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Physics Letters B 584 (2004) 133–140
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Nonlinearly realized local scale invariance: gravity and matter

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Received 5 January 2004; received in revised form 15 January 2004; accepted 16 January 2004

Editor: M. Cvetič

Abstract

That the scalar field theories with no dimensional couplings possess local scale invariance (LSI) via the curvature gauging is utilized to show that the Goldstone boson, released by the spontaneous LSI breakdown, is swallowed by the spacetime curvature in order to generate Newton's constant in the same spirit as the induction of vector boson masses via spontaneous gauge symmetry breaking. For Einstein gravity to be reproduced correctly, the Goldstone boson of spontaneous LSI breaking must be endowed with ghost dynamics. The matter sector, taken to be the standard model spectrum, gains full LSI property with the physical Higgs boson acting as the Goldstone boson released by LSI breakdown at the weak scale. The pattern of particle masses is identical to that of the standard model. There are unitary LSI gauges in which either the Goldstone ghost from gravity sector or the Higgs boson from matter sector is eliminated from the spectrum. The heavy right-handed neutrinos as well as softly broken supersymmetry naturally fit into the nonlinearly realized LSI framework.

1. Introduction

The Lagrangian field theories bearing no dimensional couplings are invariant under global rescalings of coordinates and fields [1,2]. The scale invariance is blatantly violated in nature at least by the existing abundance of massive particles. Though one expects an approximate invariance in matter sector at distances sufficiently shorter than the Compton wavelengths of the particles, there is no such prescription for scaling violation in the gravity sector since Newton's constant defines the shortest length scale below which gravity becomes strong and a field-theoretic description of nature breaks down. This observation entails the possibility that the Newton's constant might in fact mark the scale of resizing invariance breakdown.

The conditions for global scale invariance does not depend on if the spacetime is flat or curved: all that is needed is to guarantee the absence of dimensional constants in the Lagrangian. One notes that rescaling of the event coordinates is equivalent to that of the metric tensor as they lead to identical effects on the event separations. Interesting effects start arising when one promotes the global invariance to a local one. In this case, even if the Lagrangian is free of any dimensional parameter, the scale invariance is not automatic at all. For fermions and bosons the global invariance guarantees the local one (in complete contradiction with local gauge invariance). For scalar fields, however, there is no local invariance even if the global one holds (similar to what happens in gauge theories). Therefore, the

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0370-2693
doi:10.1016/j.physletb.2004.01.044

local scale invariance in scalar field theories with no dimensional couplings can be achieved only by introducing an Abelian gauge field, i.e., Weyl's vector field [1]. However, it has long been known [3] and will be fully detailed in Section 2 that various operators involving Weyl's gauge field are equivalent to certain combinations of the curvature tensors. This then suggests that the spacetime curvature acts as the gauge field of local rescaling transformations. As will be analyzed in Section 3 this observation will lead to a full restoration of the local resizing invariance with a nonlinear sigma model such that the Einstein–Hilbert is generated in the same way as the formation of vector boson masses in spontaneously broken gauge theories. The Goldstone boson released by spontaneous breakdown of local scale symmetry assumes ghost character if the Einstein–Hilbert term is to come out correctly. The local scale invariance is a highly restrictive symmetry in that no local operators other than Weyl gravity, Einstein–Hilbert term and cosmological constant (dressed by the nonlinear sigma model field) are allowed.

Matter sector will be analyzed in Section 4 within in a fully scale-invariant framework in which masses of the particles will be related to electroweak breaking rather than the resizing invariance breaking. It will be shown that, it is possible to go to unitary gauges for local scale invariance where (i) either gravity sector is described by Weyl plus Einstein gravity with a cosmological constant, and the matter sector is precisely that of the standard model with yet-to-be discovered Higgs boson, (ii) or the gravity sector is a scalar-tensor theory with now-physical Goldstone ghost, and the matter sector is precisely what has been established by experiment and what is predicted by standard model with an important difference: there is no Higgs boson to search for. Either gauge has observable consequences. In addition, heavy right-handed neutrinos, needed to induce tiny masses for active flavors, can be directly incorporated into the locally scale invariant scheme.

