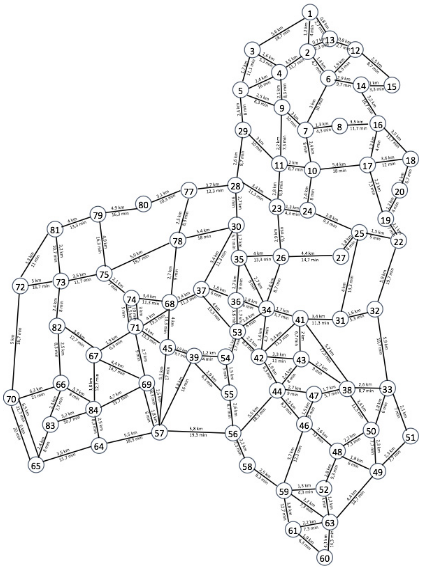

Fig. 1. The Trebon zone network: on each edge the distance (km) and the traveling time (minutes) are depicted.

The design cost of each edge is computed by multiplying the length (in meters) of the track that should be reconditioned by the cost of paving for 1 meter. The estimated cost per meter of a 3 meters wide path are: 115€ to turn it into an asphalt surface and 75€ for gravel one if starting from dirt road. Different scenarios arise depending on the kind of upgrading work to be done: either pave only the portions with bad quality surfaces or repave the entire track. E.g. edge 4-9 consists of field/forest path (0.6 km), gravel (0.9 km) and asphalt (0.6 km). If only the bad quality surface is rebuilt to gravel, the costs will be 44,460€. If only the bad quality surface is rebuilt to asphalt, the costs will be 66,660 €. If the whole surface is paved to asphalt, the costs will be 166,650 €.

Regarding rewards at successive traversals, at the second and at the third traversal the marginal reward is one fourth of the previous one, i.e., $a_{ij}^{k}=\phi^{ij}(k)-\phi^{ij}(k-1)=1/4\ a_{ij}^{k-1}$ with $k=2,3\ \forall[i,j]\in E$, and $d_i^{k}=\varphi^{i}(k)-\varphi^{i}(k-1))=1/4\ d_i^{k-1}$ with $k=2,3\ \forall i\in N$. The next ones ($k\geq 4$) rapidly tend to zero as $a_{ij}^{k}=\max\{0,\log(a_{ij}^{k-1})\}$ and $d_i^{k}=\varphi\max\{0,\log(d_i^{k-1})\}$.

4.2. Computational results

The model was coded in AMPL and solved by ILOG Cplex 12.2 on a quad core laptop with i7 processor. The connectivity constrains are introduced dynamically: for each instance, we solve a sequence of ILP problems with an increasing set of constraints: iteratively, an integer solution is found with respect to the current subset of constraints. If the resulting solution is not connected, the set of nodes not connected to the origin-destination path is recorded in set S. For each node in S the associated connectivity cut is added with respect to the edge cut (S, N/S), and then the new problem solved with a warm start (the previous solution provides an upper bound). We refer to the solution of each ILP problem in this sequence as to a macro iteration. We considered two origin destination pairs, 22-70 and 74-49, and for each pair we created nine instances, obtained by different values of the maximum duration $T\in\{4h, 5.5h, 7h\}$ and different levels of budget $B\in\{0€, 1.000.000€, 2.000.000€\}$. Since time translates into distance due to the average speed hypothesis, it can be used to model different classes of cyclist, such as families with children (4 h ride per day), adults (5:30 h), and trained cyclists (6 h).

