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Weak magnetism effects in the direct Urca processes in cooling neutron stars ☆

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Abstract

In the mean field approximation, we study the effects of weak magnetism and pseudoscalar interaction in the neutrino energy losses caused by the direct Urca processes on relativistic nucleons in the degenerate baryon matter. Our formula for the neutrino energy losses incorporates the effects of nucleon recoil, parity violation, weak magnetism, and pseudoscalar interaction. For numerical testing of our formula, we use a self-consistent relativistic model of the multicomponent baryon matter. We found that the effects of weak magnetism approximately double the neutrino emissivity, while the pseudoscalar interaction slightly suppresses the energy losses, approximately by 10%.

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The direct Urca processes, $n \to p + l + \bar{\nu}_l$, $p + l \to n + \nu_l$, where l is either an electron or a muon, are the most powerful neutrino reactions by which neutron stars lose their energy. In spite of widely accepted importance of these reactions, the relevant neutrino energy losses are not well investigated yet. By modern scenarios, the central density of the star can be up to eight times larger than the nuclear saturation density, what implies a substantially relativistic motion of nucleons [1]. The relevant equation of state of the matter is usually calculated in the relativistic approach [2], while the energy losses are still calculated by the non-relativistic formula obtained by Lattimer et al. [3] more than ten years ago. Some aspects of this problem was studied by Leinson and Pérez [4], who have estimated relativistic effects of baryon recoil and parity violation in the direct Urca processes. The above relativistic effects increase substantially the neutrino emissivity in comparison with the known non-relativistic prediction.

In the present Letter we derive a relativistic expression for neutrino energy losses caused by direct Urca processes on nucleons by taking into account also the effects of weak magnetism and pseudoscalar interaction. It is known that weak magnetism plays an important role in core collapse supernovae by increasing mean free paths of antineutrinos [5,6]. Our goal is to

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study the role of weak magnetism in radiation of neutrinos and antineutrinos at the long-cooling epoch of neutron star.

We employ the Walecka-type relativistic model of baryon matter [7], where the baryons interact via exchange of σ, ω, and ρ mesons, and perform the calculation of the neutrino energy losses in the mean field approximation. This approximation is widely used in the theory of relativistic nuclear matter, and allows to calculate in a self-consistent way the composition of the matter together with energies, and effective masses of the baryons. The Lagrangian density, which includes the interaction of a nucleon field Ψ with a scalar field σ, a vector field ω_μ and an isovector field $\mathbf{b}_\mu$ of ρ-meson is of the form[1,2]

$$\begin{aligned}\mathcal{L} = \overline{\Psi}\big[\gamma_\mu\big(i\partial^\mu - g_{\omega B}\omega^\mu - \tfrac{1}{2}g_{\rho B}\mathbf{b}^\mu\cdot\boldsymbol{\tau}\big)\\ - \big(M_B - g_{\sigma B}\sigma\big)\big]\Psi\\ - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\\ + \tfrac{1}{2}m_\rho^2\mathbf{b}_\mu\mathbf{b}^\mu + \tfrac{1}{2}\big(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\big)\\ - U(\sigma) + \bar{l}\big(i\gamma_\mu\partial^\mu - m_l\big)l.\end{aligned} \quad (1)$$

Here Ψ are the Dirac spinor fields for nucleons, $\mathbf{b}_\mu$ is the isovector field of ρ-meson. We denote as τ the isospin operator, which acts on the nucleons of the bare mass M. The leptons are represented only by electrons and muons, $l = e^-$, μ^-, which are included in the model as noninteracting particles. The field strength tensors for the ω and ρ mesons are $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu$, respectively. The potential $U(\sigma)$ represents the self-interaction of the scalar field and is taken to be of the form

$$U(\sigma) = \tfrac{1}{3}bM(g_{\sigma N}\sigma)^3 + \tfrac{1}{4}c(g_{\sigma N}\sigma)^4. \quad (2)$$

In what follows we consider the mean field approximation widely used in the theory of relativistic nuclear matter. In this approximation, the meson fields are replaced with their expectation values

$$\begin{aligned}\sigma &\to \langle\sigma\rangle \equiv \sigma_0,\\ \omega^\mu &\to \langle\omega^\mu\rangle \equiv \omega_0\delta_{\mu 0},\\ \mathbf{b}^\mu &\to \langle\mathbf{b}^\mu\rangle \equiv (0,0,\rho_0)\delta_{\mu 0}.\end{aligned} \quad (3)$$

In this case only the nucleon fields must be quantized. This procedure yields the following linear Dirac equation for the nucleon

$$\begin{aligned}\big(i\partial_\mu\gamma^\mu - g_\omega\gamma^0\omega_0 - \tfrac{1}{2}g_\rho\gamma^0\rho_0\tau_3 - (M - g_\sigma\sigma_0)\big)\\ \times\Psi(x) = 0.\end{aligned} \quad (4)$$

Here and below we denote as τ_3, and $\tau_\pm = (\tau_1 \pm i\tau_2)/2$ the components of isospin operator, which act on the isobaric doublet $\Psi(x)$ of nucleon field.

