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# Unusual gauged supergravities from type IIA and type IIB orientifolds

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**Abstract**

We analyse different $\mathcal{N}=4$ supergravities coupled to six vector multiplets corresponding to low-energy descriptions of the bulk sector of $T_6/\mathbb{Z}_2$ orientifolds with $p$-brane in IIB ($p$ odd) and in IIA ($p$ even) superstrings. When fluxes are turned on, a gauging emerges corresponding to some non-semisimple Lie algebra related to nilpotent subalgebras $N_p \subset \mathrm{so}(6,6)$, with dimension $h_{N_p} = 15 + (p-3)(9-p)$. The non-metric axions have Stueckelberg couplings that induce a spontaneous breaking of gauge symmetries. In four cases the gauge algebra is non-Abelian with a non-commutative structure of the compactification torus, due to fluxes of NS–NS and R–R forms.



## 1. Introduction

Effective four-dimensional supergravity theories obtained by superstring compactifications on certain six-dimensional manifolds are not only distinct by the number of supersymmetries preserved by the background, but also by the duality symmetries which act linearly on the vector fields. Although in general, theories with the same amount of supersymmetries are related by a (non-local) symplectic change of the duality basis acting on the electric and magnetic field strengths [1], after some isometries are gauged, that in theories with $\mathcal{N}>1$ also amounts to the generation of a scalar potential, such change of basis is no longer allowed, and different gaugings describe genuinely different vacua [2–4].

The simplest manifestation of this phenomenon is perhaps given by two different gaugings of $\mathcal{N}=8$ four-dimensional supergravity [2]: the SO(8) gauging [5], corresponding to M-theory on $AdS_4 \times S_7$, and the $\mathcal{N}=8$ spontaneously broken supergravity dimensionally reduced à la Scherk–Schwarz [6] on $\mathcal{M}_4 \times T_7$. In the former case the gauge algebra is a subalgebra of $\mathrm{sl}(8,\mathbb{R}) \subset \mathrm{e}_{7,7}$, while in the latter example the "flat algebra" is a subalgebra of $(\mathrm{e}_6 + \mathrm{so}(1,1)) + T_{27} \subset \mathrm{e}_{7,7}$ [7].

Similar manifestations also appear in $\mathcal{N}=4$ supergravities describing $T_6/\mathbb{Z}_2$ orientifolds, where the $\mathbb{Z}_2$ projection is a combination of the world-sheet par-

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ity $\Omega$ and geometric inversions of $9-p$ directions of the compactification six-torus [8–12]. Indeed, in the two extremal cases of IIB orientifolds with $p=3$ and $p=9$ one is led to completely different low-energy supergravities. In the former case the fifteen Peccei–Quinn symmetries of the $C_{MNPQ}$ R–R scalars do not rotate the twelve vectors $B_{\mu i}$ and $C_{\mu i}$, and thus can be gauged [13–16] yielding a twelve-dimensional Abelian gauge algebra. On the other hand, the $p=9$ case corresponds to the $T_6$ reduction of the $\mathcal{N}=1$ ten-dimensional type I superstring. The fifteen Peccei–Quinn symmetries of the $C_{MN}$ R–R scalars now rotate the twelve vectors $\mathcal{G}^i_\mu$ and $C_{\mu i}$

$$\delta C_{\mu i} = \xi_{ij}\mathcal{G}^j_\mu, \tag{1}$$

and no gauging is thus possible. The other orientifolds with $3<p<9$ appear as intermediate cases of these two, with the twelve vectors originating in part by the metric $G_{MN}$, in part by the NS–NS $B$-field, and in part by the R–R $C$-forms [17].