2. From global to local scale invariance

The global scale invariance (GSI) of a physical system refers to its immunity to resizing of coordinates and fields [2] by constant amounts. In general, Lagrangian field theories with no dimensional couplings possess GSI. For definiteness, consider a real scalar field $\phi(x)$ described by the diffeomorphic invariant

$$-\int d^4x\,\sqrt{-g}\left[g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi+\lambda\phi^4\right], \tag{1}$$

where λ is a dimensionless parameter, and $g_{\mu\nu}(x)$ is the spacetime metric with determinant $g\equiv\det(g_{\mu\nu})$ and signature $(-,+,+,+)$. This action is invariant under the resizings $x_\mu\to e^{\omega_0}x_\mu$ (or equivalently $g_{\mu\nu}\to e^{2\omega_0}g_{\mu\nu}$ due to diffeomorphism invariance) and $\phi\to e^{d_\phi\omega_0}\phi$ when ω_0 is constant and $d_\phi=-1$. However, this very symmetry property depends crucially on the global nature of ω_0. Indeed, the action above is not invariant under local resizings

$$\begin{aligned}&\phi(x)\to e^{d_\phi\omega(x)}\phi(x),\\ &g_{\mu\nu}(x)\to e^{2\omega(x)}g_{\mu\nu}(x)\end{aligned} \tag{2}$$

due to the inhomogeneous terms generated by its kinetic part. Clearly, local resizings are not unitary transformations since conformal weight d_ϕ of ϕ and the conformal factor $\omega(x)$ are both real. For the action to possess local scale invariance (LSI) one has to, in analogy with gauge theories, promote ∇_μ to a gauge-covariant derivative $\mathcal{D}_\mu\equiv\nabla_\mu+d_\phi A_\mu$ with $A_\mu\to A_\mu-\nabla_\mu\omega$ so that $\mathcal{D}_\mu\phi\to e^{d_\phi\omega}\mathcal{D}_\mu\phi$ under the transformations in Eq. (2). This procedure, known as Weyl gauging [1], makes the action Eq. (1) locally scale invariant at the expense of introducing an extra vector field into the spectrum

$$-\int d^4x\,\sqrt{-g}\left[\frac{1}{4d_\phi^2}\,g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}+g^{\mu\nu}\mathcal{D}_\mu\phi\mathcal{D}_\nu\phi+\lambda\phi^4\right], \tag{3}$$

where $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$. Obviously, A_μ has nothing to do with electromagnetism or some other local unitary symmetry principle. Instead, it must be, if ever, related to gravity since the local symmetry that A_μ implements concerns the point-dependent resizing of the spacetime coordinates. This viewpoint is further supported by the observations made in [4], that is, the specific structure made out of the vector boson

$$\nabla_\mu A_\nu-A_\mu A_\nu-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}A_\alpha A_\beta \tag{4}$$

transforms in exactly the same way as[1]

$$-\frac{1}{2}\left(R_{\mu\nu}-\frac{1}{6}Rg_{\mu\nu}\right) \tag{5}$$

though this is not of much help for reinterpreting the vector boson sector as a gravitational effect since the specific structure (4) can arise in an action only as an irrelevant operator. However, it still gives a clue to eliminating A_μ from the system using appropriate combinations of curvature tensors and the scalar field. Indeed, the A_μ-dependent part of the scalar kinetic term transforms as

$$\begin{aligned}&\sqrt{-g}\left[g^{\mu\nu}\mathcal{D}_\mu\phi\mathcal{D}_\nu\phi-\nabla_\mu\phi\nabla_\nu\phi\right]\\&\quad\to\sqrt{-g}\left[g^{\mu\nu}\mathcal{D}_\mu\phi\mathcal{D}_\nu\phi-\nabla_\mu\phi\nabla_\nu\phi\right.\\&\qquad\left.+d_\phi\big(-\nabla_\mu\nabla_\nu\omega+(2+d_\phi)\nabla_\mu\omega\,\nabla_\nu\omega\big)\phi^2\right]\end{aligned} \tag{6}$$

which is nothing but the transformation property of

$$\sqrt{-g}\,\zeta_c R\phi^2 \tag{7}$$

provided that $\zeta_c = 1/6$ and $d_\phi = -1$. This simple result, which might have also been guessed from [3], implies the similarity relation

$$\begin{aligned}&\sqrt{-g}\left[g^{\mu\nu}\mathcal{D}_\mu\phi\mathcal{D}_\nu\phi+\lambda\phi^4\right]\\&\quad\sim\sqrt{-g}\left[g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi+\zeta_c R\phi^2+\lambda\phi^4\right]\end{aligned} \tag{8}$$

which provides a firm foundation for the viewpoint that *the Ricci scalar is the gauge field of the LSI*. Indeed, the kinetic term of the action Eq. (1) gains exact invariance under the local resizings via the Ricci gauging $g^{\mu\nu}\nabla_\mu\nabla_\nu \to g^{\mu\nu}\nabla_\mu\nabla_\nu - \zeta_c R$ which is similar to the construction of the gauge-covariant derivative. Physically, the curvature scalar acts as a connection field for restoring the change in the scalar kinetic term under local resizing of the coordinates.