The stationary and uniform condensate fields equally shift the effective masses

$$M^* = M - g_\sigma\sigma_0, \quad (5)$$

but lead to different potential energies of the proton and neutron

$$U_{\mathrm{n}} = g_\omega\omega_0 - \tfrac{1}{2}g_\rho\rho_0, \qquad U_{\mathrm{p}} = g_\omega\omega_0 + \tfrac{1}{2}g_\rho\rho_0, \quad (6)$$

thus creating the energy gap $U_{\mathrm{n}} - U_{\mathrm{p}} = -g_\rho\rho_0$ between possible energies of protons and neutrons.

The exact solutions of Eq. (4) can be found separately for protons and for neutrons. In our case of a stationary and uniform system, solutions are the spinor plane waves

$$\psi_{\mathrm{n}}(x) = N_{\mathrm{n}}u_{\mathrm{n}}(P)\exp(-iE_{\mathrm{n}}t + i\mathbf{pr}), \quad (7)$$

$$\psi_{\mathrm{p}}(x) = N_{\mathrm{p}}u_{\mathrm{p}}(P')\exp\big(-iE_{\mathrm{p}}t + i\mathbf{p}'\mathbf{r}\big), \quad (8)$$

where the neutron and proton energies are given by

$$E_{\mathrm{n}}(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M^{*2}} + U_{\mathrm{n}},$$

and

$$E_{\mathrm{p}}(\mathbf{p}') = \sqrt{\mathbf{p}'^2 + M^{*2}} + U_{\mathrm{p}}.$$

The free-like spinors $u_{\mathrm{n}}(P)$ and $u_{\mathrm{p}}(P')$ are constructed from the kinetic momenta of the neutron and the proton

$$P^\mu = (E_{\mathrm{n}} - U_{\mathrm{n}}, \mathbf{p}) = \Big(\sqrt{\mathbf{p}^2 + M^{*2}}, \mathbf{p}\Big), \quad (9)$$

$$P'^\mu = \big(E_{\mathrm{p}} - U_{\mathrm{p}}, \mathbf{p}'\big) = \Big(\sqrt{\mathbf{p}'^2 + M^{*2}}, \mathbf{p}'\Big). \quad (10)$$

[1] In principle, the pion fields should be also included in the model. However, the expectation value of the pion field equals zero, giving no contribution to the mean fields. Therefore, only non-redundant terms are exhibited in the Lagrangian density.

[2] In what follows we use the system of units $\hbar = c = 1$ and the Boltzmann constant $k_B = 1$. Summation over repeated Greek indexes is assumed.

In what follows we denote by $\varepsilon=\sqrt{\mathbf{p}^2+M^{*2}}$, $\varepsilon'=\sqrt{\mathbf{p}'^2+M^{*2}}$ the kinetic energy of the neutron and the proton, respectively. So the normalization factors are

$$N_{\rm n}=\frac{1}{\sqrt{2\varepsilon}},\qquad N_{\rm p}=\frac{1}{\sqrt{2\varepsilon'}},\tag{11}$$

and the single-particle energies can be written as $E_{\rm n}(\mathbf{p})=\varepsilon+U_{\rm n}$, and $E_{\rm p}(\mathbf{p}')=\varepsilon'+U_{\rm p}$.

We consider massless neutrinos of energy and momentum $k_1=(\omega_1,\mathbf{k}_1)$ with $\omega_1=|\mathbf{k}_1|$. The energy-momentum of the final lepton $l=e^-,\ \mu^-$ of mass m_l is denoted as $k_2=(\omega_2,\mathbf{k}_2)$ with $\omega_2=\sqrt{\mathbf{k}_2^2+m_l^2}$. In the lowest order in the Fermi weak coupling constant G_F, the matrix element of the neutron beta decay is of the form

$$\begin{aligned}\langle f|(S-1)|i\rangle=-i\,\frac{G_FC}{\sqrt{2}}\,&N_{\rm n}N_{\rm p}\bar{u}_l(k_2)\gamma_\mu(1+\gamma_5)\\ &\times\nu(-k_1)_{\rm p}\langle P'|J^\mu(0)|P\rangle_{\rm n}\\ &\times(2\pi)^4\delta(E_{\rm n}-E_{\rm p}-\omega_1-\omega_2)\\ &\times\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}_1-\mathbf{k}_2),\end{aligned}\tag{12}$$

were $C=\cos\theta_C=0.973$ is the Cabibbo factor. The effective charged weak current in the medium consists of the polar vector and the axial vector, $J^\mu(x)=V^\mu(x)+A^\mu(x)$.

