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New results concerning the exponential stability of delayed neural networks with impulses

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ABSTRACT

Employing the matrix measure approach and Lyapunov function, the author studies the global exponential stability of delayed neural networks with impulses in this paper. Some novel and sufficient conditions are given to guarantee the global exponential stability of the equilibrium point for such delayed neural networks with impulses. Finally, three examples are given to show the effectiveness of the results obtained here.

1. Introduction

In recent years, delayed neural networks proposed by Marcus and Westervelt [1] in 1989 have been investigated by many researchers [2–8] and successfully applied in signal processing, pattern recognition, moving objects speed detection, quadratic optimization, robotics and control.

On the other hand, the most neural networks widely studied and used can be classified as either continuous or discrete. However, there are also many neural networks, which is neither purely continuous-time nor purely discrete-time ones; these are called impulsive neural networks. This third category of neural networks display a combination of both characteristics of continuous-time and discrete-time systems, which is an appropriate description of the phenomena of abrupt qualitative dynamical changes of essentially continuous-time systems. In 2008, Luo and Cui in [9] studied the global asymptotic stability of delay bi-directional associative memory neural networks with impulses by Lyapunov functional and matrix theory. Wen and Sun in [10] investigated the existence and uniqueness of an equilibrium point for delayed Cohen–Grossberg bidirectional associative memory neural networks with impulses based on the nonsmooth analysis method. Recently, the stability analysis for delayed neural networks with impulses has attracted considerable attention and some progresses have been made, see [11–14] and the references cited therein. In [15,16], the stability of cellular neural networks and dynamical neural networks is discussed by using matrix measure. Matrix measure can have positive as well as negative values, whereas a norm can assume only nonnegative values. Thus the results obtained using matrix measure are more precise than the ones using norms. Motivated by the above mentioned works, a natural and interesting idea is to study the stability of neural networks with impulses by using the matrix measure method. To the best of our knowledge, the exponential stability of delayed neural networks with impulses by the matrix measure approach is seldom discussed.

The purpose of this paper is to investigate the global exponential stability of delayed neural networks with impulses by constructing a new Lyapunov function and applying the matrix measures approach and delay differential inequality.

This paper is organized as follows. In Section 2, we introduce the delayed neural networks with impulses model and some preliminaries. The global exponential stability of the zero solution of delayed neural networks with impulses are analyzed in Section 3. In Section 4, three examples are given to illustrate our results. A conclusion is drawn in Section 5.

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2. Preliminaries

Consider the following impulsive neural networks systems with time delays

$$\begin{cases}\dot{x}(t) = Ax(t) + Bf(x(t)) + Cg(x(t-\tau(t))), & t \neq t_k,\ t \geq 0,\\ \Delta x(t_k) = D_k x(t_k), & k \in N,\\ x(t) = \phi(t), & -\tau \leq t \leq 0,\end{cases} \tag{2.1}$$

where t_k denotes the moment when impulsive control occurs, $t_k < t_{k+1}$, $\lim_{t\to\infty} t_k = \infty$; $x \in R^n$ is the state vectors of system (2.1); $f, g : R^n \to R^n$ are two nonlinear functions; $x(t-\tau(t)) = (x_1(t-\tau(t)), \ldots, x_n(t-\tau(t)))^T$, $\tau(t)$ is the transmission delay such that $0 < \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \omega < 1$, $t \geq 0$, where τ, ω are constants; At time instants t_k, jumps in the state variable x are denoted by $\Delta x(t_k) = x(t_k^+) - x(t_k)$, where $x(t_k^+) = \lim_{t\to t_k^+} x(t)$. $A, B, C, D_K \in R^{n\times n}$, and $\phi \in C([-\tau, 0], R^n)$.

For $x \in R^n$, the vector norm $\|\cdot\|_p$ $(p = 1, 2, \infty)$ is defined as

$$\|x\|_1 = \sum_{i=1}^n |x_i|, \qquad \|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2}, \qquad \|x\|_\infty = \max_{1\leq i\leq n} |x_i|.$$

Let $x(t, \phi)$ denote the solution of system (2.1) through $(0, \phi)$. We have the following definition:

Definition 2.1. The zero solution of system (2.1) is said to be globally exponentially stable if for any solution $x(t, \phi)$ with the initial condition $\phi \in C([-\tau, 0], R^n)$, there exist constants $\alpha > 0$ and $M > 1$ such that

$$\|x(t, \phi)\|_P \leq M\|\phi\|_p e^{-\alpha t}, \quad t \geq 0,$$

where $\|\phi\|_p = \max_{-\tau\leq s\leq 0} \|\phi(s)\|_p$.

