Physics Letters B 764 (2017) 31–37

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

One-loop radiative correction to the triple Higgs coupling in the Higgs singlet model

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ARTICLE INFO

Article history:
Received 2 August 2016
Received in revised form 30 September 2016
Accepted 6 November 2016
Available online 10 November 2016
Editor: G.F. Giudice

ABSTRACT

Though the 125 GeV Higgs boson is consistent with the standard model (SM) prediction until now, the triple Higgs coupling can deviate from the SM value in the physics beyond the SM (BSM). In this paper, the radiative correction to the triple Higgs coupling is calculated in the minimal extension of the SM by adding a real gauge singlet scalar. In this model there are two scalars h and H and both of them are mixing states of the doublet and singlet. Provided that the mixing angle is set to be zero, namely the SM limit, h is the pure left-over of the doublet and its behavior is the same as that of the SM at the tree level. However the loop corrections can alter h-related couplings. In this SM limit case, the effect of the singlet H may show up in the h-related couplings, especially the triple h coupling. Our numerical results show that the deviation is sizable. For $\lambda_{\Phi S} = 1$ (see text for the parameter definition), the deviation $\delta_{hhh}^{(1)}$ can be 40%. For $\lambda_{\Phi S} = 1.5$, the $\delta_{hhh}^{(1)}$ can reach 140%. The sizable radiative correction is mainly caused by three reasons: the magnitude of the coupling $\lambda_{\Phi S}$, light mass of the additional scalar and the threshold enhancement. The radiative corrections for the hVV, hff couplings are from the counter-terms, which are the universal correction in this model and always at O(1%). The hZZ coupling, which can be precisely measured, may be a complementarity to the triple h coupling to search for the BSM. In the optimal case, the triple h coupling is very sensitive to the BSM physics, and this model can be tested at future high luminosity hadron colliders and electron–positron colliders.

1. Introduction

The standard model (SM) has been extensively tested, especially the deviations for the gauge sector are strongly constrained by the electro-weak precision measurements from the Large Electron–Positron Collider (LEP) [1], Tevatron and the Large Hadron Collider (LHC). However, the Yukawa sector and the scalar sector are two sectors which are still not well probed. Since the discovery of the Higgs boson at the LHC in 2012 [2,3], the most important task is to measure the properties of the scalar accurately. The measurements will help us understand the nature of the electro-weak symmetry breaking mechanism (EWSB) [4–7]. If there exists new physics beyond the SM (BSM), it is believed that it is related with the Higgs couplings more or less. The Higgs boson is a door to the unknown new world.

Current measurements of the Higgs couplings with gauge bosons tend to be the SM values. At the same time Higgs couplings with the third generation fermions are inferred from the Higgs production processes at the LHC, which are also consistent with those in the SM. Usually for the model construction, the Higgs couplings with fermions and gauge bosons will have the SM limit at the electro-weak scale. However the triple Higgs coupling can deviate from the SM value largely in this limit. Such feature of the triple Higgs coupling has been studied extensively in the two Higgs doublet model (THDM) [8–10], inert Higgs doublet model (IHDM) [11], Higgs triplet model (HTM) [12] and models with an additional heavy neutrino [13].

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http://dx.doi.org/10.1016/j.physletb.2016.11.007

Searching for BSM physics is one of the most important goals of high energy physics. The most direct way is to increase the energy of the colliders and see whether there are new heavy resonances, while it is always hard or even impossible to construct the very high energy colliders because of the limitations from the expanses, technologies and so on. However there are other methods to achieve this goal. The new heavy particles will leave footprints at the electro-weak scale through loop effects. We may have indirect signals for the BSM through some physical quantities which are sensitive to the heavy particles.