When fluxes are turned on [18–30] (see [31] for a comprehensive review), a very rich structure emerges depending on $p$. In particular, for $4<p<9$, the $p-3$ graviphotons $\mathcal{G}^i_\mu$ always gauge "non-Abelian" isometries when the $H$-flux of the $B$-field strength is non-vanishing. This is a new manifestation of a non-commutative structure of the compactification torus in the presence of a non-trivial NS–NS background. For each case, there is a non-injective homomorphism $\iota$ between the gauge group $\mathcal{G}_g$, under which the gauge fields transform in the adjoint representation, and its realisation $\mathcal{G}'_g$ in terms of isometries of the scalar manifold, which is fixed by the scalar–vector minimal couplings:

$$\begin{aligned}
&\mathcal{G}_g \xrightarrow{\iota} \mathcal{G}'_g \subset \mathrm{Isom}(\mathcal{M}_{\mathrm{scal}}),\\
&\mathcal{G}'_g \equiv \mathcal{G}_g/\mathrm{Ker}(\iota), \quad \text{with } \mathrm{Ker}(\iota)\neq\emptyset.
\end{aligned} \tag{2}$$

Elements in $\mathrm{Ker}(\iota)$ are central charges in the gauge algebra $\mathbb{G}_g$ of $\mathcal{G}_g$ whose action is trivial on the scalar fields, and amounts to a pure *gauge* transformation on the vector fields. In some cases, the closure of $\mathcal{G}'_g$ requires additional conditions on the fluxes.

The structure of the gauge algebras for the IIB orientifolds with $p=7$ and $p=5$, originally outlined in [17], where also the salient features of the underlying (ungauged) supergravities were exposed, is here summarised in Section 2. Section 3 contains new results on the gauge algebras emerging from IIA orientifolds ($p$ even). Finally, in Section 4 our conclusions are drawn.

## 2. The gauge algebra of IIB orientifolds with fluxes

We recall here the gauge algebras of IIB orientifolds with $p=7$ and $p=5$, first exploited in [17]. To fix the notation, it is convenient to split the six-torus as

$$T_6 = T_{p-3}\times T_{9-p}, \tag{3}$$

with indices $i,j=1,\dots,p-3$ labelling coordinates along the $T_{p-3}$ subtorus, and indices $a,b=1,\dots,9-p$ labelling the coordinates in $T_{9-p}$. The $\mathbb{Z}_2$ symmetry we are implementing is a combination of world-sheet parity $\Omega$ and inversions $I_{9-p}$ of the $9-p$ coordinates $y^a$ of $T_{9-p}$. As a result, only the subgroup $\mathrm{GL}(p-3)\times\mathrm{GL}(9-p)$ of the isometries of the six-torus is perturbatively realised in the orientifold models we are interested in, and thus the decomposition (3) turns out to be the natural one.

### 2.1. The $T_4\times T_2$ model

In this model the bulk gauge fields and the non-metric axions, invariant under the $\Omega I_4$ projection, are:

$$\begin{aligned}
&\mathcal{G}^i_\mu, \quad B_{\mu a}, \quad C_{\mu a}, \quad C^i_\mu = \epsilon^{ijkl}C_{\mu jkl},\\
&C_0, \quad B_{ia}, \quad C_{ia}, \quad C_{ijab} = C_{ij}\epsilon_{ab}, \quad C_{ijk\ell}.
\end{aligned} \tag{4}$$

We shall focus on the effect of the fluxes

$$F_{ija}, \quad H_{ija}, \quad G_{ijkab}, \tag{5}$$

where $F_{ija}$, $H_{ija}$ are the R–R and NS–NS three-form fluxes while $G_{ijkab}$ is the flux of the five-form field strength, whose effect was not considered in our previous analysis [17]. For our purposes it is convenient to collect the $B_{\mu a}$ and $C_{\mu a}$ vectors as well as the $B_{ia}$ and $C_{ia}$ scalars and the fluxes $H_{ija}$ and $F_{ija}$ into $\mathrm{SO}(2,2)$ covariant quantities: $A^\Lambda_\mu$, $\Phi^\Lambda_i$ and $H^\Lambda_{ij}$ ($\Lambda=1,\dots,4$). The $C^i_\mu$ vectors decouple completely so that the active gauge algebra $\mathbb{G}_g$ of $\mathcal{G}_g$ is eight-dimensional with connection

$$\Omega_g = X_i\mathcal{G}^i_\mu + X_\Lambda A^\Lambda_\mu, \tag{6}$$

and with the following structure constants

$$[X_i, X_j] = H_{ij}^{\Lambda} X_{\Lambda},$$
$$[X_i, X_{\Lambda}] = [X_{\Lambda}, X_{\Sigma}] = 0. \tag{7}$$