In this paper, we always assume that

(H1) $f, g : R^n \to R^n$ are Lipschitz continuous functions: there exist positive constants l_1, l_2 such that

$$\|f(x) - f(y)\|_p \leq l_1\|x - y\|_p, \qquad \|g(x) - g(y)\|_p \leq l_2\|x - y\|_p, \quad \forall x, y \in R^n.$$

Now, we introduce the concept of matrix measure.

Definition 2.2 ([17]). The matrix measure of a real square matrix $A = (a_{ij})_{n\times n}$ is as follows:

$$\mu_p(A) = \lim_{\varepsilon\to 0^+} \frac{\|I + \varepsilon A\|_p - 1}{\varepsilon},$$

where $\|\cdot\|_p$ is an induced matrix norm on $R^{n\times n}$, I is the identity matrix, and $p = 1, 2, \infty$.

When the matrix norm

$$\|A\|_1 = \max_j \sum_{i=1}^\infty |a_{ij}|, \qquad \|A\|_2 = \sqrt{\lambda_{\max}(A^T A)}, \qquad \|x\|_\infty = \max_i \sum_{j=1}^n |a_{ij}|,$$

we can obtain the matrix measure

$$\mu_1(A) = \max_j \left\{a_{jj} + \sum_{i=1,\ i\neq j} |a_{ij}|\right\}, \qquad \mu_2(A) = \frac{1}{2}\lambda_{\max}(A + A^T), \qquad \mu_\infty = \max_i \left\{a_{ii} + \sum_{j=1,\ j\neq i} |a_{ij}|\right\},$$

where $\lambda_{\max}(A + A^T)$ denotes the maximum eigenvalue of matrix $A + A^T$, A^T is the transpose matrix of A.

The following lemmas will be used later.

Lemma 2.1 ([17]). *The matrix measure $\mu_p(\cdot)$ defined in Definition 2.2 has the following properties:*

(i) $-\|A\|_p \leq \mu_p(A) \leq \|A\|_p, \quad \forall A \in R^{n\times n}$;

(ii) $\mu_p(\alpha A) = \alpha\mu_p(A), \quad \forall \alpha > 0,\ \forall A \in R^{n\times n}$;

(iii) $\mu_p(A + B) \leq \mu_p(A) + \mu_p(B), \quad A, B \in R^{n\times n}$.

Lemma 2.2 ([18]). *Suppose $p > q \geq 0$ and $u(t)$ satisfies the scalar impulsive differential inequality*

$$\begin{cases}D^+u(t) \leq -pu(t) + q(\sup_{t-\tau\leq s\leq t} u(s)), & t \neq t_k,\ t \geq t_0,\\ u(t_k^+) \leq \alpha_k u(t_k^-) & u(t) = \phi(t),\ t \in [t_0 - \tau, t_0]\end{cases}$$

where $u(t)$ is continuous at $t \neq t_k$, $t \geq t_0$, $\phi \in PC([t_0 - \tau, t_0], R)$. Then

$$u(t) \leq \prod_{t_0<t_k\leq t} \theta_k e^{-\mu(t-t_0)} \left(\sup_{t_0-\tau\leq s\leq t_0} \phi(s) \right),$$

where $\theta_k = \max\{1, |\alpha_k|\}$ *and* $\mu > 0$ *is a solution of the inequality* $\mu - p + qe^{\mu\tau} \leq 0$.

3. Main results

In this paper, we always assume that $\beta := \sup_k \|I + D_k\|_p$.
Now we are in the position to establish our main results.