The minimal extension of the SM in the scalar sector is to add a real gauge singlet. The Higgs singlet model (HSM) has been studied exhaustively in a lot of papers. For example, Refs. [14,15] studied a model which includes a Z_2 symmetry spontaneously breaking real Higgs singlet and the author considered the theoretical and phenomenological constraints of this model. Ref. [16] explored the resonant di-Higgs production in the 14 TeV hadron collider with an additional intermediate, heavy mass Higgs boson. Ref. [17] considered two scenarios: there was (no) mixing between the SM Higgs and the singlet. Then, authors analyzed the constraints from electro-weak precision observables, LHC Higgs phenomenology and dark matter phenomenology. Ref. [18] analyzed direct and indirect constraints on the parameter regions and the prospects for observing the decay of the heavier state into a pair of the 125 GeV Higgs. Refs. [19,20] discussed the electro-weak phase transition (EWPT) in this model. Ref. [21] calculated all one-loop scalar vertices in the effective potential approach. Ref. [22] emphasized the heavy-to-light Higgs boson decay at the electro-weak next-to-leading (NLO) order. Ref. [23] focused on the one-loop radiative corrections in the HSM and performed the numerical calculations for the hZZ, hWW, $hf\bar{f}$, $h\gamma\gamma$, $h\gamma Z$, hgg couplings, but not for triple h coupling, which is the main topic in this paper.

In the following, we will make a careful analysis of the triple h coupling up to one-loop level in this model in the SM limit. There will be an universal deviation from the SM predictions for the hVV, $hf\bar{f}$ couplings arising from the wave-function renormalization constant δZ_h. The numerical results show that the universal correction is small. For the triple h coupling, there are still hHH, $hhHH$ couplings (see Appendix A) in this limit. When the mass of the additional scalar is [90, 150] GeV and the coupling $\lambda_{\Phi S}$ is order one, the radiative correction to the triple h coupling can be 40% or even larger in the vicinity of double Higgs production. It may be measured at future hadron colliders and electron–positron colliders.

This paper is organized as follows. In Sec. 2, we give a detailed description of the model including the theoretical constraints on the parameter space and the radiative correction to the triple h & hZZ couplings. In Sec. 3, we present the numerical results. Sec. 4 is devoted to the conclusions and discussions. Feynman rules, related Feynman diagrams and calculational details are collected in the Appendix.

2. Model

We introduce a real additional gauge singlet S with hyper-charge $Y = 0$ besides the SM Higgs doublet Φ. Then, we can write the scalar potential $V(\Phi, S)$ as

$$V(\Phi,S) = -m_\Phi^2\Phi^\dagger\Phi + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \mu_{\Phi S}\Phi^\dagger\Phi S + \lambda_{\Phi S}\Phi^\dagger\Phi S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4. \tag{1}$$

Evidently, the singlet doesn't have any Yukawa interactions or gauge interactions with the SM fields. The scalar fields Φ, S in the unitary gauge can be parameterized as

$$\Phi = \begin{bmatrix} 0 \\ \frac{v+h_1}{\sqrt{2}} \end{bmatrix} (v \approx 246\ \text{GeV}),\ S = h_2 + v_S.$$

Without loss of generality, we can set v_S to be zero by shifting the S, namely the redefinition of the field. After EWSB, the two tadpoles are $-T_{h_1} = v(\lambda_\Phi v^2 - m_\Phi^2)$, $-T_{h_2} = t_S + \frac{\mu_{\Phi S}}{2}v^2$. T_{h_1}, T_{h_2} are the coefficients in front of the fields h_1, h_2 in the Lagrangian. At tree level, $T_{h_1} = 0$, $T_{h_2} = 0$. Then $m_\Phi^2 = \lambda_\Phi v^2$, $t_S = -\frac{\mu_{\Phi S}}{2}v^2$. Mass terms of the scalar fields are

$$\begin{aligned} \mathscr{L}_{mass} &= -\frac{1}{2}\begin{bmatrix} h_1 & h_2 \end{bmatrix}\begin{bmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \\ M_{11}^2 &= 2\lambda_\Phi v^2,\ M_{12}^2 = \mu_{\Phi S} v,\ M_{22}^2 = 2m_S^2 + \lambda_{\Phi S} v^2. \end{aligned} \tag{2}$$