Having done with the scalar sector, what remains to analyze is the A_μ kinetic term in Eq. (3). This term does obviously possess exact LSI. On the other hand, in the gravitational sector there is one and only one resizing invariant object

$$\sqrt{-g}\,W_{\mu\nu\lambda}{}^{\rho}W^{\mu\nu\lambda}{}_{\rho}, \tag{9}$$

where the Weyl tensor

$$\begin{aligned}W_{\mu\nu\lambda}{}^{\rho} &= R_{\mu\nu\lambda}{}^{\rho}\\&\quad-\frac{1}{2}\big(g_{\mu\lambda}R^{\rho}_{\nu}-g_{\nu\lambda}R^{\rho}_{\mu}+g^{\rho}_{\nu}R_{\mu\lambda}-g^{\rho}_{\mu}R_{\lambda\nu}\big)\\&\quad+\frac{1}{6}R\big(g_{\mu\lambda}g^{\rho}_{\nu}-g_{\nu\lambda}g^{\rho}_{\mu}\big)\end{aligned} \tag{10}$$

is the traceless part of the Riemann tensor $R_{\mu\nu\lambda}{}^{\rho}$ and satisfies all of its properties except the Bianchi identity. In addition, it is conformal invariant for the given index positions. Clearly, with the same logic that lead to Eq. (8), the A_μ kinetic term is equivalent to Eq. (9). In this sense Weyl gravity in Eq. (9) serves as 'the kinetic term' of the spacetime curvature—the gauge field of the LSI.

The programme of promoting the global conformal invariance to a local symmetry principle, in the light of gauge-gravity equivalence relations derived above, ends by embedding the scalar field theory in Eq. (1) into the action

$$\begin{aligned}\int d^4x\,\sqrt{-g}\Bigg[&-\frac{\gamma}{4d_\phi^2}W_{\mu\nu\lambda}{}^{\rho}W^{\mu\nu\lambda}{}_{\rho}\\&-\left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi+\zeta_c R\phi^2+\lambda\phi^4\right)\Bigg],\end{aligned} \tag{11}$$

where γ is a dimensionless constant. In conclusion the scalar field theory in Eq. (1) gains full LSI via the curvature gauging. The Weyl contribution, which satisfies the equivalence relation

$$W_{\mu\nu\lambda}{}^{\rho}W^{\mu\nu\lambda}{}_{\rho}\equiv 2g^{\mu\alpha}g^{\nu\beta}R_{\mu\nu}R^{\alpha\beta}-\frac{2}{3}R^2 \tag{12}$$

after using the Gauss–Bonnet theorem, is a higher derivative contribution since the Riemann curvature is already quadratic in ∇_μ.

3. Gravitational sector

Consider the locally rescaling invariant Abelian gauge theory in Eq. (3). This local invariance can be broken in various ways one of which being an explicit mass term for A_μ. Indeed, the action for a massive

[1] The curvature tensors are defined as $R = g^{\mu\nu}R_{\mu\mu}$, $R_{\mu\nu} = g^{\rho\lambda}R_{\mu\rho\nu\lambda}$, and

$$R_{\mu\nu\lambda}{}^{\rho} = \partial_\nu\Gamma^{\rho}{}_{\mu\lambda}-\partial_\mu\Gamma^{\rho}{}_{\nu\lambda}+\Gamma^{\alpha}{}_{\mu\lambda}\Gamma^{\rho}{}_{\alpha\nu}-\Gamma^{\alpha}{}_{\nu\lambda}\Gamma^{\rho}{}_{\alpha\mu}$$

with the connection coefficients

$$\Gamma^{\rho}{}_{\mu\nu}=\Gamma^{\rho}{}_{\nu\mu}=\frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\nu\sigma}+\partial_\nu g_{\mu\sigma}-\partial_\sigma g_{\mu\nu}).$$

vector boson

$$\int d^4x\,\sqrt{-g}\left[-\frac{1}{4d_\phi^2}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} - \frac{1}{2}M_A^2 g^{\mu\nu}A_\mu A_\nu\right] \tag{13}$$