Theorem 3.1. *Assume that* (H1) *holds and* $\rho = \sup_{k\in N}\{t_k - t_{k-1}\} < \infty$. *If* $\beta + \frac{l_2}{1-\omega}\tau\|C\|_p < 1$ *and*

$$-l_1\|B\|_p - \frac{l_2}{1-\omega}\|C\|_p < \mu_p(A) < -l_1\|B\|_p - \frac{l_2}{1-\omega}\|C\|_p - \frac{\ln\left(\beta + \frac{l_2}{1-\omega}\tau\|C\|_p\right)}{\rho}, \tag{3.1}$$

then the zero solution of system (2.1) *is globally exponentially stable.*

Proof. Let the Lyapunov function be in the form of

$$V(x(t)) = \|x(t)\|_p + \frac{l_2}{1-\omega}\|C\|_p \int_{t-\tau(t)}^{t} \|x(s)\|_p \mathrm{d}s.$$

Then the upper right-hand derivative of $V(x(t))$ with respect to time along the solution of Eq. (2.1) is as follows:

$$\begin{aligned}
D^+V(x(t)) &= \overline{\lim_{h\to 0^+}} \frac{\|x(t+h)\|_p - \|x(t)\|_p}{h} + \frac{l_2}{1-\omega}\|C\|_p(\|x(t)\|_p - (1-\dot{\tau}(t))\|x(t-\tau(t))\|_p) \\
&\leq \overline{\lim_{h\to 0^+}} \frac{\|x(t+h)\|_p - \|x(t)\|_p}{h} + \frac{l_2}{1-\omega}\|C\|_p\|x(t)\|_p - l_2\|C\|_p\|x(t-\tau(t))\|_p \\
&= \overline{\lim_{h\to 0^+}} \frac{\|x(t) + h\dot{x}(t) + o(h)\|_p - \|x(t)\|_p}{h} + \frac{l_2}{1-\omega}\|C\|_p\|x(t)\|_p - l_2\|C\|_p\|x(t-\tau(t))\|_p \\
&= \overline{\lim_{h\to 0^+}} \frac{\|x(t) + h[Ax(t) + Bf(x(t)) + Cg(x(t-\tau))] + o(h)\|_p - \|x(t)\|_p}{h} \\
&\quad + \frac{l_2}{1-\omega}\|C\|_p\|x(t)\|_p - l_2\|C\|_p\|x(t-\tau(t))\|_p \\
&\leq \overline{\lim_{h\to 0^+}} \frac{\|x(t) + hAx(t)\|_p - \|x(t)\|_p}{h} + \|Bf(x(t))\|_p + \|Cg(x(t-\tau))\|_p \\
&\quad + \frac{l_2}{1-\omega}\|C\|_p\|x(t)\|_p - l_2\|C\|_p\|x(t-\tau(t))\|_p \\
&\leq \overline{\lim_{h\to 0^+}} \frac{\|I + hA\|_p - 1}{h}\|x(t)\|_p + \|B\|_p\|f(x(t))\|_p + \|C\|_p\|g(x(t-\tau))\|_p \\
&\quad + \frac{l_2}{1-\omega}\|C\|_p\|x(t)\|_p - l_2\|C\|_p\|x(t-\tau(t))\|_p.
\end{aligned} \tag{3.2}$$

By Assumption (H1), we have

$$\|f(x(t))\|_p \leq l_1\|x(t)\|_p, \qquad \|g(x(t-\tau))\|_p \leq l_2\|x(t-\tau(t))\|_p. \tag{3.3}$$

Substituting inequalities (3.3) into the right-hand side of inequality (3.2) yields

$$\begin{aligned}
D^+V(x(t)) &\leq \mu_p(A)\|x(t)\|_p + l_1\|B\|_p\|x(t)\|_p + l_2\|C\|_p\|x(t-\tau(t))\|_p + \frac{l_2}{1-\omega}\|C\|_p\|x(t)\|_p - l_2\|C\|_p\|x(t-\tau(t))\|_p \\
&= \left(\mu_p(A) + l_1\|B\|_p + \frac{l_2}{1-\omega}\|C\|_p\right)\|x(t)\|_p \\
&= \lambda\|x(t)\|_p \leq \lambda V(x(t)), \quad t \in (t_{k-1}, t_k],\ k = 1, 2, \ldots,
\end{aligned}$$

where $\lambda = \mu_p(A) + l_1\|B\|_p + \frac{l_2}{1-\omega}\|C\|_p > 0$ (by (3.1)), $t_0 = 0$. Then we have

$$V(x(t)) \leq V(x(0))e^{\lambda t}, \quad t \in [0, t_1], \tag{3.4}$$

and

$$V(x(t)) \le V(x(t_{k-1}^{+}))e^{\lambda(t-t_{k-1})}, \quad t \in (t_{k-1}, t_k],\ k = 2, 3, \ldots. \tag{3.5}$$