After diagonalizing the mass matrix, we get the following expressions

$$\begin{aligned} &\mathscr{L}_{mass} = -\frac{1}{2}\begin{bmatrix} h & H \end{bmatrix}\begin{bmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{bmatrix}\begin{bmatrix} h \\ H \end{bmatrix},\ \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} h \\ H \end{bmatrix} \\ &m_h^2 = \cos^2\alpha M_{11}^2 + \sin^2\alpha M_{22}^2 - \sin 2\alpha M_{12}^2,\ m_H^2 = \sin^2\alpha M_{11}^2 + \cos^2\alpha M_{22}^2 + \sin 2\alpha M_{12}^2 \\ &\tan 2\alpha = \frac{2M_{12}^2}{M_{22}^2 - M_{11}^2} = \frac{2\mu_{\Phi S} v}{2m_S^2 - (2\lambda_\Phi - \lambda_{\Phi S})v^2}. \end{aligned} \tag{3}$$

In the above expressions, we use m_H instead of m_S to avoid the confusion with the parameter in the Lagrangian. s_α, c_α, $s_{2\alpha}$ are the simplified notations for $\sin\alpha$, $\cos\alpha$, $\sin 2\alpha$. From now on, we will choose the parameters m_h^2, m_H^2, α, $\lambda_{\Phi S}$, λ_S, μ_S, v as the inputs. According to the definitions of m_h^2, m_H^2, $\tan 2\alpha$ in Eq. (2) and Eq. (3), λ_Φ, m_S^2, $\mu_{\Phi S}$ can be expressed by the new inputs as

$$\lambda_\Phi = \frac{1}{2v^2}(c_\alpha^2 m_h^2 + s_\alpha^2 m_H^2),\ m_S^2 = \frac{c_\alpha^2 m_H^2 + s_\alpha^2 m_h^2}{2} - \frac{1}{2}\lambda_{\Phi S} v^2,\ \mu_{\Phi S} = \frac{s_{2\alpha}}{2v}(m_H^2 - m_h^2). \tag{4}$$

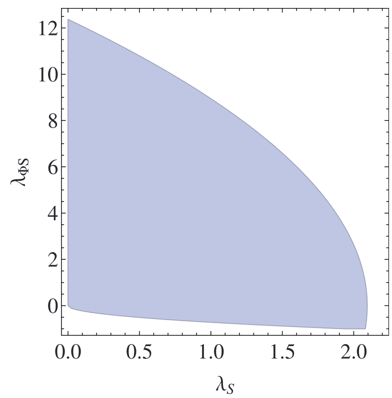


Fig. 1. The allowed parameter space (blue area) of λ_S, $\lambda_{\Phi S}$ in the SM limit. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2.1. Constraints on the parameter space

In the SM limit ($\alpha \to 0$), H will decouple from the fermions and gauge bosons because of the scaling factor s_α. Thus, it will evade all the present experimental constraints. But the SM Higgs will still couple with H by the interacting vertices hHH, $hhHH$. And this makes great influence on the triple h coupling which is discussed later. All the analyses below will be carried out under the SM limit assumption, namely $\alpha = 0$. When Φ, S are very large, the scalar potential will become $V(\Phi, S) = \lambda_\Phi(\Phi^\dagger\Phi)^2 + \lambda_{\Phi S}\Phi^\dagger\Phi S^2 + \lambda_S S^4$. It must be bounded from below, so we have

$$\lambda_\Phi > 0, \lambda_S > 0, \lambda_{\Phi S} > -2\sqrt{\lambda_\Phi\lambda_S}. \tag{5}$$