Our goal now is to derive the nucleon matrix element of the charged weak current in the medium. Consider first the polar-vector contribution. The Lagrangian density (1) ensures a conserved isovector current [8]

$$\mathbf{T}^\mu=\tfrac12\bar\psi\gamma^\mu\boldsymbol{\tau}\psi+\mathbf{b}_\nu\times\mathbf{B}^{\nu\mu},\qquad\partial_\mu\mathbf{T}^\mu=0.\tag{13}$$

Besides the directly nucleon contribution this current includes also the contribution of the isovector field $\mathbf{b}^\mu$, which obeys the field equations

$$\partial_\nu\mathbf{B}^{\nu\mu}+m_\rho^2\mathbf{b}^\mu=\tfrac12 g_\rho\bar\psi\gamma^\mu\boldsymbol{\tau}\psi,\qquad\partial^\nu\mathbf{b}_\nu=0.\tag{14}$$

By the use of Eq. (14) the condition $\partial_\mu\mathbf{T}^\mu=0$ may be transformed as

$$i\partial_\mu(\bar\psi\gamma^\mu\boldsymbol{\tau}\psi)=g_\rho\mathbf{b}_\mu\times\bar\psi\gamma^\mu\tau\psi.\tag{15}$$

In the mean field approximation this gives

$$i\partial_\mu(\bar\psi\gamma^\mu\tau_+\psi)=-g_\rho\rho_0\bar\psi\gamma^0\tau_+\psi.\tag{16}$$

By introducing the covariant derivative

$$D_\mu=\left(\frac{\partial}{\partial t}-ig_\rho\rho_0,\nabla\right),\tag{17}$$

we can recast the Eq. (16) to the following form

$$D_\mu(\bar\psi\gamma^\mu\tau_+\psi)=0.\tag{18}$$

At the level of matrix elements this can be written as

$$\bar{u}_{\rm p}(P')q_\mu\gamma^\mu u_{\rm n}(P)=0,\tag{19}$$

where q_μ is the kinetic momentum transfer

$$q^\mu=(E_{\rm n}-E_{\rm p}+g_\rho\rho_0,\mathbf{p}-\mathbf{p}')=(\varepsilon-\varepsilon',\mathbf{p}-\mathbf{p}').\tag{20}$$

Thus the matrix element of the transition current is orthogonal to the *kinetic* momentum transfer, but not to the total momentum transfer from the nucleon.[3] Note that this effect originates not from a special form (13) of the conserved isovector current in the medium but is caused by the energy gap between the proton and neutron spectrums. This follows directly from the Dirac equation (4), which ensures the Eq. (19) with

$$q^\mu=P^\mu-P'^\mu=(E_{\rm n}-E_{\rm p}-U_{\rm n}+U_{\rm p},\mathbf{p}-\mathbf{p}'),\tag{21}$$

which coincides with Eq. (20) because $U_{\rm n}-U_{\rm p}=-g_\rho\rho_0$.

By the use of the isovector current (13) one can construct the conserved electromagnetic current in the medium

$$\begin{aligned}&J^\mu_{\rm em}=\frac12\,\bar\psi\gamma^\mu\psi+T_3^\mu+\frac{1}{2M}\,\partial_\nu(\overline{\Psi}\lambda\sigma^{\mu\nu}\Psi),\\ &\partial_\mu J^\mu_{\rm em}=0.\end{aligned}\tag{22}$$

The last term in Eq. (22) is the Pauli contribution, where $2\sigma^{\mu\nu}=\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu$ and

$$\lambda=\lambda_{\rm p}\tfrac12(1+\tau_3)+\lambda_{\rm n}\tfrac12(1-\tau_3).\tag{23}$$

In the mean field approximation, we replace the magnetic formfactors of the nucleon with anomalous magnetic moments of the proton and the neutron, $\lambda_{\rm p}=1.7928$ and $\lambda_{\rm n}=-1.9132$.

By the conserved-vector-current theory (CVC), the nucleon matrix element of the charged vector weak current is given by

$${}_{\rm p}\langle P'|V^\mu|P\rangle_{\rm n}={}_{\rm p}\langle P'|J^\mu_{\rm em}|P\rangle_{\rm p}-{}_{\rm n}\langle P'|J^\mu_{\rm em}|P\rangle_{\rm n}.\tag{24}$$

[3] Some consequences of this fact for the weak response functions of the medium are discussed in [4].