does obviously vary with $A_\mu \to A_\mu - \nabla_\mu\omega$. Is it possible to restore the LSI? The answer to this question is provided by the fact that a vector boson can never acquire a mass unless the spectrum contains an exactly massless scalar particle. An additional fact is that every spontaneously broken continuous symmetry releases a massless scalar [5], and if the symmetry under concern refers to a local invariance these scalars are swallowed [6] by the vector bosons to develop their longitudinal polarization states as required of a massive vector boson. En passant, one notes that masslessness of the requisite scalar field is a key property needed for both generating a mass for the vector boson and preserving the LSI of the interactions. Letting $U(x)$ be the scalar field sought for and f be the scale of spontaneous LSI breakdown, the massive Abelian gauge model of Eq. (13) gains full LSI via the embedding

$$\int d^4x\,\sqrt{-g}\left[-\frac{1}{4d_U^2}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} - \frac{1}{2}f^2\left(g^{\mu\nu}\mathcal{D}_\mu U\mathcal{D}_\nu U + \frac{1}{2}\lambda f^2U^4\right)\right], \tag{14}$$

where $U(x) \to e^{d_U\omega(x)}U(x)$ under local resizings, and it can be parameterized as $U(x) = e^{\pi(x)/f}$ where $\pi(x)$ is the Goldstone boson released by the spontaneous LSI breakdown: $\pi(x) \to \pi(x) + f\omega(x)$. This action is unique in that it includes all possible terms allowed by LSI. Furthermore, it directly follows from Eq. (3) via the replacement $\phi(x) \to fe^{d_U\pi(x)/f}$. Consequently, the LSI, which is explicitly broken by the gauge boson mass, can be realized nonlinearly by widening the spectrum with a nonlinear sigma model field $U(x)$. However, the two actions, Eqs. (13) and (14), are physically identical since one can always go to the unitary gauge $U(x) = 1$ using $\omega(x) = -\pi(x)/f$ in which case Eq. (14) reduces to Eq. (13) with $M_A^2 = d_U^2 f^2$ and $\lambda f^4/4$ representing an additional LSI breaking source. Hence, restoration of the resizing symmetry in Eq. (13) does not lead to any physical novelty. Despite this, however, the Goldstone boson formalism is a highly powerful tool for elucidating the ultraviolet physics. First, by becoming strongly coupled at energies $\sim 4\pi M_A$, it enables one to determine the scale and symmetries of the ultraviolet completion. Next, it enables one to determine the size and structure of the higher-dimensional operators by simple power counting.

That the LSI can be restored using a nonlinear sigma model has important implications for the gravitational sector. Indeed, in the same spirit that the gauge-gravity correspondence relations derived in the last section have bridged the Weyl-gauged scalar theory in Eq. (3) to the gravitational action in Eq. (11), the gravitational equivalent of Eq. (14) can be readily written down as

$$S[g_{\mu\nu}, U] = \int d^4x\,\sqrt{-g}\left[-\frac{\gamma}{4d_U^2}W_{\mu\nu\lambda}{}^\rho W^{\mu\nu\lambda}{}_\rho - \frac{1}{2}\kappa f^2\left(\zeta_c RU^2 + g^{\mu\nu}\nabla_\mu U\nabla_\nu U + \frac{1}{2}\lambda f^2U^4\right)\right], \tag{15}$$

where one may visualize Ricci scalar as the 'mass term' and Weyl contribution as the 'kinetic term' under the curvature gauging. This Ricci-gauged nonlinear sigma model reveals certain important aspects of the gravitational interactions:

- Phenomenologically, the LSI breaking scale must be well inside the Planckian territory:

$$f^2 = \frac{M_{\rm Pl}^2}{\zeta_c}, \tag{16}$$

  where $M_{\rm Pl} = (8\pi G_{\rm Newton})^{-1/2}$ is the reduced Planck mass. Saying differently, the invariance under local resizing of coordinates and fields must be spontaneously broken around $(6\pi G_{\rm Newton}^{-1})^{1/2}$ beyond which the theory must be completed (by string theory).
- The transition from Eq. (13) to Eq. (14) makes it clear that the overall sign of the sigma model Lagrangian is fixed by the sign of the gauge boson mass term. In fact, it has to be negative to avoid tachyonic behavior for A_μ, and this very fact guarantees that the corresponding Goldstone boson has positive kinetic energy. These observa-

tions hold also for $S[g_{\mu\nu}, U]$ which reduces to

$$\int d^4x\,\sqrt{-g}\left[-\frac{\gamma}{4d_U^2}W_{\mu\nu\lambda}{}^{\rho}W^{\mu\nu\lambda}{}_{\rho} - \frac{1}{2}\kappa f^2\left(\zeta_c R + \frac{1}{2}\lambda f^2\right)\right] \qquad (17)$$