On the other hand, there are $15 + (p-3)(9-p)$ (twenty-three in this case) scalar axions, whose associated solvable subalgebra [32–35] of so(6, 6) is [17]

$$\left[T_0, T_{\Lambda}^i\right] = \mathscr{M}_{\Lambda}{}^{\Lambda'} T_{\Lambda'}^i,$$
$$\left[T_{\Lambda}^i, T_{\Lambda'}^j\right] = \eta_{\Lambda\Lambda'} T^{ij}, \tag{8}$$

with the remaining commutators vanishing. The realisation $\mathscr{G}_g'$ of the gauge algebra in terms of isometries of the scalar manifold is achieved through the following identification of its generators:

$$X_i' = -H_{ij}^{\Lambda} T_{\Lambda}^j + G_{ijkab} T^{jk},$$
$$X'^{\Lambda} = \tfrac{1}{2} H_{ij}^{\Lambda} T^{ij}. \tag{9}$$

Notice that the presence of the five-form flux $G_{ijkab}$ does not affect the structure of the gauge algebra but amounts to an additional term in the covariant derivative of $C_{ij}$:

$$D_\mu C_{ij} = \partial_\mu C_{ij} - \tfrac{1}{2} H_{ij\Lambda} A_\mu^{\Lambda} - \mathscr{G}_\mu^k G_{kijab} + \tfrac{1}{2} \mathscr{G}_\mu^k H_{k[i}^{\Lambda} \Phi_{j]\Lambda}. \tag{10}$$

In general, the identification of the gauge generators with isometries does not guarantee automatically that the gauge algebra $\mathbb{G}_g'$ be compatible with $\mathbb{G}_g$. Indeed, in the case at hand, one can show that the expressions (9) for the generators of $\mathbb{G}_g'$ reproduces the structure (7) of $\mathbb{G}_g$ only if the following condition on the fluxes is fulfilled:

$$H_{ij}^{\Lambda} H_{\Lambda}^{ij} = 0. \tag{11}$$

This is consistent with the fact that the theory contains seven-branes ($p = 7$). Interestingly enough, this condition also allows a lift of the $\mathscr{N} = 4$ theory to a truncation of a $\mathscr{N} = 8$ gauge algebra [36].

### 2.2. *The $T_2 \times T_4$ model*

In this example [17] the twelve vector fields and the non-metric axions which are invariant under the orientifold projection are:

$$\mathscr{G}_\mu^i, \quad B_{\mu a}, \quad C_{\mu i}, \quad C_\mu^a = \epsilon^{abcd} C_{\mu bcd},$$
$$C_{ab}, \quad B_{ia}, \quad C_i^a = \epsilon^{abcd} C_{ibcd}, \quad C_{\mu\nu}, \quad C_{ij}. \tag{12}$$

Also in this case the $C_{\mu i}$ decouple, so that the active gauge algebra is ten-dimensional, with connection

$$\Omega_g = \mathscr{G}_\mu^i X_i + B_{a\mu} X^a + C_\mu^a X_a. \tag{13}$$

We shall consider only the effect of the NS–NS and R–R three-form fluxes $H_{ija} = \epsilon_{ij} H_a$ and $F_{iab}$. They appear as structure constants in the gauge algebra

$$[X_i, X_j] = \epsilon_{ij} H_a X^a,$$
$$\left[X_i, X^a\right] = F_i{}^{ab} X_b, \tag{14}$$

with the remaining commutators vanishing.[1]

Turning to the scalar sector, the generators $T$, $T^{ia}$, $T_a^i$ and $T^{ab}$ of the twenty-three-dimensional solvable algebra $N_5$ associated to the relevant axionic non-metric scalars obey the commutation relations

$$\left[T^{ia}, T^{bc}\right] = \epsilon^{abcd} T_d^i,$$
$$\left[T^{ia}, T_b^j\right] = \epsilon^{ij} \delta_b^a. \tag{15}$$

One is thus led to the following identifications

$$X_i' = -F_i{}^{ab} T_{ab} + H_a T_i^a,$$
$$X_a' = -H_a T,$$
$$X'^a = F_i{}^{ab} T_b^i, \tag{16}$$

of the gauge generators with the isometries of the solvable algebra. However, they reproduce now only a contracted version of $\mathbb{G}_g$ as given in (14). Indeed, as we have already stated, the groups $\mathscr{G}_g$ and $\mathscr{G}_g'$ are related by the non-injective homomorphism (2), where now $\mathrm{Ker}(\iota)$ is generated by the three central charges $X_a$ orthogonal to $X_a'$.