Thus, in the mean field approximation, we obtain

$$
{}_{\mathrm{p}}\langle P'|V^{\mu}(0)|P\rangle_{\mathrm{n}} = \bar{u}_{\mathrm{p}}(P')\left[\gamma^{\mu} + \frac{\lambda_{\mathrm{p}} - \lambda_{\mathrm{n}}}{2M}\sigma^{\mu\nu}q_{\nu}\right]u_{\mathrm{n}}(P). \tag{25}
$$

The second term in Eq. (25), describes the weak magnetism effects. By the use of Eq. (4) we find

$$
{}_{\mathrm{p}}\langle P'|q_{\mu}V^{\mu}(0)|P\rangle_{\mathrm{n}} = 0. \tag{26}
$$

Consider now the axial-vector charged current. This current is responsible for both the np transitions and the pion decay. In the limit of chiral symmetry, $m_{\pi} \to 0$, the axial-vector current must be conserved. In the medium with ρ meson condensate, this implies

$$
\lim_{m_{\pi}\to 0} D_{\mu}A^{\mu}(x) = 0, \tag{27}
$$

where the covariant derivative D_{μ} is defined by Eq. (17). At the finite mass of a pion, m_{π}, the axial-vector charged current is connected to the field $\pi_{-}(x) = (\pi_1 + i\pi_2)/\sqrt{2}$ of π^{-} meson. For a free space, this relation is known as the hypothesis of partial conservation of the axial current (PCAC). In the medium the PCAC takes the form

$$
D_{\mu}A^{\mu}(x) = m_{\pi}^2 f_{\pi}\pi_{-}(x), \tag{28}
$$

where $m_{\pi} = 139$ MeV is the mass of π-meson, and f_{π} is the pion decay constant.

With allowing for interactions of the pions with nucleons and ρ mesons the Lagrangian density for the pion field is of the form [8]

$$
\mathcal{L}_{\pi} = \frac{1}{2}\left[(\partial_{\mu}\boldsymbol{\pi} - g_{\rho}\mathbf{b}_{\mu}\times\boldsymbol{\pi})\cdot(\partial^{\mu}\boldsymbol{\pi} - g_{\rho}\mathbf{b}^{\mu}\times\boldsymbol{\pi}) - m_{\pi}^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right] + ig_{\pi}\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi. \tag{29}
$$

In the mean field approximation this results in the following equation for the field of π^{-} meson

$$
\left((i\partial^0 + g_{\rho}\rho_0)^2 - (i\nabla)^2 - m_{\pi}^2\right)\pi_{-}(x) = -\sqrt{2}\, ig_{\pi}\bar{\psi}_{\mathrm{p}}\gamma_5\psi_{\mathrm{n}}, \tag{30}
$$

where g_{π} is the pion–nucleon coupling constant.

For the nucleon transition of our interest, the Eq. (28) gives

$$
{}_{\mathrm{p}}\langle P'|q_{\mu}A^{\mu}(0)|P\rangle_{\mathrm{n}} = im_{\pi}^2 f_{\pi}\,{}_{\mathrm{p}}\langle P'|\pi_{-}(0)|P\rangle_{\mathrm{n}}. \tag{31}
$$

Here the right-hand side can be calculated by the use of Eq. (30). We obtain

$$
{}_{\mathrm{p}}\langle P'|q_{\mu}A^{\mu}(0)|P\rangle_{\mathrm{n}} = -\frac{\sqrt{2}m_{\pi}^2 f_{\pi}g_{\pi}}{m_{\pi}^2 - q^2}\bar{u}_{\mathrm{p}}(P')\gamma_5 u_{\mathrm{n}}(P). \tag{32}
$$

This equation allows to derive the nucleon matrix element of the axial-vector charged current. Really, to construct the axial-vector matrix element of the charged current, caused by the nucleon transition, we have only two independent pseudovectors, consistent with invariance of strong interactions under T_2 isospin transformation, namely: $\bar{u}_{\mathrm{p}}(P')\gamma^{\mu}\gamma_5 u_{\mathrm{n}}(P)$, and $\bar{u}_{\mathrm{p}}(P')q^{\mu}\gamma_5 u_{\mathrm{n}}(P)$. This means that the matrix element of the axial-vector charged current is of following general form

$$
{}_{\mathrm{p}}\langle P'|A^{\mu}(0)|P\rangle_{\mathrm{n}} = C_A\bar{u}_{\mathrm{p}}(P')(\gamma^{\mu}\gamma_5 + F_q q^{\mu}\gamma_5)u_{\mathrm{n}}(P). \tag{33}
$$

Here, in the mean field approximation, we set $C_A \simeq 1.26$, while F_q is the form-factor to be chosen to satisfy the Eq. (32), which now reads

$$
C_A(-2M^* + F_q q^2)\bar{u}_{\mathrm{p}}(P')\gamma_5 u_{\mathrm{n}}(P) = -\frac{\sqrt{2}m_{\pi}^2 f_{\pi}g_{\pi}}{m_{\pi}^2 - q^2}\bar{u}_{\mathrm{p}}(P')\gamma_5 u_{\mathrm{n}}(P). \tag{34}
$$