in the unitary gauge, $U = 1$. The first term is the Weyl contribution which always possesses LSI like the kinetic term of massive vector boson A_μ. The last term is nothing but the cosmological constant

$$\Lambda = \kappa\lambda\left(\frac{M_{\rm Pl}^2}{2\zeta_c}\right)^2 \qquad (18)$$

whose sign is determined by that of $\kappa\lambda$, and whose size is naturally Planckian. On the other hand, the term proportional to the curvature scalar reproduces the Einstein–Hilbert term if and only if $\kappa = -1$ (within the conventions mentioned in footnote 1). This, however, implies that the Goldstone boson $\pi(x)$ assumes negative kinetic energy, i.e., it behaves as a ghost [7]. In other words the unitary gauge, $U(x) = 1$, is not necessarily the energetically preferred state; at finite $\pi(x)$ there may exist states with lower energy unless the nonlinearities neutralize the ghost dynamics. This unwanted aspect of $S[g_{\mu\nu}, U]$, however, is not special to the nonlinear sigma model. In fact, even the unitary gauge action Eq. (17) contains ghosts due to the Weyl contribution which is quartic in the derivatives [8]. Consequently, the resizing invariant action in Eq. (15) contains ghosts from both nonlinear sigma model and Weyl contribution. In a way this is expected: the spacetime curvature swallows a Goldstone ghost to generate the Newton's constant because it already includes ghost degrees of freedom. The implications of two coexisting ghost sectors as well as their mutual effects on the gravity loops require a separate analysis to extend [8] to the framework of Eq. (15).

- The Goldstone boson picture is particularly useful in determining the structure and size of the higher dimension operators. Given the unitary gauge action in Eq. (17), in principle, one may add as many higher dimension operators as possible provided that the general covariance is respected. However, from the window of the nonlinear sigma model $S[g_{\mu\nu}, U]$, each such operator has to comply with the LSI requirements. The volume element $d^4x\,\sqrt{-g}$, though diffeomorphically invariant, changes with the rescalings of the metric tensor. This implies that the Lagrangian of Eq. (15) does not admit any additional operator structure no matter what combinations of curvature tensors and sigma model field are considered. In fact, only the operators involving powers of $\sqrt{-g}$ times the Lagrangian possess LSI. For instance, operators of the form $[\sqrt{-g}\,(\zeta_c R U^2 + g^{\mu\nu}\nabla_\mu U\nabla_\nu U)]^n$ or $[\sqrt{-g}\,W_{\mu\nu\lambda}{}^{\rho}W^{\mu\nu\lambda}{}_{\rho}]^n$ are automatically invariant. However, all such operators are in obvious conflict with general covariance since the determinant of the metric tensor as well as d^4x are densities rather than tensors and hence the only covariant combination is $d^4x\,\sqrt{-g}$. All these no-go cases enforce the inference that the higher order interactions are allowed to arise only in a nonlocal fashion, i.e., in a way involving only the powers of $S[g_{\mu\nu}, U]$ itself. For example, a functional dependence of the form $e^{\alpha S[g_{\mu\nu}, U]}$ would generate higher order nonlocal interactions in a way respecting LSI, general covariance and the action principle.

In conclusion, the Einstein–Hilbert term $(1/2)M_{\rm Pl}^2 R$ can be viewed as arising from the spontaneous breakdown of the LSI at the Planck scale. The Goldstone boson released by the spontaneous breakdown gains ghosty dynamics in accord with the ghost degrees of freedom contained in the Weyl contribution. The nonlinearly realized LSI is a highly restrictive symmetry in that it allows no operator structure other than those contained in $S[g_{\mu\nu}, U]$; in particular, higher dimension operators can arise only in a nonlocal way.

4. Matter sector

The Goldstone ghost, released by the spontaneous breakdown of the local resizing invariance, is swallowed by the spacetime curvature, the gauge field of the LSI, so as to generate the Newton's constant. In the matter sector, which comprises at least the known fermions and vector bosons, there is no field to gauge the resizing invariance. In principle, somehow naively, one might envision all the mass parameters in the mat-

ter sector as spurions with appropriate conformal dimensions so that the LSI always holds. This view is similar to that of [9], and essentially requires each mass parameter to be dressed by some nonlinear sigma model field. Then the main problem is to determine the origin and role of this field for enabling matter sector to gain exact LSI. First of all, the scale of LSI breakdown is enormously large compared to even the heaviest particle, the top quark, hence the existing pattern of particle masses must follow from the spontaneous breakdown of some other symmetry. Next, experimental results on various relations among the masses and couplings of vector bosons and fermions suggest that symmetries of the standard model of electroweak interactions must be kept as the basic machinery. In the standard model, masses of the intermediate vector bosons needed to complete the Fermi theory are envisioned to correspond to the unitary gauge of a linear sigma model, the Higgs sector, with local $SU(2)_L \times U(1)_Y$ invariance. The standard matter, made up of three families of quarks and leptons, $SU(2)_L$ and $U(1)_Y$ gauge bosons and the Higgs doublet, can be coupled to gravity as