Further constraints we should consider are the so-called perturbative unitarity. S-wave amplitude a_0 should satisfy the relation $|\mathrm{Re}(a_0)| < \frac{1}{2}$, where a_0 is given by $a_0 = \frac{1}{16\pi s}\int_{-s}^{0} dt\mathscr{M}(t)$. Here, s, t are the Mandelstam variables as usual, and $\mathscr{M}$ is the scattering amplitude. According to the Goldstone equivalence theorem, massive vector boson is dominated by the longitudinal polarization at high energy. So we need only to consider the two-to-two scattering processes with initial and final states: $W_L^+W_L^-$, Z_LZ_L, Z_Lh, Z_LH, hh, HH, hH. Similar analyses have been discussed in many papers [24,25]. This is a 7×7 matrix, but it will be reduced into a 4×4 matrix in the SM limit. A subtlety one may caution is an extra $\frac{1}{\sqrt{2}}$ for the same initial and final states, which is often ignored in many papers. After some trivial calculations (see Appendix B), we have the constraints from perturbative unitarity

$$\lambda_\Phi < 4\pi\,, \lambda_{\Phi S} < 4\pi\,, 3\lambda_\Phi + 6\lambda_S + \sqrt{(3\lambda_\Phi - 6\lambda_S)^2 + 4\lambda_{\Phi S}^2} < 8\pi\,. \tag{6}$$

In the SM limit, $\lambda_\Phi = \frac{m_h^2}{2v^2}$. Together with the bounded constraints in Eq. (5), we get the following parameter space in Fig. 1. The interesting feature is that there are no constraints on m_H, $\mu_{\Phi S}$.

2.2. One-loop radiative correction to the triple h & hZZ couplings in the SM limit

We will calculate the deviation of the triple h coupling from the SM value originated from one-loop radiative correction in the SM limit. During the calculations, we adopt the conventions from Ref. [26]. There is no doubt that the loop particles must be the additional scalar H. To gauge the deviation from the SM value, we define $\delta_{hhh}^{(1)}$ as

$$\delta_{hhh}^{(1)} \equiv \frac{\lambda_{hhh}^{(\mathrm{HSM})} - \lambda_{hhh}^{(\mathrm{SM})}}{\lambda_{hhh}^{(\mathrm{SM},tree)}}. \tag{7}$$

In the following, we will present the numerical results for $\delta_{hhh}^{(1)}$ for the chosen model parameters.

Similarly, the deviation of the hZZ coupling from the SM value originated from one-loop radiative correction in the SM limit is defined as

$$\delta_{hZZ}^{(1)} \equiv \frac{\lambda_{hZZ}^{(\mathrm{HSM})} - \lambda_{hZZ}^{(\mathrm{SM})}}{\lambda_{hZZ}^{(\mathrm{SM},tree)}}. \tag{8}$$

The analytical expressions can be found in Appendix D.

3. Numerical results

In this section, we will do some numerical evaluations of $\delta_{hhh}^{(1)}$ for different model parameters. We set $m_h = 125$ GeV, $v = 246$ GeV as in the SM. The deviation of $\delta_{hhh}^{(1)}$ is mainly determined by $\lambda_{\Phi S}$, m_H, $\sqrt{p^2}$, where one of the Higgs boson with momentum p is off-shell. The dominant contribution is from the triangle diagram which is proportional to $\lambda_{\Phi S}^3$. We choose the allowed value of $\lambda_{\Phi S} = 1, 1.5,$

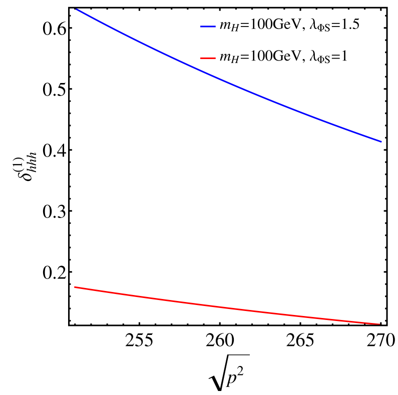


Fig. 2. $\delta^{(1)}_{hhh}$ defined in Eq. (7) as a function of $\sqrt{p^2}$ for $m_H = 100$ GeV, $\lambda_{\Phi S} = 1$ (red), 1.5 (blue) respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