Moreover, no further constraints are to be imposed on the fluxes, that however satisfy $H_3 \wedge F_3 = 0$ identically, at all consistent with the fact that the model would now include D5-branes. Also this model can be lifted to a gauged $\mathscr{N} = 8$ theory [36].

## 3. Type IIA orientifolds

We now turn to the description of gauge algebras of IIA orientifolds with fluxes, for the three different cases $p = 8$, 6 and 4. Their spectra and ungauged low-energy supergravities have already been discussed in [17].

[1] Indices are lowered and raised with the $\epsilon_{ij}$ and $\epsilon_{abcd}$ tensors.

### 3.1. The $T_5 \times T_1$ model

Aside from the four-dimensional graviton $g_{\mu\nu}$, and the geometric moduli $g_{ij}$ and $g_{99}$ of $T_5 \times T_1$, the massless bosonic spectrum consists of

$$\begin{array}{lllll} \text{scalars (axionic):} & C_i, & B_{i9}, & C_{ij9}, & C_{\mu\nu 9}, \\ \text{vector fields:} & \mathscr{G}^i_\mu, & C_{i9\mu}, & C_\mu, & B_{9\mu}, \end{array} \tag{17}$$

while only the $H_{ij9}$ and $G_{ijk9}$ fluxes for the NS–NS $B$-field and R–R three-form potential are allowed by the orientifold projection.

The gauge group $\mathscr{G}_g$ is generated by the algebra $\mathbb{G}_g = \{X_i, X, X^{i9}, X^9\}$, with connection

$$\Omega^g = \mathscr{G}^i_\mu X_i + C_\mu X + C_{i9\mu} X^{i9} + B_{9\mu} X^9. \tag{18}$$

When fluxes are turned on, they appear as structure constants in the commutators

$$\begin{aligned} &[X_i, X] = -H_{ij9} X^{j9}, \\ &[X_i, X_j] = H_{ij9} X^9 + G_{ijk9} X^{k9}, \end{aligned} \tag{19}$$

from which we deduce that the generators $\{X^9, X^{i9}\}$ are central charges. The form of the algebra (19) then suggests that the field strength of the vector fields present non-Abelian couplings

$$\begin{aligned} &\mathscr{F}^i_{\mu\nu} = \partial_\mu \mathscr{G}^i_\nu - \partial_\nu \mathscr{G}^i_\mu, \\ &F_{i9\mu\nu} = \partial_\mu C_{i9\nu} - \partial_\nu C_{i9\mu} + \mathscr{G}^k_\mu C_\nu H_{ki9} \\ &\qquad - \mathscr{G}^k_\nu C_\mu H_{ki9} - \mathscr{G}^k_\mu \mathscr{G}^\ell_\nu G_{k\ell i9}, \\ &F_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu, \\ &\mathscr{H}_{9\mu\nu} = \partial_\mu B_{9\nu} - \partial_\nu B_{9\mu} - \mathscr{G}^k_\mu \mathscr{G}^\ell_\nu H_{k\ell 9}, \end{aligned} \tag{20}$$

as is confirmed by a supergravity inspection.