$$\int d^4x\,\sqrt{-g}\big[-g^{\mu\nu}(\mathcal{D}_\mu H)^\dagger \mathcal{D}_\nu H - \zeta_c R H^\dagger H - \lambda\big(H^\dagger H\big)^2 + \Delta\mathcal{L}\big], \tag{19}$$

where $\mathcal{D}_\mu$ represents covariant derivative with respect to both $SU(2)_L$ and $U(1)_Y$ gauge groups, and $\Delta\mathcal{L}$ stands for gauge boson and fermion kinetic terms including the Yukawa couplings of fermions to the Higgs doublet. The Higgs field can be parameterized as

$$H = \frac{1}{\sqrt{2}} U_{\mathrm{SM}}(x) \begin{pmatrix} 0 \\ \phi_0(x) \end{pmatrix}, \tag{20}$$

where $U_{\mathrm{SM}}(x)$ is a general $SU(2)_L$ element which comprises charged and neutral Goldstone degrees of freedom. Note that these Goldstone bosons, in any parametrization of the Higgs doublet, do not couple to the curvature scalar [10]. This implies that Goldstone bosons are not Ricci gaugeable, or that they remain intact to resizing transformations, or that the mechanisms which generate Newton's constant and the electroweak scale are entirely independent. Consequently, it is the norm of the Higgs doublet $\phi_0(x)$ that is sensitive to varying system size.

The matter action possesses exact LSI thanks to the presence of no dimensionful parameter and thanks to proper Ricci gauging of the Higgs kinetic term. Therefore, the direct sum of the two actions, Eqs. (15) and (19), provides a locally resizing invariant description of gravity and matter. It is clear that the Higgs sector cannot realize spontaneous $SU(2)_L$ and $U(1)_Y$ breaking except for cases in which the curvature scalar develops a constant negative value at the right scale (presumably in a higher-dimensional context [11]). Then what is the meaning of a constant ϕ_0 background? How does it permeate the space so as to provide already observed masses for fermions and vector bosons? It is useful to answer these questions from the angle of LSI and gauge invariance, and possible gauge fixing thereof. First of all, the three Goldstone modes contained in $U_{\mathrm{SM}}(x)$ generate the requisite helicity states for relevant gauge bosons and fermions with a general $SU(2)_L \times U(1)_Y$ rotation. This procedure does not interfere with the LSI requirements since Goldstone bosons are blind to the spacetime curvature. In this gauge, the unitary gauge, mass of each flavor is proportional to $\phi_0(x)$ that can always be parameterized as

$$\phi_0(x) = M_0 e^{h(x)/M_0}, \tag{21}$$

where M_0 stands for the characteristic scale of $\phi_0(x)$ and $h(x)$ for its inhomogeneity. With this very form of $\phi_0(x)$ the Higgs sector of Eq. (19) becomes a replica of the $U(x)$ dependent terms in Eq. (15): they have, respectively, the mass scales M_0 and $M_{\mathrm{Pl}}/\sqrt{\zeta_c}$, and the sigma model fields $e^{\pi(x)/f}$ and $e^{h(x)/M_0}$. Indeed, after inserting Eq. (21) for $\phi_0(x)$, the standard model Lagrangian acts as possessing a Goldstone mode $h(x)$ released by LSI breakdown at M_0. Indeed, it is $e^{h(x)/M_0}$ that couples to the curvature scalar—the gauge field of the LSI. However, this is just a similarity since the scale of spontaneous $SU(2)_L \times U(1)_Y$ breakdown has already been fixed by experiment to be $M_0 \simeq 250$ GeV in which case the pattern of fermion and vector boson masses is the one predicted by standard model. It is worthy of emphasizing that M_0 does not follow from the minimization of the Higgs potential; it is the experiment itself which forces M_0 to a nonzero value whereby implying to a spontaneous breakdown of $SU(2)_L \times U(1)_Y$. This scheme corresponds precisely to that of [9] in that the whole system respects LSI since all-dimensional parameters of the Lagrangian

are dressed by $e^{h(x)/M_0}$ in matter sector, and by $e^{\pi(x)/f}$ in the gravity sector. Here one recalls an important difference between the gravity and matter sectors: while $h(x)$ is a true scalar field $\pi(x)$ is a ghost though both transform as a Goldstone boson under local resizings.