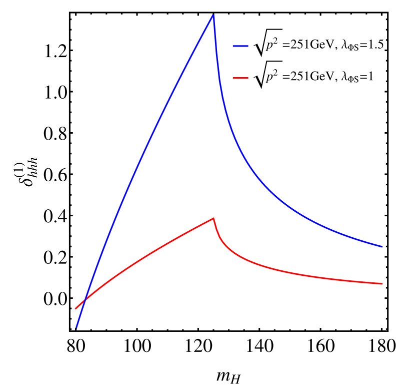


Fig. 3. $\delta^{(1)}_{hhh}$ defined in Eq. (7) as a function of m_H for $\sqrt{p^2} = 251$ GeV, $\lambda_{\Phi S} = 1$ (red), 1.5 (blue) respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

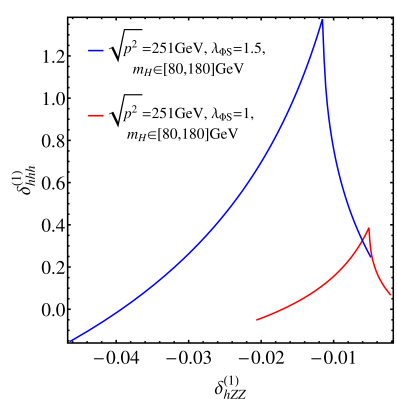


Fig. 4. The plot of triple h and hZZ couplings for $\sqrt{p^2} = 251$ GeV, $m_H \in [80, 180]$ GeV, $\lambda_{\Phi S} = 1$ (red), 1.5 (blue) respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

respectively, and study the dependence on m_H, $\sqrt{p^2}$ using LoopTools [27]. Behaviors of $\delta^{(1)}_{hhh}$ are shown in Fig. 2, Fig. 3. It is easy to see that the correction to the triple h coupling is sensitive to $\lambda_{\Phi S}$, m_H, $\sqrt{p^2}$. If the coupling $\lambda_{\Phi S}$ can reach order one, the deviation can be very large for $m_H \in [90, 150]$ GeV in the vicinity of double Higgs production ($\sqrt{p^2} \approx 250$ GeV). For $\lambda_{\Phi S} = 1$, the $\delta^{(1)}_{hhh}$ can be 40%. For $\lambda_{\Phi S} = 1.5$, the $\delta^{(1)}_{hhh}$ can almost approach 140%. The sizable radiative correction is mainly caused by three reasons: order one coupling $\lambda_{\Phi S}$, light mass of the additional scalar and the threshold enhancement. In this case, the triple h coupling is very sensitive to BSM physics. Experimentally, the deviation of the triple h coupling may be probed through $gg \to h^* \to hh$ production channel at future hadron colliders [28–32]. The model may also be probed through $e^+e^- \to Z^* \to Zh^* \to Zhh$ and $e^+e^- \to \nu_e\bar{\nu}_e W^{+*}W^{-*} \to \nu_e\bar{\nu}_e h^* \to \nu_e\bar{\nu}_e hh$ production channels at future electron–positron colliders [33–35]. At a low energy electron–positron colliders with 240 GeV or so and high luminosity, $\delta^{(1)}_{hhh}$ can also be detected indirectly [36–38].

Additionally, we compare the deviations for the triple h and hZZ coupling. Numerical results are shown in Fig. 4. We can find that the $\delta^{(1)}_{hZZ}$ is very small, compared to the triple h coupling. Due to the high precision measurement of hZZ coupling, it can be complementary to the triple h coupling to search for the BSM.