Thus

$$
C_A(2M^* - F_q q^2) = \frac{\sqrt{2}m_{\pi}^2 f_{\pi}g_{\pi}}{m_{\pi}^2 - q^2}. \tag{35}
$$

In the mean field approximation, we assume that the coupling constants are independent of the momentum transfer. By setting $q^2 = 0$ in Eq. (35) we obtain the Goldberger–Treiman relation $f_{\pi}g_{\pi} = \sqrt{2}M^*C_A$. By inserting this in (35) we find

$$
F_q = -\frac{2M^*}{(m_{\pi}^2 - q^2)}. \tag{36}
$$

Thus, with taking into account Eqs. (25), (33) and (36), the total matrix element of the neutron beta decay is found to be

$$
\mathcal{M}_{fi} = -i\frac{G_F C}{\sqrt{2}}\bar{u}_l(k_2)\gamma_{\mu}(1 + \gamma_5)\nu(-k_1)
$$

$$\times \bar{u}_{\rm p}(P')\Big[C_V\gamma^\mu + \frac{1}{2M}C_M\sigma^{\mu\nu}q_\nu + C_A\big(\gamma^\mu\gamma_5 + F_q q^\mu\gamma_5\big)\Big]u_{\rm n}(P), \tag{37}$$

where, in the mean field approximation, we assume

$$C_V = 1, \qquad C_M = \lambda_p - \lambda_n \simeq 3.7, \qquad C_A = 1.26. \tag{38}$$

Note that the matrix element obtained is of the same form as that for the neutron decay in a free space, but with the total momentum transfer replaced with the kinetic momentum transfer. Due to the difference in the neutron and proton potential energy, the kinetic momentum transfer

$$q = P - P' = \big(\varepsilon - \varepsilon', \mathbf{p} - \mathbf{p}'\big) \tag{39}$$

to be used in the matrix element (37) differs from the total momentum of the final lepton pair

$$K \equiv k_1 + k_2 = \big(\varepsilon - \varepsilon' + U_{\rm n} - U_{\rm p}, \mathbf{p} - \mathbf{p}'\big). \tag{40}$$

This ensures $K^2 > 0$, while $q^2 = (\varepsilon - \varepsilon')^2 - (\mathbf{p} - \mathbf{p}')^2 < 0$.

The square of the matrix element of the reaction summed over spins of initial and final particles is found to be:

$$\begin{aligned}
|\mathcal{M}_{fi}|^2 &= 32G_F^2C^2\Big[\big(C_A^2 - C_V^2\big)M^{*2}(k_1k_2) \\
&\quad + (C_A - C_V)^2(k_1P_2)(k_2P_1) \\
&\quad + (C_A + C_V)^2(k_1P_1)(k_2P_2) \\
&\quad + 2C_M\frac{M^*}{M} \\
&\quad \times \big[2C_A\big((k_1P_1)(k_2P_2) - (k_1P_2)(k_2P_1)\big) \\
&\qquad + C_V\big((k_1k_2)\big(P_1P_2 - M^{*2}\big) \\
&\qquad - (k_1P_1 - k_1P_2)(k_2P_1 - k_2P_2)\big)\big] \\
&\quad - \frac{C_M^2}{M^2}\big[M^{*2}(k_1P_2)\big(3(k_2P_2) - (k_2P_1)\big) \\
&\qquad + M^{*2}(k_1P_1)\big(3(k_2P_1) - (k_2P_2)\big) \\
&\qquad + (k_1k_2)\big(P_1P_2 - M^{*2}\big)^2 \\
&\qquad - (k_1P_1 + k_1P_2)(k_2P_1 + k_2P_2)(P_1P_2)\big] \\
&\quad + C_A^2F_q\big(2M^* + F_q\big(M^{*2} - (P_1P_2)\big)\big) \\
&\quad \times \big[(k_1k_2)\big(M^{*2} - (P_1P_2)\big) \\
&\qquad - (k_1P_1 - k_1P_2)(k_2P_1 - k_2P_2)\big]\Big]
\end{aligned} \tag{41}$$

with $P_1 = (\varepsilon, \mathbf{p})$ and $P_2 = (\varepsilon', \mathbf{p}')$.