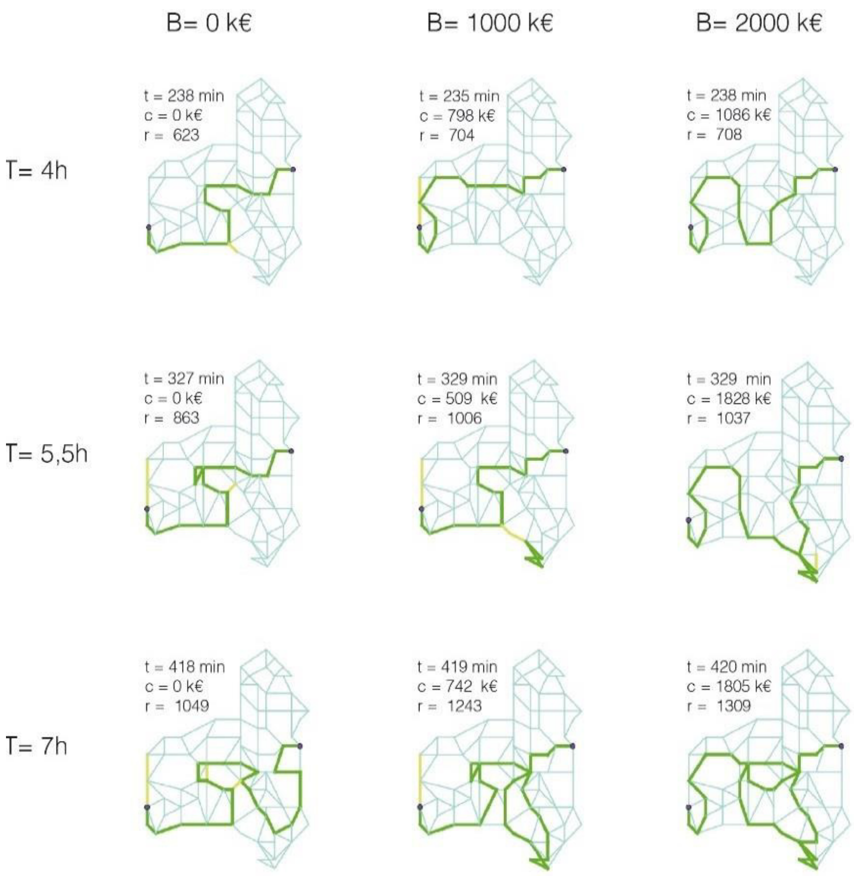

Fig. 2. Node pair 22-70. Nine scenarios are considered, with T=240, 330 and 420 minutes, and budget being 0, 1 million and 2 million euros. For each of the 9 cases the optimal solution is depicted, with edges traversed once in green and those traversed twice in yellow. Duration, cost and total reward are also provided for each solution.

The first pair is Rybnik Novy - Rimov: the two nodes are located on the right and on the left of the map in Figure 1, respectively. This pair was chosen according to the suggestions of the expert cyclists, since both locations can be reached by public transport and provide bike devoted services, such as renting and repairing facilities. The shortest path from node 22 to node 70 takes 147 minutes while the cheapest path needs no investment, since it uses roads that are already paved. In Figure 2 we draw the route of the optimal solution computed for each of the nine scenarios. The color of each edge represents the number of time the edge is traversed in both directions. For each scenario the duration, the cost, and the total reward are shown together with the picture of the optimal path. On the basis of these results note that in every scenario the available time is used almost completely, whatever the budget. However, the same does not hold for the budget: optimal solutions in the different scenarios spend from 50% to 90% of the available budget, but this percentage depends on the available time: if this is short extra edges cannot be fully exploited. Total reward increases, as expected, with resource upper bounds, but the behavior w.r.t. time and w.r.t. budget is not the same. It may be explained observing that more budget allows for new edges but they can be part of the itinerary only provided there is enough time. On the contrary, a time increment given the same budget will always be used to increase reward, in the worst case by multiple traversals. In fact, doubling the budget from 1 million to 2 million increases the reward much less (below or much below one third) than raising it from 0 to 1 million, and this effect is more evident if the available time is higher. On the other side, if an extra 90 minutes ride are allowed, thus raising maximum duration.