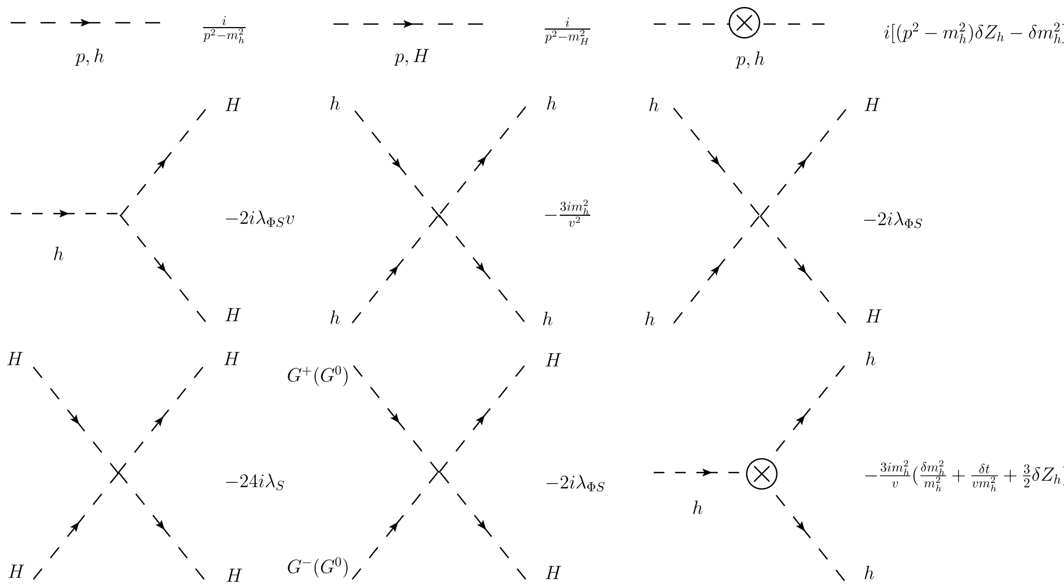
4. Conclusions

The radiative correction to the triple h coupling is calculated in the minimal extension of the SM by adding a real gauge singlet scalar. In this model there are two scalars h and H and both of them are mixing states of the doublet and singlet. Provided that the mixing angle is set to be zero, h is the pure left-over of the doublet and its behavior is the same as that in the SM at the tree level. However the radiative corrections from the singlet H can alter h-related couplings. Our numerical results show that the deviation $\delta^{(1)}_{hhh}$ is sizable. For $\lambda_{\Phi S} = 1$, the $\delta^{(1)}_{hhh}$ can be 40%. For $\lambda_{\Phi S} = 1.5$, the $\delta^{(1)}_{hhh}$ can reach 140%. The sizable radiative correction is mainly caused by three reasons: the magnitude of the coupling $\lambda_{\Phi S}$, light mass of the additional scalar and the threshold enhancement. The radiative correction for the hZZ coupling can be a complementarity to the triple h coupling because of the high precision measurement. In the optimal case, the triple h coupling is very sensitive to BSM physics, and this model can be tested at future high luminosity hadron colliders and electron–positron colliders.

Acknowledgements

We would like to thank Gang Li, Chen Shen, Chen Zhang, Yang Li and Kei Yagyu for helpful discussions. This work was supported in part by the Natural Science Foundation of China (Grants No. 11135003, No. 11635001 and No. 11375014).

Appendix A. Related Feynman rules



Appendix B. Perturbative unitarity constraints in the SM limit

In the SM, the perturbative unitarity constraints have been studied in the article [39]. For the HSM, we can write down the 7×7 two-to-two scattering matrix in the SM limit similarly. In the basis $W_L^+ W_L^-$, $\frac{1}{\sqrt{2}} Z_L Z_L$, hZ_L, $\frac{1}{\sqrt{2}} hh$, $\frac{1}{\sqrt{2}} HH$, hH, HZ_L, the explicit form is shown in the following:

$$a_0 = -\frac{1}{16\pi}\begin{bmatrix} 4\lambda_\Phi & \sqrt{2}\lambda_\Phi & 0 & \sqrt{2}\lambda_\Phi & \sqrt{2}\lambda_{\Phi S} & 0 & 0 \\ \sqrt{2}\lambda_\Phi & 3\lambda_\Phi & 0 & \lambda_\Phi & \lambda_{\Phi S} & 0 & 0 \\ 0 & 0 & 2\lambda_\Phi & 0 & 0 & 0 & 0 \\ \sqrt{2}\lambda_\Phi & \lambda_\Phi & 0 & 3\lambda_\Phi & \lambda_{\Phi S} & 0 & 0 \\ \sqrt{2}\lambda_{\Phi S} & \lambda_{\Phi S} & 0 & \lambda_{\Phi S} & 12\lambda_S & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\lambda_{\Phi S} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We can easily get three eigenvalues: 0, $-\frac{1}{8\pi}\lambda_\Phi$, $-\frac{1}{8\pi}\lambda_{\Phi S}$. Then, this matrix is reduced into a 4×4 matrix in the basis $W_L^+ W_L^-$, $\frac{1}{\sqrt{2}} Z_L Z_L$, $\frac{1}{\sqrt{2}} hh$, $\frac{1}{\sqrt{2}} HH$:

$$a_0^{red} = -\frac{1}{16\pi}\begin{bmatrix} 4\lambda_\Phi & \sqrt{2}\lambda_\Phi & \sqrt{2}\lambda_\Phi & \sqrt{2}\lambda_{\Phi S} \\ \sqrt{2}\lambda_\Phi & 3\lambda_\Phi & \lambda_\Phi & \lambda_{\Phi S} \\ \sqrt{2}\lambda_\Phi & \lambda_\Phi & 3\lambda_\Phi & \lambda_{\Phi S} \\ \sqrt{2}\lambda_{\Phi S} & \lambda_{\Phi S} & \lambda_{\Phi S} & 12\lambda_S \end{bmatrix}$$

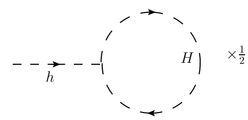
Owing to the special structure of this matrix, we get four eigenvalues:

$$-\frac{1}{8\pi}\lambda_\Phi, -\frac{1}{8\pi}\lambda_\Phi, -\frac{1}{16\pi}(3\lambda_\Phi + 6\lambda_S \pm \sqrt{(3\lambda_\Phi - 6\lambda_S)^2 + 4\lambda_{\Phi S}^2})$$

It is the same as that in Ref. [23].

Appendix C. Tadpole and self-energy of the SM Higgs from the additional scalar

Tadpole of the SM Higgs and the renormalization constant δt:



$$iT_h = \frac{i\lambda_{\Phi S} v}{16\pi^2} A_0(m_H^2)\ , \ \delta t = -T_h = -\frac{\lambda_{\Phi S} v}{16\pi^2} A_0(m_H^2)$$

Self-energy of the SM Higgs and the renormalization constants δm_h^2, δZ_h:

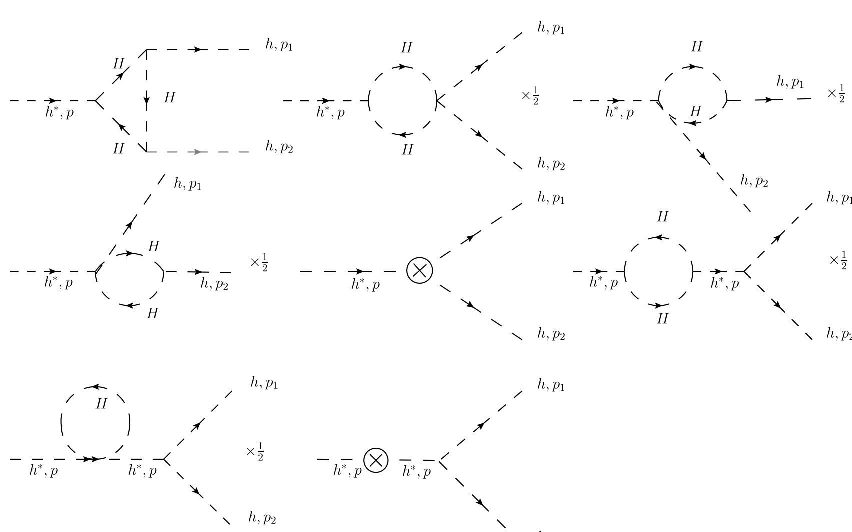
$$i\Sigma_h(p^2) = \frac{i\lambda_{\Phi S}}{16\pi^2} A_0(m_H^2) + \frac{i\lambda_{\Phi S}^2 v^2}{8\pi^2} B_0(p^2, m_H^2, m_H^2)$$