We consider the total energy which is emitted into neutrino and antineutrino per unit volume and time. Within beta equilibrium, the inverse reaction $p + l \to n + \nu_l$ corresponding to a capture of the lepton l, gives the same emissivity as the beta decay, but in neutrinos. Thus, the total energy loss Q for the Urca processes is twice more than that caused by the beta decay. Taking this into account by Fermi's "golden" rule we have

$$\begin{aligned}
Q &= 2\int \frac{d^3k_2\, d^3k_1\, d^3p\, d^3p'}{(2\pi)^{12}2\omega_2 2\omega_1 2\varepsilon 2\varepsilon'}|\mathcal{M}_{fi}|^2\omega_1 \\
&\quad \times f_{\rm n}(1 - f_{\rm p})(1 - f_l) \\
&\quad \times (2\pi)^4\delta\big(E_{\rm n}(\mathbf{p}) - E_{\rm p}(\mathbf{p}') - \omega_1 - \omega_2\big) \\
&\quad \times \delta\big(\mathbf{p} - \mathbf{p}' - \mathbf{k}_1 - \mathbf{k}_2\big).
\end{aligned} \tag{42}$$

Antineutrinos are assumed to be freely escaping. The distribution function of initial neutrons as well as blocking of final states of the proton and the lepton l are taken into account by the Pauli blocking-factor $f_{\rm n}(1 - f_{\rm p})(1 - f_l)$. The Fermi–Dirac distribution function of leptons is given by

$$f_l(\omega_2) = \frac{1}{\exp(\omega_2 - \mu_l)/T + 1}, \tag{43}$$

while the individual Fermi distributions of nucleons are of the form

$$f_{\rm n}(\varepsilon) = \frac{1}{\exp((\varepsilon + U_{\rm n} - \mu_{\rm n})/T) + 1}, \tag{44}$$

$$f_{\rm p}\big(\varepsilon'\big) = \frac{1}{\exp((\varepsilon' + U_{\rm p} - \mu_{\rm p})/T) + 1}. \tag{45}$$

By neglecting the chemical potential of escaping neutrinos, we can write the condition of chemical equilibrium as $\mu_l = \mu_{\rm n} - \mu_{\rm p}$. Then by the use of the energy conservation equation, $\varepsilon + U_{\rm n} = \varepsilon' + U_{\rm p} + \omega_2 + \omega_1$, and taking the total energy of the final lepton and antineutrino as $\omega_2 + \omega_1 = \mu_l + \omega'$ we can recast the blocking-factor as

$$\begin{aligned}
&f_{\rm n}(\varepsilon)\big(1 - f_{\rm p}\big(\varepsilon'\big)\big)\big(1 - f_l(\omega_2)\big) \\
&\quad \equiv f_{\rm n}(\varepsilon)\big(1 - f_{\rm n}\big(\varepsilon - \omega'\big)\big)\big(1 - f_l\big(\mu_l + \omega' - \omega_1\big)\big),
\end{aligned} \tag{46}$$

where $\omega' \sim T$.

Furthermore, since the antineutrino energy is $\omega_1 \sim T$, and the antineutrino momentum $|\mathbf{k}_1| \sim T$ is much smaller than the momenta of other particles, we can neglect the neutrino contributions in the energy-momentum conserving delta-functions

$$\delta\big(\varepsilon + U_{\rm n} - \varepsilon' - U_{\rm p} - \omega_1 - \omega_2\big)\delta\big(\mathbf{p} - \mathbf{p}' - \mathbf{k}_1 - \mathbf{k}_2\big) \simeq \delta\big(\varepsilon + U_{\rm n} - \varepsilon' - U_{\rm p} - \omega_2\big)\delta\big(\mathbf{p} - \mathbf{p}' - \mathbf{k}_2\big) \quad (47)$$

and perform integral over d^3p' to obtain $\mathbf{p}' = \mathbf{p} - \mathbf{k}_2$ in the next integrals.

The energy exchange in the matter goes naturally on the temperature scale $\sim T$, which is small compared to typical kinetic energies of degenerate particles. Therefore, in all smooth functions under the integral (42), the momenta of in-medium fermions can be fixed at their values at Fermi surfaces, which we denote as $p_{\rm n}$, $p_{\rm p}$ for the nucleons and p_l for leptons, respectively.

The energy of the final lepton is close to its Fermi energy $\mu_l = \mu_{\rm n} - \mu_{\rm p}$, and the chemical potentials of nucleons can be approximated by their individual Fermi energies $\mu_{\rm n} = \varepsilon_{\rm n} + U_{\rm n}$, $\mu_{\rm p} = \varepsilon_{\rm p} + U_{\rm p}$. This allows us to transform the energy-conserving δ-function as

$$\delta\Big(\varepsilon_{\rm n} - \sqrt{p_{\rm n}^2 + p_l^2 - 2p_{\rm n}p_l\cos\theta_l + M^{*2}} + U_{\rm n} - U_{\rm p} - \mu_l\Big) = \frac{\varepsilon_{\rm p}}{p_{\rm n}p_l}\,\delta\left(\cos\theta_l - \frac{1}{2p_{\rm n}p_l}\big(p_{\rm n}^2 - p_{\rm p}^2 + p_l^2\big)\right), \quad (48)$$

where θ_l is the angle between the momentum $\mathbf{p}_{\rm n}$ of the initial neutron and the momentum $\mathbf{p}_l$ of the final lepton. Notice, when the baryon and lepton momenta are at their individual Fermi surfaces, the δ-function (48) does not vanish only if $p_{\rm p} + p_l > p_{\rm n}$.