From (3.1), we have

$$0 < \lambda\rho < -\ln\left(\beta + \frac{l_2}{1-\omega}\tau\|C\|_p\right). \tag{3.6}$$

Let g be defined by

$$g(z) = (z+\lambda)\rho + \ln\left(\beta + \frac{l_2}{1-\omega}\tau\|C\|_p e^{\tau z}\right),$$

where $z \in [0, +\infty)$. By (3.6), we have $g(0) < 0$. Since $g(z) \to +\infty$ as $z \to +\infty$, and

$$g'(z) = \rho + \frac{l_2\tau^2\|C\|_p e^{\tau z}}{(1-\omega)\beta + l_2\tau\|C\|_p e^{\tau z}} > 0, \quad z \in [0, +\infty),$$

there exists unique positive constant $\alpha > 0$ such that $g(\alpha) = 0$, *i.e.*,

$$(\lambda+\alpha)\rho = -\ln\left(\beta + \frac{l_2}{1-\omega}\tau\|C\|_p e^{\tau\alpha}\right). \tag{3.7}$$

By (3.4), for $t \in [0, t_1]$, we get that

$$\begin{aligned} V(x(t)) &\le \left(\|x(0)\|_p + \frac{l_2}{1-\omega}\|C\|_p \int_{-\tau(0)}^{0} \|x(s)\|_p \mathrm{d}s\right) e^{\lambda t} \\ &\le \left(1 + \frac{l_2}{1-\omega}\tau(0)\|C\|_p\right)\|\phi\|_p e^{\lambda t_1} \\ &\le \left(1 + \frac{l_2}{1-\omega}\tau\|C\|_p\right)\|\phi\|_p e^{\lambda t_1}. \end{aligned}$$

Thus

$$\begin{aligned} \|x(t)\|_p &\le V(x(t)) \le \left(1 + \frac{l_2}{1-\omega}\tau\|C\|_p\right)\|\phi\|_p e^{\lambda\rho} \\ &= \left(1 + \frac{l_2}{1-\omega}\tau\|C\|_p\right) e^{(\lambda+\alpha)\rho}\|\phi\|_p e^{-\alpha\rho} \\ &\le \left(1 + \frac{l_2}{1-\omega}\tau\|C\|_p\right) e^{(\lambda+\alpha)\rho}\|\phi\|_p e^{-\alpha t} \\ &= M\|\phi\|_p e^{-\alpha t}, \quad t \in [0, t_1], \end{aligned} \tag{3.8}$$

where $\alpha > 0$ as in (3.7), and $M = (1 + \frac{l_2}{1-\omega}\tau\|C\|_p)e^{(\lambda+\alpha)\rho} > 1$. On the other hand, by (3.8) we have

$$\sup_{t_1-\tau(t_1)\le s\le t_1} \|x(s)\|_p \le M\|\phi\|_p e^{-\alpha(t_1-\tau(t_1))} \le M\|\phi\|_p e^{-\alpha t_1} e^{\alpha\tau}. \tag{3.9}$$

Form the second equation of system (2.1), we have

$$\|x(t_k^{+})\|_p = \|(I+D_k)x(t_k)\|_p \le \|I+D_k\|_p\|x(t_k)\|_p \le \beta\|x(t_k)\|_p, \quad k = 1, 2, \ldots. \tag{3.10}$$

Hence, by (3.7)–(3.10), we obtain

$$\begin{aligned} V(x(t_1^{+})) &= \|x(t_1^{+})\|_p + \frac{l_2}{1-\omega}\|C\|_p \int_{t_1-\tau(t_1)}^{t_1} \|x(s)\|_p \mathrm{d}s \\ &\le \beta\|x(t_1)\|_p + \frac{l_2}{1-\omega}\|C\|_p \int_{t_1-\tau(t_1)}^{t_1} \|x(s)\|_p \mathrm{d}s \\ &\le \beta M\|\phi\|_p e^{-\alpha t_1} + \frac{l_2}{1-\omega}\|C\|_p\tau(t_1)M\|\phi\|_p e^{-\alpha t_1}e^{\alpha r} \\ &\le \beta M\|\phi\|_p e^{-\alpha t_1} + \frac{l_2}{1-\omega}\|C\|_p\tau M\|\phi\|_p e^{-\alpha t_1}e^{\alpha r} \end{aligned}$$