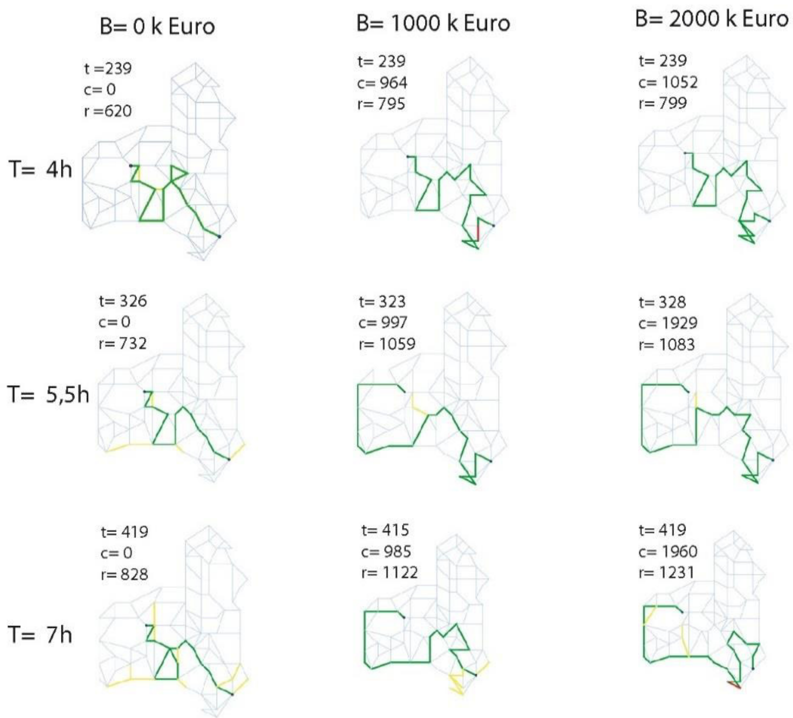

Fig. 3. The optimal solution for pair 74-49 is depicted for each of the 9 scenarios obtained with T=240, 330, 420 minutes, and B 0, 1 10^6, 2 10^6 euros. As in Figure 2, green edges are traversed once and yellow ones twice. Duration, cost and total reward are also provided for each solution.

from 5:30 to 6 hours, reward increases approximately from 2/3 to 9/11 of the increase obtained by raising maximum riding time from 4 to 5:30. Looking at the routes, the number of edges traversed twice decreases when budget increases, which can be related to the possibility of recondition a higher number of tracks whose first visit is more rewarding than the second visit of other edges already selected.

The second origin destination pair is 74-49, Zizkuv Dvorec – Jakule. Both locations are served by public transport and can be reached by train with train carriages equipped for bicycle transportation. The first one is a historical town close to a beautiful forest. The duration of the shortest path is 93 minutes, so this pair provides an example with a large feasible region, since the allotted time in all scenarios allows for several detours and alternative itineraries. Results are reported in Figure 3. Most of the observations holding for pair 22-70 hold true also for this pair, while, in some cases, the budget is almost entirely used: this is explained by the large availability of time.

Running time mainly depends on the dynamic constraint generation procedure. The value of the time and budget parameters heavily impacts on the number of constraints to be added and on the number of macro iterations needed to solve a single instance, but the cut generation policy influences both. Moreover, MACTPP is a design problem to be solved offline so the current performance can be considered satisfactory to this purpose. The average running time per instance in our test bed is about one minute, which allows to use it for scenario evaluation. Actually, we experienced that there is a sensitive performance improvement if, given an origin destination pair, we solve the 9 instances in sequence and we keep the set of violated constraints generated so far when solving the next instance, having a higher value of either T or B. This is a challenging issue which is actually under investigation. In our experiments, the number of added constraints at each macro iteration ranges from few units to some tens, and the number of macro iterations, i.e., ILP problems solved for a single instance, ranges from few units to some tens, the average being about 14, while the running time of each iteration goes from few seconds to few minutes, the first the highest for each instance, due to the warm start. It is worth mentioning that, since we keep the pool of cuts generated for the previous instances for the same o-d pair, the number of macro iterations as well as the number of new added constraints decreases when T and B increase, as opposite to the behavior observed when solving each instance from scratch.

4.3. Conclusions

We introduced a new problem in Combinatorial Optimization, namely the Most Attractive Cycle Tourist Path Problem (MACTPP), modeling the design of an origin-destination path with maximum attractiveness, subject to budget and duration constraints. The problem comes from a real application regarding the selection of the tracks to be reconditioned in order to design the most appealing itinerary for cycle tourists, given limited resources and a maximum duration. Since the path will be used for leisure the objective is to maximize the reward accumulated by traversing the arcs and the nodes of the path. These can be traversed several times, each time with a different satisfaction. This feature does not allow to solve MACTPP by existing solution approaches despite its close relations to the Orienteering Problem which MACTPP generalizes. We propose an ILP model which we solve by commercial software, with a dynamic generation of violated constraints, and we present computational results on realistic data concerning the Trebon region of South Boemia. Running time was not the focus of this work, but rather to devise and test an ILP model for the MACTP problem. Computational results confirm that we achieved our goals and that the model can be easily solved within a reasonable amount of time. Moreover, we believe that this work already provides a practical decision support tool to help decision makers setting the proper budget level for each class of users, since it allows scenario analysis. This work is the first step of a feasibility study aimed to set up a network of cycle tracks in the Trebon region. Future work will first concentrate on strengthening the ILP formulation, refining the cut generation procedure, experimenting different fading functions for marginal rewards, and extending the experimental campaign on other networks. Then, we aim at addressing the cycle tourist network design, where itineraries connecting several origin-destination pairs share some of the edges, and different classes of users are considered at the same time.

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