Further we use the particular frame with Z-axis directed along the neutron momentum $\mathbf{p}_{\rm n}$. Then

$$P_1 = (0, 0, p_{\rm n}, \varepsilon_{\rm n}),$$
$$k_1 = \omega_1(\sin\theta_\nu, 0, \cos\theta_\nu, 1),$$
$$k_2 = (p_l\sin\theta_l\cos\varphi_l, p_l\sin\theta_l\sin\varphi_l, p_l\cos\theta_\nu, \mu_l). \quad (49)$$

The energy–momentum of the final proton is defined by conservation laws:

$$P_2 = (-p_l\sin\theta_l\cos\varphi_l, -p_l\sin\theta_l\sin\varphi_l, p_{\rm n} - p_l\cos\theta_\nu, \varepsilon_{\rm p}). \quad (50)$$

Insertion of (49) and (50) in the square of the matrix element (41) yields a rather cumbersome expression, which, however, is readily integrable over solid angles of the particles.

Since we focus on the actually important case of degenerate nucleons and leptons, we may consider the neutrino energy losses to the lowest accuracy in T/μ_l. Then the remaining integration reduces to the factor

$$\int d\omega_1\omega_1^3\,d\omega'\,d\varepsilon\, f_{\rm n}(\varepsilon)\big(1 - f_{\rm n}(\varepsilon - \omega')\big) \times \big(1 - f_l(\mu_l + \omega' - \omega_1)\big) \simeq \int_{-\infty}^{\infty} d\omega'\frac{\omega'}{\exp\omega'/T - 1} \times \int_0^{\infty} d\omega_1\,\frac{\omega_1^3}{1 + \exp(\omega_1 - \omega')/T} = \tfrac{457}{5040}\pi^6T^6. \quad (51)$$

Finally the neutrino emissivity is found to be of the form:

$$\begin{aligned} Q = \frac{457\pi}{10\,080}G_F^2C^2T^6\Theta(p_l + p_{\rm p} - p_{\rm n}) &\times \Big\{\big(C_A^2 - C_V^2\big)M^{*2}\mu_l + \frac{1}{2}\big(C_V^2 + C_A^2\big) \\ &\times \big[4\varepsilon_{\rm n}\varepsilon_{\rm p}\mu_l - (\varepsilon_{\rm n} - \varepsilon_{\rm p})\big((\varepsilon_{\rm n} + \varepsilon_{\rm p})^2 - p_l^2\big)\big] \\ &+ C_VC_M\frac{M^*}{M}\big[2(\varepsilon_{\rm n} - \varepsilon_{\rm p})p_l^2 - \big(3(\varepsilon_{\rm n} - \varepsilon_{\rm p})^2 - p_l^2\big)\mu_l\big] \\ &+ C_A\left(C_V + 2\frac{M^*}{M}C_M\right)(\varepsilon_{\rm n} + \varepsilon_{\rm p}) \\ &\times \big(p_l^2 - (\varepsilon_{\rm n} - \varepsilon_{\rm p})^2\big) + C_M^2\frac{1}{4M^2} \\ &\times \big[8M^{*2}(\varepsilon_{\rm n} - \varepsilon_{\rm p})\big(p_l^2 - (\varepsilon_{\rm n} - \varepsilon_{\rm p})\mu_l\big) \\ &\quad + \big(p_l^2 - (\varepsilon_{\rm n} - \varepsilon_{\rm p})^2\big)\big(2\varepsilon_{\rm n}^2 + 2\varepsilon_{\rm p}^2 - p_l^2\big)\mu_l \\ &\quad - \big(p_l^2 - (\varepsilon_{\rm n} - \varepsilon_{\rm p})^2\big)(\varepsilon_{\rm n} + \varepsilon_{\rm p})^2 \\ &\quad \times (2\varepsilon_{\rm n} - 2\varepsilon_{\rm p} - \mu_l)\big] \\ &- C_A^2M^{*2}\Phi\big(1 + m_\pi^2\Phi\big) \\ &\times \big[\mu_l\big((\varepsilon_{\rm n} - \varepsilon_{\rm p})^2 + p_l^2\big) - 2(\varepsilon_{\rm n} - \varepsilon_{\rm p})p_l^2\big]\Big\} \end{aligned} \quad (52)$$