Turning to the scalar sector, we have shown in [17] that the solvable algebra parametrised by the (non-metric) axionic scalars is generated by

$$N_8 = \{B_{i9} T'^i + C_i T^i + C_{ij9} T^{ij}\}, \tag{21}$$

with the only non-vanishing commutator given by

$$[T^i, T'^j] = T^{ij}. \tag{22}$$

The group $\mathscr{G}'_g$ of gauge transformations on the axionic scalars is now generated by the algebra $\mathbb{G}'_g = \{X'_i, X'\}$, since in this case $\mathrm{Ker}(\iota) = \{X^9, X^{i9}\}$. The realisation of $\mathbb{G}'_g$ in the terms of isometries of the scalar manifold suggests the identifications

$$\begin{aligned} &X'_i = H_{ij9} T'^j - G_{ijk9} T^{jk}, \\ &X' = H_{ij9} T^{ij}, \end{aligned} \tag{23}$$

that reproduce the structure (19) once we set to zero the central charges.

The generators (23) induce then the following transformations on the scalars

$$\begin{aligned} &\delta\tilde{C}_{ij9} = -\xi H_{ij9} - \xi^k G_{ijk9} + \xi^k H_{k[i|9} C_{j]}, \\ &\delta B_{i9} = \xi^j H_{ji9}, \\ &\delta C_i = 0, \\ &\delta C_{\mu\nu 9} = 0, \end{aligned} \tag{24}$$

where we have found convenient to define the scalar $C_{ij9} \to \tilde{C}_{ij9} = C_{ij9} - C_{[i} B_{j]9}$. As a result, the corresponding covariant derivatives read

$$\begin{aligned} &D_\mu \tilde{C}_{ij9} = \partial_\mu \tilde{C}_{ij9} + C_\mu H_{ij9} \\ &\qquad + \mathscr{G}^k_\mu G_{ijk9} - \mathscr{G}^k_\mu H_{k[i|9} C_{j]}, \\ &D_\mu B_{i9} = \partial_\mu B_{i9} - \mathscr{G}^k_\mu H_{ki9}, \\ &D_\mu C_i = \partial_\mu C_i. \end{aligned} \tag{25}$$

### 3.2. The $T_3 \times T_3$ model

The next model we shall describe, is the $T_3 \times T_3/\mathbb{Z}_2$ orientifold of the IIA superstring. Its massless spectrum comprises, aside from the four-dimensional metric $g_{\mu\nu}$, the vector fields

$$\mathscr{G}^i_\mu, \quad C_{ij\mu}, \quad B_{a\mu}, \quad C_{ab\mu}, \tag{26}$$

the dilaton, the geometric moduli $g_{ab}$ and $g_{ij}$ of the six-torus in its $T_3 \times T_3$ decomposition, and the axionic scalars $\{C^{ab}, B_{ia}, C_{iab}, C_{k\mu\nu} = C_{ij}, C_{ijk}\}$. These latter, aside from $C_{ijk}$, parametrise a twenty-four-dimensional solvable subalgebra

$$N_6 = \{B_{ia} T^{ia} + C^{ab} T_{ab} + C^a_i T^i_a + C_{ij} T^{ij}\}, \tag{27}$$

whose structure is encoded in the non-vanishing commutators

$$\begin{aligned} &[T_{ab}, T^{ic}] = T^i_{[a} \delta^c_{b]}, \\ &[T^{ia}, T^j_b] = T^{ij} \delta^a_b. \end{aligned} \tag{28}$$

The active gauge group $\mathscr{G}_g$ is generated by the algebra $\mathbb{G}_g = \{X_i, X^a, X^{ab}\}$ with connection

$$\Omega^g = \mathscr{G}^i_\mu X_i + C_{ab\mu} X^{ab} + B_{a\mu} X^a. \tag{29}$$

We shall consider the effect of the fluxes

$$F_{ia}, \quad H_{ija}, \quad G_{ijab}, \tag{30}$$

which determine a non-Abelian gauge algebra, with commutators

$$[X_i, X_j] = H_{ija} X^a + G_{ijab} X^{ab}, \\ \left[X^a, X_i\right] = \tfrac{1}{2} F_{ib} X^{ab}. \tag{31}$$