$$= \left(\beta + \frac{l_2}{1-\omega}\tau\|C\|_p e^{\alpha\tau}\right) M\|\phi\|_p e^{-\alpha t_1}$$
$$= e^{-(\lambda+\alpha)\rho} M\|\phi\|_p e^{-\alpha t_1}. \tag{3.11}$$

Now, we shall show that

$$V(x(t_k^+)) \le e^{-(\lambda+\alpha)\rho} M\|\phi\|_p e^{-\alpha t_k}, \quad k = 1, 2, \ldots. \tag{3.12}$$

Obviously, (3.12) holds for $k = 1$ by (3.11). If we assume that it holds for $k = i$, i.e.

$$V(x(t_i^+)) \le e^{-(\lambda+\alpha)\rho} M\|\phi\|_p e^{-\alpha t_i},$$

then we have, for $t \in (t_i, t_{i+1}]$

$$\begin{aligned}\|x(t)\|_p &\le V(x(t)) \le V(x(t_i^+))e^{\lambda(t-t_i)} \le e^{-(\lambda+\alpha)\rho} M\|\phi\|_p e^{-\alpha t_i} e^{\lambda\rho}\\ &= e^{-\alpha\rho} M\|\phi\|_p e^{-\alpha t_i}\\ &\le e^{-\alpha(t-t_i)} M\|\phi\|_p e^{-\alpha t_i}\\ &= M\|\phi\|_p e^{-\alpha t}.\end{aligned} \tag{3.13}$$

On the other hand, by (3.13), we have

$$\sup_{t_{i+1}-\tau\le s\le t_{i+1}} \|x(s)\|_p \le M\|\phi\|_p e^{-\alpha(t_{i+1}-\tau(t_{i+1}))} \le M\|\phi\|_p e^{-\alpha t_{i+1}} e^{\alpha\tau}. \tag{3.14}$$

From (3.13), (3.14), (3.10) and (3.7), we get

$$\begin{aligned}V(x(t_{i+1}^+)) &= \|x(t_{i+1}^+)\|_p + \frac{l_2}{1-\omega}\|C\|_p \int_{t_{i+1}-\tau(t_{i+1})}^{t_{i+1}} \|x(s)\|_p \mathrm{d}s\\ &\le \beta\|x(t_{i+1})\|_p + \frac{l_2}{1-\omega}\|C\|_p \int_{t_{i+1}-\tau(t_{i+1})}^{t_{i+1}} \|x(s)\|_p \mathrm{d}s\\ &\le \beta M\|\phi\|_p e^{-\alpha t_{i+1}} + \frac{l_2}{1-\omega}\tau(t_{i+1})\|C\|_p M\|\phi\|_p e^{-\alpha t_{i+1}} e^{\alpha\tau}\\ &\le \beta M\|\phi\|_p e^{-\alpha t_{i+1}} + \frac{l_2}{1-\omega}\tau\|C\|_p M\|\phi\|_p e^{-\alpha t_{i+1}} e^{\alpha\tau}\\ &= \left(\beta + \frac{l_2}{1-\omega}\tau\|C\|_p e^{\alpha\tau}\right) M\|\phi\|_p e^{-\alpha t_{i+1}}\\ &= e^{-(\lambda+\alpha)\rho} M\|\phi\|_p e^{-\alpha t_{i+1}},\end{aligned}$$

which implies that (3.12) holds for $k = i + 1$, and hence (3.12) holds for each $k = 1, 2, \ldots$. Thus, for $t \in (t_k, t_{k+1}]$, we have by (3.12) that

$$\begin{aligned}\|x(t)\|_p &\le V(x(t)) \le V(x(t_k^+))e^{\lambda(t-t_k)} \le V(x(t_k^+))e^{\lambda\rho}\\ &\le e^{-(\lambda+\alpha)\rho} M\|\phi\|_p e^{-\alpha t_k} e^{\lambda\rho}\\ &= e^{-\alpha\rho} M\|\phi\|_p e^{-\alpha t_k}\\ &\le e^{-\alpha(t-t_k)} M\|\phi\|_p e^{-\alpha t_k}\\ &= M\|\phi\|_p e^{-\alpha t},\end{aligned}$$

where $k = 1, 2, \ldots$, which together with (3.8) yields that the zero solution of system (2.1) is globally exponentially stable. □

If $\tau(t) \equiv \tau$ in (2.1), then Theorem 3.1 reduces to the following corollary.