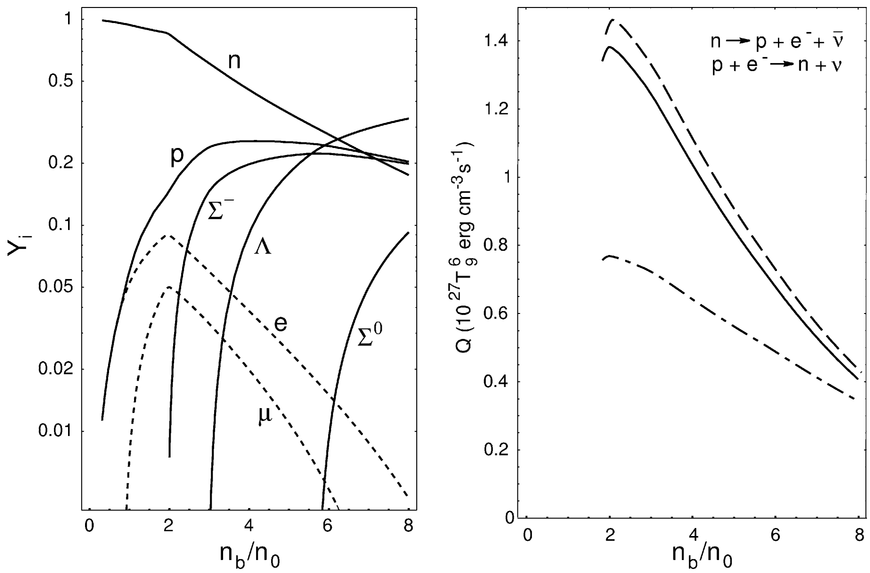

Fig. 1. The left panel shows individual concentrations for matter in beta equilibrium among nucleons, hyperons, electrons and muons as a function of the density ratio n_b/n_0. The right panel represents the neutrino emissivity of the direct Urca processes among nucleons and electrons for the matter composition represented on the left panel. The curves begin at the threshold density. The solid curve represents the total relativistic emissivity, as given by Eq. (52). The dot-dashed curve shows the relativistic emissivity without contributions of weak magnetism and pseudoscalar interaction, and the long-dashed curve is the emissivity without the pseudoscalar contribution. All the emissivities are given in units $10^{27} T_9^6$ erg cm^{-3} s^{-1}, where the temperature $T_9 = T/10^9$ K.

with $\Theta(x) = 1$ if $x \geqslant 0$ and zero otherwise. In the above, the last term, with

$$\Phi = \frac{1}{m_\pi^2 + p_l^2 - (\varepsilon_{\rm n} - \varepsilon_{\rm p})^2}, \tag{53}$$

represents the contribution of the pseudoscalar interaction. The "triangle" condition $p_{\rm p} + p_l > p_{\rm n}$, required by the step-function, is necessary for conservation of the total momentum in the reaction and exhibits the threshold dependence on the proton concentration.

Neutrino energy losses caused by the direct Urca on nucleons depend essentially on the composition of beta-stable nuclear matter. Therefore, in order to estimate the relativistic effects, we consider the model of nuclear matter, which besides nucleons includes Σ and Λ hyperons [1]. The parameters of the model are chosen as suggested in Ref. [9] to reproduce the nuclear matter equilibrium density, the binding energy per nucleon, the symmetry energy, the compression modulus, and the nucleon effective mass at saturation density $n_0 = 0.16$ fm^{-3}. The composition of neutrino-free matter in beta equilibrium among nucleons, hyperons, electrons and muons is shown on the left panel of Fig. 1 versus the baryon number density n_b, in units of n_0.

On the right panel of Fig. 1, by a solid curve we show the relativistic neutrino emissivity of reactions $n \to p + e^- + \bar{\nu}_e$, $p + e^- \to n + \nu_e$, as given by Eq. (52). Appearance of hyperons in the system suppresses the nucleon fractions and lepton abundance. Therefore at densities, where the number of hyperons is comparable with the number of protons, the relativistic emissivity reaches the maximum and then has a tendency to decrease. To inspect the contribution of weak magnetism and the pseudoscalar interaction we demonstrate two additional graphs. The long-dashed curve demonstrates the energy losses obtained from Eq. (52) by formal setting $\Phi = 0$. This eliminates the pseudoscalar contribution. The dot-dashed curve is obtained by formal replacing $\Phi = 0$ and $C_M = 0$, which eliminates both the weak magnetism and pseudoscalar contributions. A comparison of these curves demonstrates the weak magnetism effects. The contribution of the pseudoscalar interaction can be observed by comparing the total neutrino energy losses (solid curve) with the long-dashed curve, which is calculated without this contribution. We see that the weak magnetism effects approximately doubles the relativistic emissivity, while the pseudoscalar interaction only slightly suppresses the energy losses, approximately by 10%.

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