Preface

Special Issue on “Structured Matrices: Analysis, Algorithms and Applications”

The mathematical modeling of problems of the real world often leads to problems in linear algebra involving structured matrices where the entries are defined by few parameters according to a compact formula. Matrix patterns and structural properties provide a uniform means for describing different features of the problem that they model. The analysis of theoretical and computational properties of these structures is a fundamental step in the design of efficient solution algorithms.

Certain structures are encountered very frequently and reflect specific features that are common to different problems arising in diverse fields of theoretical and applied mathematics and engineering. In particular, properties of shift invariance, shared by many mathematical entities like point-spread functions, integral kernels, probability distributions, convolutions, etc., are the common feature which originates Toeplitz matrices. In fact, Toeplitz matrices, characterized by having constant entries along their diagonals, are encountered in fields like image processing, signal processing, digital filtering, queueing theory, computer algebra, linear prediction and in the numerical solution of certain difference and differential equations, just to mention a few. The interest in this class of matrices is not motivated only by the applications; in fact, Toeplitz matrices are endowed with a very rich set of mathematical properties and there exists a very wide literature dated back to the first half of the last century on their analytic, algebraic, spectral and computational properties.

Other classes of structured matrices are less pervasive in terms of applications but nevertheless they are not less important. Frobenius matrices, Hankel matrices, Sylvester matrices and Bezoutians, encountered in control theory, in stability issues, and in polynomial computations have a rich variety of theoretical properties and have been object of many studies. Vandermonde matrices, Cauchy matrices, Loewner matrices and Pick matrices are more frequently encountered in the framework of interpolation problems.

Tridiagonal and more general banded matrices and their inverses, which are semi-separable matrices, are very familiar in numerical analysis. Their extension to more general classes and the design of efficient algorithms for them has recently received much attention.

Multi-dimensional problems lead to matrices which can be represented as structured block matrices with a structure within the blocks themselves. Kronecker product
structures are typical of multi-dimensional problems where the separability of the different levels holds.

Matrix algebras provide another example where structured matrices play an important role. Circulant matrices, the tau class, the Hartley algebra are examples of trigonometric algebras having rich algebraic and computational properties. Their relations with fast discrete transforms, like FFT, make them useful tools for the design of efficient algorithms in matrix computations, especially in the preconditioning methods.

The analysis of theoretical and computational properties of matrix structures is a fundamental step in the design of efficient algorithms for matrix and polynomial computations. The large size of the problems encountered in the applications makes general algorithms unusable for their large complexity. Rather, algorithms specifically designed relying on the peculiar properties allow one to treat problems of very large dimensions. In fact, the richness of the properties of many classes of structured matrices enabled many researchers to design and analyze fast algorithms for solving structured systems.

The interest in structured matrix analysis has had a strong impetus and a growing interest in the last decades. Many international research groups moving from different fields and motivated by different needs are presently working in this research area. Impressive advances have been achieved from the theoretical point of view, especially concerning the spectral properties of Toeplitz matrices. The theoretical achievements have been in part used for the design of very effective numerical algorithms for solving different computational problems concerning structured matrices. Other computational problems apparently far from structured matrices have been reformulated in terms of matrix structures and efficiently solved by means of related algorithms. The results obtained in this way have been used for solving certain problems in different applicative areas.

This special issue collects a set of contributions in structured matrix computations which cover different aspects in this research area. Three main points of view and main research interests are involved in this collection: the theoretical analysis of structured matrices including algebraic, analytic and spectral properties; the design of fast algorithms relying on specific properties of structured matrices; the applications to problems of scientific computing.

These three interest areas have a strong interplay. Indeed, the theoretical analysis is a step which is important in itself but it is also needed for the design of specific solution algorithms and for the better understanding of applicative models. Vice-versa, the design and analysis of algorithms feeds the theoretical research with the demand of new theoretical tools. Similarly, the main motivation of algorithms is provided by applications, but, at the same time, a “physical” interpretation of an algorithm or of a theoretical property provides more insights for the analysis of theoretical and computational properties.

The papers collected in this issue have been presented at the workshop “Structured Matrices: Analysis, Algorithms and Applications”, Cortona (Italy) September 21–28, 2000, organized by D.A. Bini under the support of Istituto Nazionale di Alta
Matematica (INDAM) F. Severi, Scuola Normale Superiore of Pisa, and MURST grant 9801229483.

Dario A. Bini
Dipartimento di Matematica
Università di Pisa
via Buonarroti 2
56127 Pisa, Italy
E-mail address: bini@dm.unipi.it

Georg Heinig
Department of Mathematics and Computer Science
Kuwait University
P.O. Box 5969, 13060 Safat
Kuwait City, Kuwait

Eugene Tyrtyshnikov
Institute of Numerical Mathematics
Russian Academy of Sciences
Ul. Gubkina 8
Moscow 117333, Russia