Corollary 3.2. *Assume that* (H1) *holds and* $\rho = \sup_{k\in N}\{t_k - t_{k-1}\} < \infty$. *If* $\beta + l_2\tau\|C\|_p < 1$ *and*

$$-l_1\|B\|_p - l_2\|C\|_p < \mu_p(A) < -l_1\|B\|_p - l_2\|C\|_p - \frac{\ln(\beta + l_2\tau\|C\|_p)}{\rho}, \tag{3.15}$$

then the zero solution of system (2.1) *with* $\tau(t) \equiv \tau$ *is globally exponentially stable.*

Theorem 3.3. *Assume that* (H1) *holds and* $\theta = \inf_{k\in N}\{t_k - t_{k-1}\} > 0$. *Suppose that* $\mu_p(A) < -l_1\|B\|_p - l_2\|C\|_p$. *If there exists a positive constant*

$$\gamma > \frac{\ln(\max\{1,\beta\})}{\theta}, \tag{3.16}$$

and satisfies

$$\gamma + \mu_p(A) + l_1\|B\|_p + l_2\|C\|_p e^{\gamma\tau} \le 0, \tag{3.17}$$

then the zero solution of system (2.1) is globally exponentially stable.

Proof. Let the Lyapunov function be in the form of

$$V(x(t)) = \|x(t)\|_p.$$

Then the upper right-hand derivative of $V(x(t))$ with respect to time along the solution of Eq. (2.1) is as follows:

$$\begin{aligned} D^+V(x(t)) &= \overline{\lim_{h\to 0^+}} \frac{\|x(t) + h\dot{x}(t) + o(h)\|_p - \|x(t)\|_p}{h} \\ &= \overline{\lim_{h\to 0^+}} \frac{\|x(t) + h[Ax(t) + Bf(x(t)) + Cg(x(t-\tau))] + o(h)\|_p - \|x(t)\|_p}{h} \\ &\le \overline{\lim_{h\to 0^+}} \frac{\|x(t) + hAx(t)\|_p - \|x(t)\|_p}{h} + \|Bf(x(t))\|_p + \|Cg(x(t-\tau))\|_p \\ &\le \overline{\lim_{h\to 0^+}} \frac{\|I + hA\|_p - 1}{h}\|x(t)\|_p + \|Bf(x(t))\|_p + \|Cg(x(t-\tau))\|_p. \end{aligned} \tag{3.18}$$

Substituting inequalities (3.3) into the right-hand side of inequality (3.18) yields

$$\begin{aligned} D^+V(x(t)) &\le \mu_p(A)\|x(t)\|_p + l_1\|B\|_p\|x(t)\|_p + l_2\|C\|_p\|x(t-\tau)\|_p \\ &\le \mu_p(A)\|x(t)\|_p + l_1\|B\|_p\|x(t)\|_p + l_2\|C\|_p \sup_{t-\tau\le s\le t}\|x(s)\|_p \\ &= (\mu_p(A) + l_1\|B\|_p)V(x(t)) + l_2\|C\|_p \sup_{t-\tau\le s\le t} V(x(s)) \\ &:= -aV(x(t)) + b\sup_{t-\tau\le s\le t} V(x(s)), \quad t \ne t_k,\ t\ge 0, \end{aligned} \tag{3.19}$$

where $a = -(\mu_p(A) + l_1\|B\|_p)$, $b = l_2\|C\|_p$. On the other hand, we have from (2.1) that

$$V(t_k^+) = \|x(t_k^+)\|_p = \|(I + D_k)x(t_k)\|_p \le \|I + D_k\|_p\|x(t_k)\|_p \le \beta V(t_k).$$