The locally resizing invariant description of matter and gravity, Eq. (15) plus Eq. (19), consists of two mass scales $M_{\rm Pl}/\sqrt{\zeta_c}$ and M_0 which respectively correspond to the spontaneous LSI and $SU(2)_L \times U(1)_Y$ breakdowns. Though they are of different origins, either of these two scales can be rendered a hard LSI breaking source by using the invariance under LSI transformations in close similarity to the fact that the freedom of $SU(2)_L$ rotations eliminated all three Goldstone bosons from the standard spectrum and hence revealed the physical particle spectrum. It is convenient to discuss two distinct unitary gauge choices:

• *Unitary LSI gauge: gravity sector.* This possibility has already been discussed in the last section. With a local resizing transformation $\omega(x) = -\pi(x)/f$ the LSI action Eq. (15) can be reduced to that in Eq. (17) which includes the Einstein–Hilbert term, the Weyl gravity and the cosmological constant. The Weyl gravity is expected to be important only at short distances since its contribution to the static gravitational potential varies as $e^{-2r/M_{\rm Pl}}/r$. The cosmological constant turns out to be $\mathcal{O}(M_{\rm Pl}^4)$ naturally; however, its experimental value is known to be 120 orders of magnitude smaller. Possible understanding of this discrepancy, for which there is no intention in this work, might come from the modification of the gravitational laws at far infrared rather than at ultraviolet.
It is clear that in this gauge the particle spectrum of the matter sector remains unchanged. In other words, $h(x)$ is the physical Higgs boson to be searched for at the LHC. The main difference from the standard picture is that the Higgs boson has a direct coupling to the curvature scalar so that its invisible width is enhanced due to graviton emission.

• *Unitary LSI gauge: matter sector.* If one performs a local resizing transformation with $\omega(x) = -h(x)/M_0$ then $h(x)$ gets completely eliminated from Eq. (19) leaving thus no Higgs boson to search for. In other words, the gauge bosons and fermions as well as their couplings are precisely the ones predicted by the standard model and measured at the LEP detectors; however, there is no physical Higgs boson—it has been used up for fixing the LSI to a specific gauge. Obvious enough, in the absence of a fundamental scalar, the tiny number $M_0/M_{\rm Pl}$, though remains unexplained, is radiatively stable, i.e., there is no gauge hierarchy problem all. These observations can in fact be tested in near future: in case the LHC fails to detect a Higgs boson signal this particular LSI gauge might be favored.
Clearly, in this gauge the gravitational sector is described by a scalar-tensor theory rather than a pure tensor theory. However, the scalar field $U(x)$, unlike Brans–Dicke type models, is not responsible for generating the Newton's constant because it is already there. Moreover, the matter sector already feeds rather small but hard $\mathcal{O}(M_0)$ contributions to Newton's constant and the cosmological constant. The fate of the Goldstone boson $\pi(x)$ is determined by its interactions with gravity and matter in that its effective mass as well as couplings to gravity and matter are all affected at the loop level. Being a highly interesting possibility, one notes that in case $\pi(x)$ is forced to condense with a linearly-growing-in-time vacuum expectation value then the resulting lump of $\pi(x)$ can fill in the universe as a nondiluting fluid which is indistinguishable from the cosmological constant [12].

Having done with the electroweak breaking and associated unitary LSI gauges, it is timely to discuss the neutrino masses. The see-saw mechanism provides a viable framework for generating rather tiny neutrino masses [13]. The right-handed neutrino, a standard model singlet, weighs near the Planck scale, and its integration out of the spectrum gives a mass $\mathcal{O}(M_0^2/M_R)$ to active flavors in agreement with data. Unlike the masses of charged fermions and gauge bosons, the mass term of the right-handed neutrino $M_R \nu_R^T \nu_R^c +$ h.c. can be incorporated into the LSI framework via $U(x)$ dressing: $M_R U \nu_R^T \nu_R^c +$ h.c. where now M_R, like $M_{\rm Pl}$, is envisioned to follow from the spontaneous breakdown of the local resizing symmetry.

In the discussions above matter sector has been restricted to standard model spectrum. However, this is not necessary. In fact, the minimal model must be extended at least for generating enough CP violation to create the baryon asymmetry of the universe. When the Higgs sector is extended to two distinct $SU(2)_L$ doublets, for instance, one cannot eliminate all the

Higgs bosons from the spectrum; there is always at least one CP-even boson, heavy or light, to be seen at collider searches. On the other hand, low-energy supersymmetry offers another viable extension of the minimal model. In this case, the hidden sector fields which acquire vacuum expectation values at the intermediate scale to generate $\mathcal{O}(\mathrm{TeV})$ soft masses can be included into the LSI framework just like the mass terms for the right-handed neutrinos.