As a result the field strengths of the vector fields read

$$\begin{aligned} \mathscr{H}_{a\mu\nu} &= \partial_\mu B_{a\nu} - \partial_\nu B_{a\mu} - \mathscr{G}^i_\mu \mathscr{G}^j_\nu H_{ija}, \\ F_{ij\mu\nu} &= \partial_\mu C_{ij\nu} - \partial_\nu C_{ij\mu}, \\ F_{ab\mu\nu} &= \partial_\mu C_{ab\nu} - \partial_\nu C_{ab\mu} - \mathscr{G}^i_\mu \mathscr{G}^j_\nu G_{ijab} \\ &\quad - \tfrac{1}{2}\mathscr{G}^i_\mu F_{i[a} B_{b]\nu} + \tfrac{1}{2}\mathscr{G}^i_\nu F_{i[a} B_{b]\mu}, \\ \mathscr{F}^i_{\mu\nu} &= \partial_\mu \mathscr{G}^i_\nu - \partial_\nu \mathscr{G}^i_\mu. \end{aligned} \tag{32}$$

The group $\mathscr{G}'_g$ of gauge transformations on the axionic scalars is generated by the algebra $\mathbb{G}'_g = \{X'_i, X'^a, X'^{ab}\}$, and is realised in terms of isometries of the scalar manifold by the identifications

$$\begin{aligned} X'_i &= -\tfrac{1}{4}\epsilon^{abc} F_{ia} T_{bc} + H_{ija} T^{ja} + \tfrac{1}{2} G_{ijab} T^j_c, \\ X'^a &= \tfrac{1}{4}\epsilon^{abc} G_{bcij} T^{ij} + \tfrac{1}{4}\epsilon^{abc} F_{ib} T^i_c, \\ X'^{ab} &= \tfrac{1}{4}\epsilon^{abc} H_{ijc} T^{ij}. \end{aligned} \tag{33}$$

An explicit calculation of their commutators, then shows that the algebra $\mathbb{G}'_g$ reproduces the structure (31) of $\mathbb{G}_g$ if the following conditions on the fluxes are met

$$V^c = \epsilon^{ijk}\epsilon^{abc} F_{ia} H_{jkb} = 0, \tag{34}$$

that also imply the useful relation

$$\epsilon^{abc} F_{[i|a} H_{j]kb} = -\tfrac{1}{2} F_{ka} H_{ijb}. \tag{35}$$

The identifications (33) induce the following gauge transformations on the axionic scalars

$$\begin{aligned} \delta C^a_i &= \tfrac{1}{4}\epsilon^{abc}\xi_b F_{ic} + \tfrac{1}{2}\epsilon^{abc}\xi^j G_{jibc} + \tfrac{1}{4}\epsilon^{abc}\xi^j F_{ja} B_{ic}, \\ \delta C_a &= -\tfrac{1}{2}\xi^i F_{ia}, \\ \delta B_{ia} &= \xi^j H_{jia}, \\ \delta C_{ij} &= \tfrac{1}{4}\epsilon^{abc}\xi_{ab} H_{ijc} + \tfrac{1}{4}\epsilon^{abc}\xi_a G_{ijbc} - \xi^k H_{k[i|a} C^a_{j]}, \end{aligned} \tag{36}$$

that generate the minimal couplings

$$\begin{aligned} D_\mu C^a_i &= \partial_\mu C^a_i - \tfrac{1}{4}\epsilon^{abc} B_{b\mu} F_{ic} \\ &\quad - \tfrac{1}{2}\epsilon^{abc}\mathscr{G}^j_\mu G_{jibc} - \tfrac{1}{4}\epsilon^{abc}\mathscr{G}^j_\mu F_{ja} B_{ic}, \\ D_\mu C_a &= \partial_\mu C_a + \tfrac{1}{2}\mathscr{G}^i_\mu F_{ia}, \\ D_\mu B_{ia} &= \partial_\mu B_{ia} - \mathscr{G}^j_\mu H_{jia}, \\ D_\mu C_{ij} &= \partial_\mu C_{ij} - \tfrac{1}{4}\epsilon^{abc} C_{ab\mu} H_{ijc} \\ &\quad - \tfrac{1}{4}\epsilon^{abc} B_{a\mu} G_{ijbc} + \mathscr{G}^k_\mu H_{k[i|a} C^a_{j]}. \end{aligned} \tag{37}$$

### 3.3. The $T_1 \times T_5$ model

Finally, we consider the $T_1 \times T_5$ orientifold. The relevant bosonic fields are

$$\begin{aligned} &\text{scalars (axionic):} && C_{abc}, \quad B_{4a}, \quad C_{a\mu\nu} = C^b, \quad C_4, \\ &\text{vector fields:} && \mathscr{G}^4_\mu, \quad C_\mu, \quad C_{4a\mu}, \quad B_{a\mu}, \end{aligned} \tag{38}$$

while the allowed fluxes for the NS–NS $B$-field and R–R one-form and three-form potentials are $H_{abc}$, $F_{ab}$ and $G_{4abc}$.