$$\delta m_h^2 = \mathrm{Re}\Sigma_h(m_h^2), \delta Z_h = -\mathrm{Re}\frac{\partial \Sigma_h(p^2)}{\partial p^2}|_{p^2=m_h^2} = -\frac{\lambda_{\Phi S}^2 v^2}{8\pi^2} DB_0(m_h^2, m_H^2, m_H^2)$$

$$DB_0(m_h^2, m_H^2, m_H^2) \equiv \frac{dB_0(p^2, m_H^2, m_H^2)}{dp^2}|_{p^2=m_h^2} = \int_0^1 dx \frac{x(1-x)}{m_H^2 - x(1-x)m_h^2}$$

Appendix D. Calculational details for the one-loop radiative correction

One-loop radiative correction for the triple h coupling:

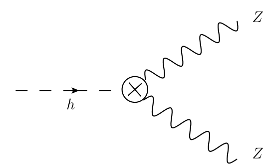


Assuming the Higgs bosons with momentum p_1, p_2 are on shell, while the Higgs boson with momentum p is off shell, that is $p_1^2 = p_2^2 = m_h^2$, $p^2 \neq m_h^2$. We can get the following analytical expression for the triple h coupling in the SM limit, which is the deviation from the SM prediction.

$$\delta_{hhh}^{(1)} \equiv \frac{\lambda_{hhh}^{(\mathrm{HSM})} - \lambda_{hhh}^{(\mathrm{SM})}}{\lambda_{hhh}^{(\mathrm{SM},tree)}} = -\frac{\lambda_{\Phi S}^3 v^4}{6\pi^2 m_h^2} C_0(p^2, m_h^2, m_h^2, m_H^2, m_H^2, m_H^2) + \frac{\lambda_{\Phi S}^2 v^2}{24\pi^2 m_h^2}[B_0(m_h^2, m_H^2, m_H^2) - B_0(p^2, m_H^2, m_H^2)]$$
$$- \frac{\lambda_{\Phi S}^2 v^2}{8\pi^2} \frac{B_0(p^2, m_H^2, m_H^2) - B_0(m_h^2, m_H^2, m_H^2)}{p^2 - m_h^2} - \frac{\lambda_{\Phi S}^2 v^2}{16\pi^2} \frac{\partial B_0(p^2, m_H^2, m_H^2)}{\partial p^2}\Big|_{p^2=m_h^2}$$

$\lambda_{hhh}^{(\mathrm{HSM})}$, $\lambda_{hhh}^{(\mathrm{SM})}$ are the coefficients in front of the h^3 vertex up to one-loop level in the HSM and SM respectively, but $\lambda_{hhh}^{(\mathrm{SM},tree)}$ is the tree level coefficient in the SM. If there is an imaginary part in $\delta_{hhh}^{(1)}$, we just extract the real part. Because the imaginary part is not observable at this order due to the interference with tree level amplitude.

Similarly, we get the one-loop radiative correction for the hZZ coupling:



$$\delta_{hZZ}^{(1)} \equiv \frac{\lambda_{hZZ}^{(\mathrm{HSM})} - \lambda_{hZZ}^{(\mathrm{SM})}}{\lambda_{hZZ}^{(\mathrm{SM},tree)}} = \frac{\delta Z_h}{2} = -\frac{\lambda_{\Phi S}^2 v^2}{16\pi^2} DB_0(m_h^2, m_H^2, m_H^2).$$

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