Noting that $a = -(\mu_p(A) + l_1\|B\|_p) > l_2\|C\|_p = b \ge 0$, it follows from Lemma 2.2 that

$$V(x(t)) \le \prod_{0<t_k\le t}\max\{1,\beta\}e^{-\gamma t}\sup_{-\tau\le s\le 0} V(\phi(s)) = \prod_{0<t_k\le t}\max\{1,\beta\}e^{-\gamma t}\|\phi\|_p, \quad t > 0, \tag{3.20}$$

where $\gamma > 0$ is a solution of the inequality

$$\gamma - a + be^{\gamma\tau} = \gamma + \mu_p(A) + l_1\|B\|_p + l_2\|C\|_p e^{\gamma\tau} \le 0.$$

Thus, by (3.20), we have

$$\|x(t)\|_p = V(x(t)) \le \max\{1,\beta\}^{(\frac{t}{\theta}+1)}e^{-\gamma t}\|\phi\|_p = \max\{1,\beta\}^{-(\gamma - \frac{\ln(\max\{1,\beta\})}{\theta})t}\|\phi\|_p. \tag{3.21}$$

By condition (3.16) and Definition 2.1, it can be concluded that the zero solution of system (2.1) is globally exponentially stable. □

4. Illustrative examples

The following three illustrative examples will demonstrate the effectiveness of our results.

Example 4.1. Consider the simple one-neuron delayed neural network with impulse [19] as follows:

$$\begin{cases} \dot{x}(t) = -3x(t) + 1.5\sin(x(t)) + \sin x(t-\tau(t)), & t \ne t_k,\ t \ge 0, \\ x(t_k^+) = \gamma_k x(t_k), & k \in 1, 2, \ldots, \\ x(t) = \cos(t), & -\tau \le t \le 0, \end{cases} \tag{4.1}$$

where $t_k - t_{k-1} = 1, \tau = 1, \gamma_k = (-1)^k(\frac{e^{0.224}+4}{5})^{\frac{1}{2}},\ k \in Z_+$. It is easy to check that condition (iii) of Theorem 1 in [18] is not satisfied. Hence, the Theorem 1 in [18] is invalid here. Now we show that the zero solution of system (4.1) is globally exponentially stable with $\tau = 1$.

Obviously, $l_1 = l_2 = 1(f(x) = g(x) = \sin x)$. Matrix $A = (-3), B = (1.5),\ C = (1)$ and $D_k = (\gamma_k - 1)$. It can be easily verified that $\mu_2(A) = -3, \|B\|_2 = 1.5, \|C\|_2 = 1$. So, $\lambda = \mu_2(A) + l_1\|B\|_2 + l_2\|C\|_2 = -0.5 < 0$. Moreover, we have $\beta = \sup_k \beta_k = \sup_k \|I + D_k\|_2 = (\frac{e^{0.224}+4}{5})^{\frac{1}{2}} = 1.0248$. Let

$$\theta = \inf_{k\in N}\{t_k - t_{k-1}\} = 1,$$

then $\gamma = 0.2345$ satisfy (3.16) and (3.17), that is,

$$\gamma > \frac{\ln(\max\{1, \beta\})}{\theta} = \frac{\ln(1.0248)}{1} = 0.0245$$

and

$$\gamma + \mu_2(A) + l_1\|B\|_2 + l_2\|C\|_2 e^{r\tau} = 0.2345 - 3 + 1.5 + e^{0.2345} = -0.0012 < 0.$$

Hence, it follows from Theorem 3.3 that the zero solution of (4.1) is globally exponentially stable with approximate exponential convergence rate $\gamma - \frac{\ln(\max\{1,\beta\})}{\theta} = 0.2345 - 0.0245 = 0.21$ (by (3.21)).

Remark 4.1. In [19], the zero solution of (4.1) is globally exponentially stable with $\tau = 0.5$ and the approximate exponential convergence rate is 0.05 (<0.21). However, the criterion is invalid for $\tau \geq 1$. Hence, our results are more feasible than those given in [19].