The active gauge group $\mathscr{G}_g$ is generated by the gauge algebra $\mathbb{G}_g = \{X_4, X, X^a\}$ with connection

$$\Omega_g = \mathscr{G}^4_\mu X_4 + C_\mu X + B_{a\mu} X^a, \tag{39}$$

is now purely Abelian, even when fluxes are turned on.

On the other hand, the generators of the group $\mathscr{G}'_g$ are not linearly independent, and have the following expressions

$$\begin{aligned} X'_4 &= G_{4abc} T^{abc}, \\ X' &= H_{abc} T^{abc}, \\ X'^a &= F_{bc} T^{abc}, \end{aligned} \tag{40}$$

in terms of the generators of the solvable algebra

$$N_4 = \left\{B_{4a} T^a + C^a T_a + C^{ab} T_{ab}\right\}, \\ \left[T_{ab}, T^c\right] = T_{[a}\delta^c_{b]}, \tag{41}$$

parametrised by the (non-metric) axionic scalars.

Under the action of $\mathscr{G}'_g$ these scalars transform as[2]

$$\begin{aligned} \delta C_{abc} &= \xi_{[a} F_{bc]} + \xi H_{abc} + \xi^4 G_{4abc}, \\ \delta B_{4a} &= 0, \\ \delta \tilde{C}^a &= 0, \end{aligned} \tag{42}$$

[2] We have here defined the scalar $\tilde{C}^a = C^a - C^{ab} B_{4a}$, as suggested by a direct supergravity analysis.

with the only non-trivial covariant derivative given by

$$D_\mu C_{abc} = \partial_\mu C_{abc} - B_{i[a} F_{bc]} - C_\mu H_{abc} - \mathscr{G}_\mu^4 G_{4abc}. \tag{43}$$

## 4. Conclusions

In this Letter we have studied the algebraic structure of four-dimensional $T_6/\mathbb{Z}_2$ orientifolds, extending the analysis in [17]. In the IIA case the active gauge algebras have dimensions twelve, nine and seven for $p = 8$, 6 and 4, and their consistency implies the condition $F_2 \wedge H_3 = 0$ (for $p \neq 4$). While in the $p = 8$ case it is trivially satisfied, for $p = 6$ it implies a constraint on the fluxes, in analogy with the $p = 7$ case in type IIB [17].

Aside from the $p = 4$ orientifold, the active gauge algebras are typically non-Abelian when fluxes are turned on, and, for $p = 8$ and 5, they are central extensions of the solvable algebras $N_p$ generated by the Peccei–Quinn symmetries of the (non-metric) axionic scalars.

Furthermore, an interesting structure emerges as far as the graviton gauge fields $\mathscr{G}_\mu^i$ are concerned. Their generators $X_i$ do not commute ($p \neq 3, 4, 9$) when $H$-fluxes are turned on,

$$\begin{aligned}
&p = 5 \quad [X_i, X_j] = \epsilon_{ij} H_a X^a,\\
&p = 6 \quad [X_i, X_j] = H_{ija} X^a + G_{ijab} X^{ab},\\
&p = 7 \quad [X_i, X_j] = H_{ij}^{\Lambda} X_\Lambda,\\
&p = 8 \quad [X_i, X_j] = H_{ij9} X^9 + G_{ijk9} X^{k9},
\end{aligned} \tag{44}$$

independently of our choices of the R–R fluxes. Since the $X_i$ are four-dimensional remnants of torus translations, this signals the non-commutative nature of the torus [37,38] in the presence of $H$-fluxes for the NS–NS $B$-field.

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