All the discussions above have been restricted to the classical action without a mention of the quantum effects. This has been necessitated by the consistency of the discussion since a combined analysis of matter and gravity, in the absence of a quantum theoretic description for the latter, can be performed only at classical level. Indeed, the quantum effects in the matter sector lead to an explicit breakdown of the rescaling invariance [14]. In this sense, resizing invariance, global or local, is an anomalous symmetry. However, one keeps in mind that a fully quantum theoretic description of gravity plus matter might modify or put this problem into a different status.

5. Conclusions

There is a manifold of inferences one can draw from the analysis of gravity and matter in the text. The Goldstone ghost, released by the spontaneous breakdown of the local resizing invariance, is swallowed by the spacetime curvature, the gauge field of the LSI, in order to generate the Newton's constant. This procedure parallels precisely the generation of the vector boson masses in gauge theories with spontaneous symmetry breaking. For the matter sector, in particular the standard model, the LSI forbids any explicit mass parameter for the Higgs field, and the physical Higgs boson turns out to act as the Goldstone boson of spontaneous LSI breakdown at the electroweak scale. The total action, comprising gravity and matter sectors, possesses exact LSI and its physical spectrum can be revealed by going to appropriate unitary gauges. There are two options: either the gravitational sector is given by Weyl plus Einstein gravity with a cosmological term and the matter sector is exactly that of the standard model, or the gravitational sector is augmented by the now-physical nonlinear sigma model field and the matter sector is that of the standard model with one exception, there is no Higgs boson to search for. The heavy right-handed neutrino can be directly included in the LSI framework, and the matter sector can be replaced by extended models like two-doublet models or supersymmetry.

Acknowledgements

The author gratefully acknowledges the discussions with Misha Voloshin about Ref. [10].

References

[1] H. Weyl, Space–Time–Matter, Dover, New York, 1952.
[2] F. Gursey, Nuovo Cimento 3 (1956) 98;
J. Wess, Nuovo Cimento 18 (1960) 1086;
K.G. Wilson, Phys. Rev. 179 (1969) 1499;
D.J. Gross, J. Wess, Phys. Rev. D 2 (1970) 753.
[3] F. Gursey, Ann. Phys. (N.Y.) 24 (1963) 211;
R. Penrose, Proc. R. Soc. A 284 (1965) 204.
[4] A. Iorio, L. O'Raifeartaigh, I. Sachs, C. Wiesendanger, Nucl. Phys. B 495 (1997) 433, hep-th/9607110.
[5] J. Goldstone, Nuovo Cimento 19 (1961) 154;
J. Goldstone, A. Salam, S. Weinberg, Phys. Rev. 127 (1962) 965.
[6] P.W. Higgs, Phys. Lett. 12 (1964) 132;
F. Englert, R. Brout, Phys. Rev. Lett. 13 (1964) 321.
[7] R.D. Pisarski, Phys. Rev. D 28 (1983) 2547.
[8] K.S. Stelle, Phys. Rev. D 16 (1977) 953.
[9] J.D. Bekenstein, A. Meisels, Phys. Rev. D 22 (1980) 1313.
[10] M.B. Voloshin, A.D. Dolgov, Sov. J. Nucl. Phys. 35 (1982) 120, Yad. Fiz. 35 (1982) 213 (in Russian).
[11] D.A. Demir, M. Shifman, Phys. Rev. D 65 (2002) 104002, hep-ph/0112090.
[12] N. Arkani-Hamed, talk at SUSY 03;
N. Arkani-Hamed, H.C. Cheng, M.A. Luty, S. Mukohyama, hep-th/0312099;
N. Arkani-Hamed, P. Creminelli, S. Mukohyama, M. Zaldarriaga, hep-th/0312100.
[13] M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D.Z. Freedman (Eds.), Supergravity, North-Holland, Amsterdam, 1979;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe, 1979, KEK Report: 79-18;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[14] L. Parker, Phys. Rev. D 7 (1973) 976;
I.T. Drummond, G.M. Shore, Phys. Rev. D 19 (1979) 1134;
N.D. Birrell, J.G. Taylor, J. Math. Phys. 21 (1980) 1740;
N.D. Birrell, P.C.W. Davies, Phys. Rev. D 22 (1980) 322.