Example 4.2. Consider the follow neural networks system with impulse and delayed

$$\begin{cases} \dot{x}(t) = Ax(t) + Bf(x(t)) + Cg(x(t-0.1)), & t \neq t_k,\ t \geq 0 \\ \Delta x(t_k) = D_k x(t_k), & k \in N \\ x(t) = \phi(t), & -0.1 \leq t \leq 0 \end{cases} \tag{4.2}$$

where $f(t) = (f_1(t), f_2(t))^T, g(t) = (g_1(t), g_2(t))^T, f_i(t) = g_i(t) = \tanh(t),\ i = 1, 2$, and

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 4.5 \end{bmatrix}, \qquad C = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4.0 \end{bmatrix}.$$

$\phi = (\phi_1, \phi_2) \in C([-0.1, 0], R^2),\ \phi_1(t) \equiv 0.3, \phi_2(t) \equiv 0.4$, and $D_k = \begin{bmatrix} -0.5 & 0.0 \\ 0.0 & -0.5 \end{bmatrix}$ for each $k = 1, 2, \ldots$.

Obviously, $l_1 = l_2 = 1, \tau = 0.1$. It can be easily verified that $\mu_2(A) = -1$, $\|B\|_2 = 6.9099$ and $\|C\|_2 = 4.0094$. So, $\lambda = \mu_2(A) + l_1\|B\|_2 + l_2\|C\|_2 = 9.9193 > 0$. Moreover, we have $\beta = \sup_k \beta_k = \sup_k \|I + D_k\|_2 = 0.5$ and $\beta + l_2\tau\|C\|_2 = 0.9009 < 1$.

Choose the impulsive interval $\rho = \sup_{k\in N}\{t_k - t_{k-1}\} = 0.01$, then

$$\lambda + \frac{\ln(\beta + l_2\tau\|C\|_2)}{\rho} = 9.9193 + \frac{\ln(0.9009)}{0.01} = 9.9193 - 10.4361 = -0.5168 < 0,$$

which implies that the condition (3.1) holds. So, it follows from Corollary 3.2 that the zero solution of (4.2) is globally exponentially stable.

Example 4.3. Consider the follow neural networks system with impulse and delayed

$$\begin{cases} \dot{x}(t) = Ax(t) + Bf(x(t)) + Cg(x(t-1)), & t \neq t_k,\ t \geq 0 \\ \Delta x(t_k) = D_k x(t_k), & k \in N \\ x(t) = \phi(t), & -1 \leq t \leq 0 \end{cases} \tag{4.3}$$

where

$$f(t) = (f_1(t), f_2(t))^T, \qquad g(t) = (g_1(t), g_2(t))^T, \qquad f_i(t) = g_i(t) = \frac{1}{2}(|t+1| - |t-1|), \quad i = 1, 2, \text{ and}$$

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.9 & -0.8 \\ -0.05 & 0.15 \end{bmatrix}.$$

$\phi = (\phi_1, \phi_2) \in C([-1, 0], R^2),\ \phi_1(t) \equiv 0.2, \phi_2(t) \equiv 0.3$, and $D_k = \begin{bmatrix} 0.015 & 0.0 \\ 0.0 & 0.015 \end{bmatrix}$ for each $k = 1, 2, \ldots$.

Obviously, $l_1 = l_2 = 1, \tau = 1$. It can be easily verified that $\mu_2(A) = -3$, $\|B\|_2 = 1.4142$, $\|C\|_2 = 1.212$, and $\beta = sup_k\{1, \|I + D_k\|_2\} = 1.015$. Let

$$\theta = \inf_{k\in N}\{t_k - t_{k-1}\} = 0.1,$$

then $\gamma = 0.161$ satisfy (3.16) and (3.17), that is,

$$\gamma > \frac{\ln(\max\{1, \beta\})}{\theta} = \frac{\ln(1.015)}{0.1} = 0.1489$$

and

$$\gamma - \mu_2(A) + l_1\|B\|_2 + l_2\|C\|_2 e^{\gamma\tau} = \gamma - 1.5858 + 1.212e^{\gamma} = -0.0011 < 0.$$

Hence, it follows from Theorem 3.3 that the zero solution of (4.3) is globally exponentially stable.

5. Conclusion

In this paper, we have obtained the global exponential stability of delayed neural networks with impulses by constructing a new Lyapunov function and applying the matrix measures approach and delay differential inequality. As we know, recently a few authors have studied the stability of delayed neural networks with impulses by using the matrix measure method. Some previous known results have been extended and improved. Finally, three examples have been presented to illustrate that our results are more feasible.

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